Question Bank

1. UNIT – I (SET , RELATION, FUNCTION)

2. Draw venn diagram showing: (i) $A\cup B \subset A\cup C$ but $B\not\subset C$, (ii) $A\cap B \subset A\cap C$ but $B\not\subset C$.
3. Let A = {1, 2, 3, 4}, and R is a relation defined by "a divides b". Write R as a set of ordered pair, draw directed graph. Also find R^{-1}
4. Let R be a binary relation defined as $R = \{(a, b) \in R^2 : a-b < 3\}$, determine whether R is reflexive, symmetric and transitive.
5. Let A = {1, 2, 3, 4, 5, 6}, construct pictorial description of the relation R on A defined as $R = \{(a, b): (a-b)^2 \in A\}$.
6. Let A = {1, 2, 3, 4}, give an example of a mapping which is (i) neither symmetric nor antisymmetric, (ii) anti-symmetric and reflexive but not transitive, (iii) transitive and reflexive but nit anti-symmetric.
7. Let R be a relation on the set A = {a, b, c} defined by R = {(a, b), (b, c), (d, c), (d, a), (a, d), (d, d)}. Write the relation matrix of R and find (i) reflexive closure of R, (ii) symmetric closure of R and (iii) transitive closure of R.
8. Show that a relation R defined on the set of real numbers as (a, b) R (c, d) iff $a^2 + b^2 = c^2 + d^2$. Show that R is an equivalence relation.
9. An inventory consists of a list of 115 items, each marked "available" of "unavailable". There are 60 available items. Show that there are at least two available items in the list exactly four items apart.
10. List all possible functions from A to B, A = {a, b, c}, B = {0, 1}. Also indicate in each case whether the function is one-to-one, is onto and one-to-one-onto.
11. Let A = {1, 2, 3}, B = {p, q} and C = {a, b}. Let $f: A \to B$ is f = {(1, p), (2, p), (3, a)} and $g: B \to C$ is given by {(p, b), (q, b)}. Find *gof* and show it pictorially.
12. If *f* is function from A to B and *g* is function B to C and both *f* and *g* are onto. Show that *gof* is also onto. Is *gof* one-to-one if both *f* and *g* are one-to-one.
13. Let *f, g* and *h*: $R \to R$ be defined by (R is the set of real numbers)
$f(x) = x + 2$, $g(x) = (1 + x^2)^{-1}$, $h(x) = 3$.

Compute $f^{-1}g(x)$ and $hf(g f^{-1})(hf(x))$.
14. Show that the function *f* and *g* both of which are from $N \times N$ to N given by $f(x, y) = x + y$ and $g(x, y) = xy$ are onto but not one-one.

15. Show that the function $f(x) = k$, where k is a constant, is primitive recursive.
16. State and prove pigeonhole principle.

2. UNIT – II (PROPOSITIONAL LOGIC)

1. Make a truth table for the following:
(i) $(p\vee q) \wedge r$ (ii) $(p\vee \sim q) \Rightarrow r$ (iii) $(p \downarrow q) \wedge (p \downarrow r)$
2. State and prove De Morgan's law for logic.
3. Is $((p\vee \sim q) \wedge(\sim p\vee \sim q)) \vee q$ a tautology?

4. The pierce arrow $\downarrow$ (NOR) is a logical operation defined as $p \downarrow q \equiv \sim (p \vee q)$, Prove that (i) $\sim p \equiv p \downarrow q$ and (ii) $(p \wedge q) \equiv (p \downarrow p) \downarrow (q \downarrow q)$.
5. Prove the following: (i) $p \vee (\sim p \wedge q) \equiv (p \vee q)$, (ii) $p \wedge (\sim p \vee q) \equiv (p \wedge q)$
6. Consider the following conditional statement:
If the flood destroy my house or the fires destroy my house, then my insurance company will pay me.
Write the converse, inverse and contrapositive of the statement.
7. Given the following statements as premises, all referring to an arbitrary meal:
If he takes coffee, he does not drink milk.
He eats crackers only if he drinks milk.
He does not take soup unless he eats crackers.
At noon today, he had coffee.
Whether he took soup at noon today? If so what is the correct conclusion.
8. There are two restaurants next to each other. One has a sign says "Good food is not cheap" and other has a sign that says "Cheap food is not good". Are the signs saying the same thing?

9. Is the following argument valid?
If taxes are lowered, then income rise
Income rise
$\therefore$ *Taxes are lowered*

10. Write the following statement in symbolic form using quantifiers:
(i) All students have taken a course in mathematics.
(ii) Some students are intelligent, but not hardworking.

11. Let p(x): x is mammal and q (x): x is animal. Translate the following in English:
$(\forall x)(q(x) \wedge (\sim p(x)))$

12. Let A = {1, 2, 3, 4, 5}, determine the truth value of the following:
(i) $(\forall x \in A)(x + 3 = 10)$, (ii) $(\exists x \in A)(x + 3 < 5)$.

13. Write the negation of the following statement:
$\exists\ x \in R\ x > 3 \Rightarrow x_2 > 9$

14. Prove the following or provide a counter example:
$A \cup B \subseteq A \cup B \Rightarrow A = B$

15. Prove or disprove the statement that if x and y are real numbers: $(x_2 = y_2) \Leftrightarrow (x = y)$.

16. Let n be an integer, prove that n_2 is an odd then n is odd.
17. Use the method of contradiction to prove that $\sqrt{5}$ is not a rational number.

3. UNIT – III (COMBINOTRICS)

1. Find the reccurence relation with initial condition for the following:
(i) 2, 10, 50, 250, ……. (ii) 1, 1, 3, 5, 8. 13. 21, …..

2. Solve $a_n - 3\, a_{n-1} = 2$, $n \geq 2$, with $a_0 = 1$.
3. Solve $a_n - 2\, a_{n-1} - 3\, a_{n-2} = 0$, $n \geq 2$, with $a_0 = 3$, $a_1 = 1$.
4. Solve $a_{n+2} - 2\, a_{n+1} + a_n = 2_n$; $a_0 = 2$, $a_1 = 1$.
5. Solve the following using the initial condition as s(0) = s(1) = 1
s(k) – 9 s(k -1) + 8 s(k-2) = 9k + 1.
6. Determine the generating function of the following numeric function
$a_n = 2_n$, if n is even
= - 2_n, if n is odd
7. Find the closed form for the generating function for the following
(i) 1, 0, -1, 0, 1, 0, -1, 0, 1, …..
(ii) 0, 3, 3, 3, 3, …… .
8. Use induction to that
(i) 2 + 4 + 6 + …. + 2n = n_2 + n, for $n \geq 1$
(ii) $11_n - 4_n$ is divisible by 7, for $n \geq 1$
(iii) 2n > n_2, for $n \geq 5$
9. What is the number of solutions of the equation x + y + z + w = 20, if x, y, z and w are nonnegative integers.
10. How many solutions are there to the equation a = b + c +d + e + f = 21, where each variable is non-negative integer such that (i) $1 \leq x$, (ii) all variables are ≥ 2.
11. Prove using counting argument C (n, r) = C (n-1, r) + C (n-1, r-1).
12. Find the generating function to select 10 candy bars from large supplies of six different kind.
13. Find the generating function for the number of ways to select (with repetition allowed) r objects from a collection of n distinct objects.
14. In how many different ways can eight identical balls be distributed among three children if each receives at least two balls and no more than four balls?
15. Find the number of ways that 9 students can be seated in the room so that there is at least one student in each of the five rows.

4. UNIT – IV (ALGEBRAIC STRUCTURE)

1. Let A = {a, b}. Which of the following tables define a semi- group? Which define monoid onA
* a b * a b * a b
a b a a b b a a b
b a b b a a b b a
2. Let (A, ∗) be a semi group, further more for every a and b in A, if a ≠ b, then $a\ b\ b\ a * \neq *$
(i) Show that for every a in A $a * a = a$
(ii) Show that for every a, b in A $a * b * a = a$
(iii) Show that for every a, b, c in A $a * b * c = a * c$
3. Differentiate between semigroup and subgroup with example.
4. Define a group. Let S = {0, 1, 2, 3, 4, 5, 6, 7} & * denote “multiplication modulo 8 i.e.
$x * y = (xy) \bmod 8$.
5. Let G be the set of all non zero real no’s & let

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$a * b = ab$ Show that (G,*) is an abelian group.

6. Let G be a group & a & b be the elements of G. Then prove that

(i) $(a) = a$ $_{-1\ -1}$ (ii) $(\ )_{-1\ -1\ -1}$ $ab = b\ a$

7. Show that G = $\{1, -1, i, -i\}$ where $i = -1$ is an abelian group with respect to multiplication as a binary operation.

8. Let G_1 and G_2 be subgroup of a groupG

(i) Show that $_{1\ 2}\ G \cap G$ is also a subgroup of G

(ii) Is $_{1\ 2}\ G \cup G$ is always a subgroup of G.

9. If a cyclic group G is generated by an element a of order n, then a_n is a genertor of G, iff the greatest divisor of m & n is 1, i.e. iff m & n are relatively prime.

10. Show that the group {1,2,3,4,5,6 X_7) is a cyclic group. How many generators are there.

11. Define permutation group. Let A = $\{1,2,3,4,5\}$. Find $(13) \circ (245) \circ (23)$

12. What do you mean by group isomorphism. Give example with example.

13. Let g be a group & let $g \in G$. Define a function $\hat{g} : G \to G$ by $\hat{g}(x) = gxg$ $_{-1}$. Show that

(i) $\hat{g}$ is a homomorphism

(ii) $\hat{g}$ is a One to One

(iii) $\hat{g}$ is Onto.

14. Let G be the group of real no.'s under addition 7 let $G_{/}$ be the group of positive real no's under multiplication. Prove that mapping $f : G \to G_{/}$ defined by $f(a) = 2_a$ is homomorphism.

15. Show that the additive group Z_4 is isomorphic to the multiplicative group of non zero element of Z_5.

16. If (R, +) is a ring with unity, then show that, for all a $\in R$

(i) (-1).a = -a

(ii) (-1).(-1) = 1

17. If R is a ring such that a_2 = a $\forall$ a $\in$R Prove that

(i) a + a = 0 $\forall$ a $\in$R i.e. each element of R is its own additive inverse

(ii) a + b =0 $\Rightarrow$ a = b

(iii) R is a commutative ring.

18. Show that the set R of real no. with composition $\circ$&$*$ by $a \circ b = a + b + 1$&

$a * b = ab + a + b$ is a ring. Determine 0 element 7 1- element of the ring.

19. Show that in a field F

(i) $(-a)b = -(ab),\ a, b \in F$ (ii) $(-a)(-b)_{-1} = ab_{-1}$ (iii) $(-a_{-1})_{-1} = a$

5. UNIT – V (GRAPH THEORY

1. What do you mean by graph isomorphism, show it by example?

2. Show that the given graph are isomorphic.

3. Define walk, path & trial and also from the given graph find which of the following sequences paths, simple paths, are cycle and simple cycle.

(a) be_7b
(b) $de_3ce_2be_5ee_4d$
(c) $ae_6de_3ce_2be_5e$
4. Prove that if the graph has n vertices and vertex 'u' is connected to vertex 'w' then there exist a path from u to w of length no more than n.
5. Prove that the maximum number of edges in a simple graph with n vertices is n(n-1)/2.
6. Define Eulerian Graph and prove that a non empty connected graph G is Eulerian iff its vertices are all of even degree.
7. Differentiate between Eulerian graph & Hamiltonian graph with example.
8. (i) Determine whether the following graph contain Eulerian circuit. If it does, then find an Eulerian circuit.
9. Determine whether the given graph has Hamiltonian circuit. If it does, find such a circuit.
10. Prove that if G is connected graph with n vertices & (n-1) edges then G is a tree.

11. Prove that the chromatic number of a tree is always 2 & chromatic polynomial is $\lambda(\lambda-1)_{n-1}$
12. Prove these are equivalent
(i) A graph G is 2- colourable.
(ii) G is bipartite
(iii) Every cycle of G is of even length.