Data Structures

Trees

Chapter 5 Trees: Outline

- Introduction
  - Representation Of Trees
- Binary Trees
- Binary Tree Traversals
- Additional Binary Tree Operations
- Threaded Binary Trees
- Heaps
- Binary Search Trees
- Selection Trees
- Forests

Introduction (1/8)

- A ***tree*** structure means that the data are organized so that items of information are related by branches
- Examples:

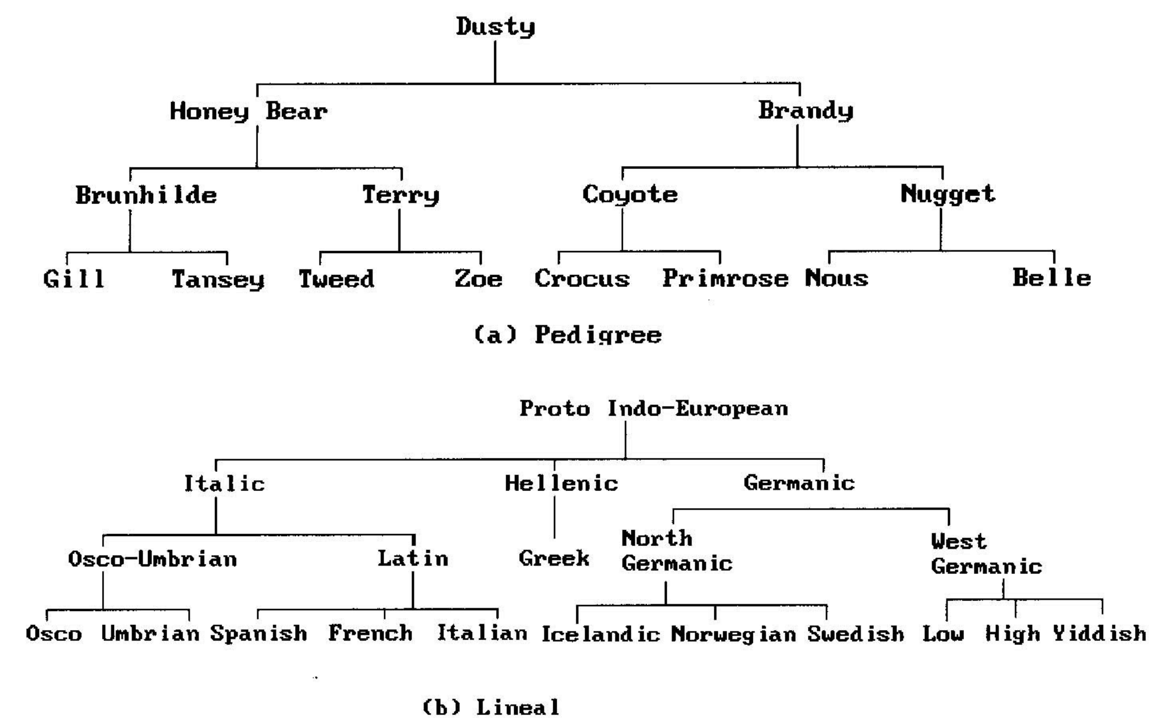


Figure 5.1: Two types of genealogical charts

Introduction (2/8)

- **Definition** (recursively): A *tree* is a finite set of one or more nodes such that
  - There is a specially designated node called *root*.
  - The remaining nodes are partitioned into n>=0 disjoint set $T_1,\ldots,T_n$, where each of these sets is a tree. $T_1,\ldots,T_n$ are called the *subtrees* of the root.
- Every node in the tree is the root of some subtree

Introduction (3/8)

- Some **Terminology**
  - *node*: the item of information plus the branches to each node.
  - *degree*: the number of subtrees of a node
  - *degree* of a tree: the maximum of the degree of the nodes in the tree.
  - *terminal* nodes (or *leaf*): nodes that have degree zero
  - *nonterminal* nodes: nodes that don’t belong to terminal nodes.
  - *children*: the roots of the subtrees of a node X are the *children* of X
  - *parent*: X is the *parent* of its children.

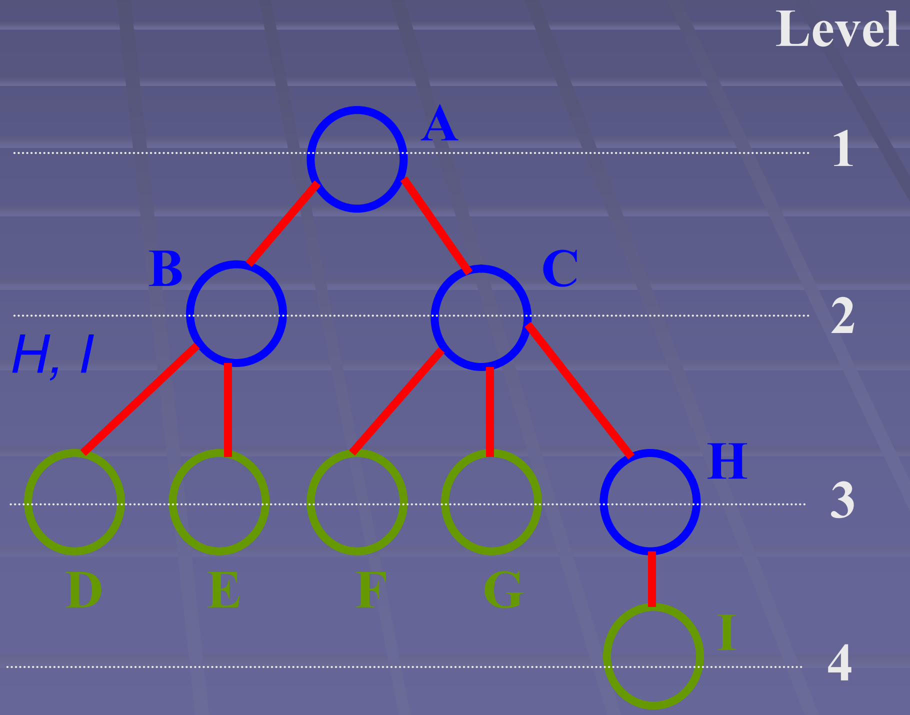
Introduction (4/8)

- Some **Terminology** (cont'd)
  - *siblings*: children of the same parent are said to be siblings.
  - *Ancestors* of a node: all the nodes along the path from the root to that node.
  - The *level* of a node: defined by letting the root be at level one. If a node is at level *l*, then it children are at level *l+1*.
  - *Height* (or *depth*): the maximum level of any node in the tree

Introduction (5/8)

- Example
  - *A* is the *root* node
  - *B* is the *parent* of D and E
  - *C* is the *sibling* of B
  - *D* and *E* are the *children* of B
  - *D, E, F, G, I* are *external nodes*, or *leaves*
  - *A, B, C, H* are *internal nodes*
  - The *level* of *E* is *3*
  - The *height (depth)* of the tree is *4*
  - The *degree* of node *B* is *2*
  - The *degree* of the tree is *3*
  - The *ancestors* of node *I* is *A, C, H*
  - The *descendants* of node *C* is *F, G, H, I*

Property: (# edges) = (#nodes) - 1



Introduction (6/8)

- Representation Of Trees
  - List Representation
    - we can write of Figure 5.2 as a list in which each of the subtrees is also a list

(A (B (E (K, L), F), C (G), D (H (M), I, J)))

    - The root comes first, followed by a list of sub-trees

Figure 5.3: Possible list representation for trees

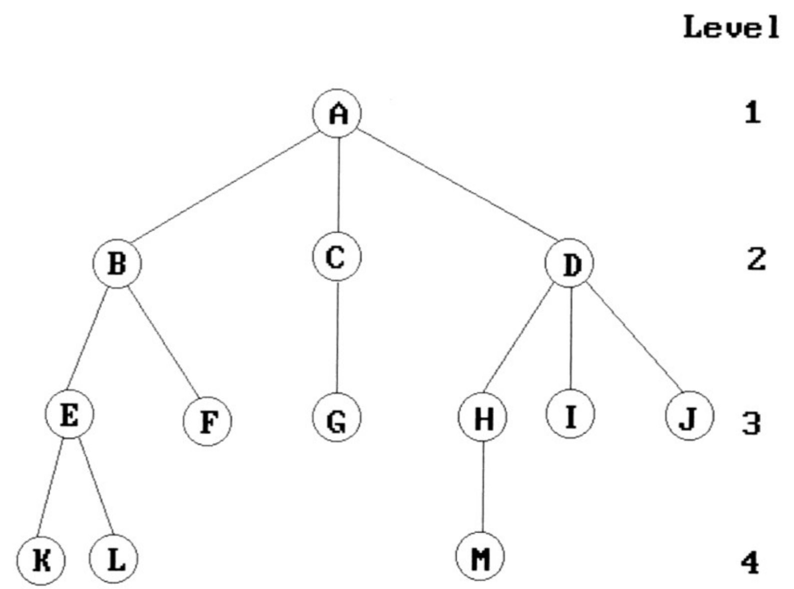


Figure 5.2: A sample tree

Introduction (7/8)

- Representation Of Trees (cont'd)
  - Left Child-Right Sibling Representation

data	
left child	right sibling

Figure 5.4: Left child-right sibling node structure

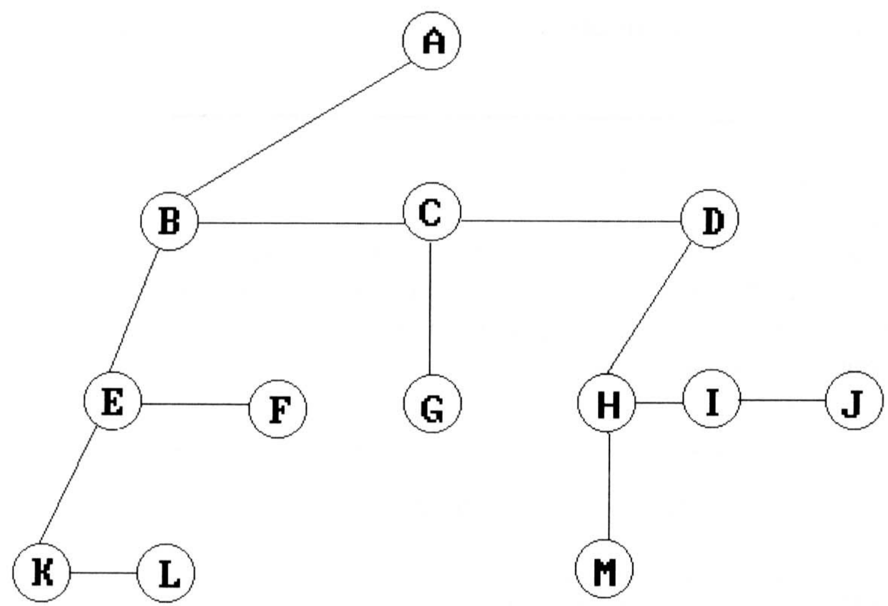


Figure 5.5: Left child-right sibling representation of a tree

Introduction (8/8)

- Representation Of Trees (cont'd)
  - Representation As A Degree Two Tree

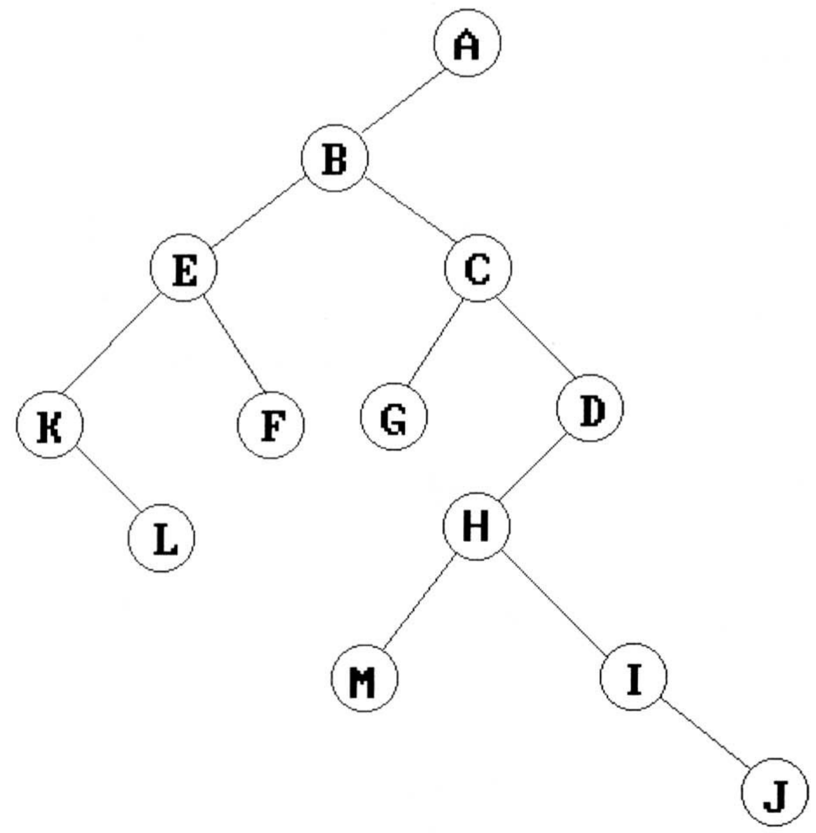
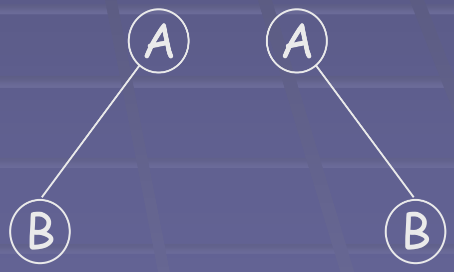


Figure 5.6: Left child-right child tree representation of a tree

Binary Trees (1/9)

- Binary trees are characterized by the fact that any node can have at most two branches
- **Definition** (recursive):
  - A *binary tree* is a finite set of nodes that is either empty or consists of a root and two disjoint binary trees called the left subtree and the right subtree
- Thus the left subtree and the right subtree are distinguished



- Any tree can be transformed into binary tree
  - by left child-right sibling representation

Binary Trees (2/9)

- The abstract data type of binary tree

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structure *Binary_Tree* (abbreviated *BinTree*) is
objects: a finite set of nodes either empty or consisting of a root node, left *Binary_Tree*, and right *Binary_Tree*.
functions:
for all *bt*,*bt*1,*bt*2 ∈ *BinTree*, *item* ∈ *element*

BinTree Create()	::=	creates an empty binary tree
Boolean IsEmpty(*bt*)	::=	**if** (*bt* == empty binary tree) **return** *TRUE* **else return** *FALSE*
BinTree MakeBT(*bt*1, *item*, *bt*2)	::=	**return** a binary tree whose left subtree is *bt*1, whose right subtree is *bt*2, and whose root node contains the data *item*.
BinTree Lchild(*bt*)	::=	**if** (IsEmpty(*bt*)) **return** error **else return** the left subtree of *bt*.
element Data(*bt*)	::=	**if** (IsEmpty(*bt*)) **return** error **else return** the data in the root node of *bt*.
BinTree Rchild(*bt*)	::=	**if** (IsEmpty(*bt*)) **return** error **else return** the right subtree of *bt*.

Structure 5.1: Abstract data type *Binary_Tree*

Binary Trees (3/9)

- Two special kinds of binary trees:
  (a) *skewed tree,* (b) *complete binary tree*
  - The all leaf nodes of these trees are on two adjacent levels

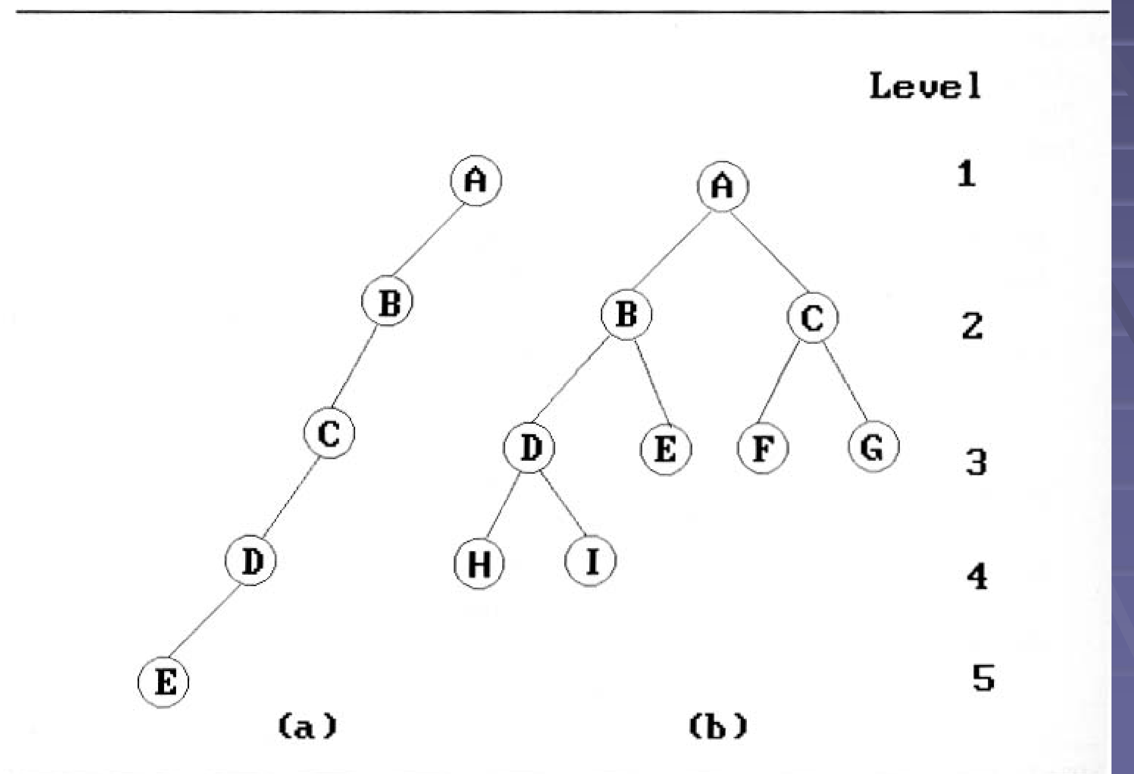


Figure 5.9: Skewed and complete binary trees

Binary Trees (4/9)

- Properties of binary trees
  - **Lemma 5.1 [*Maximum number of nodes*]:**

1. The maximum number of nodes on level *i* of a binary tree is 2^{i-1}, $i \geq 1$.
2. The maximum number of nodes in a binary tree of depth *k* is 2^k-1, $k \geq 1$.

  - **Lemma 5.2 [*Relation between number of leaf nodes and degree-2 nodes*]:**

    For any nonempty binary tree, T, if n_0 is the number of leaf nodes and n_2 is the number of nodes of degree 2, then $n_0 = n_2 + 1$.

- These lemmas allow us to define full and complete binary trees

Binary Trees (5/9)

- Definition:
  - A *full binary tree* of depth *k* is a binary tree of death *k* having 2^k-1 nodes, $k \geq 0$.
  - A binary tree with n nodes and depth k is complete *iff* its nodes correspond to the nodes numbered from 1 to n in the full binary tree of depth k.
  - From Lemma 5.1, the height of a complete binary tree with *n* nodes is $\lceil \log_2(n+1) \rceil$

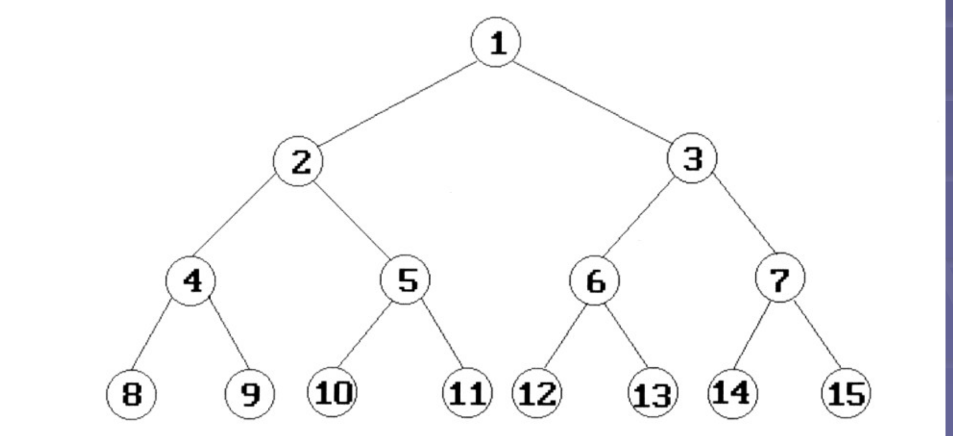


Figure 5.10: Full binary tree of depth 4 with sequential node numbers

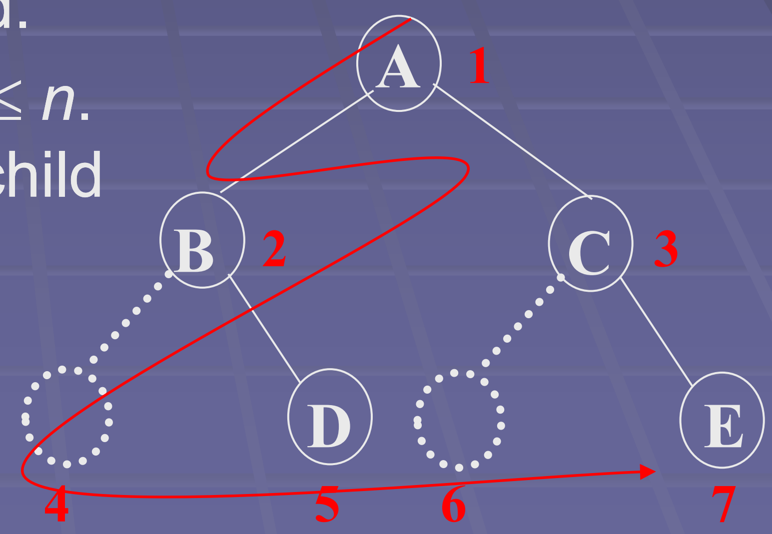
Binary Trees (6/9)

- Binary tree representations (using array)
  - **Lemma 5.3:** If a complete binary tree with n nodes is represented sequentially, then for any node with index i, $1 \leq i \leq n$, we have

1. *parent(i)* is at $\lfloor i/2 \rfloor$ if $i \neq 1$.
   If $i = 1$, i is at the root and has no parent.
2. *LeftChild(i)* is at $2i$ if $2i \leq n$.
   If $2i > n$, then i has no left child.
3. *RightChild(i)* is at $2i+1$ if $2i+1 \leq n$.
   If $2i+1 > n$, then i has no left child

[1]	[2]	[3]	[4]	[5]	[6]	[7]
A	B	C	—	D	—	E

$\underbrace{\text{A}}_{\text{Level 1}}$ $\underbrace{\text{B C}}_{\text{Level 2}}$ $\underbrace{\text{— D — E}}_{\text{Level 3}}$



Binary Trees (7/9)

- Binary tree representations (using array)
  - Waste spaces: in the worst case, a skewed tree of depth *k* requires 2^k-1 spaces. Of these, only *k* spaces will be occupied
  - Insertion or deletion of nodes from the middle of a tree requires the movement of potentially many nodes to reflect the change in the level of these nodes

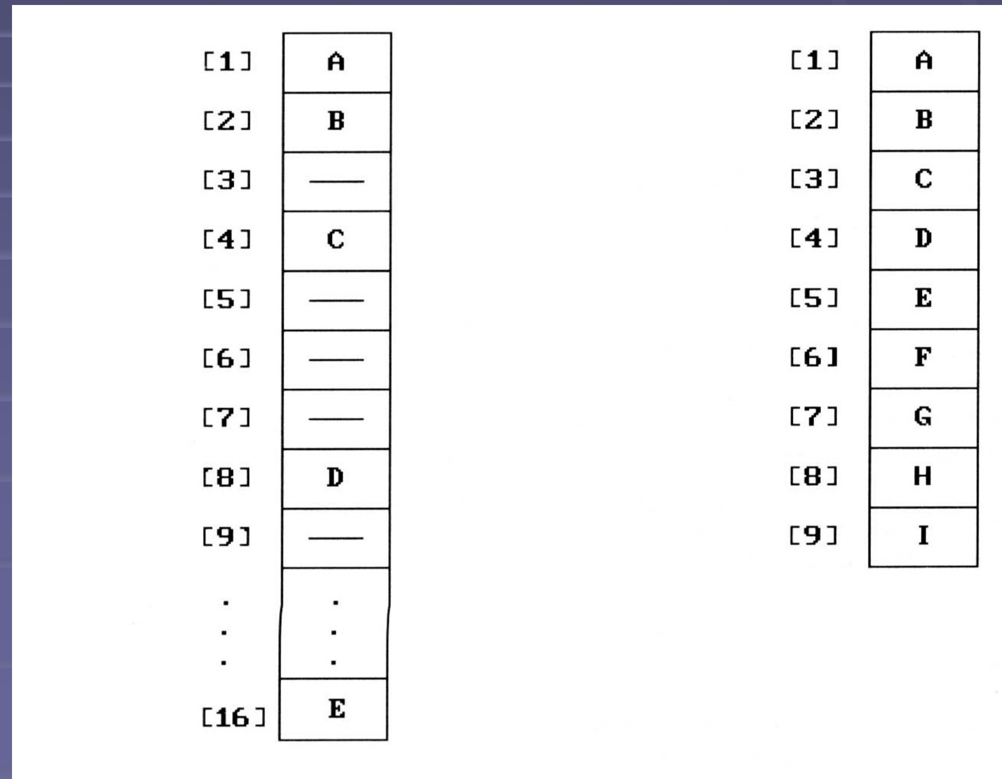


Figure 5.11: Array representation of binary trees of Figure 5.9

Binary Trees (8/9)

- Binary tree representations (using link)

```
typedef struct node *tree_pointer;
typedef struct node {
        int data;
        tree_pointer left_child, right_child;
        };
```

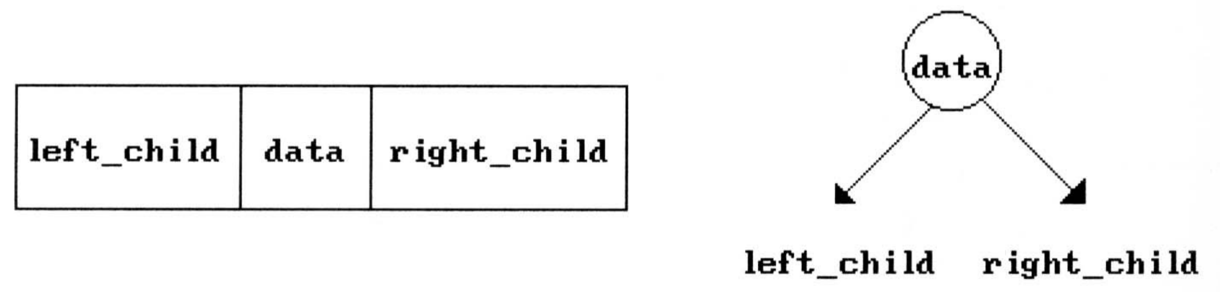


Figure 5.12: Node representation for binary trees

Binary Trees (9/9)

- Binary tree representations (using link)

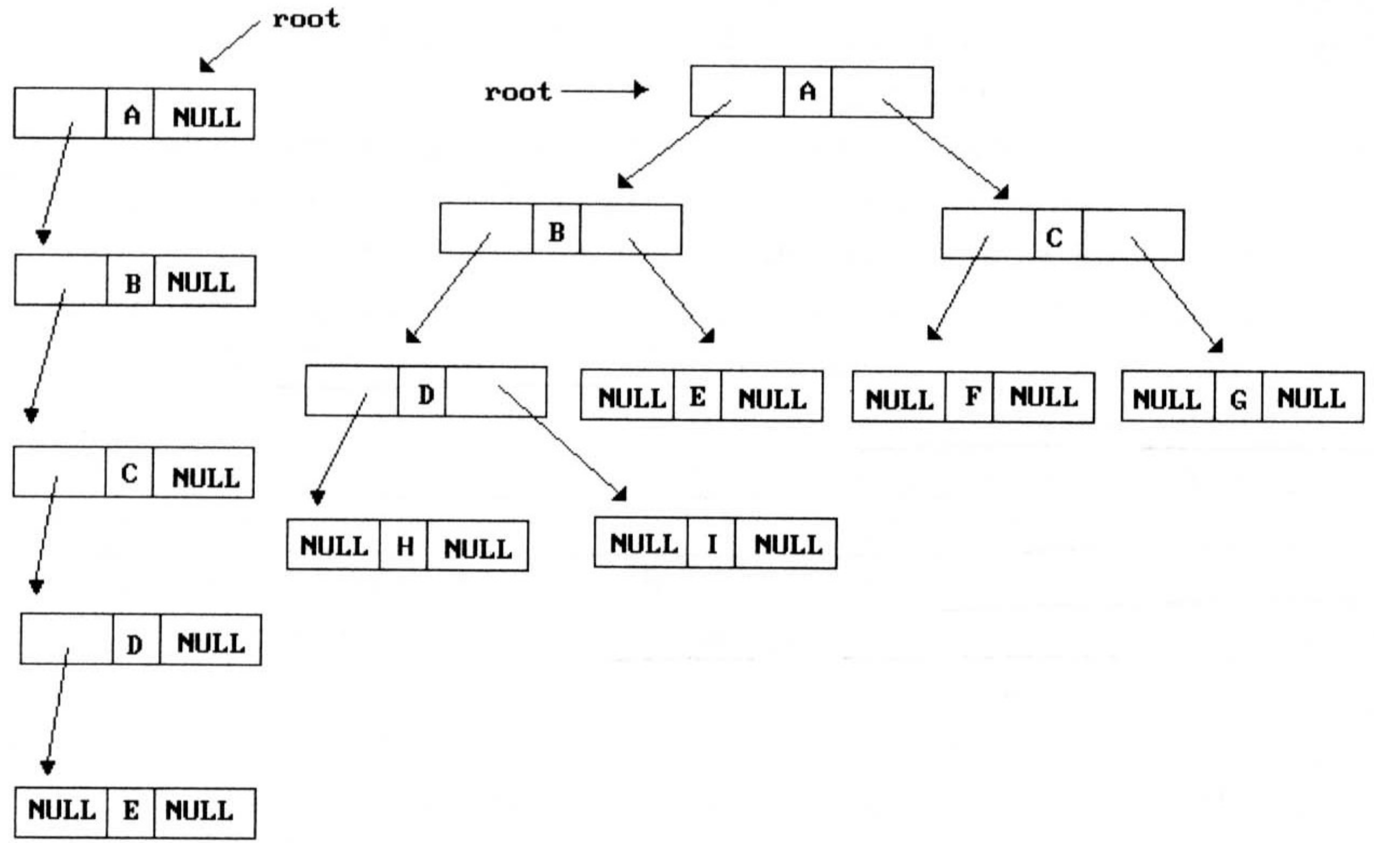


Figure 5.13: Linked representation for the binary trees of Figure 5.9

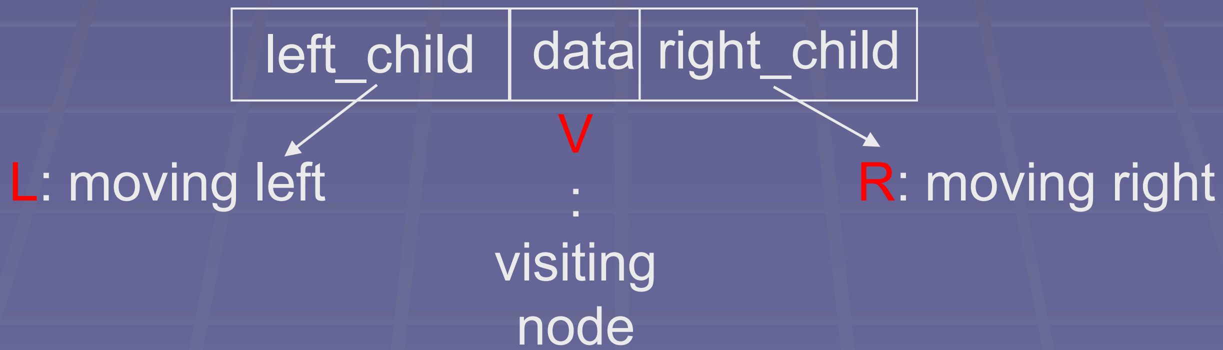
Binary Tree Traversals (1/9)

- How to traverse a tree or visit each node in the tree exactly once?
  - There are six possible combinations of traversal

LVR, LRV, VLR, VRL, RVL, RLV

  - Adopt convention that we traverse left before right, only 3 traversals remain

LVR (inorder), LRV (postorder), VLR (preorder)



Binary Tree Traversals (2/9)

- Arithmetic Expression using binary tree
  - inorder traversal (infix expression)

    A / B * C * D + E
  - preorder traversal (prefix expression)

    + * * / A B C D E
  - postorder traversal (postfix expression)

    A B / C * D * E +
  - level order traversal

    + * E * D / C A B

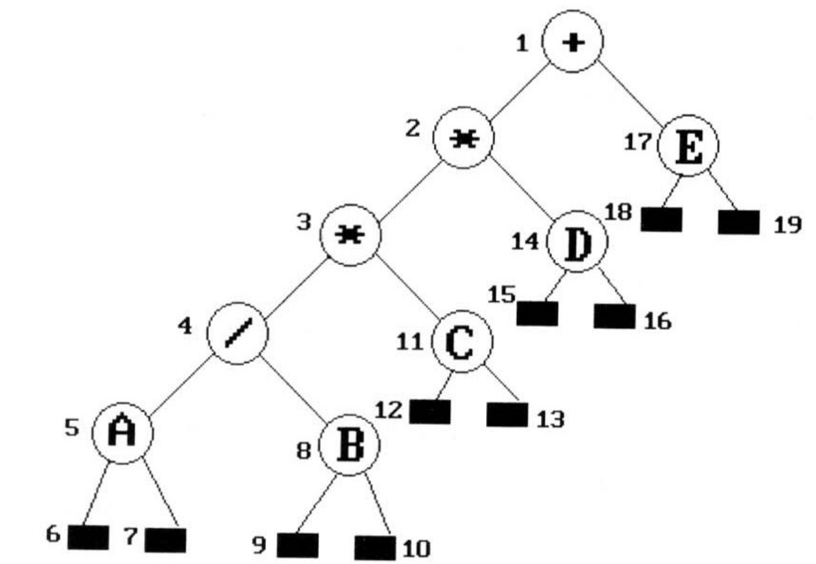


Figure 5.15: Binary tree with arithmetic expression

Binary Tree Traversals (3/9)

- Inorder traversal (*LVR*) (recursive version)

output: A / B * C * D + E

```
void inorder(tree_pointer ptr)
/* inorder tree traversal */
{
    if (ptr) {
        inorder(ptr->left_child);
        printf("%d",ptr->data);
        inorder(ptr->right_child);
    }
}
```

L
V
R

Program 5.1: Inorder traversal of a binary tree

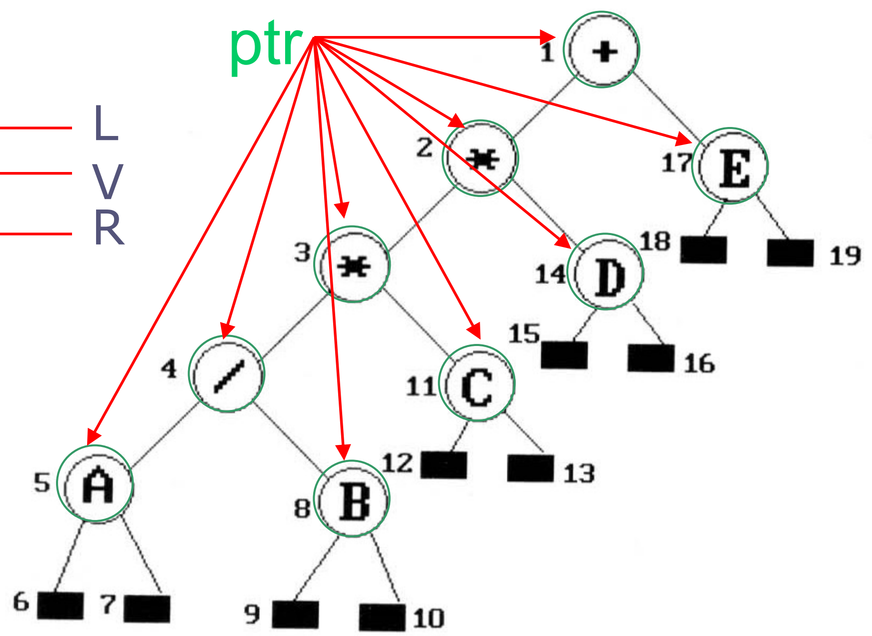


Figure 5.15: Binary tree with arithmetic expression

Binary Tree Traversals (4/9)

- Preorder traversal (*VLR*) (recursive version)

output: + * * / A B C D E

```
void preorder(tree_pointer ptr)
/* preorder tree traversal */
{
   if (ptr) {
      printf("%d",ptr->data);
      preorder(ptr->left_child);
      preorder(ptr->right_child);
   }
}
```

V
L
R

Program 5.2: Preorder traversal of a binary tree

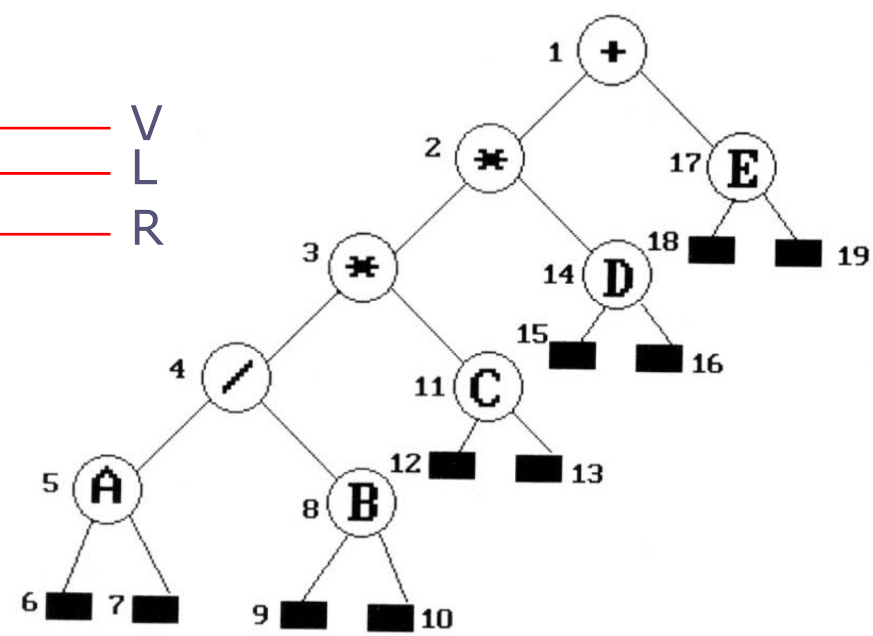


Figure 5.15: Binary tree with arithmetic expression

Binary Tree Traversals (5/9)

- Postorder traversal (*LRV*) (recursive version)

output: A B / C * D * E +

```
void postorder(tree_pointer ptr)
/* postorder tree traversal */
{
   if (ptr) {
      postorder(ptr->left_child);
      postorder(ptr->right_child);
      printf("%d",ptr->data);
   }
}
```

L
R
V

Program 5.3: Postorder traversal of a binary tree

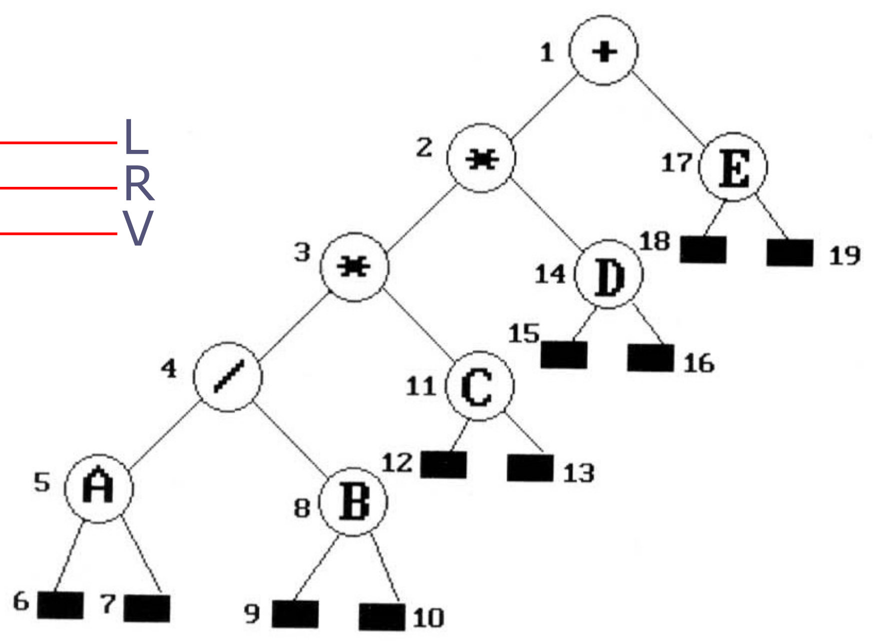


Figure 5.15: Binary tree with arithmetic expression

Binary Tree Traversals (6/9)

- Iterative inorder traversal
  - we use a stack to simulate recursion

```
void iter_inorder(tree_pointer node)
{
   int top = -1; /* initialize stack */
   tree_pointer stack[MAX_STACK_SIZE];
   for (;;) {
      for(; node; node = node->left_child)
        add(&top, node); /* add to stack */
      node = delete(&top); /* delete from stack */
      if (!node) break; /* empty stack */
      printf("%d", node->data);
      node = node->right_child;
   }
}
```

L

V

R

Program 5.4: Iterative inorder traversal

5	8	11	14	17
A	B	C	D	E

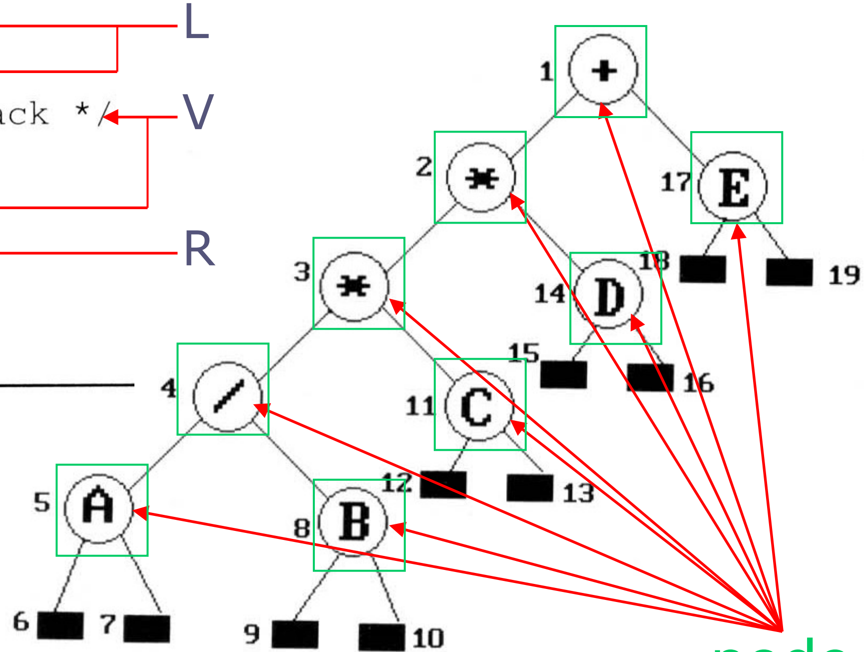


Figure 5.15: Binary tree with arithmetic expression

output: A / B * C * D + E

Binary Tree Traversals (7/9)

- Analysis of inorder2 (Non-recursive Inorder traversal)
  - Let n be the number of nodes in the tree
  - Time complexity: O(n)
    - Every node of the tree is placed on and removed from the stack exactly once
  - Space complexity: O(n)
    - equal to the depth of the tree which (skewed tree is the worst case)

Binary Tree Traversals (8/9)

- Level-order traversal
  - method:
    - We visit the root first, then the root’s left child, followed by the root’s right child.
    - We continue in this manner, visiting the nodes at each new level from the leftmost node to the rightmost nodes
  - This traversal requires a queue to implement

Binary Tree Traversals (9/9)

- Level-order traversal (using queue)

```
void level_order(tree_pointer ptr)
/* level order tree traversal */
{
   int front = rear = 0;
   tree_pointer queue[MAX_QUEUE_SIZE];
   if (!ptr) return; /* empty tree */
   addq(front, &rear, ptr);
   for (;;) {
      ptr = deleteq(&front, rear);
      if (ptr) {
         printf("%d",ptr->data);
         if(ptr->left_child)
            addq(front,&rear,ptr->left_child);
         if (ptr->right_child)
            addq(front,&rear,ptr->right_child);
      }
      else break;
   }
}
```

FIFO

Program 5.5: Level order traversal of a binary tree

output: + * E * D / C A B

2	17	3	14	4	11	5	8
*	E	*	D	/	C	A	B