

Asynchronous Sequential Logic

UNIT-5

Introduction

- **Synchronous Sequential Circuits**

The change of internal state occurs in response to the **synchronized clock pulse**.

Memory elements are **clocked flip-flops**.

- **Asynchronous Sequential Circuits**

The change of internal state occurs when there is a **change in the input variables**.

Memory elements are **unclocked flip-flops or time-delay elements**.

Introduction

- **Synchronous Sequential Circuits**

Timing problems are eliminated by triggering all flip-flops with pulse edge.

- **Asynchronous Sequential Circuits**

Care must be taken to ensure that each new state is stable even though a feedback path exists.

Higher speed , More economical

Introduction

• Asynchronous Sequential Circuits

When an input variable changes in value, the secondary variables do not change instantaneously.

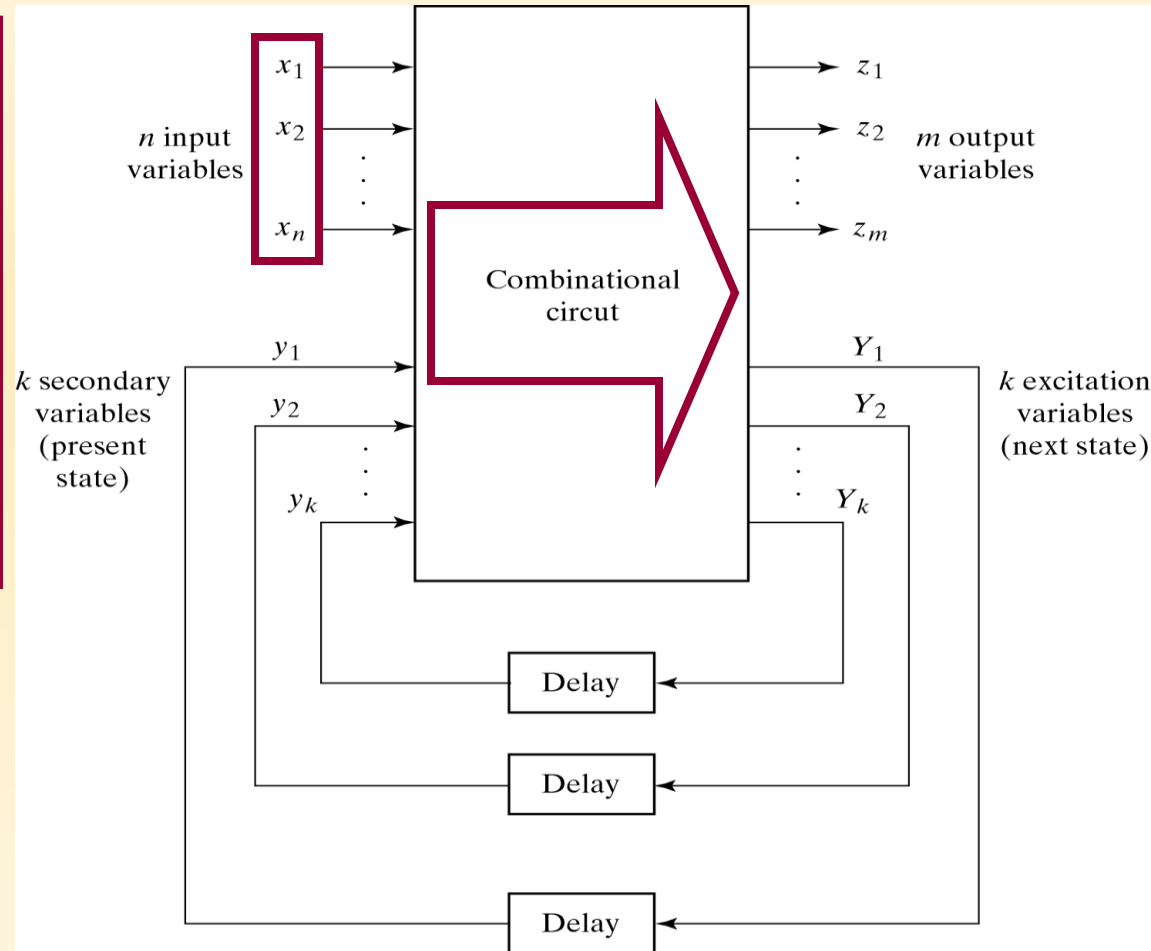


Fig. 9-1 Block Diagram of an Asynchronous Sequential Circuit

Introduction

• Asynchronous Sequential Circuits

In steady-state condition, the y 's and the Y 's are the same, but during transition they are not.

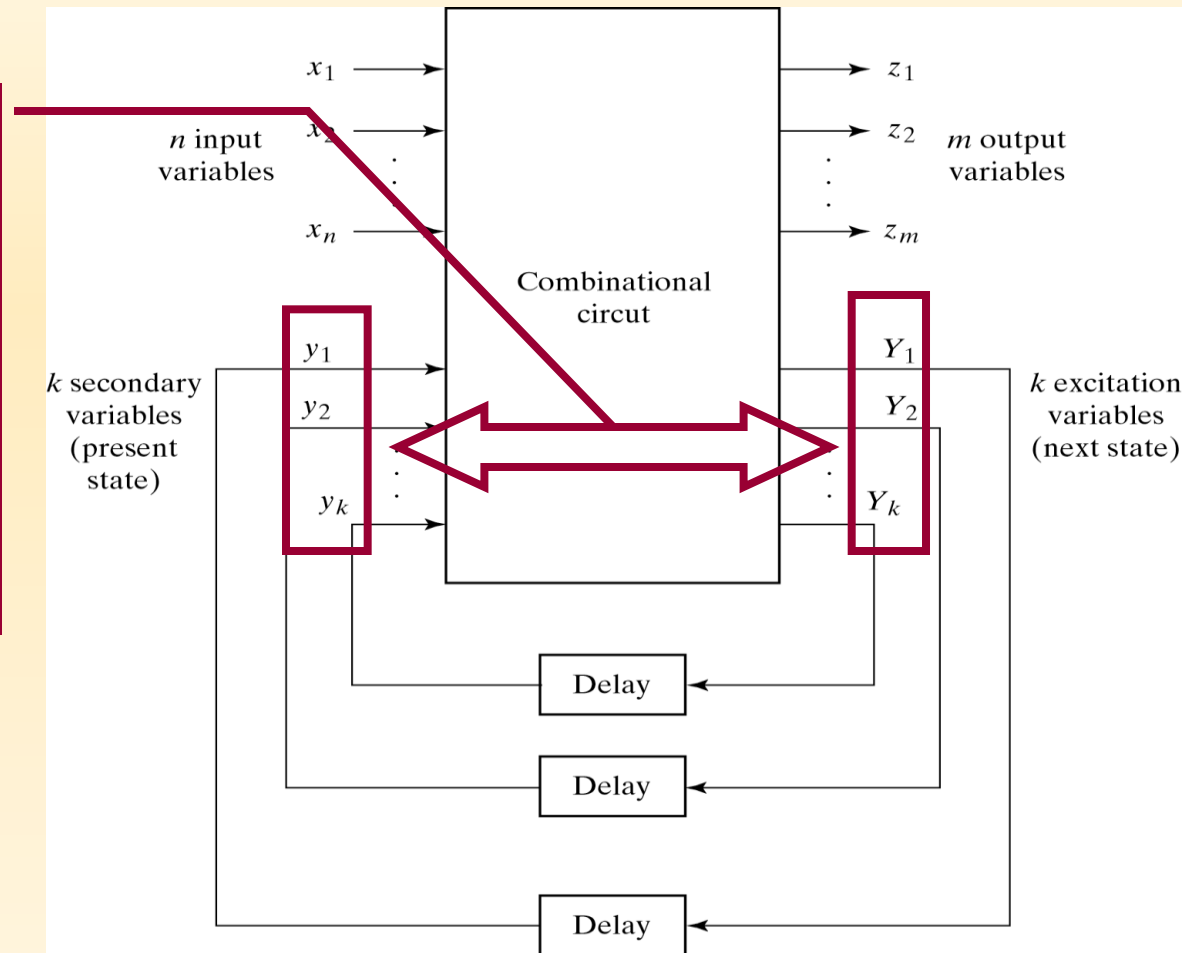


Fig. 9-1 Block Diagram of an Asynchronous Sequential Circuit

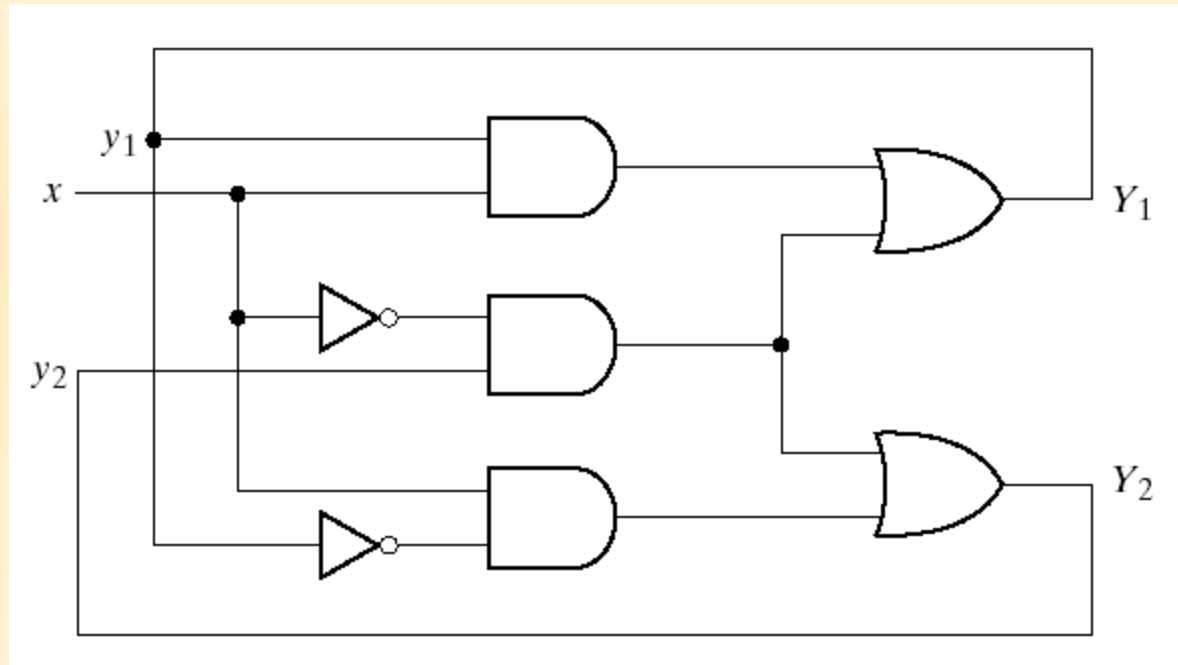
Introduction

- **fundamental mode**

fundamental mode : Only one input variable can change at any one time and the time between two input changes must be longer than the time it takes the circuit to reach a stable state.

Analysis Procedure

Transition Table



$$Y_1 = xy_1 + x'y_2$$

$$Y_2 = xy'_1 + x'y_2$$

Analysis Procedure

Transition Table

$$Y_1 = xy_1 + x'y_2$$

$$Y_2 = xy'_1 + x'y_2$$

	x	
y ₁ y ₂	0	1
00	0	0
01	1	0
11	1	1
10	0	1

(a) Map for
 $Y_1 = xy_1 + x'y_2$

	x	
y ₁ y ₂	0	1
00	0	1
01	1	1
11	1	0
10	0	0

(b) Map for
 $Y_2 = xy'_1 + x'y_2$

	x	
y ₁ y ₂	0	1
00	00	01
01	11	01
11	11	10
10	00	10

(c) Transition table

Analysis Procedure

Transition Table

For a state to be stable, the value of Y must be the same as that of $y = y_1y_2$

	x	
	0	1
y_1y_2		
00	00	01
01	11	01
11	11	10
10	00	10

(c) Transition table

Analysis Procedure

Transition Table

The Unstable states,
 $Y \neq y$

		x	
		0	1
$y_1 y_2$	00	00	01
	01	11	01
	11	11	10
	10	00	10

(c) Transition table

Analysis Procedure

Transition Table

Consider the square for $x = 0$ and $y = 00$. It is stable.

x changes from 0 to 1.

The circuit changes the value of Y to 01. The state is unstable.

The feedback causes a change in y to 01. The circuit reaches stable.

	$x=0$	$x=1$
$y_1 y_2 = 00$	00	01
$y_1 y_2 = 01$	11	01
$y_1 y_2 = 11$	11	10
$y_1 y_2 = 10$	00	10

(c) Transition table

Analysis Procedure

Transition Table

In general, if a change in the input takes the circuit to an unstable state, y will change until it reaches a stable state.

	$x=0$	$x=1$
$y_1 y_2 = 00$	00	01
$y_1 y_2 = 01$	11	01
$y_1 y_2 = 11$	11	10
$y_1 y_2 = 10$	00	10

(c) Transition table

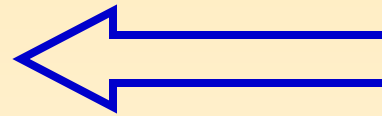
Analysis Procedure

Flow Table

	x	
	0	1
a	a	b
b	c	b
c	c	d
d	a	d

Flow Table

Flow table whose states are designated by letter symbols and whose inputs and outputs are binary values



	x	
$y_1 y_2$	0	1
00	00	01
01	11	01
11	11	10
10	00	10

(c) Transition table

Analysis Procedure

Flow Table

It is called primitive flow table because it has only one stable state in each row.

	x	
	0	1
a	a	b
b	c	b
c	c	d
d	a	d

	$x_1 x_2$			
	00	01	11	10
a	$a, 0$	$a, 0$	$a, 0$	$b, 0$
b	$a, 0$	$a, 0$	$b, 1$	$b, 0$

(b) Two states with two inputs and one output

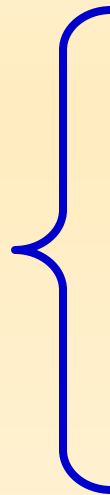
It is a flow table with more than one stable state in the same row.

Analysis Procedure

Flow Table

		x	
		0	1
y ₁ y ₂	00	00	01
	01	11	01
	11	11	10
	10	00	10

(c) Transition table



		x ₁ x ₂			
		00	01	11	10
y	0	0	0	0	1
	1	0	0	1	1

(a) Transition table
 $Y = x_1x_2' + x_1y$

		x ₁ x ₂			
		00	01	11	10
y	0	0	0	0	0
	1	0	0	1	0

(b) Map for output
 $z = x_1x_2y$

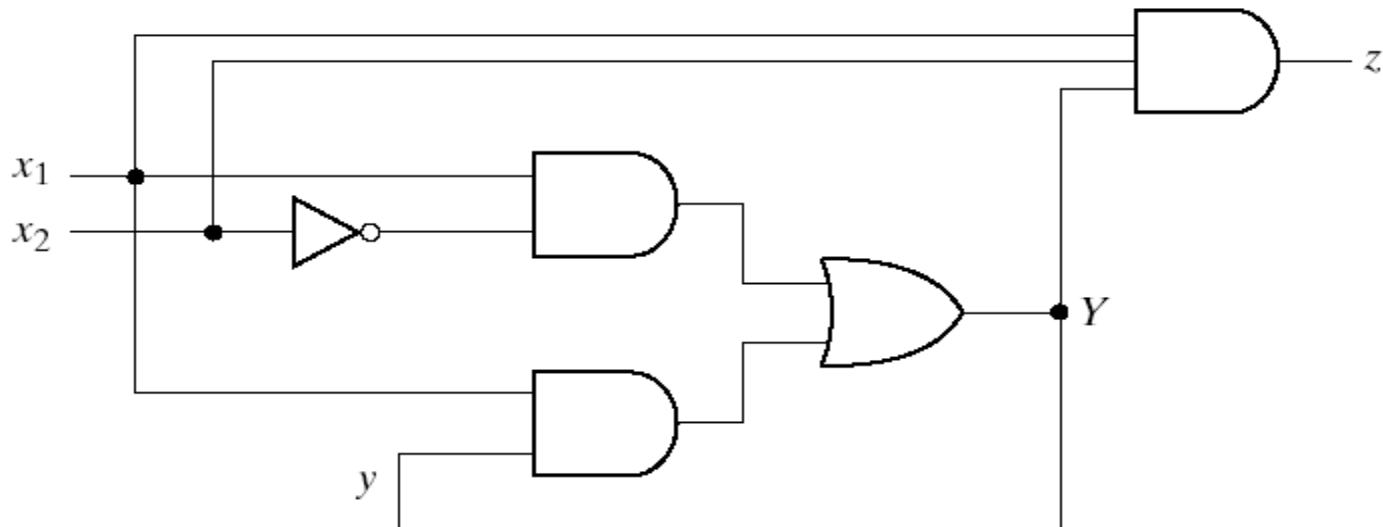
Analysis Procedure

		$x_1 x_2$			
		00	01	11	10
y	0	0	0	0	1
	1	0	0	1	1

(a) Transition table
 $Y = x_1x'_2 + x_1y$

		$x_1 x_2$			
		00	01	11	10
y	0	0	0	0	0
	1	0	0	1	0

(b) Map for output
 $z = x_1x_2y$



Analysis Procedure

Race Conditions

Noncritical Race:

Two or more binary state variables change value in response to a change in an input variable. The possible transitions could be

00	11	
00	01	11
00	10	11

It is a **noncritical** race. The final stable state that the circuit reaches does not depend on the order in which the state variables change.

Analysis Procedure

Race Conditions

Critical Race:

State variables change from 00 to 11. The possible transition could be

00	11	
00	01	11
00	10	

It is a **critical** race. The final stable state depends on the order in which the state variables change.

		x	
		0	1
$y_1 y_2$	00	00	11
	01		11

(b) Possible transitions:

Analysis Procedure

Race Conditions

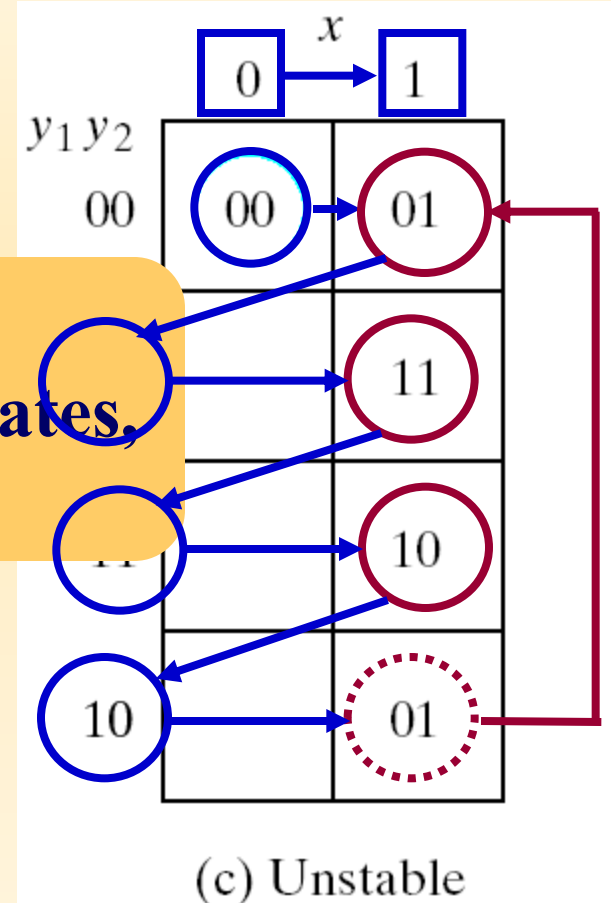
Cycle

It starts at state 00

the
0 to

When a circuit goes through a unique sequence of unstable states, it is said to have a *cycle*.

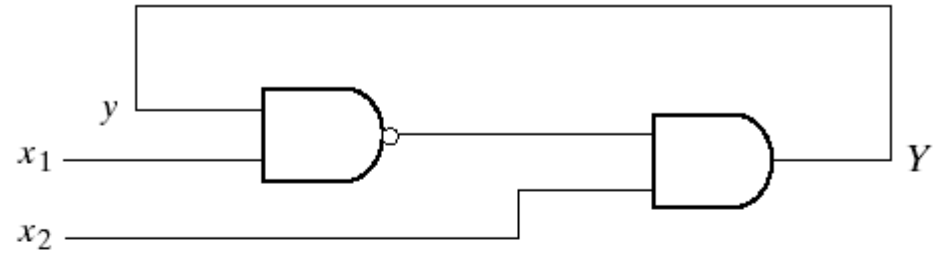
The sequence is as follows,



Analysis Procedure

Stability Consideration

Column 11 has no stable state. With input $x_1 x_2 = 11$, Y and y are never the same.



(a) Logic diagram

This will cause instability.

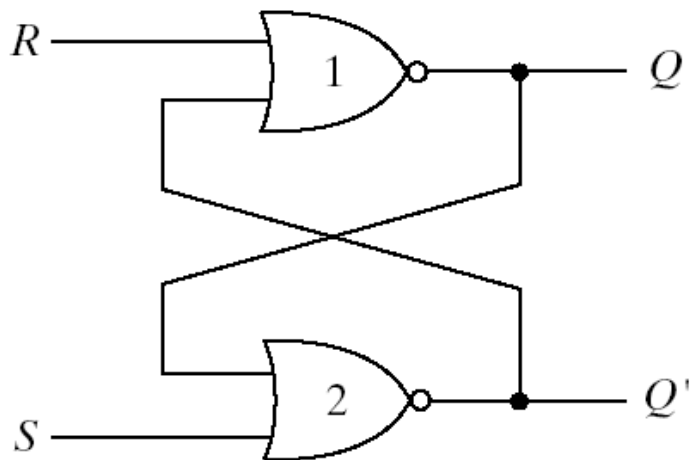
	$x_1 x_2$			
y	00	01	11	10
0	0	1	1	0
1	0	1	0	0

(b) Transition table

Circuits with Latches

SR Latch

The circuit diagram and truth table of the *SR* latch are shown as follows,



(a) Crossed-coupled circuit

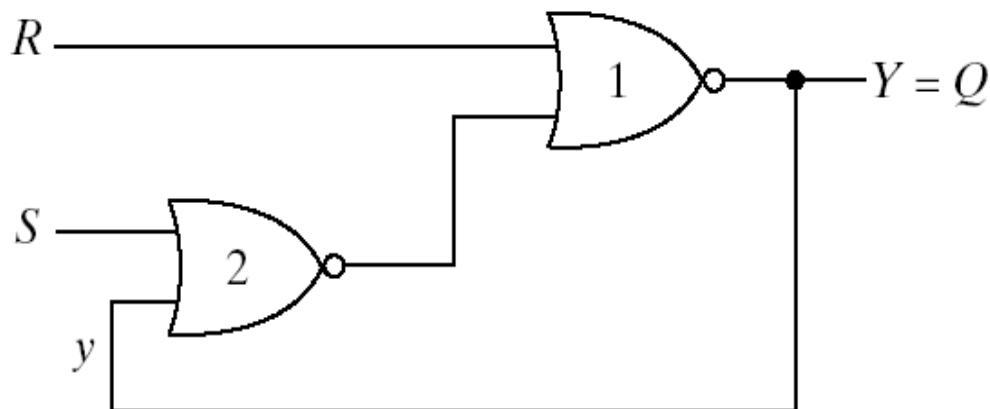
S	R	Q	Q'	
1	0	1	0	
0	0	1	0	(After $SR = 10$)
0	1	0	1	
0	0	0	1	(After $SR = 01$)
1	1	0	0	

(b) Truth table

Circuits with Latches

SR Latch

The circuit diagram of the *SR* latch can be redrawn as follows,



(c) Circuit showing feedback

	00	01	11	10
0	0	0	0	1
1	1	0	0	1

$$Y = SR' + R'y$$

$$Y = S + R'y \text{ when } SR = 0$$

(d) Transition table

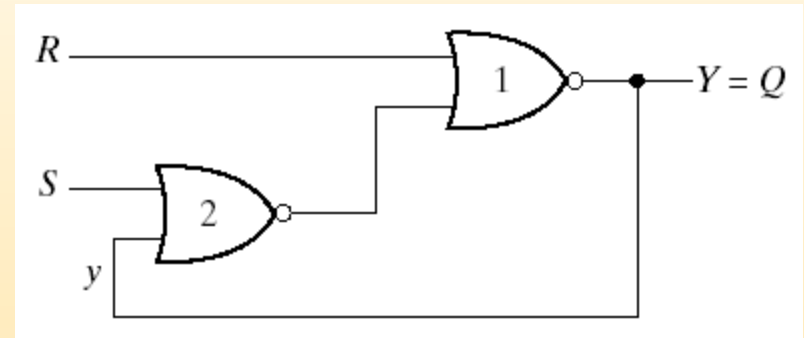
Circuits with Latches

SR Latch

$$Y = [(S + y)' + R]'$$
$$= (S + y)R' = SR' + R'y$$

$$\left. \begin{array}{l} SR' + SR = S(R' + R) = S \\ SR = 0 \end{array} \right\} SR' = S$$

$$\Rightarrow Y = SR' + R'y = S + R'y \quad \text{when } SR=0$$



Circuits with Latches

Analysis Example

$$Y_1 = S_1 + R'_1 y_1 = x_1 y_2 + (x_1 + x_2) y_1$$

$$Y_2 = S_2 + R'_2 y_2 = x_1 x_2 + (x_2 + y'_1) y_2$$

There is a critical race condition

		$x_1 x_2$			
		00	01	11	10
$y_1 y_2$	00	00	00	01	00
	01	01	01	11	11
	11	00	11	11	10
	10	00	10	11	10

Circuits with Latches

Latch Excitation Table

A table that lists the required inputs S and R for each of the possible transitions from y to Y

The first two columns list the four possible transitions from y to Y .

The next two columns specify the required input values that will result in the specified transition.

y	Y	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	1

(b) Latch excitation table

Circuits with Latches

Implementation Example

	x_1x_2			
	00	01	11	10
y				
0	0	0	0	1
1	0	0	1	1

(a) Transition table
 $Y = x_1x'_2 + x_1y$

y	Y	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	1

(b) Latch excitation table

	x_1x_2			
	00	01	11	10
y				
0	0	0	0	1
1	0	0	X	X

(c) Map for $S = x_1x'_2$

	x_1x_2			
	00	01	11	10
y				
0	X	X	X	0
1	1	1	0	0

(d) Map for $R = x'_1$

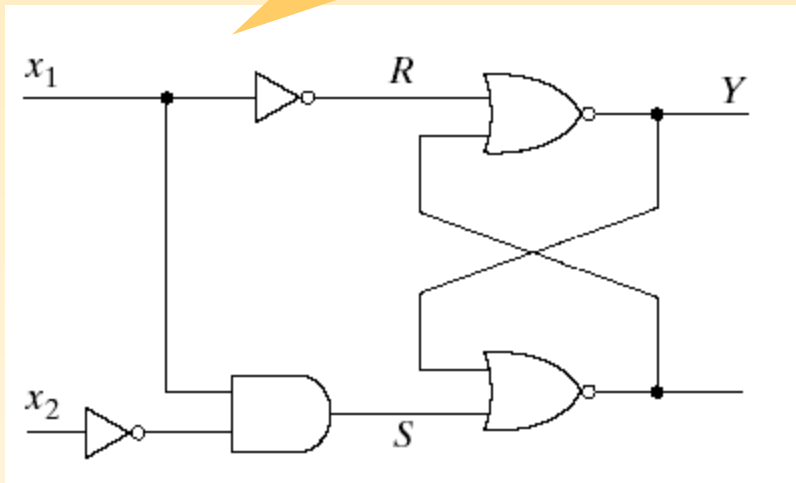


Circuits with Latches

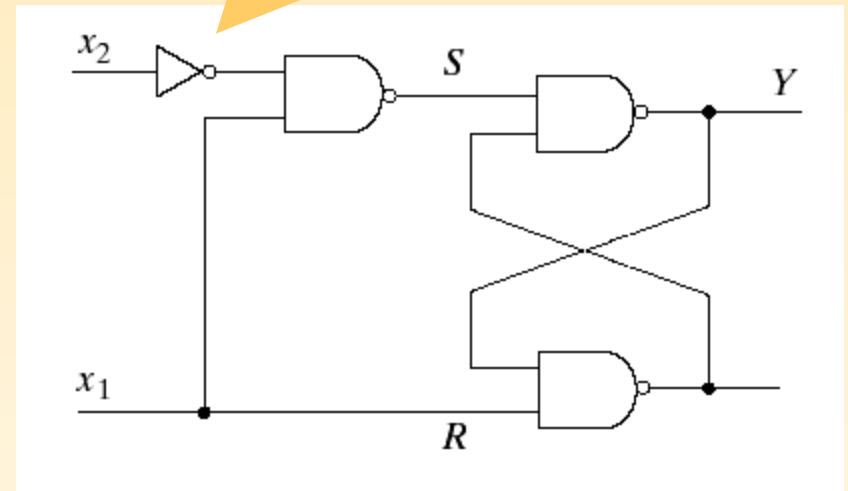
Implementation Example

$$S = x_1x'_2 \quad R = x'_1$$

Circuit with
NOR latch



Circuit with
NAND latch



Design Procedure

Design Example

Design a gated latch circuit with two inputs T (enable) and D (data), and one output Q .

Gated-Latch
Total States

State	Inputs		Output	comments
	D	G	Q	
a	0	1	0	$D = Q$ because $G = 1$
b	1	1	1	$D = Q$ because $G = 1$
c	0	0	0	After state a or d
d	1	0	0	After state c
e	1	0	1	After state b or f
f	0	0	1	After state e

Design Procedure

Design Example

State	Inputs		Output
	D	G	Q
a	0	1	0
b	1	1	1
c	0	0	0
d	1	0	0
e	1	0	1
f	0	0	1



	DG			
	00	01	11	10
a	$c, -$	$a, 0$	$b, -$	$-, -$
b	$-, -$	$a, -$	$b, 1$	$e, -$
c	$c, 0$	$a, -$	$-, -$	$d, -$
d	$c, -$	$-, -$	$b, -$	$d, 0$
e	$f, -$	$-, -$	$b, -$	$e, 1$
f	$f, 1$	$a, -$	$-, -$	$e, -$

D

Two or more rows in the primitive flow table can be merged into one row if there are **non-conflicting** states and outputs in each of the columns.

Reduction of Primitive Flow Table

	DG			
	00	01	11	10
a	c, -	a, 0	b, -	-, -
b	-, -	a, -	b, 1	e, -
c	c, 0	a, -	-, -	d, -
d	c, -	-, -	b, -	d, 0
e	f, -	-, -	b, -	e, 1
f	f, 1	a, -	-, -	e, -



	DG			
	00	01	11	10
a	c, -	a, 0	b, -	-, -
c	c, 0	a, -	-, -	d, -
d	c, -	-, -	b, -	d, 0

	DG			
	00	01	11	10
b	-, -	a, -	b, 1	e, -
e	f, -	-, -	b, -	e, 1
f	f, 1	a, -	-, -	e, -

Design Procedure

Reduction of the Primitive Flow Table

DG

	00	01	11	10
<i>a, c, d</i>	c , 0	a , 0	<i>b</i> , -	d , 0
<i>b, e, f</i>	f , 1	<i>a</i> , -	b , 1	e , 1



DG

	00	01	11	10
<i>a</i>	a , 0	a , 0	<i>b</i> , -	a , 0
<i>b</i>	b , 1	<i>a</i> , -	b , 1	b , 1



DG

	00	01	11	10
<i>a</i>	<i>c</i> , -	a , 0	<i>b</i> , -	- , -
<i>c</i>	c , 0	<i>a</i> , -	- , -	<i>d</i> , -
<i>d</i>	<i>c</i> , -	- , -	<i>b</i> , -	d , 0

DG

	00	01	11	10
<i>b</i>	- , -	<i>a</i> , -	b , 1	<i>e</i> , -
<i>e</i>	<i>f</i> , -	- , -	<i>b</i> , -	e , 1
<i>f</i>	f , 1	<i>a</i> , -	- , -	<i>e</i> , -

Design Procedure

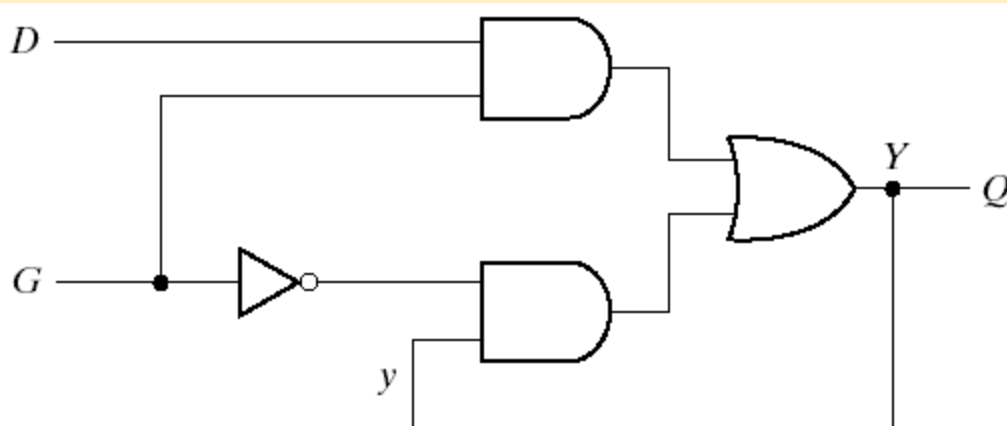
Transition Table and Logic Diagram

		<i>DG</i>			
<i>y</i>		00	01	11	10
0		0	0	1	0
1		1	0	1	1

(a) $Y = DG + G'y$

		<i>DG</i>			
<i>y</i>		00	01	11	10
0		0	0	1	0
1		1	0	1	1

(b) $Q = Y$



Design Procedure

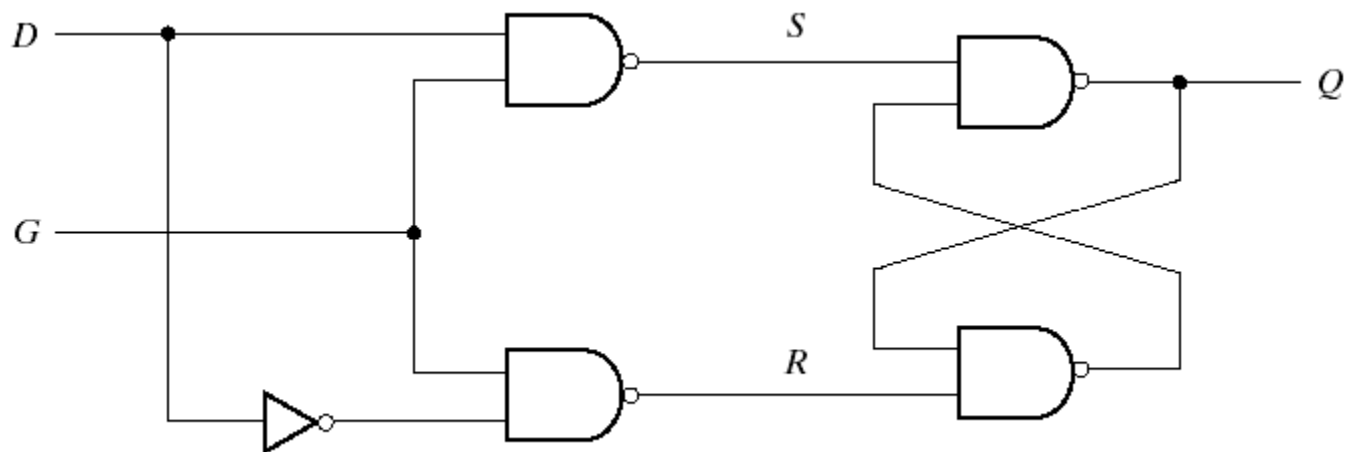
Circuit With *SR* Latch

	<i>DG</i>			
<i>y</i>	00	01	11	10
0	0	0	1	0
1	X	0	X	X

(a) $S = DG$

	<i>DG</i>			
<i>y</i>	00	01	11	10
0	X	X	0	X
1	0	1	0	0

$R = D'G$



Design Procedure

Assigning Output to Unstable States

1. Assign a **0** to an output variable associated with an unstable state that is a transient state between two stable states that have a **0** in the corresponding output variable.
2. Assign a **1** to an output variable associated with an unstable state that is a transient state between two stable states that have a **1** in the corresponding output variable.
3. Assign a **don't-care** condition to an output variable associated with an unstable state that is a transient state between two stable states that have **different values** in the corresponding output variable.

Reduction of State and Flow Tables

Equivalent States

Two states are equivalent if for each possible input, they give exactly the same output and go to the same next states or to equivalent next states.

The characteristic of equivalent states is that if (a,b) imply (c,d) and (c,d) imply (a,b) , then both pairs of states are equivalent.

Reduction of State and Flow Tables

Implication Table

Two states are equivalent if for each possible input, they give exactly the same output and go to the same next states or to equivalent next states.

The characteristic of equivalent states is that if (a,b) imply (c,d) and (c,d) imply (a,b) , then both pairs of states are equivalent.

Reduction of State and Flow Tables

Implication

Without the first

Example

$S_n \backslash X_1 X_2$	00	01	11	10
A	D/0	D/0	F/0	A/0
B	C/1	D/0	E/1	F/0
C	C/1	D/0	E/1	A/0
D	D/0	B/0	A/0	F/0
E	C/1	F/0	E/1	A/0
F	D/0	D/0	A/0	F/0
G	G/0	G/0	A/0	A/0
H	B/1	D/0	E/1	A/0

S_{n+1}/Z_n

B	×						
C	×	AF					
D	BD AF	×	×				
E	×	DF AF	DF	×			
F	√	×	×	BD	×		
G	DG AF	×	×	BG AF	×	DG AF	
H	×	BC AF	BC	×	BC DF	×	×
	A	B	C	D	E	F	G

Without the last

Reduction of State and Flow Tables

Implied Pairs

example

$S_n \backslash X_1 X_2$	00	01	11	10
A	D/0	D/0	F/0	A/0
B	C/1	D/0	E/1	F/0
C	C/1	D/0	E/1	A/0
D	D/0	B/0	A/0	F/0
E	C/1	F/0	E/1	A/0
F	D/0	D/0	A/0	F/0
G	G/0	G/0	A/0	A/0
H	B/1	D/0	E/1	A/0

S_{n+1}/Z_n

B	×						
C	×	AF					
D	BD AF	×	×				
E	×	DF AF	DE	×			
F	√	×	×	BD	×		
G	DG AF	×	×	BG	×	×	DG AF
H	×	BC AF	BC	×	BC DF	×	×
	A	B	C	D	E	F	G

Reduction of State and Flow Tables

The equivalent states: [A, F], [B, H], [B, C], [C, H].

Denoted by A

Denoted by B

Combined into [B,C,H]

group of equivalent states
 [A, F], [B, C, H],
 [D], [E], [G]

$S_n \backslash X_1 X_2$	00	01	11	10
A	D/0	D/0	A/0	A/0
B	C/1	D/0	E/1	A/0
D	D/0	B/0	A/0	A/0
E	B/1	A/0	E/1	A/0
G	G/0	G/0	A/0	A/0

S_{n+1}/Z_n

B	×						
C	×	AF					
D	BD AF	×	×				
E	×	DF AF	DF	×			
F	√	×	×	BD	×		
G	DG AF	×	×	BG AF	×	×	DG AF
H	×	BC AF	BC	×	BC DF	×	×
	A	B	C	D	E	F	G

Race-Free State Assignment

The primary objective in choosing a proper binary state assignment is the **prevention of critical races.**

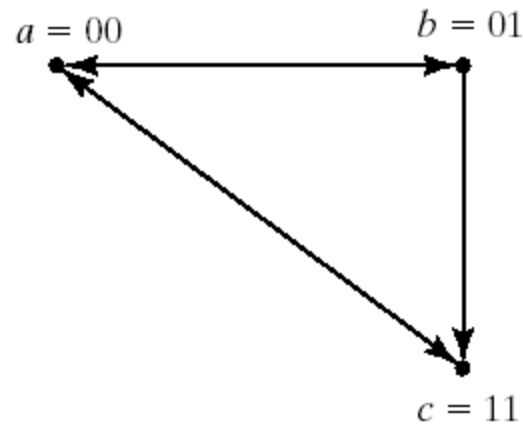
Critical races can be avoided by making a binary state assignment in such a way that **only one variable changes** at any given time when a state transition occurs in the flow table.

Race-Free State Assignment

Three-Row Flow-Table Example

		$x_1 x_2$			
		00	01	11	10
00	a	a	b	c	a
01	b	a	b	b	c
11	c	a	c	c	c

(a) Flow table

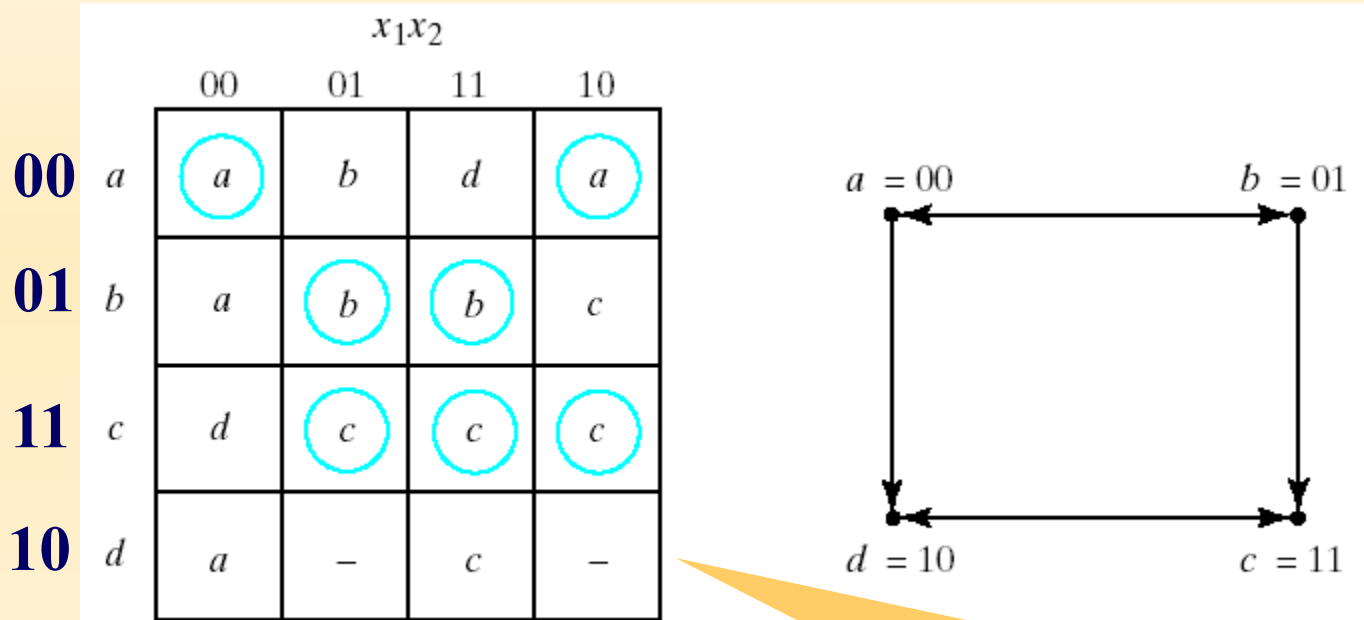


(b) Transition diagram

This assignment will cause a critical race during the transition from a to c .

Race-Free State Assignment

Three-Row Flow-Table Example



The transition from a to c must now go through d , thus avoiding a critical race.

Race-Free State Assignment

Multiple-Row Method

In the multiple-row assignment, each state in the original flow table is replaced by two or more combinations of state variables.

		$y_2 y_3$			
		00	01	11	10
y_1	0	a_1	b_1	c_1	d_1
	1	c_2	d_2	a_2	b_2

(a) Binary assignment

Note that a_2 is adjacent to d_2 , c_1 , b_2 .

Race-Free State Assignment

Multiple-Row Method

	00	01	11	10
<i>a</i>	<i>b</i>	<i>a</i>	<i>d</i>	<i>a</i>
<i>b</i>	<i>b</i>	<i>d</i>	<i>b</i>	<i>a</i>
<i>c</i>	<i>c</i>	<i>a</i>	<i>b</i>	<i>c</i>
<i>d</i>	<i>c</i>	<i>d</i>	<i>d</i>	<i>c</i>

(a) Flow table



	00	01	11	10
000 = <i>a</i> ₁	<i>b</i> ₁	<i>a</i> ₁	<i>d</i> ₁	<i>a</i> ₁
111 = <i>a</i> ₂	<i>b</i> ₂	<i>a</i> ₂	<i>d</i> ₂	<i>a</i> ₂
001 = <i>b</i> ₁	<i>b</i> ₁	<i>d</i> ₂	<i>b</i> ₁	<i>a</i> ₁
110 = <i>b</i> ₂	<i>b</i> ₂	<i>d</i> ₁	<i>b</i> ₂	<i>a</i> ₂
011 = <i>c</i> ₁	<i>c</i> ₁	<i>a</i> ₂	<i>b</i> ₁	<i>c</i> ₁
100 = <i>c</i> ₂	<i>c</i> ₂	<i>a</i> ₁	<i>b</i> ₂	<i>c</i> ₂
010 = <i>d</i> ₁	<i>c</i> ₁	<i>d</i> ₁	<i>d</i> ₁	<i>c</i> ₁
101 = <i>d</i> ₂	<i>c</i> ₂	<i>d</i> ₂	<i>d</i> ₂	<i>c</i> ₂

(b) Flow table

The original
flow table

Hazards

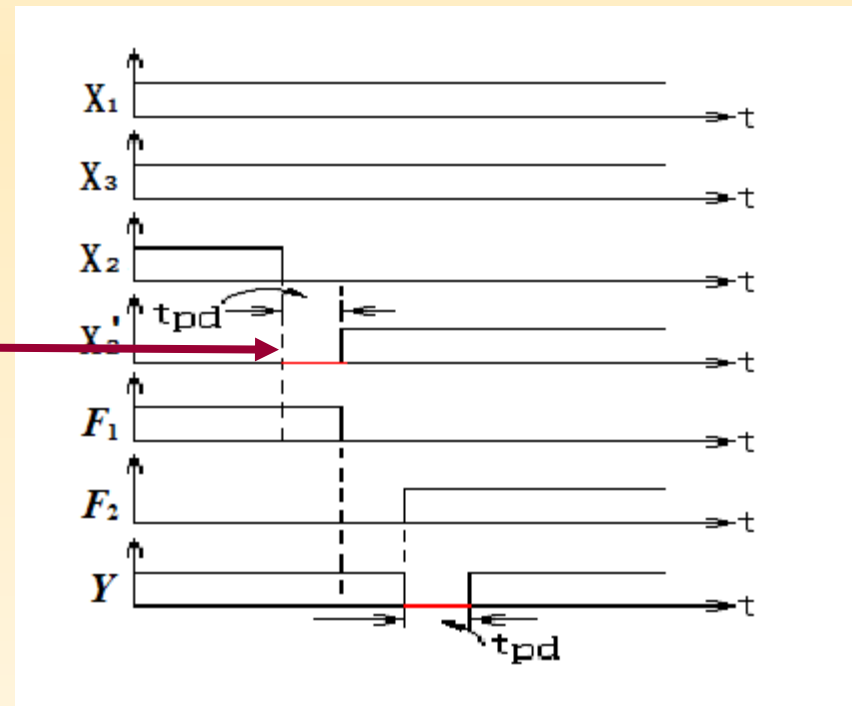
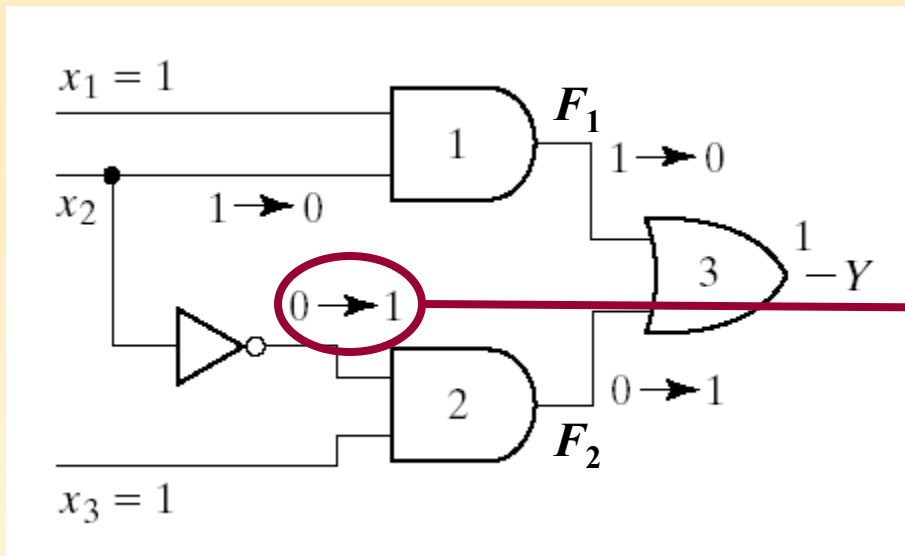
Hazards are unwanted switching transients that may appear at the output of a circuit because different paths exhibit different propagation delays.

Hazards occur in combinational circuits, where they may cause a temporary false-output value.

When hazards occur in sequential circuits, it may result in a transition to a wrong stable state.

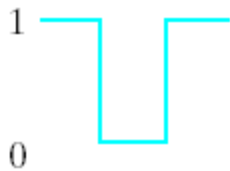
Hazards

Hazards in Combinational Circuits

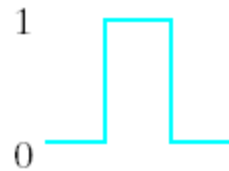


Hazards

Hazards in Combinational Circuits



(a) Static 1-hazard



(b) Static 0-hazard



(c) Dynamic hazard

Hazards

Hazards in Combinational Circuits

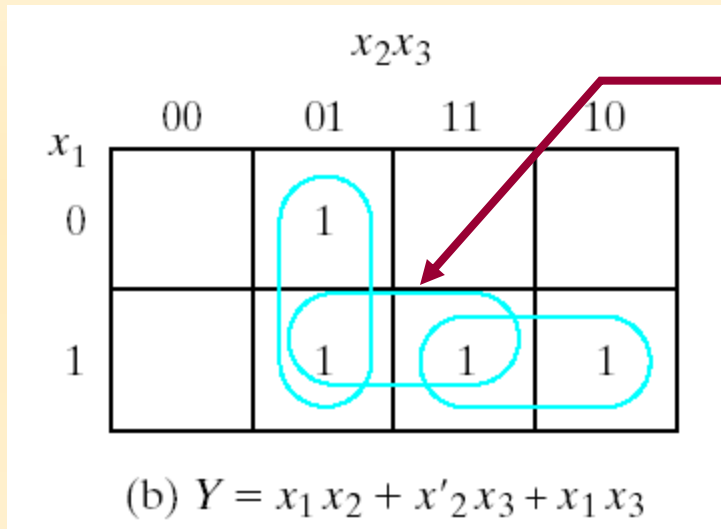
		x_2x_3			
		00	01	11	10
x_1	0		1		
	1		1	1	1

(a) $Y = x_1x_2 + x'_2x_3$

The hazard exists because the change of input results in a different product term covering the two minterms.

Hazards

Hazards in Combinational Circuits



The remedy for eliminating a hazard is to enclose the two minterms in question with another product term that overlap both grouping.

Hazards

Hazards in Combinational Circuits

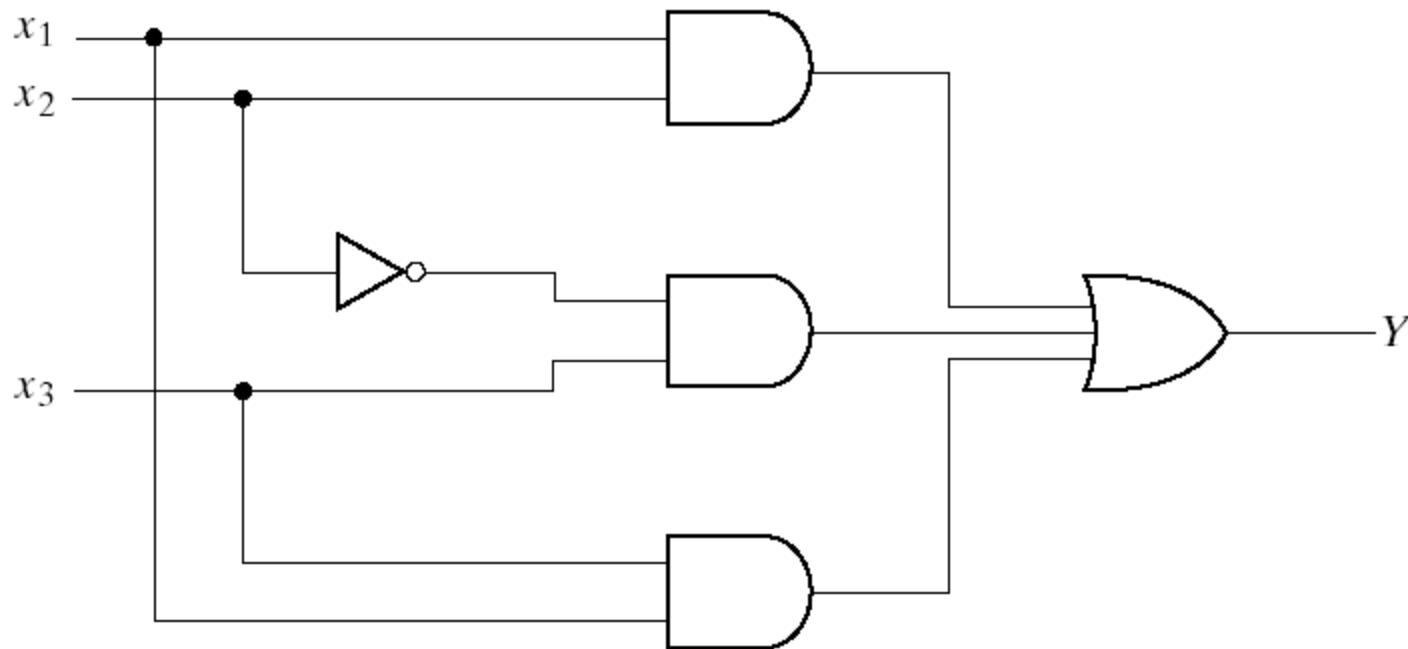


Fig. 9-36 Hazard-Free Circuit

Hazards

Implementation with SR Latches

Consider a NAND SR Latch
Boolean functions for

$$S = AB + CD$$

$$R = A'C$$

$$S = (AB + CD)' = (AB)' (CD)'$$

$$R = (A'C)'$$

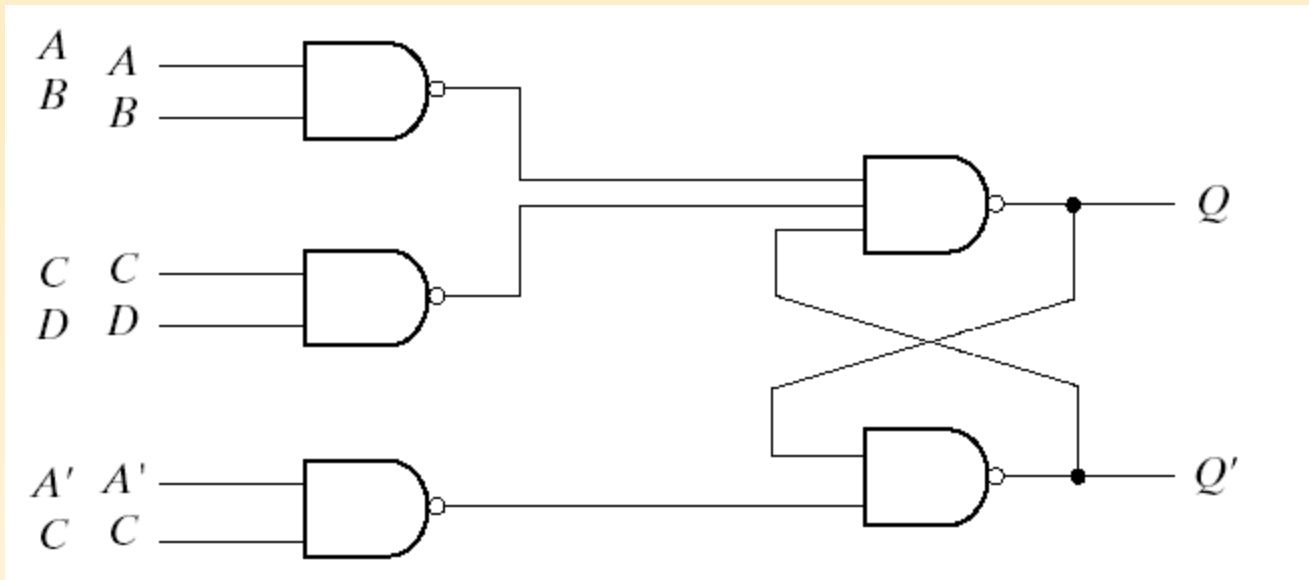
$$Q = (Q'S)' = [Q' (AB)' (CD)']'$$

Since this is a NAND latch,
we apply the complemented
values to the inputs:

Hazards

Implementation with SR Latches

$$Q = (Q'S)' = [Q' (AB)' (CD)']'$$



Design Example

The Recommended Procedure

1. State the design specifications
2. Derive a primitive flow table
3. Reduce the flow table by merging the rows
4. Make a race-free binary state assignment
5. Obtain the transition table and output map
6. Obtain the logic diagram using SR latch

Design Example

Design Specifications

It is necessary to design a negative-edge-triggered flip-flop. The circuit has two inputs, T (toggle) and C (clock), and one output, Q .

Design Example

Primitive Flow Table

Specification of
Total States

State	Inputs		Output	Comments
	T	C	Q	
a	1	1	0	Initial input is 0
b	1	0	1	After state a
c	1	1	1	Initial input is 1
d	1	0	0	After state c
e	0	0	0	After state d or f
f	0	1	0	After state e or a
g	0	0	1	After state b or h
h	0	1	1	After state g or c

Primitive Flow Table

Design Example

Primitive Flow Table

	TC			
	00	01	11	10
a	- , -	f , -	a , 0	b , -
b	g , -	- , -	c , -	b , 1
c	- , -	h , -	c , 1	d , -
d	e , -	- , -	a , -	d , 0
e	e , 0	f , -	- , -	d , -
f	e , -	f , 0	a , -	- , -
g	g , 1	h , -	- , -	b , -
h	g , -	h , 1	c , -	- , -

Design Example

Implication Table

<i>b</i>	<i>a, c</i> ×							
<i>c</i>	×	<i>b, d</i> ×						
<i>d</i>	<i>b, d</i> ×		×	<i>a, c</i> ×				
<i>e</i>	<i>b, d</i> ×	<i>e, g</i> × <i>b, d</i> ×	<i>f, h</i> ×	✓				
<i>f</i>	✓	<i>e, g</i> × <i>a, c</i> ×	<i>f, h</i> × <i>a, c</i> ×	✓	✓			
<i>g</i>	<i>f, h</i> ×	✓	<i>b, d</i> ×	<i>e, g</i> × <i>b, d</i> ×		×	<i>e, g</i> × <i>f, h</i> ×	
<i>h</i>	<i>f, h</i> × <i>a, c</i> ×	✓	✓	<i>d, e</i> × <i>c, f</i> ×	<i>e, g</i> × <i>f, h</i> ×		×	✓
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	

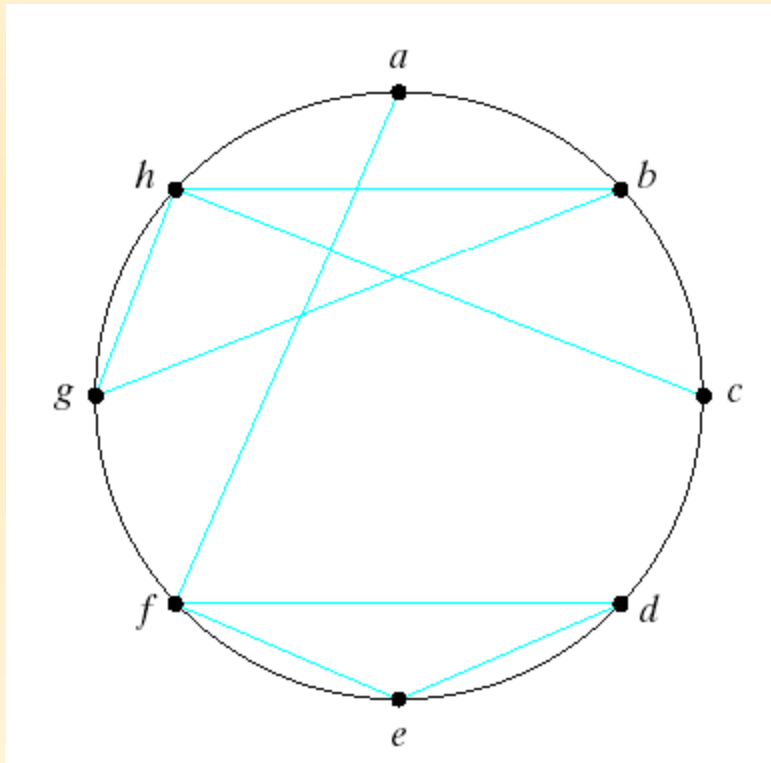
Merging the Flow Table

The compatible pairs:

(a, f) (b, g) (b, h)
 (c, h) (d, e) (d, f)
 (e, f) (g, h)

Design Example

Merging the Flow Table



The maximal compatible set:

(a, f) (b, g, h) (c, h)

(d, e, f)

Design Example

Merging the Flow Table

Reduced flow table

	TC			
	00	01	11	10
<i>a, f</i>	<i>e</i> , -	<i>f</i> , 0	<i>a</i> , 0	<i>b</i> , -
<i>b, g, h</i>	<i>g</i> , 1	<i>h</i> , 1	<i>c</i> , -	<i>b</i> , 1
<i>c, h</i>	<i>g</i> , 1	<i>h</i> , 1	<i>c</i> , 1	<i>d</i> , -
<i>d, e, f</i>	<i>e</i> , 0	<i>f</i> , 0	<i>a</i> , -	<i>d</i> , 0

(a)

	TC			
	00	01	11	10
<i>a</i>	<i>d</i> , -	<i>a</i> , 0	<i>a</i> , 0	<i>b</i> , -
<i>b</i>	<i>b</i> , 1	<i>b</i> , 1	<i>c</i> , -	<i>b</i> , 1
<i>c</i>	<i>b</i> , -	<i>c</i> , 1	<i>c</i> , 1	<i>d</i> , -
<i>d</i>	<i>d</i> , 0	<i>d</i> , 0	<i>a</i> , -	<i>d</i> , 0

(b)

Design Example

State Assignment and Transition Table

		<i>TC</i>			
		00	01	11	10
<i>a</i> :	y_1y_2				
	$a = 00$	10	00	00	01
	$b = 01$	01	01	11	01
	$c = 11$	01	11	11	10
<i>d</i> :					
	$d = 10$	10	10	00	10

(a) Transition table

		<i>TC</i>			
		00	01	11	10
y_1y_2					
	00	0	0	0	<i>X</i>
	01	1	1	1	1
	11	1	1	1	<i>X</i>
	10	0	0	0	0

(b) Output map $Q = y_2$

Design Example

Logic Diagram

		<i>TC</i>			
		00	01	11	10
<i>y</i> ₁ <i>y</i> ₂					
00		1	0	0	0
01		0	0	1	0
11		0	X	X	X
10		X	X	0	X

(a) $S_1 = y_2 TC + y_2' TC'$

		<i>TC</i>			
		00	01	11	10
<i>y</i> ₁ <i>y</i> ₂					
00		0	X	X	X
01		X	X	0	X
11		1	0	0	0
10		0	0	1	0

(b) $R_1 = y_2 TC' + y_2' TC$

		<i>TC</i>			
		00	01	11	10
<i>y</i> ₁ <i>y</i> ₂					
00		0	0	0	1
01		X	X	X	X
11		X	X	X	0
10		0	0	0	0

(c) $S_2 = y_1' TC'$

		<i>TC</i>			
		00	01	11	10
<i>y</i> ₁ <i>y</i> ₂					
00		X	X	X	0
01		0	0	0	0
11		0	0	0	1
10		X	X	X	X

(d) $R_2 = y_1 TC'$

Design Example

Logic Diagram

