Compression & Huffman Codes

Compression

- Definition
  - Reduce size of data
    (number of bits needed to represent data)
- Benefits
  - Reduce storage needed
  - Reduce transmission cost / latency / bandwidth

Sources of Compressibility

- Redundancy
  - Recognize repeating patterns
  - Exploit using
    - Dictionary
    - Variable length encoding
- Human perception
  - Less sensitive to some information
  - Can discard less important data

Types of Compression

- Lossless
  - Preserves all information
  - Exploits redundancy in data
  - Applied to general data
- Lossy
  - May lose some information
  - Exploits redundancy & human perception
  - Applied to audio, image, video

Effectiveness of Compression

- Metrics
  - Bits per byte (8 bits)
    - 2 bits / byte ⇒ ¼ original size
    - 8 bits / byte ⇒ no compression
  - Percentage
    - 75% compression ⇒ ¼ original size

Effectiveness of Compression

- Depends on data
  - Random data ⇒ hard
    - Example: 1001110100 ⇒ ?
  - Organized data ⇒ easy
    - Example: 1111111111 ⇒ 1×10
- Corollary
  - No universally best compression algorithm

Effectiveness of Compression

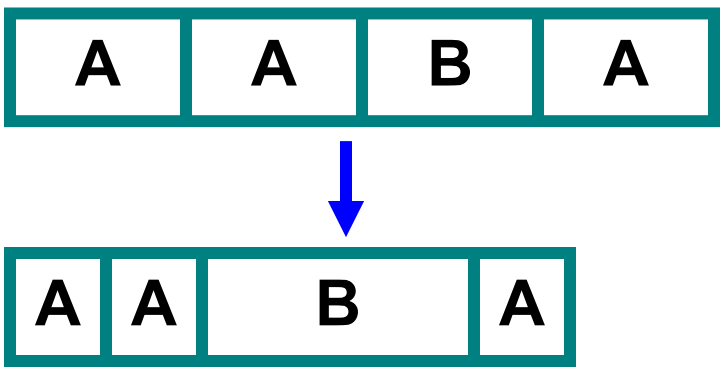
- **Lossless Compression is not always possible**
  - If compression is always possible (alternative view)
    - Compress file (reduce size by 1 bit)
    - Recompress output
    - Repeat (until we can store data with 0 bits)

Lossless Compression Techniques

- LZW (Lempel-Ziv-Welch) compression
  - Build pattern dictionary
  - Replace patterns with index into dictionary
- Run length encoding
  - Find & compress repetitive sequences
- Huffman codes
  - Use variable length codes based on frequency

Huffman Code

- **Approach**
  - **Variable length encoding of symbols**
  - **Exploit statistical frequency of symbols**
  - **Efficient when symbol probabilities vary widely**
- **Principle**
  - **Use fewer bits to represent frequent symbols**
  - **Use more bits to represent infrequent symbols**



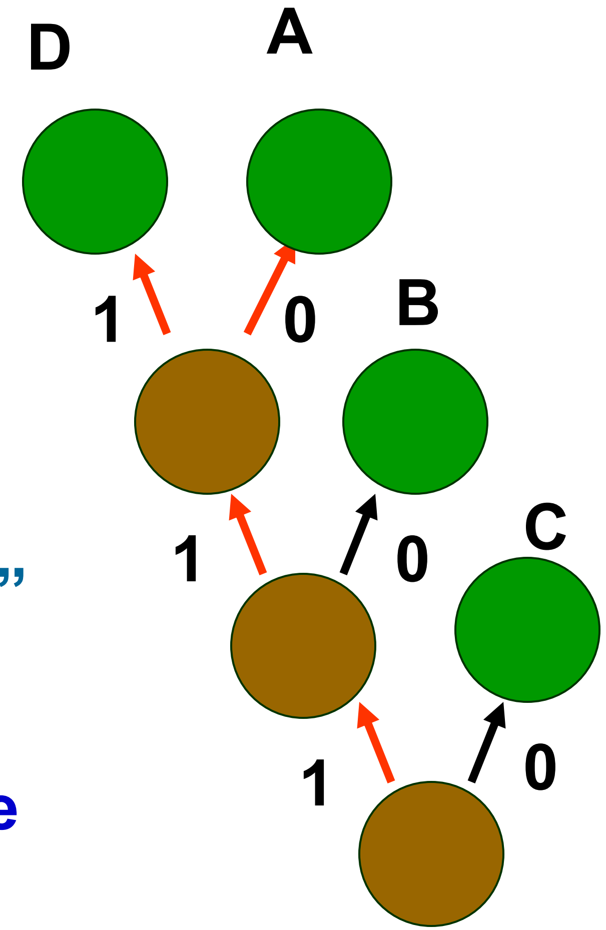
Huffman Code Example

Symbol	A	B	C	D
Frequency	13%	25%	50%	12%
Original Encoding	00	01	10	11
	2 bits	2 bits	2 bits	2 bits
Huffman Encoding	110	10	0	111
	3 bits	2 bits	1 bit	3 bits

- **Expected size**
  - Original $\Rightarrow$ $1/8 \times 2 + 1/4 \times 2 + 1/2 \times 2 + 1/8 \times 2 = 2$ bits / symbol
  - Huffman $\Rightarrow$ $1/8 \times 3 + 1/4 \times 2 + 1/2 \times 1 + 1/8 \times 3 = 1.75$ bits / symbol

Huffman Code Data Structures

- Binary (Huffman) tree
  - Represents Huffman code
  - Edge ⇒ code (0 or 1)
  - Leaf ⇒ symbol
  - Path to leaf ⇒ encoding
  - Example
    - A = “110”, B = “10”, C = “0”
- Priority queue
  - To efficiently build binary tree



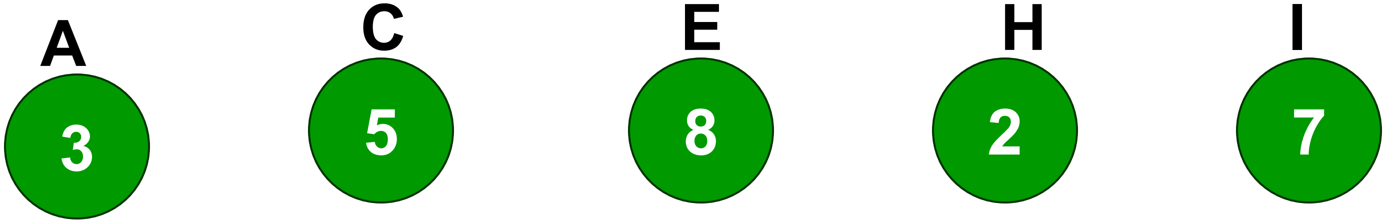
Huffman Code Algorithm Overview

- Encoding
  - Calculate frequency of symbols in file
  - Create binary tree representing “best” encoding
  - Use binary tree to encode compressed file
    - For each symbol, output path from root to leaf
    - Size of encoding = length of path
  - Save binary tree

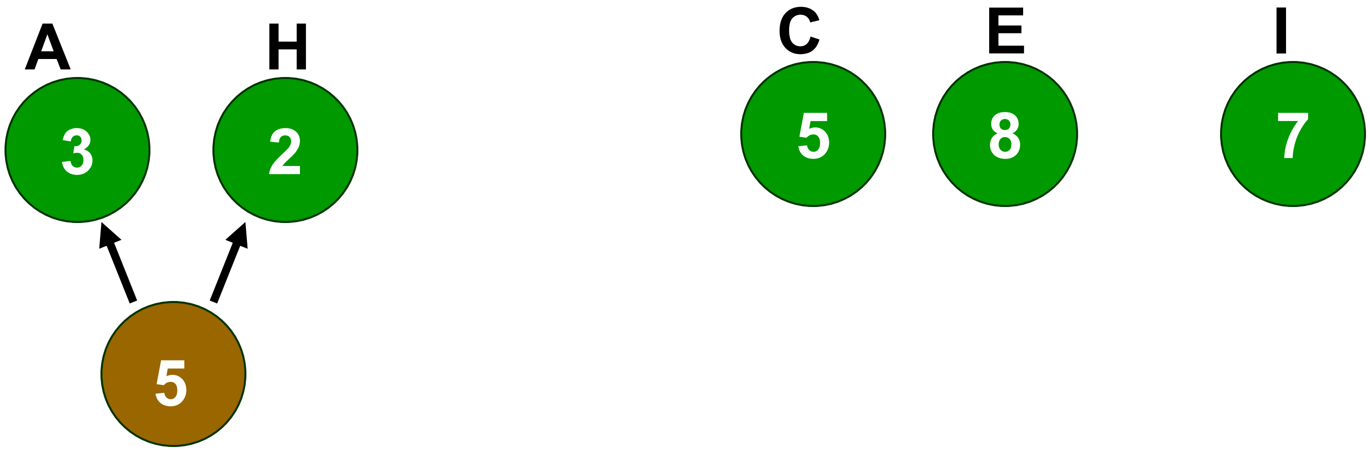
Huffman Code – Creating Tree

- Algorithm
  - Place each symbol in leaf
    - Weight of leaf = symbol frequency
  - Select two trees L and R (initially leafs)
    - Such that L, R have lowest frequencies in tree
  - Create new (internal) node
    - Left child $\Rightarrow$ L
    - Right child $\Rightarrow$ R
    - New frequency $\Rightarrow$ frequency(L) + frequency(R)
  - Repeat until all nodes merged into one tree

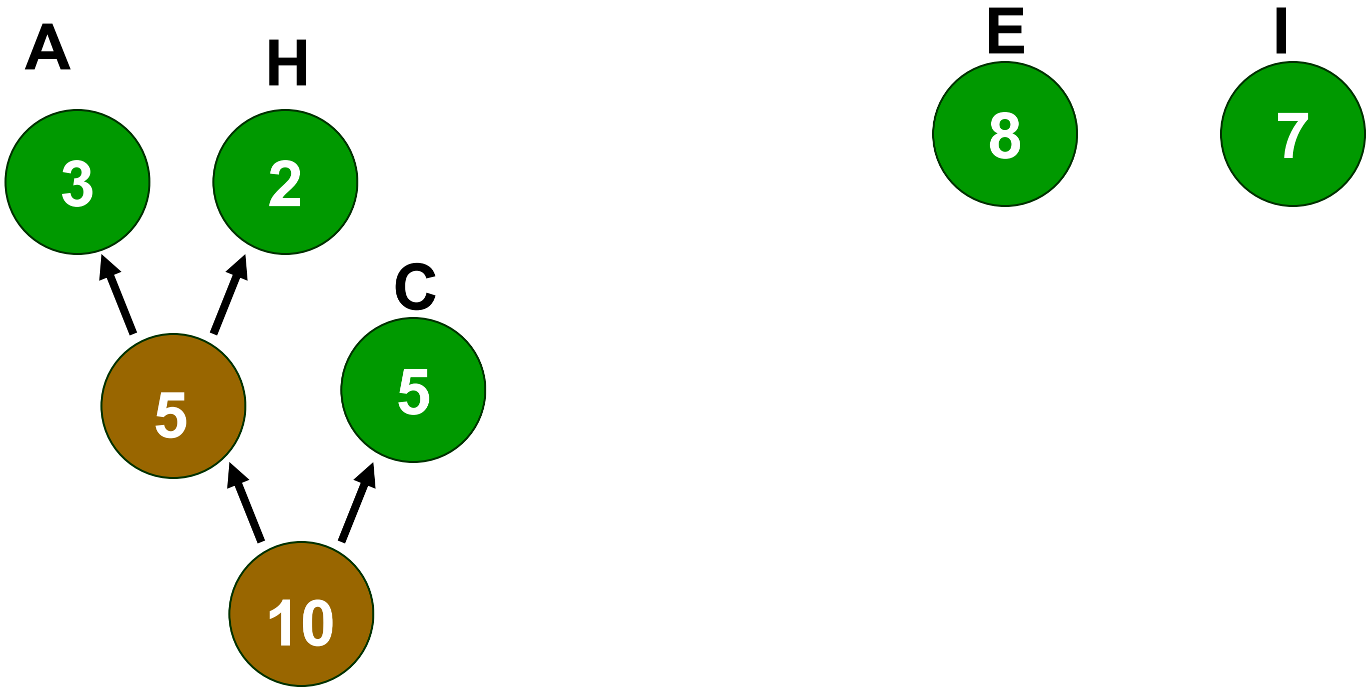
Huffman Tree Construction 1



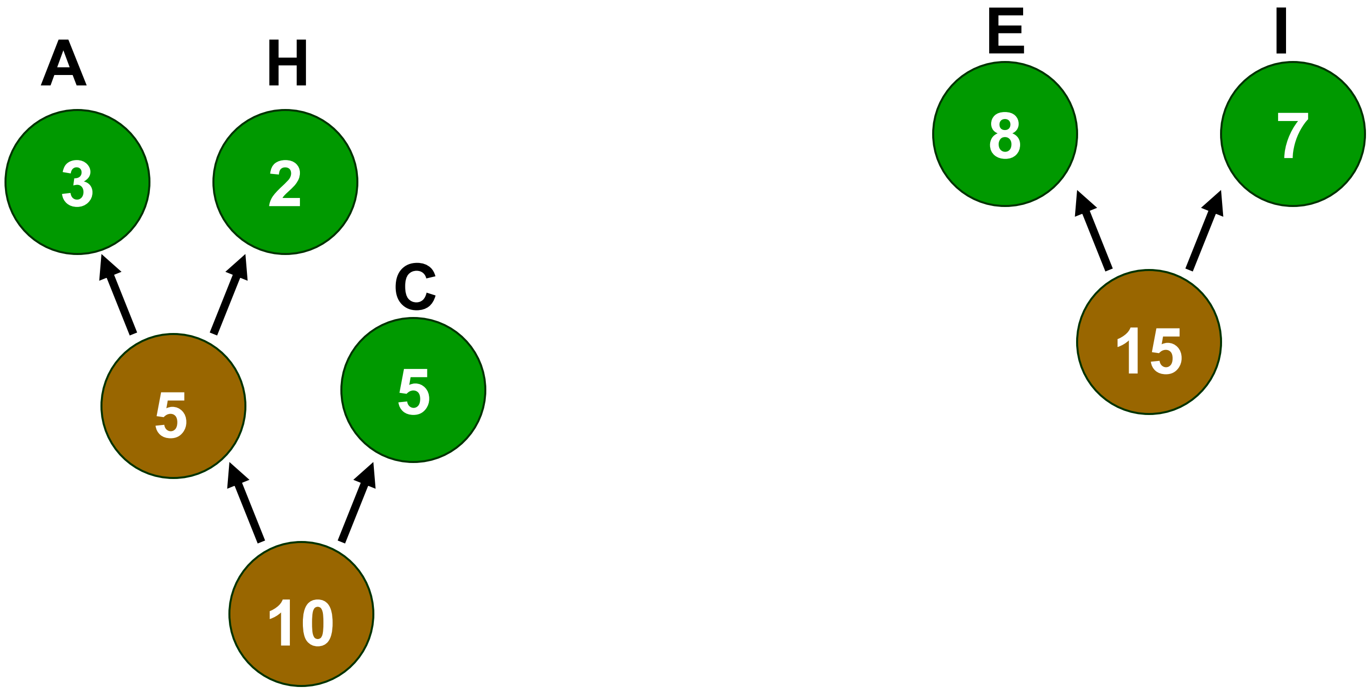
Huffman Tree Construction 2



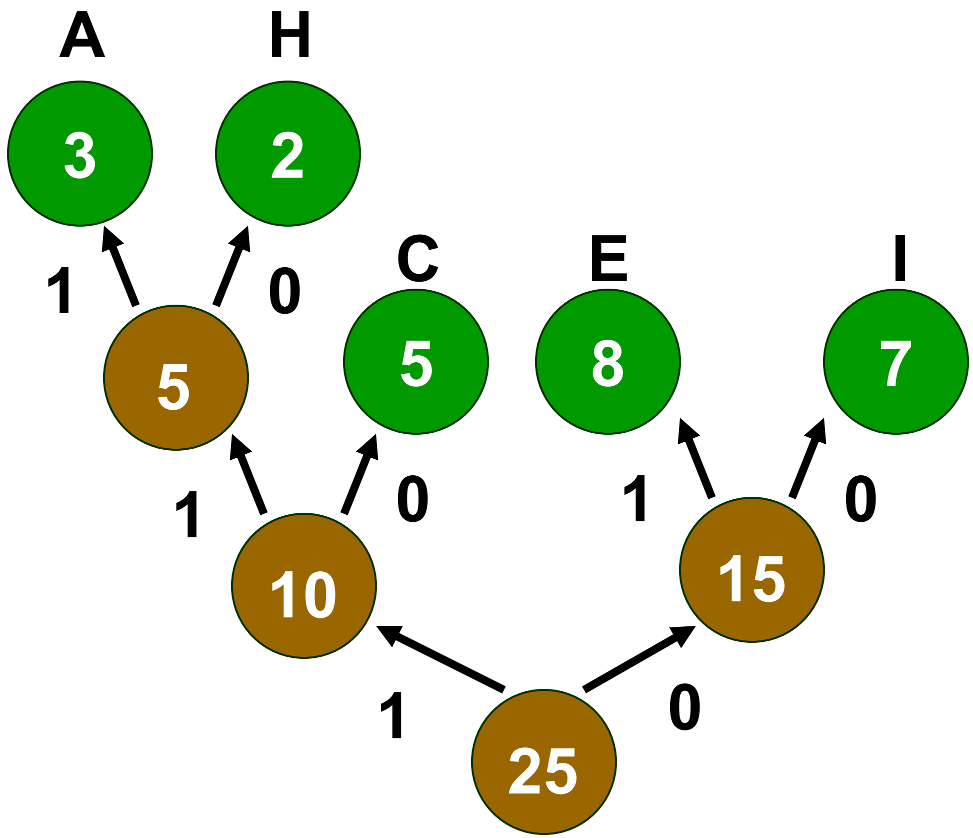
Huffman Tree Construction 3



Huffman Tree Construction 4



Huffman Tree Construction 5



E	=	01
I	=	00
C	=	10
A	=	111
H	=	110

Huffman Coding Example

- Huffman code

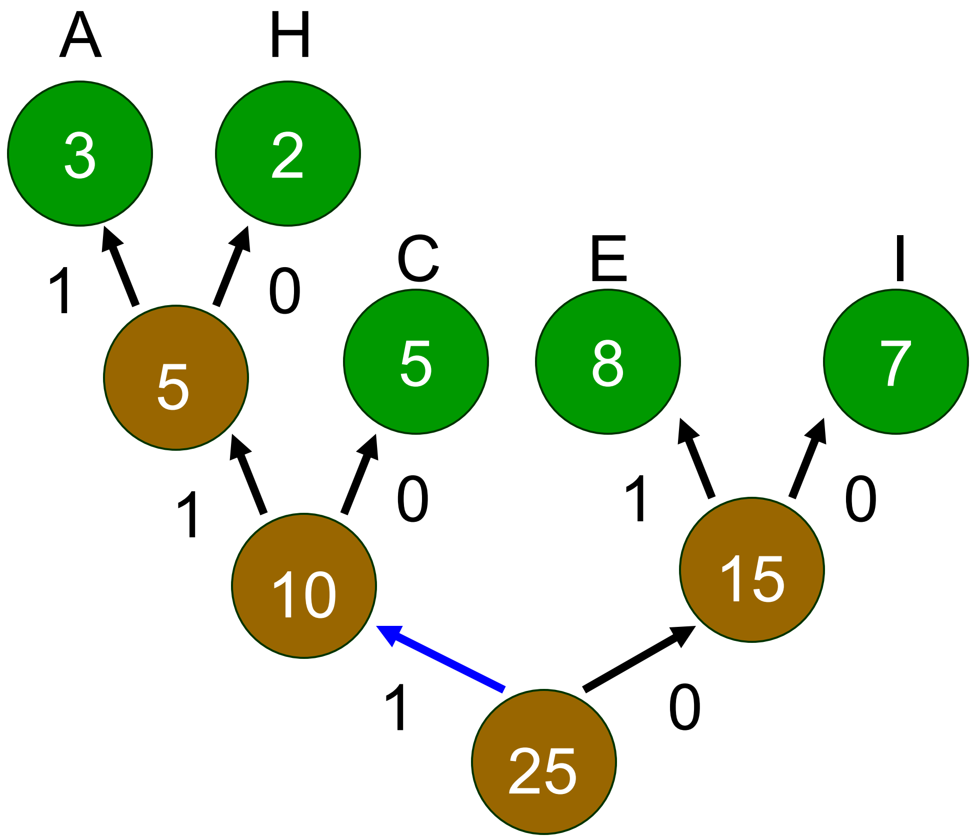
E	=	01
I	=	00
C	=	10
A	=	111
H	=	110

- Input
  - ACE
- Output
  - (111)(10)(01) = 1111001

Huffman Code Algorithm Overview

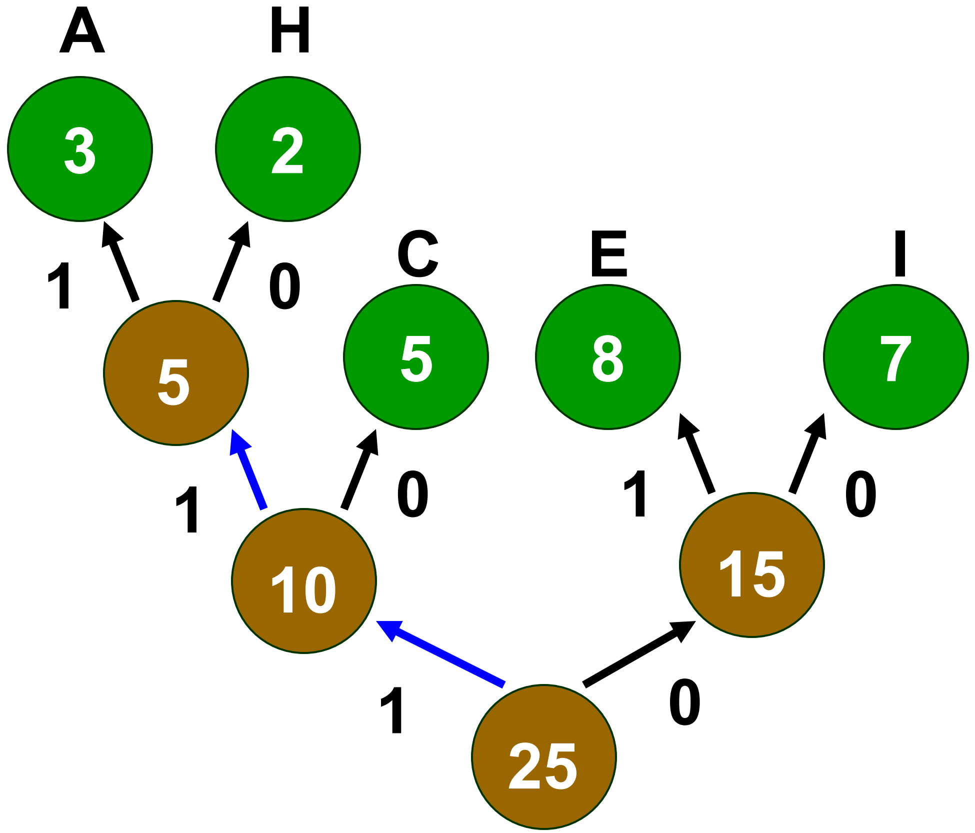
- Decoding
  - Read compressed file & binary tree
  - Use binary tree to decode file
    - Follow path from root to leaf

Huffman Decoding 1



1111001

Huffman Decoding 2



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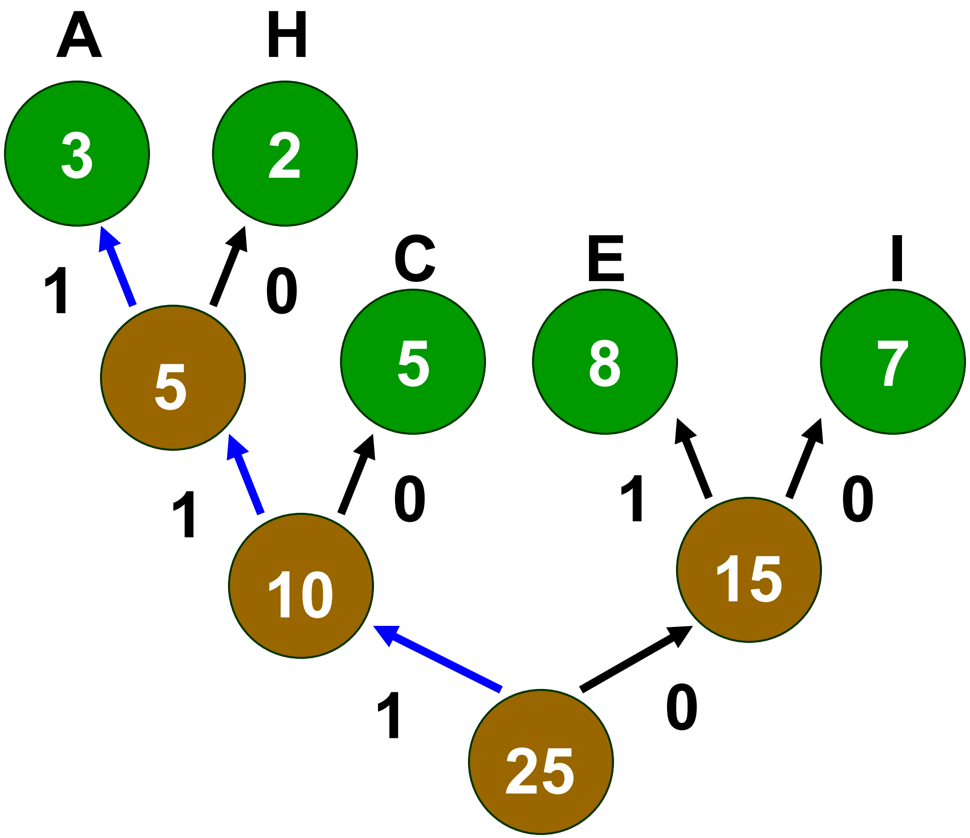
Huffman Decoding 3



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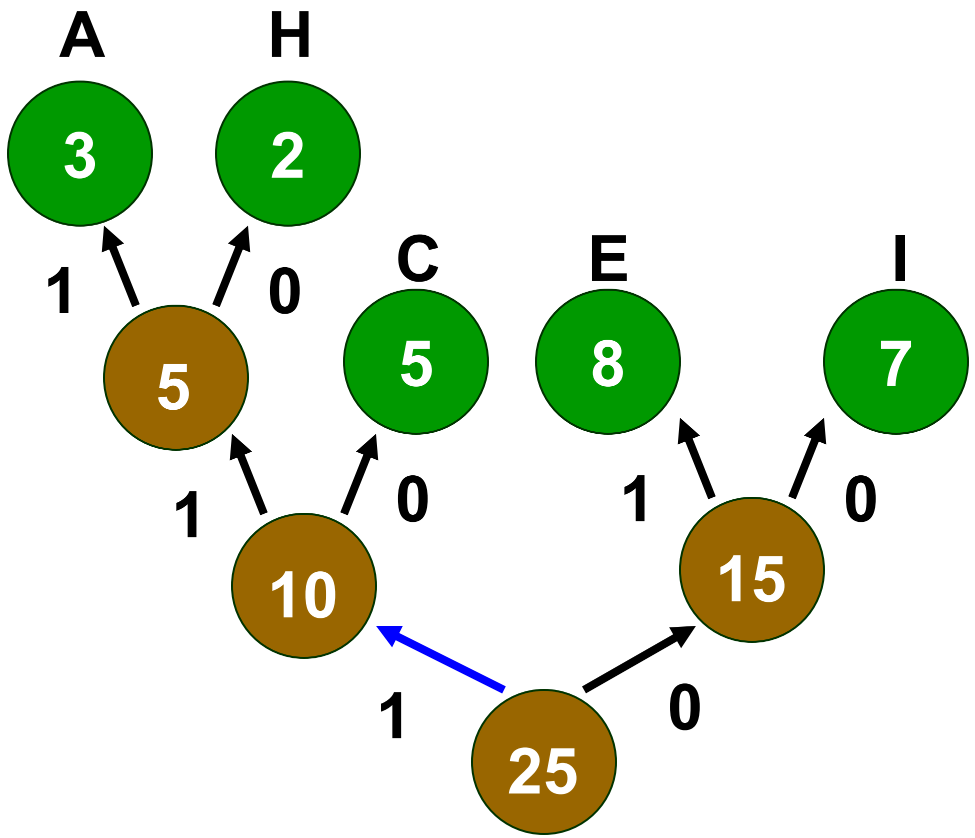
A

Huffman Decoding 4



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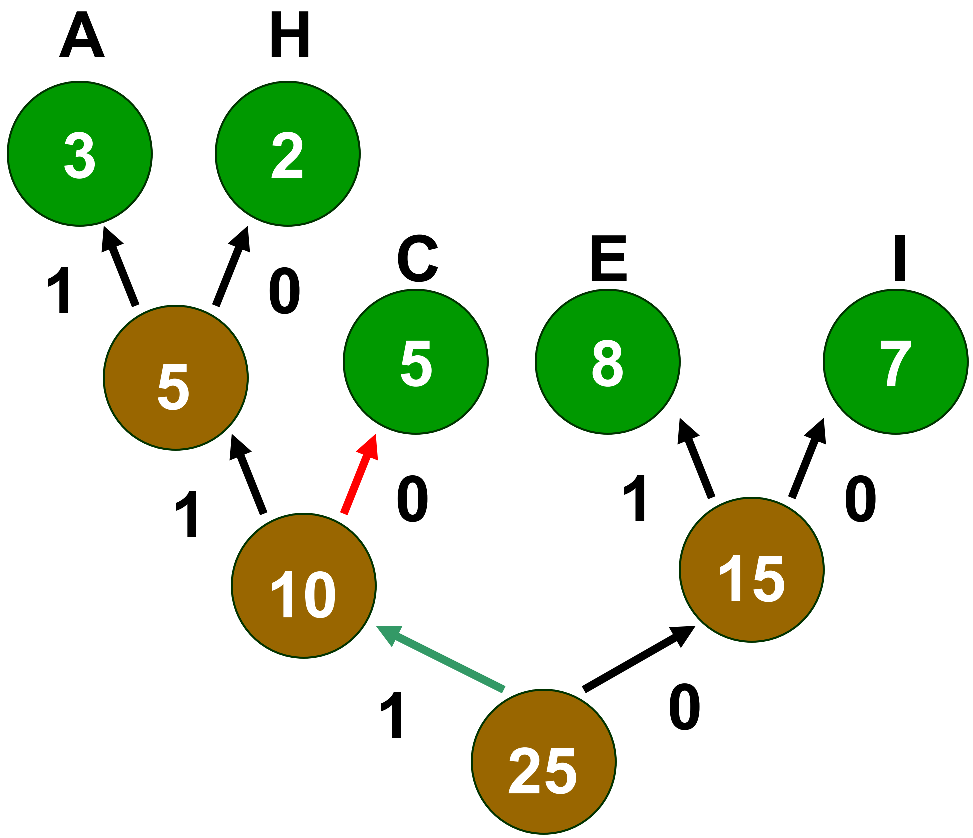
A

Huffman Decoding 5



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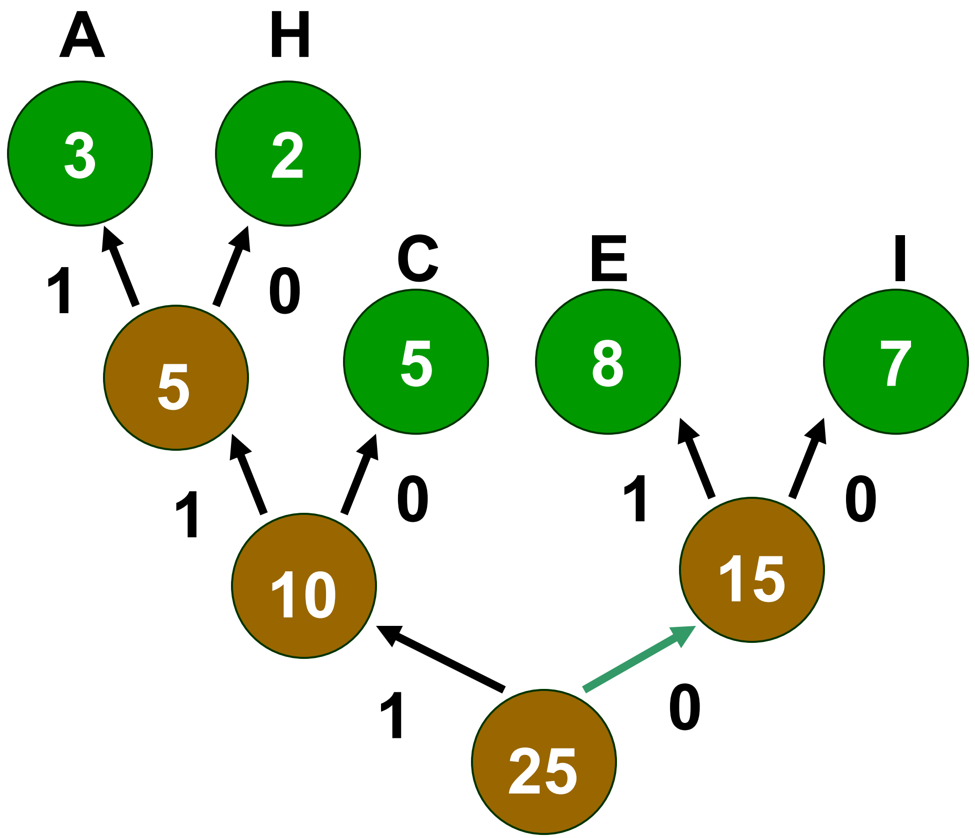
AC

Huffman Decoding 6



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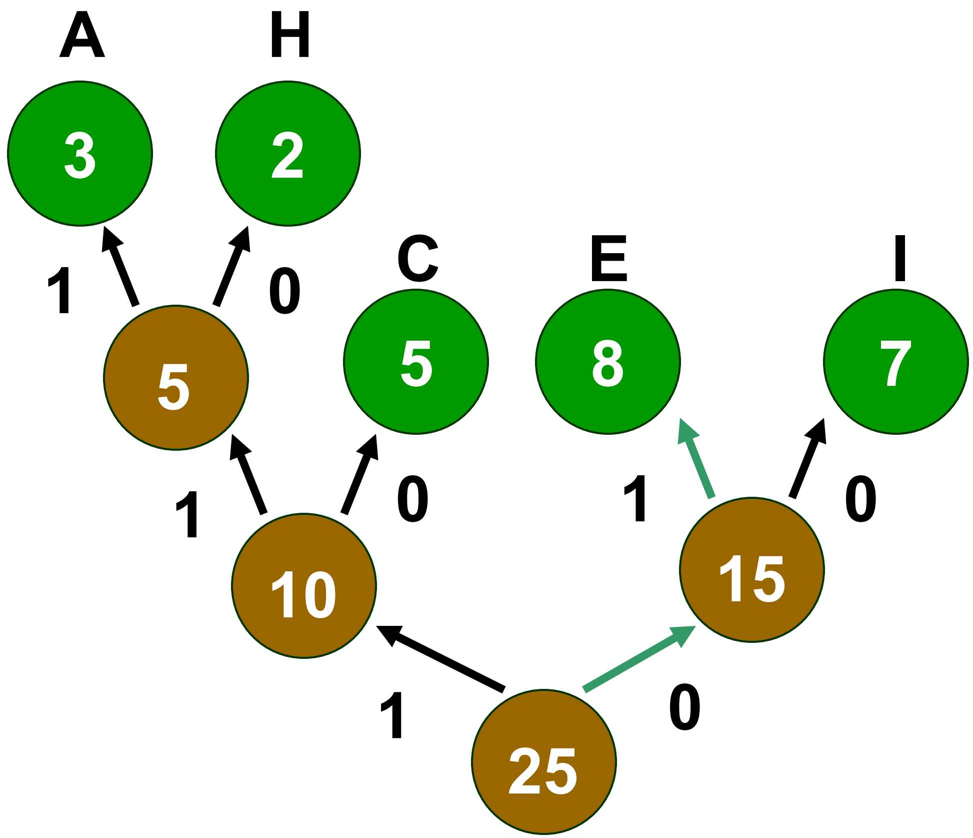
AC

Huffman Decoding 7



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ACE

Huffman Code Properties

- Prefix code
  - No code is a prefix of another code
  - Example
    - Huffman(“I”) ⇒ 00
    - Huffman(“X”) ⇒ 001 // not legal prefix code
  - Can stop as soon as complete code found
  - No need for end-of-code marker
- Nondeterministic
  - Multiple Huffman coding possible for same input
  - If more than two trees with same minimal weight

Huffman Code Properties

- Greedy algorithm
  - Chooses best local solution at each step
  - Combines 2 trees with lowest frequency
- Still yields overall best solution
  - Optimal prefix code
  - Based on statistical frequency
- Better compression possible (depends on data)
  - Using other approaches (e.g., pattern dictionary)