

Inverse Fourier Transform

Fourier Transform and Its Inverse

- Writing the exponential function in (4) as a product of exponential functions, we have

$$(5) \quad f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(v) e^{-i w v} dv \right] e^{i w x} dw.$$

The expression in brackets is a function of w , is denoted by $\hat{f}(w)$ and is called the **Fourier transform** of f ; writing $v = x$, we have

$$(6) \quad \hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i w x} dx.$$

With this, (5) becomes

$$(7) \quad f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(w) e^{iwx} dw$$

and is called the **inverse Fourier transform** of $\hat{f}(w)$

- Another notation for the Fourier transform is

so that

$$\hat{f} = \mathcal{F}(f), \quad f = \mathcal{F}^{-1}(\hat{f}).$$

- The process of obtaining the Fourier transform

$$\mathcal{F}(f) = \hat{f}$$

from a given f is also called the **Fourier transform** or the *Fourier transform method*.

Table of Fourier Transforms

$f(x)$	$\hat{f}(\omega)$
1. $\frac{1}{x^2 + a^2} \quad (a > 0)$	$\frac{\pi}{a} e^{-a \omega }$
2. $H(x)e^{-ax} \quad (\operatorname{Re} a > 0)$	$\frac{1}{a + i\omega}$
3. $H(-x)e^{ax} \quad (\operatorname{Re} a > 0)$	$\frac{1}{a - i\omega}$
4. $e^{-a x } \quad (a > 0)$	$\frac{2a}{\omega^2 + a^2}$
5. e^{-x^2}	$\sqrt{\pi} e^{-\omega^2/4}$
6. $\frac{1}{2a\sqrt{\pi}} e^{-x^2/(2a)^2} \quad (a > 0)$	$e^{-a^2\omega^2}$
7. $\frac{1}{\sqrt{ x }}$	$\sqrt{\frac{2\pi}{ \omega }}$
8. $e^{-a x /\sqrt{2}} \sin\left(\frac{a}{\sqrt{2}} x + \frac{\pi}{4}\right) \quad (a > 0)$	$\frac{2a^3}{\omega^4 + a^4}$
9. $H(x + a) - H(x - a)$	$\frac{2 \sin \omega a}{\omega}$