Projections and Perspectives

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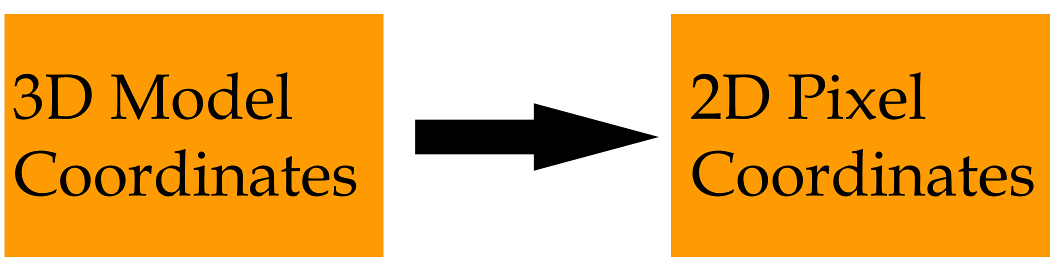
- We have to display our 3D model
- Screen is 2D
- Transformation from 3D model coordinates to 2D pixels coordinates.



Projections

- Representation of a 3D scene, for a virtual observer
- One viewpoint:
  - position of the observer
- One direction of view:
  - direction where the observer is looking
- One "up vector":
  - vertical for the observer

Different projections

- Parallel projections:
  - no shortening due to distance
  - several kinds, depending on orientation:
    - isometric, cavalier,…
- Perspective projections:
  - shortening of objects in the distance
  - several kind, depending on orientation:
    - one, two, three vanishing points

Parallel Projection Matrix

- Parallel projection onto $z=0$ plane:

  $x'=x,\ y'=y,\ w'=w$
- Matrix for this projection:

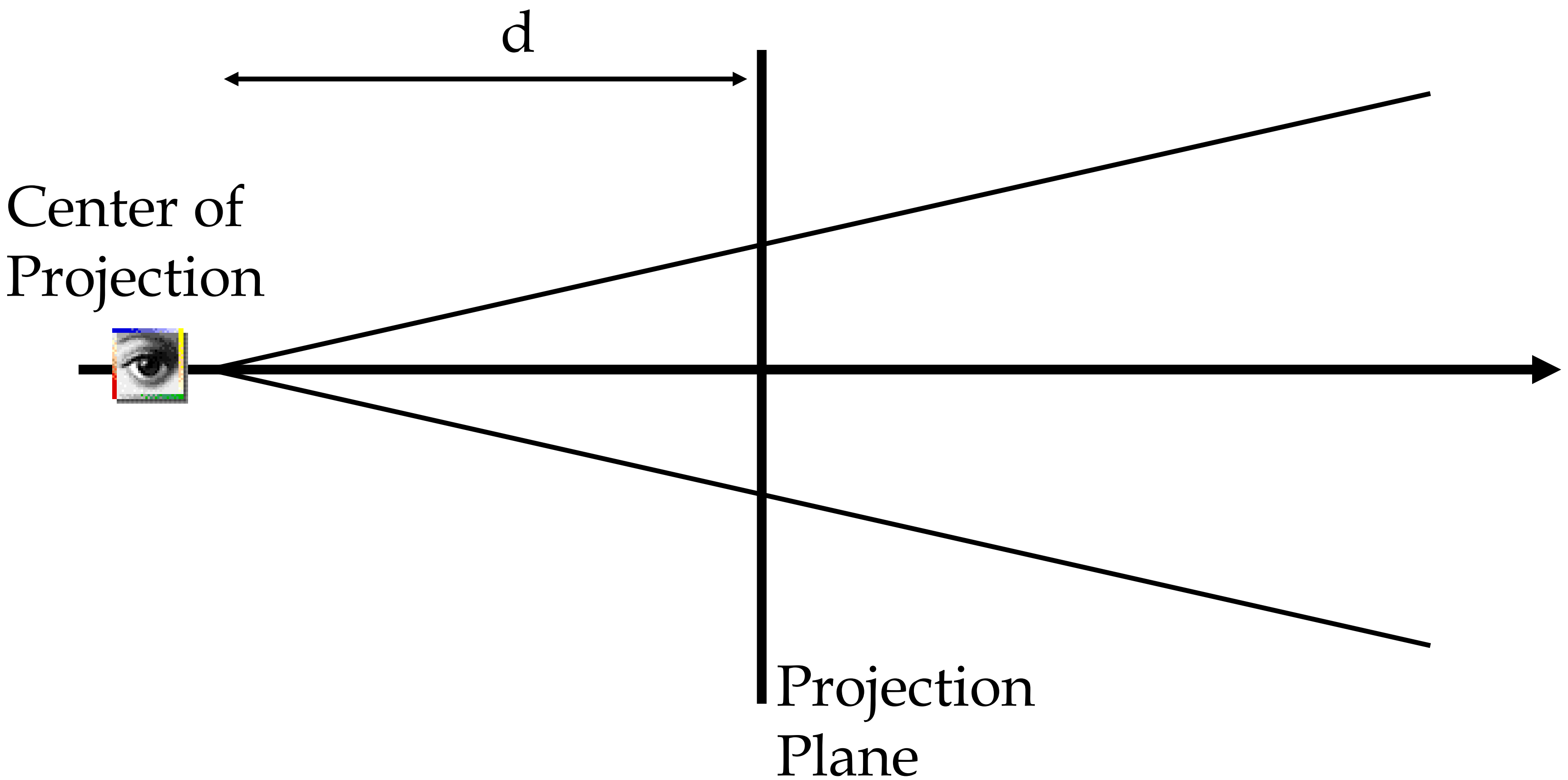
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Perspective Projection Matrix

- Projection onto plane *z=0,* with center of projection at *z=-d*:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix}$$

Distance to projection plane

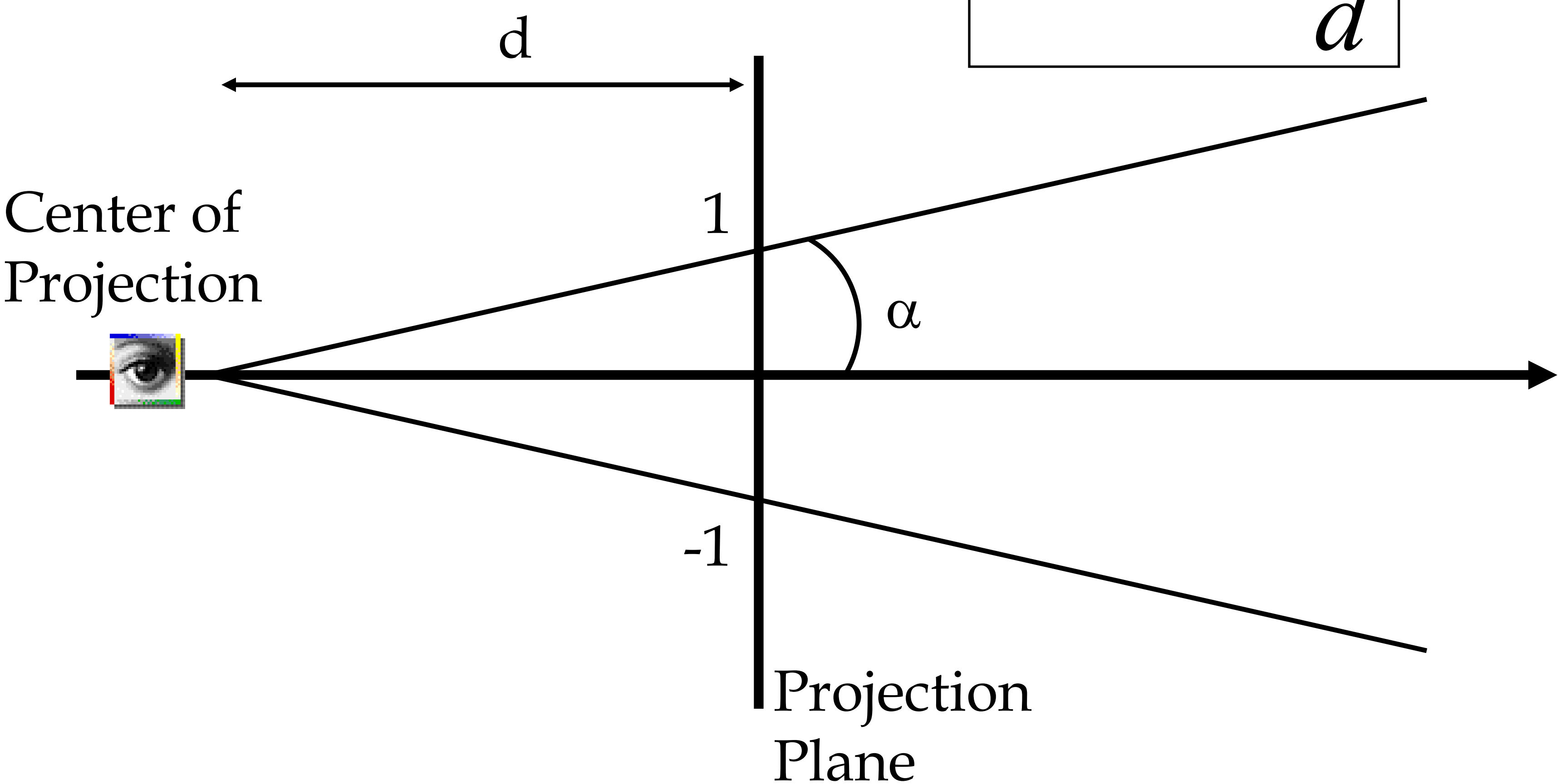


Field of view

- Distance to projection plane not intuitive
- Easier notion: field of view
- FOV= angle, in degrees
- Expresses how wide is my vision

Field of view

$$\boxed{\tan\alpha = \frac{1}{d}}$$



Homogeneous coordinates

- Essential for perspective projection
- Note the shortening of distances uses w

$$w' = \frac{z}{d} + w$$

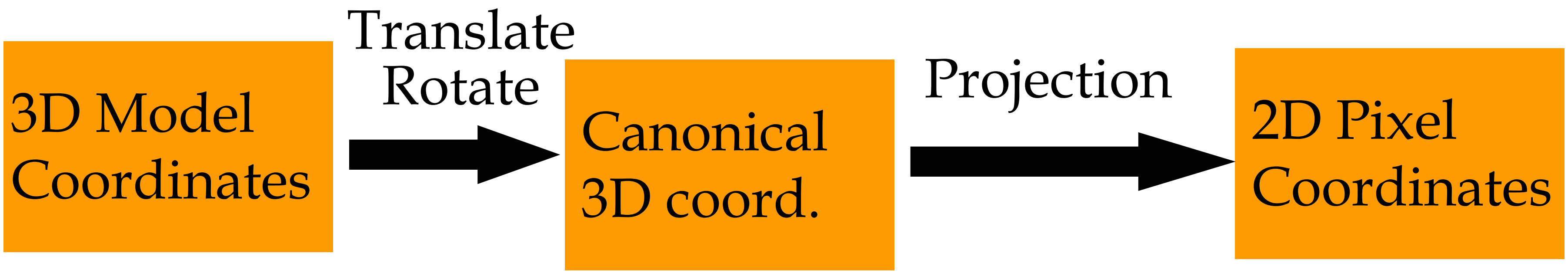
$$\frac{x'}{w'} = \frac{x}{\frac{z}{d} + w}$$

- Impossible to do without homogeneous

Other viewpoints

- If we are viewing from another point?
- Translations:
  - until viewpoint is at origin
- Rotations:
  - until direction of viewing is on z-axis
- Back to the previous case

Canonical Coordinates



- After translation, before projection: *canonical 3D coordinates*.

Projections: discussion

- Projections have several goals:
  - exactitude (*e.g.* plans for architects)
  - realism (view of a scene, VR)
  - visualization
  - artistic view: non-linear perspectives (*e.g.* Rubber Soul)
- An important part of realism in 3D rendering