The Map Method

- Simplification of Boolean Expression
  - Minimum # of terms, minimum # of literals
  - To reduce complexity of digital logic gates
  - The simplest expression is not unique
- Methods:
  - Algebraic minimization ⇒ lack of specific rules
    - Section 2.4
  - Karnaugh map or K-map
    - Combination of 2, 4, … adjacent squares

Logic circuit ⇔ Boolean function ⇔ Truth table ⇔ K-map
⇔ Canonical form (sum of minterms, product of maxterms)
⇔ (Simplifier) standard form (sum of products, product of sums)

The Map Method

A Karnaugh map is a graphical tool for assisting in the general simplification procedure.

Two-Variable Maps

- 2 variables → 4 minterms → 4 squares.

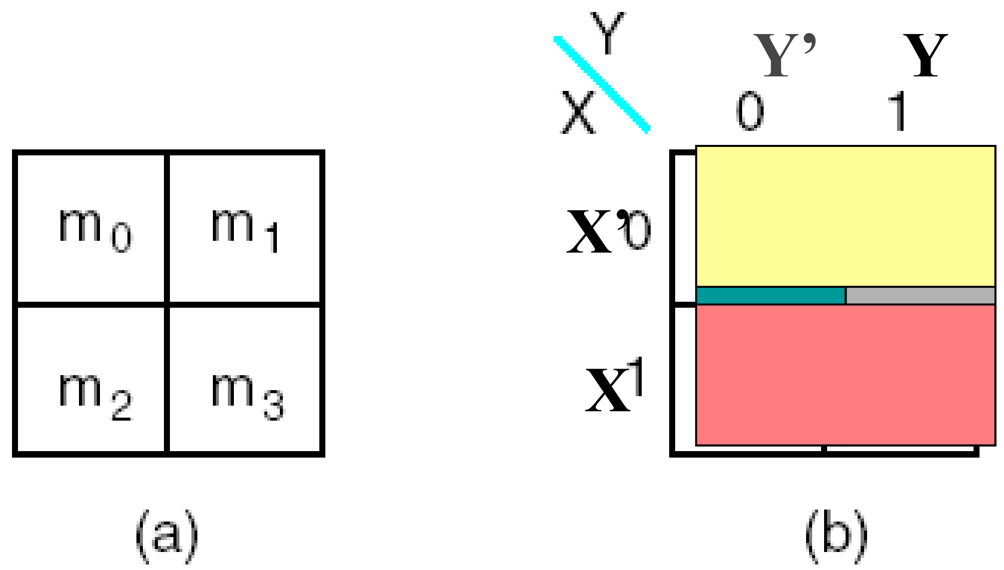


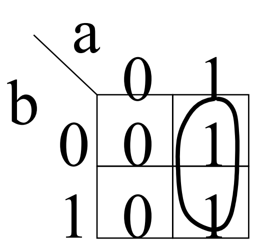
Fig. 2-8 Two-Variable Map

Rules for K-Maps

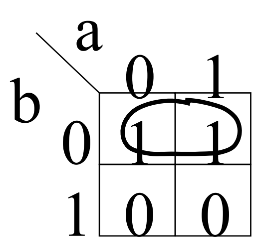
- We can reduce functions by circling 1's in the K-map
- Each circle represents minterm reduction
- Following circling, we can deduce minimized and-or form.
- Rules to consider
  - Every cell containing a 1 must be included at least once.
  - The largest possible "power of 2 rectangle" must be enclosed.
  - The 1's must be enclosed in the smallest possible number of rectangles.

Two-Variable Maps (Cont.)

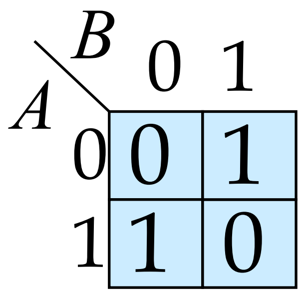
- <u>Two variable maps:</u>



$f = a$



$g = b'$



$F = AB' + A'B$

Two-variable Map

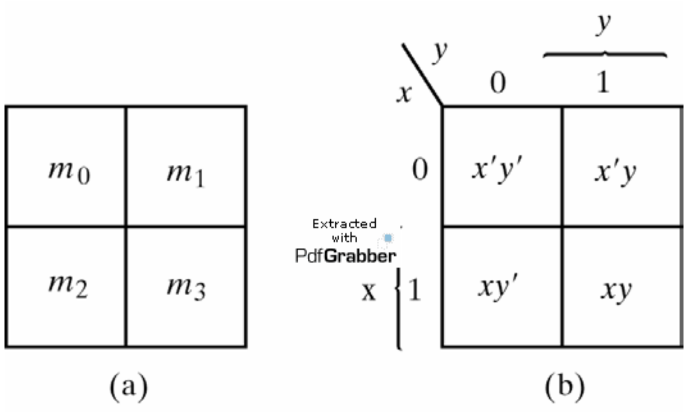


Fig. 3-1 Two-variable Map

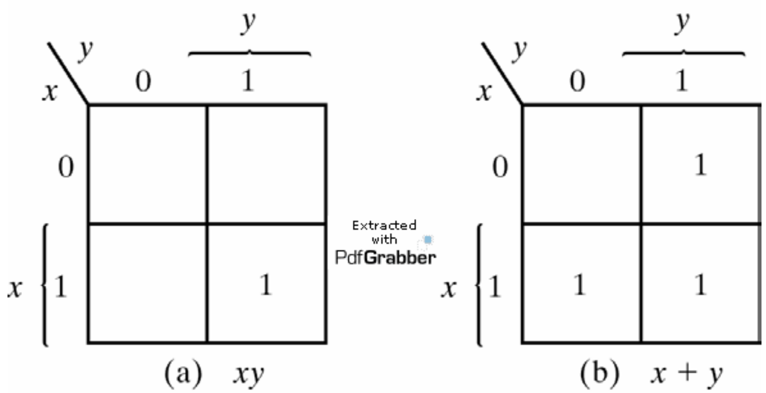
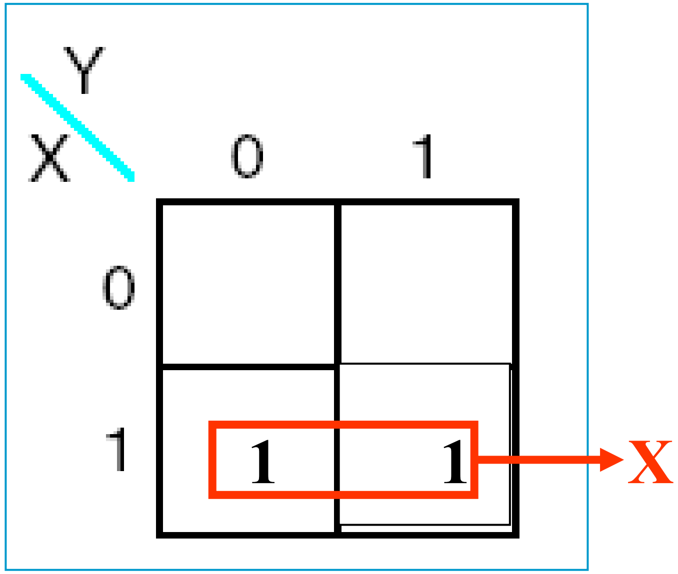


Fig. 3-2 Representation of Functions in the Map

$$m_1 + m_2 + m_3 = x'y + xy' + xy = x + y$$

2-Variable Map Simplification Example (1)

- Example: F(X,Y) = XY’ + XY
- From the map, we see that
  F (X,Y) = X.
  Note: There are implied 0s in other boxes.



- This can be justified using algebraic manipulations:

$$
\begin{aligned}
F(X,Y) &= XY' + XY \\
&= X(Y' + Y) \\
&= X.1 \\
&= X
\end{aligned}
$$

2-Variable Map Simplification Example (2)

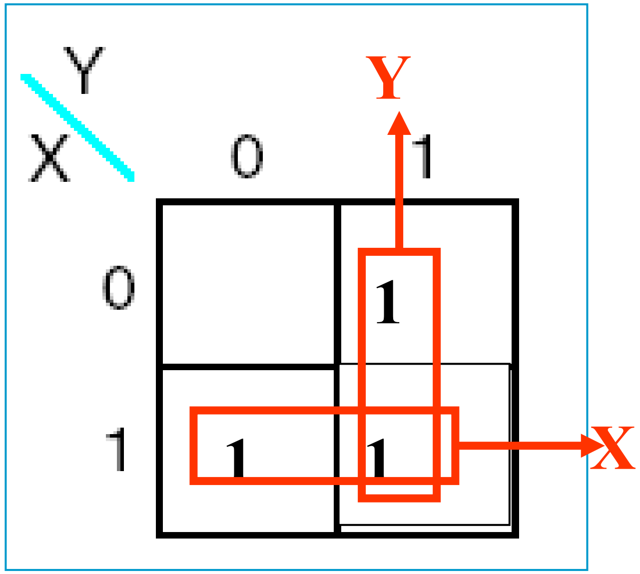
- **Example:**
  G(x,y) = m1 + m2 + m3

G(x,y) = m1 + m2 + m3

= X'Y + XY' + XY

From the map, we can see that

G = X + Y



2-Variable Map Simplification Example (3)

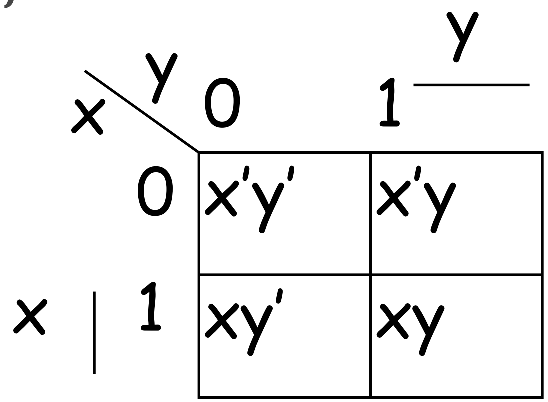
- **Example:**

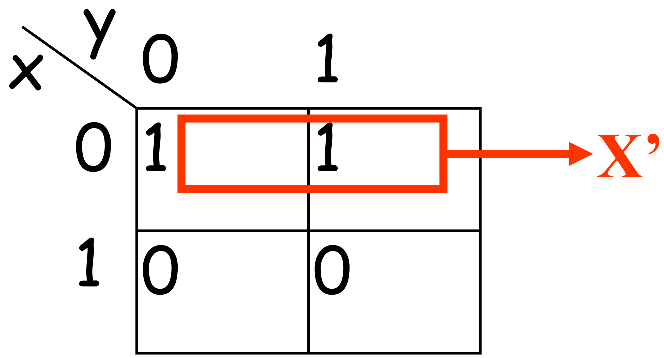
  $F = \Sigma(m0, m1)$

x	y	F
0	0	1
0	1	1
1	0	0
1	1	0

Using algebraic manipulations

$$
\begin{aligned}
F &= \Sigma(m0, m1) \\
&= x'y + x'y' \\
&= x'(y + y') \\
&= x'
\end{aligned}
$$





Three-Variable Maps

- 3 variables→ 8 minterms (m0 – m7).

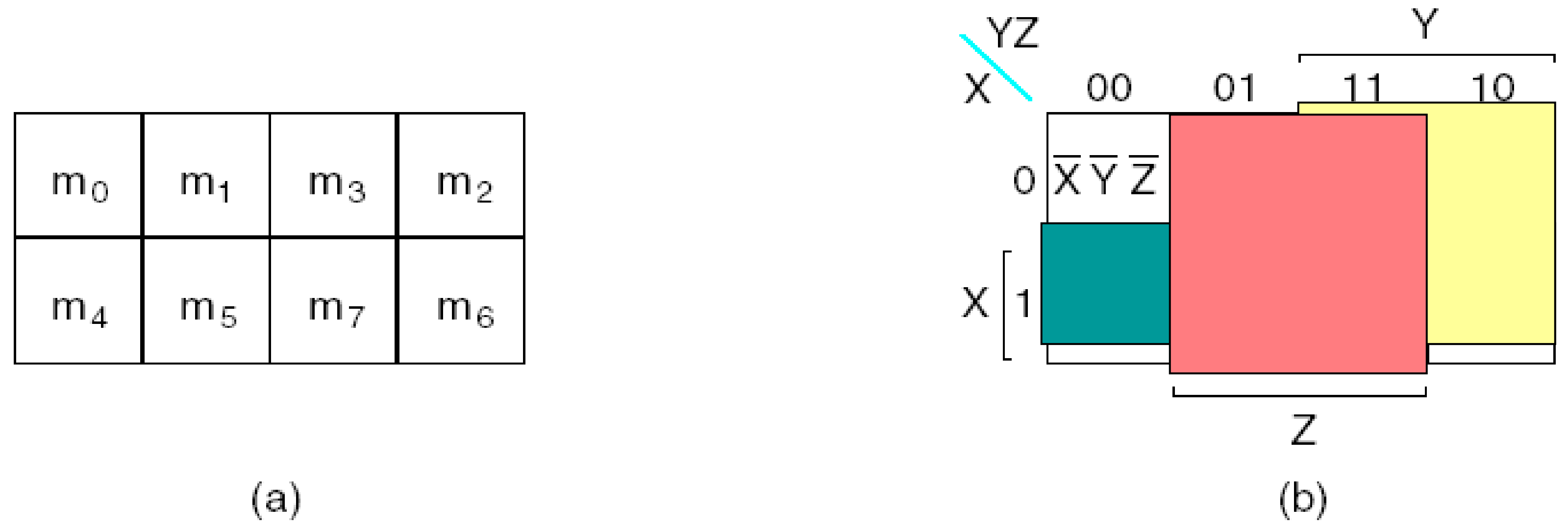


Fig. 2-10 Three-Variable Map

How can we locate a minterm square on the map?

→ Use figure (a) OR → use column # and row # from figure (b)

E.g. m_5 is in row 1 column 01 ($5_{10} = 101_2$)

Q. Show the area representing X'? Y'? Z'?

Three-Variable map

- 8 minterms for 3 binary variables
- Any two adjacent squares differ by only one variable

$$m_0 + m_2 = x'y'z' + x'yz' = x'z'(y' + y) = x'z'$$
$$m_4 + m_6 = xy'z' + xyz' = xz' + (y' + y) = xz'$$

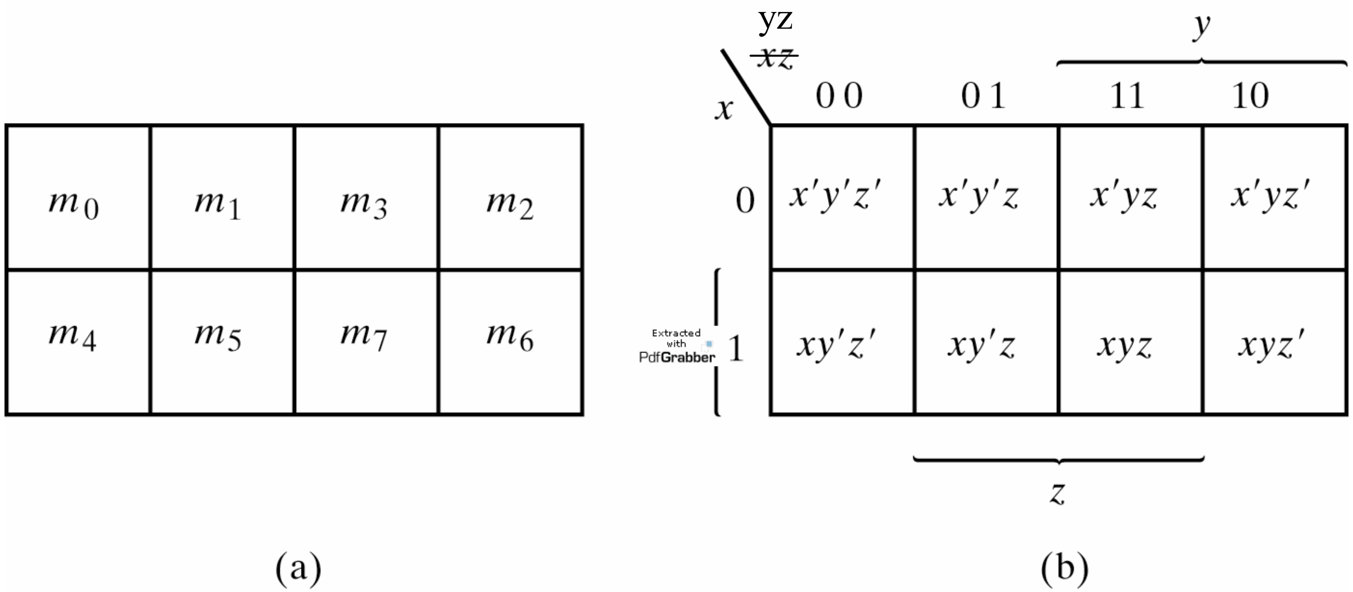


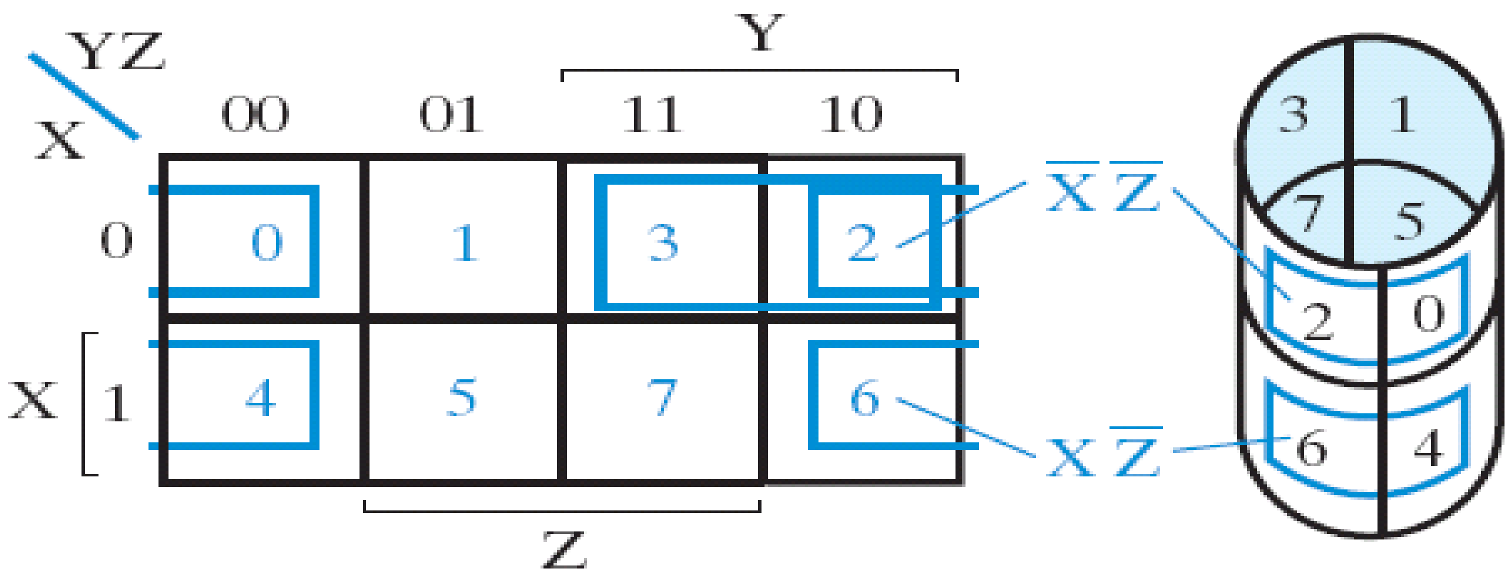
Fig. 3-3 Three-variable Map

Three-Variable Maps (Cont.)

- By combining squares *in powers of 2*, we reduce number of literals in a product term, reducing the literal cost, thereby reducing the other two cost criteria.
- On a 3-variable K-Map:
  - One square represents a minterm with three variables
  - Two adjacent squares represent a product term with two variables
  - Four “adjacent” terms represent a product term with one variables
  - Eight “adjacent” terms is the function of all ones (logic 1).

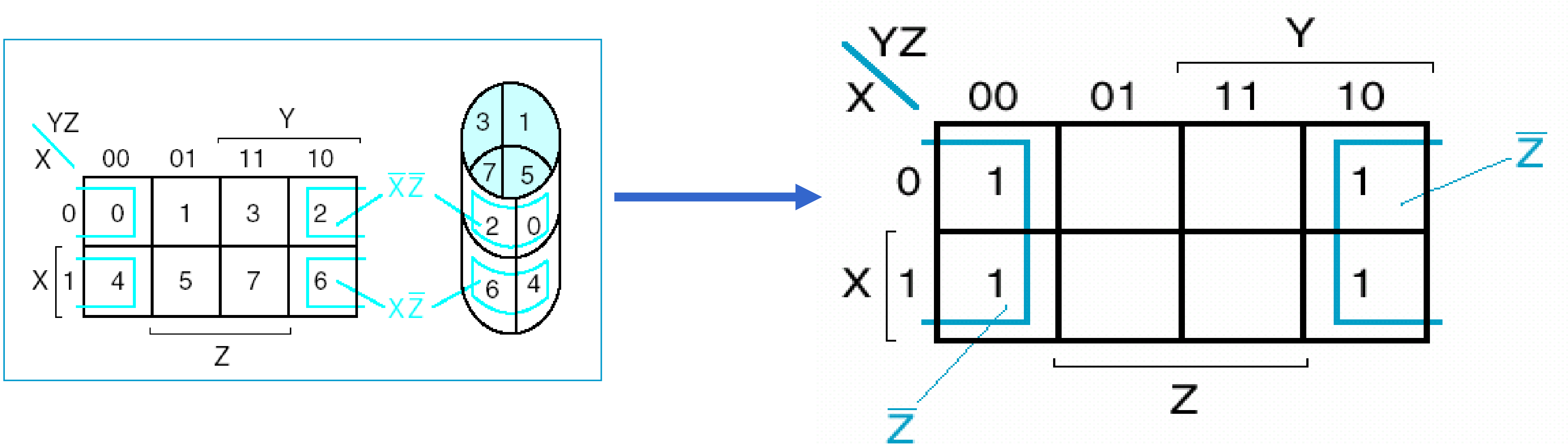
3-Variable Map Simplification Example (1)

- **Adjacent Squares**
  - m0+m2 = XYZ + XYZ = XZ(Y+Y) = XZ
  - m4+m6 = XYZ + XYZ = XZ(Y+Y) = ZX



Note that Z' wraps from left edge to right edge.

3-Variable Map Simplification Example (2)



- $F = X'Y'Z' + X'YZ' + XY'Z' + XYZ'$

  $= Z' (X'Y' + X'Y + XY' + XY)$

  $= Z' (X' (Y'+Y) + X (Y'+Y))$

  $= Z' (X'+ X)$

  $= Z'$

3-Variable Map Simplification Example (3)

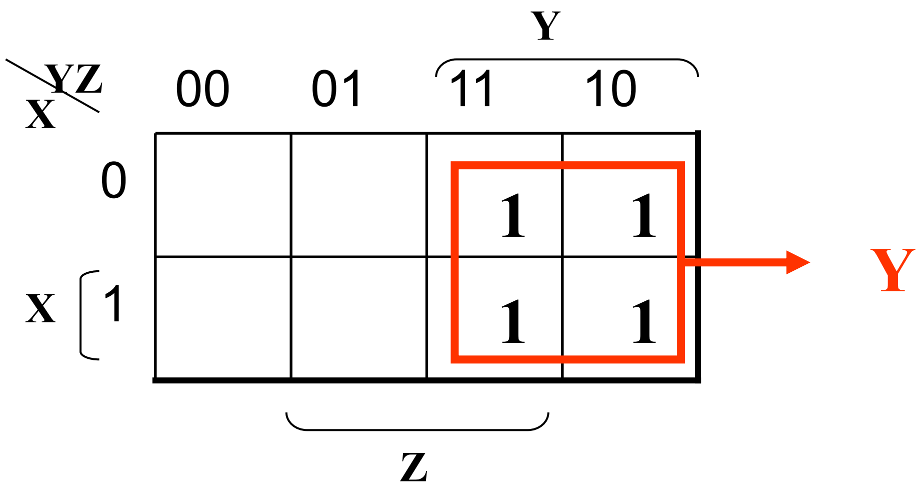
- $F = AB'C' + AB'C + ABC + ABC' + A'B'C + A'BC'$

A \ BC	00	01	11	10
0	0	1	0	1
1	1	1	1	1

$F = A + B'C + BC'$

3-Variable Map Simplification Example (4)

- **Example:**
  F (x, y, z)= Σm (2, 3, 6, 7)



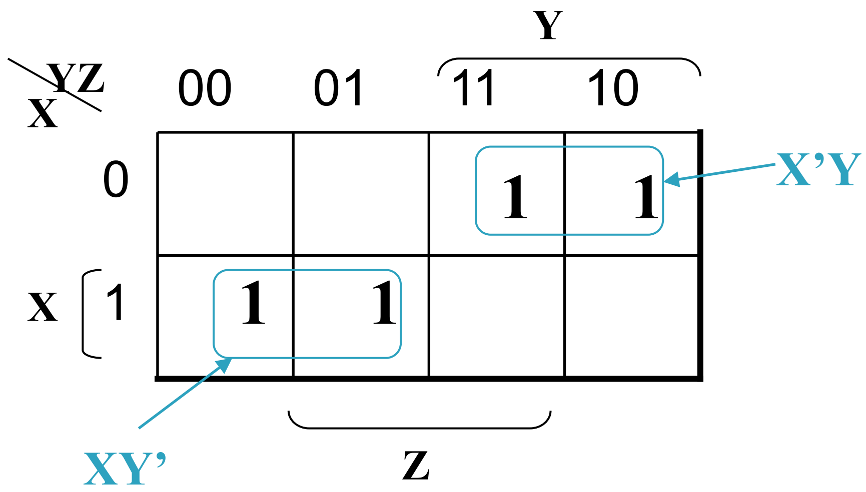
- **Applying the Minimization Theorem three times:**

$$F(x, y, z) = \overline{x}\,y\,z + x\,y\,z + \overline{x}\,y\,\overline{z} + x\,y\,\overline{z}$$
$$= yz + y\,\overline{z}$$
$$= y$$

- **Thus the four terms that form a 2 × 2 square correspond to the term "y".**

3-Variable Map Simplification Example (5)

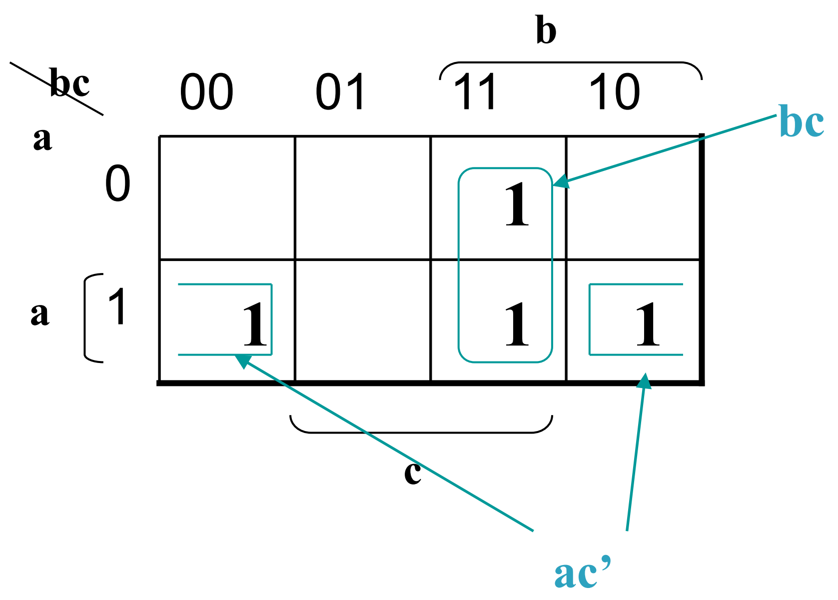
- **Example: Simplify**
  F (x, y, z)= Σm (2, 3, 4, 5)



$F(X,Y,Z) = X'Y + XY'$

3-Variable Map Simplification Example (6)

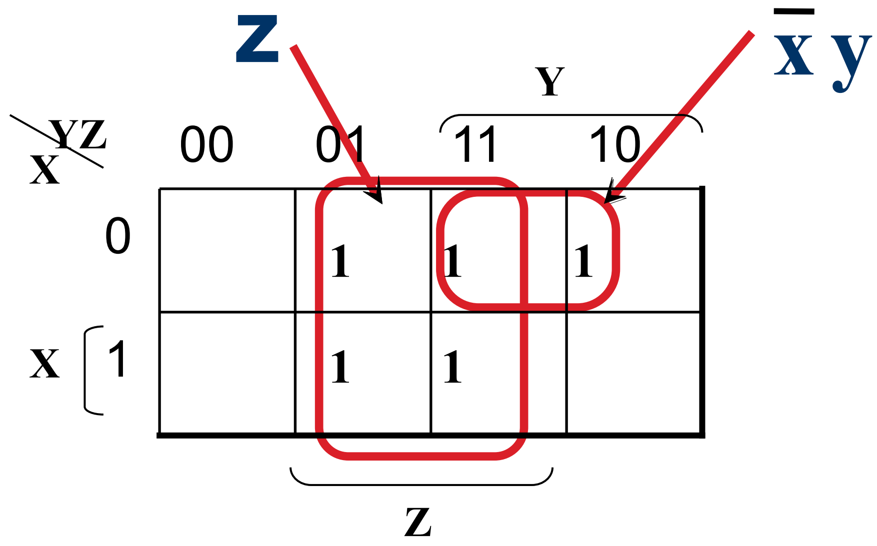
- **Example: Simplify**
  G (a, b, c)= Σm (3, 4, 6, 7)



$G\ (a,b,c) = bc + ac'$

3-Variable Map Simplification Example (7)

- Example: Simplify
  F(X, Y, Z) = X'Z + X'Y + XY'Z + YZ
  F(X, Y, Z) = Σm (1, 2, 3, 5, 7)



- In general, as more squares are combined, we obtain a product term with fewer literals.
- Overlap is allowed.

$$F(x, y, z) = z + \bar{x}\,y$$

Examples 3-1 and 3-2

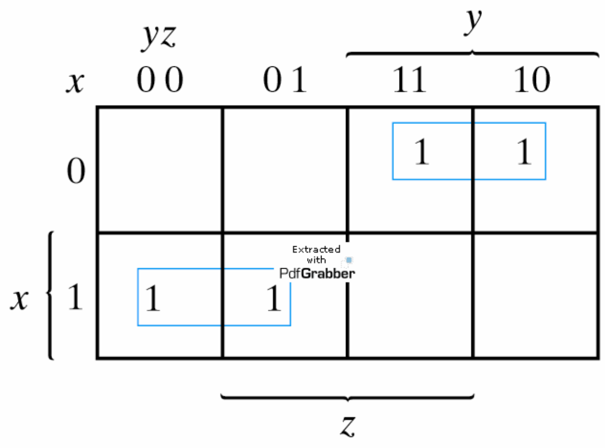


Fig. 3-4 Map for Example 3-1; $F(x, y, z) = \Sigma(2, 3, 4, 5) = x'y + xy'$

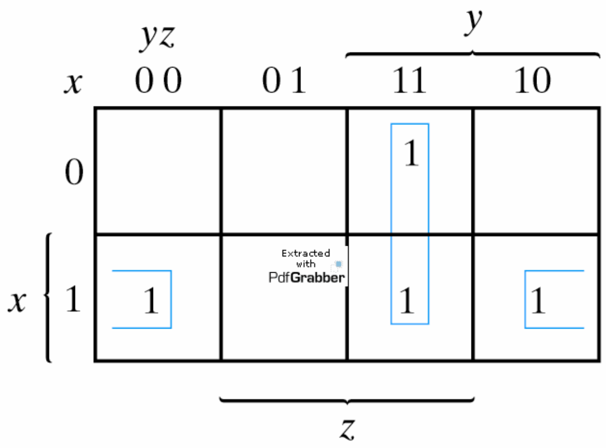


Fig. 3-5 Map for Example 3-2; $F(x, y, z) = \Sigma(3, 4, 6, 7) = yz + xz'$

Examples 3-3 and 3-4

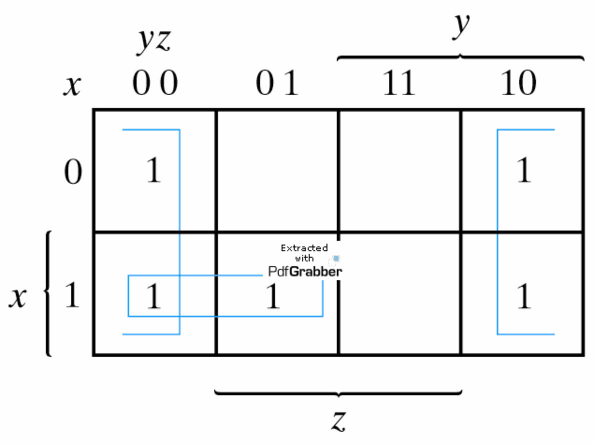


Fig. 3-6 Map for Example 3-3; $F(x, y, z) = \Sigma(0, 2, 4, 5, 6) = z' + xy'$

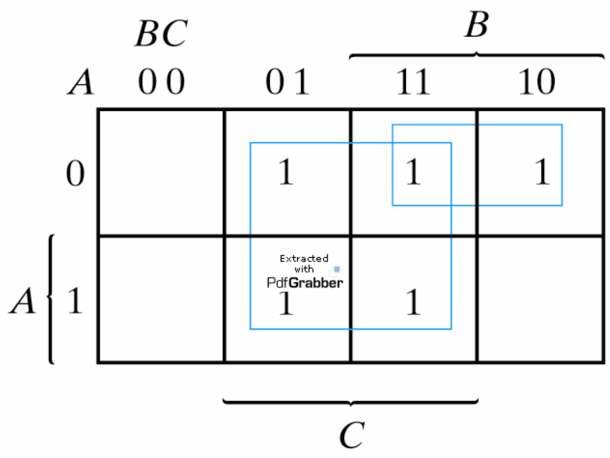


Fig. 3-7 Map for Example 3-4; $A'C + A'B + AB'C + BC = C + A'B$

One square represents one minterm, giving a term of three literals

- Two adjacent squares represent a term of two literals
- Four adjacent squares represent a term of one literal

Four-Variable Map

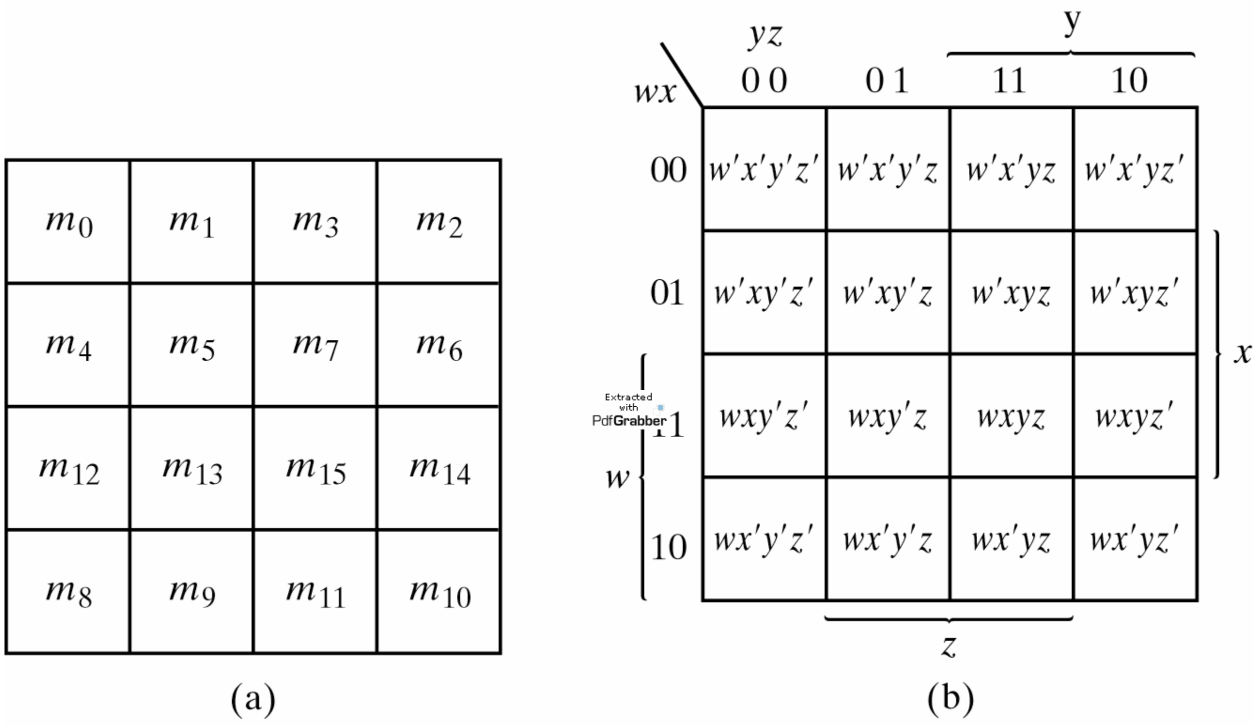
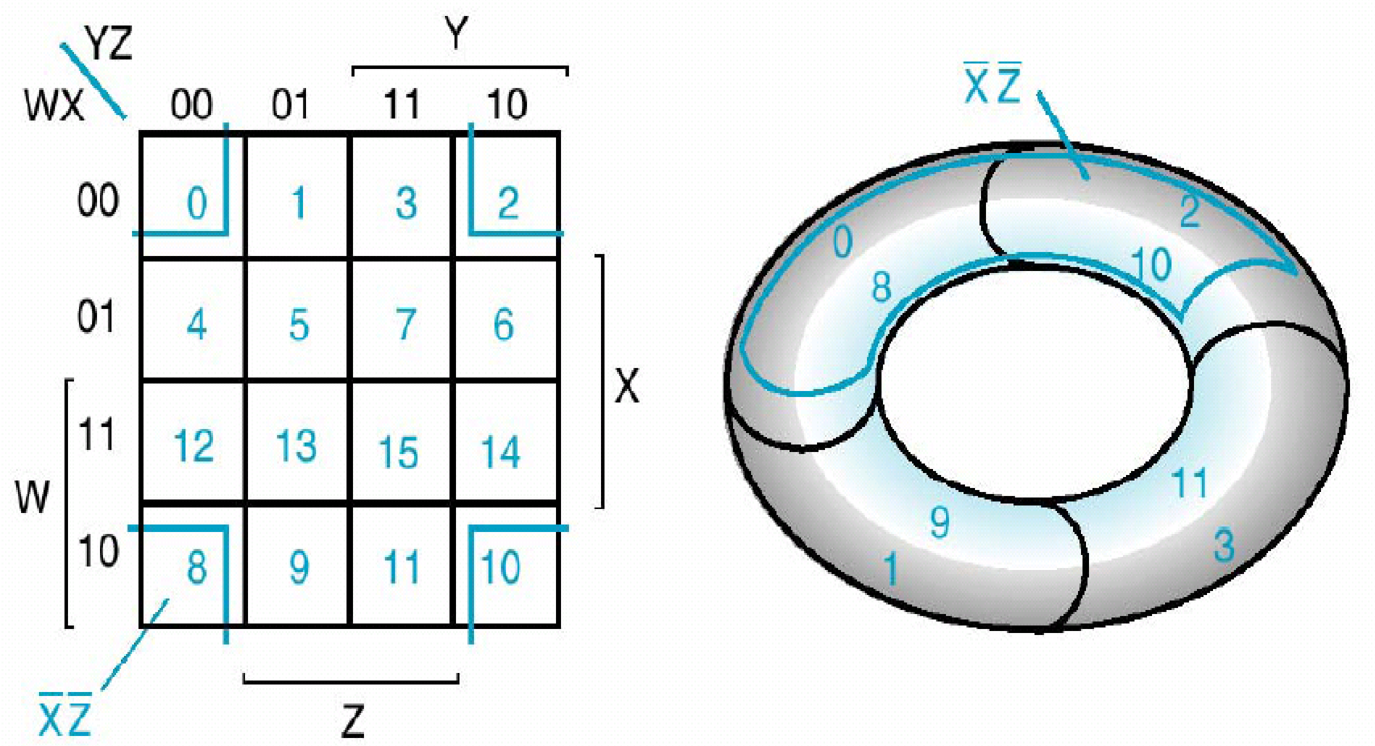


Fig. 3-8 Four-variable Map

- Two adjacent squares represent a term of three literals
- Four adjacent squares represent a term of two literals
- Eight adjacent squares represent a term of one literal

The larger the number of squares combined, the smaller the number of literals in the term

Flat Map Vs Torus



Examples 3-5 and 3-6

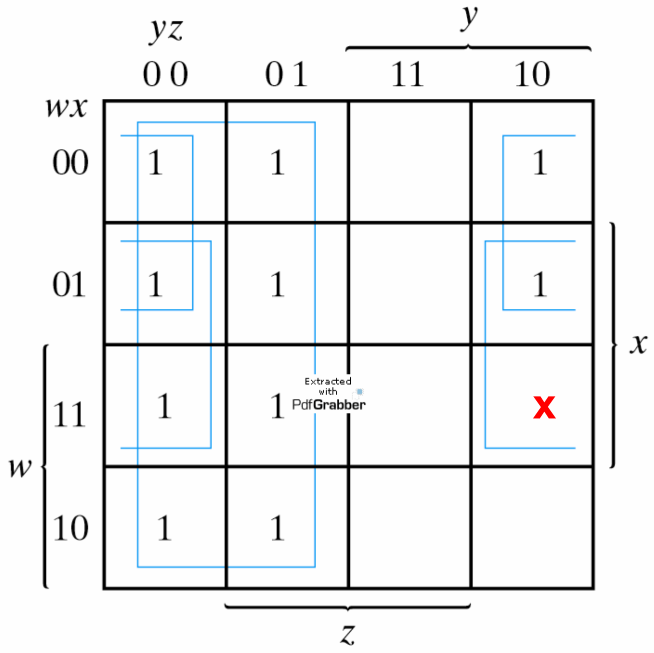


Fig. 3-9 Map for Example 3-5; $F(w, x, y, z)$
$= \Sigma\,(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14) = y' + w'z' + xz'$

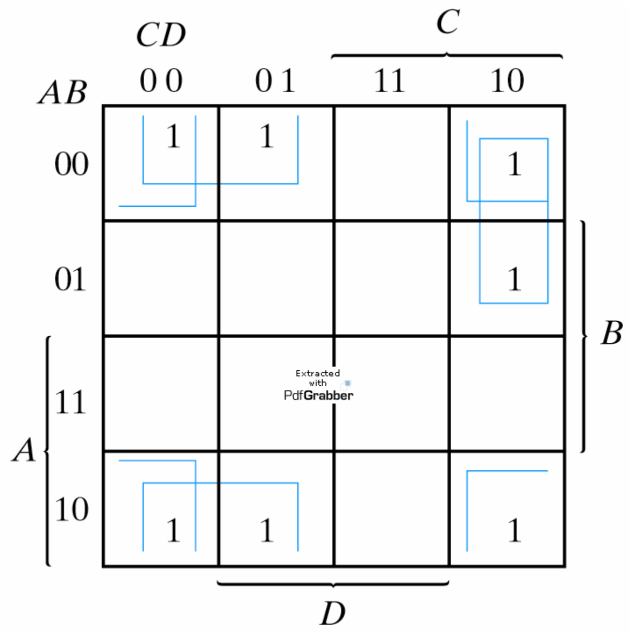
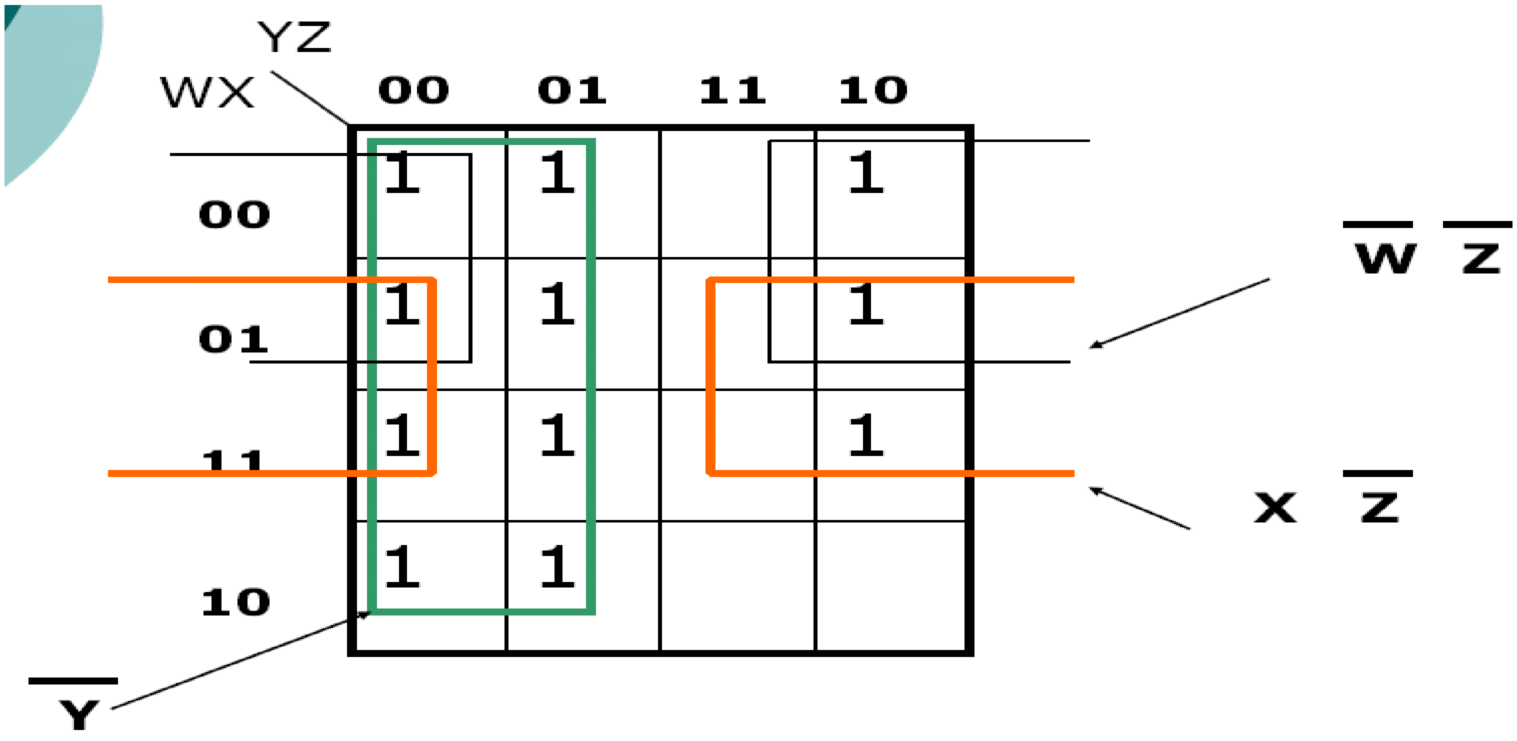


Fig.3-10 Map for Example 3-6; $A'B'C' + B'CD' + A'BCD'$
$+ AB'C' = B'D' + B'C' + A'CD'$

4-Variable Map Simplification Example

- $F = \sum m(0,1,2,4,5,6,8,9,12,13,14)$



$$F = Y' + XZ' + W'Z'$$

4-Variable Map Simplification Example

- $F = \sum m(0,2,4,5,6,7,10,13,15)$
- Do it and show it to me next time!