Spheres

Warm Up

Problem of the Day

Lesson Presentation

Warm Up

1. Find the surface area of a square pyramid whose base is 3 m on a side and whose slant height is 5 m.
  39 m^2

2. Find the surface area of a cone whose base has a radius of 10 in. and whose slant height is 14 in. Use 3.14 for π.
  753.6 in^2

Problem of the Day

Find the slant height of the cone with the following measurements:

The area of its base is one-third of its total surface area. The radius is 4 cm.

8 cm

Learn to find the volume and surface area of spheres.

Vocabulary
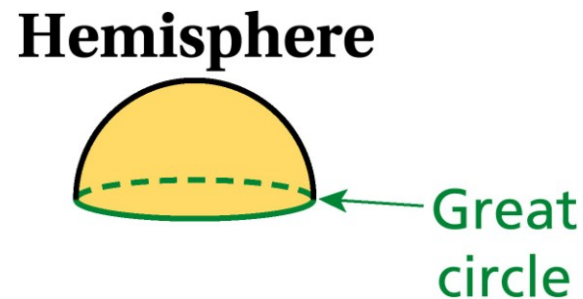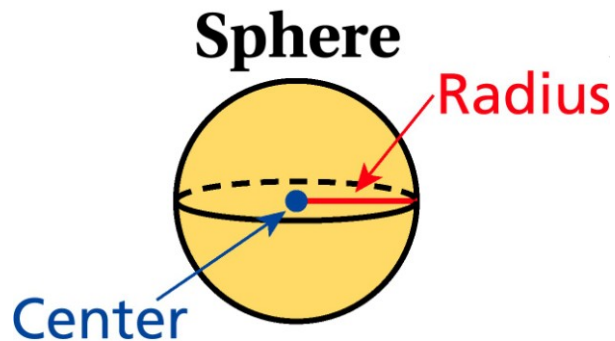
sphere

hemisphere

great circle

A **sphere** is the set of points in three dimensions that are a fixed distance from a given point, the center. A plane that intersects a sphere through its center divides the two halves or **hemispheres**. The edge of a hemisphere is a **great circle**.

Sphere

Radius

Center

Hemisphere

Great circle

The volume of a hemisphere is exactly halfway between the volume of a cone and a cylinder with the same radius *r* and height equal to *r.*
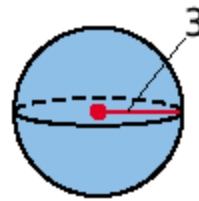
Spheres

VOLUME OF A SPHERE

Words	Numbers	Formula
The volume V of a sphere is $\frac{4}{3}\pi$ times the cube of the radius r.	$V = \left(\frac{4}{3}\right)\pi(3^3)$ $= \frac{108}{3}\pi$ $= 36\pi$ $\approx 113.1 \text{ units}^3$	$V = \left(\frac{4}{3}\right)\pi r^3$

Spheres

Additional Example 1: Finding the Volume of a Sphere

Find the volume of a sphere with radius 9 cm, both in terms of π and to the nearest tenth of a unit.

$$V = \left(\frac{4}{3}\right)\pi r^3 \qquad \textit{Volume of a sphere}$$

$$= \left(\frac{4}{3}\right)\pi (9)^3 \qquad \textit{Substitute 9 for r.}$$

$$= 972\pi \text{ cm}^3 \approx 3{,}052.1 \text{ cm}^3$$

Try This: Example 1

Find the volume of a sphere with radius 3 m, both in terms of π and to the nearest tenth of a unit.

$$V = \left(\frac{4}{3}\right)\pi r^3 \qquad \textit{Volume of a sphere}$$

$$= \left(\frac{4}{3}\right)\pi(3)^3 \qquad \textit{Substitute 3 for r.}$$

$$= 36\pi \text{ cm}^3 \approx 113.0 \text{ m}^3$$

The surface area of a sphere is four times the area of a great circle.

SURFACE AREA OF A SPHERE

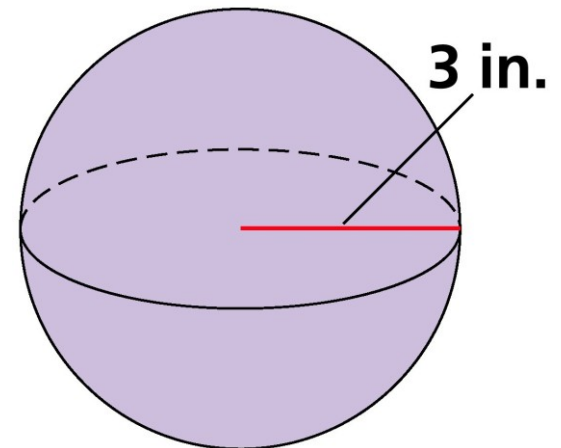| Words	Numbers	Formula
The surface area S of a sphere is 4π times the square of the radius r.	$S = 4\pi(2^2)$ $= 16\pi$ $\approx 50.3 \text{ units}^2$	$S = 4\pi r^2$

Additional Example 2: Finding Surface Area of a Sphere

Find the surface area, both in terms of π and to the nearest tenth of a unit.

$S = 4\pi r^2$ *Surface area of a sphere*

$= 4\pi(3^2)$ *Substitute 3 for r.*

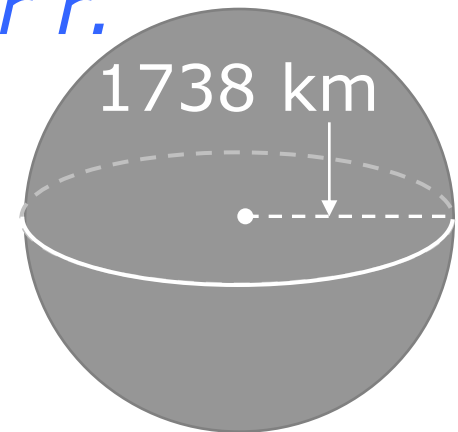$= 36\pi$ in$^2 \approx 113.0$ in^2

3 in.

<u>Try This</u>: Example 2

The moon has a radius of 1738 km. Find the surface area, both in terms of π and to the nearest tenth.

$S = 4\pi r^2$ *Surface area of a sphere*

$= 4\pi(1738^2)$ *Substitute 1738 for r.*

$= 12{,}082{,}576\pi \text{ km}^2$

$\approx 37{,}939{,}288.6 \text{ km}^2$

1738 km

Additional Example 3: Comparing Volumes and Surface Areas

Compare the volumes and surface areas of a sphere with radius 42 cm with that of a rectangular prism measuring 44 cm × 84 cm × 84 cm.

Sphere:

Rectangular Prism:

$$V = \left(\frac{4}{3}\right)\pi r^3 = \left(\frac{4}{3}\right)\pi(42^3)$$

$$V = lwh$$

$$\approx \left(\frac{4}{3}\right)\left(\frac{22}{7}\right)74{,}088$$

$$= (44)(84)(84)$$

$$\approx 310{,}464 \text{ cm}^3$$

$$= 310{,}464 \text{ cm}^3$$

Additional Example 3 Continued

Compare the volumes and surface areas of a sphere with radius 42 cm with that of a rectangular prism measuring 44 cm × 84 cm × 84 cm.

Sphere: **Rectangular Prism:**

$S = 4\pi r^2 = 4\pi(42^2)$ $S = 2lw + 2lh + 2wh$

$\quad = 7{,}056\pi$ $S = 2(44)(84) + 2(44)(84)$

$\approx 7{,}056\left(\dfrac{22}{7}\right) \approx 22{,}176 \text{ cm}^2$ $+ 2(84)(84)$

$\qquad\qquad\qquad\qquad\qquad = 28{,}896 \text{ cm}^2$

The sphere and the prism have approximately the same volume, but the prism has a larger surface area.

Try This: Example 3

Compare the volume and surface area of a sphere with radius 21 mm with that of a rectangular prism measuring 22 × 42 × 42 mm.

Sphere:

$$V = \left(\frac{4}{3}\right)\pi r^3 = \left(\frac{4}{3}\right)\pi(21^3)$$

$$\approx \left(\frac{4}{3}\right)\left(\frac{22}{7}\right)9261$$

$$\approx 38{,}808 \text{ mm}^3$$

Rectangular Prism:

$$V = lwh$$

$$= (22)(42)(42)$$

$$= 38{,}808 \text{ mm}^3$$

Try This: Example 3 Continued

Compare the volume and surface area of a sphere with radius 21 mm with that of a rectangular prism measuring 22 × 42 × 42 mm.

Sphere: **Rectangular Prism:**

$S = 4\pi r^2 = 4\pi(21^2)$ $S = 2lw + 2lh + 2wh$

$\quad = 1764\pi$ $S = 2(22)(42) + 2(22)(42)$

$\quad \approx 1764\left(\dfrac{22}{7}\right) \approx 5544 \text{ mm}^2$ $\quad\quad + 2(42)(42)$

$\quad\quad = 7224 \text{ mm}^2$

The sphere and the prism have approximately the same volume, but the prism has a larger surface area.

Lesson Quiz: Part 1

Find the volume of each sphere, both in terms of π and to the nearest tenth. Use 3.14 for π.

1. $r = 4$ ft   85.3π ft^3, 267.8 ft^3

2. $d = 6$ m   36π m^3, 113.0 m^3

Find the surface area of each sphere, both in terms of π and to the nearest tenth. Use 3.14 for π.

3. $r = 22$ in   1936π in^2, 6079.0 in^2

4. $d = 1.5$ mi   2.25π mi^2, 7.1 mi^2

Lesson Quiz: Part 2

5. A basketball has a circumference of 29 in. To the nearest cubic inch, what is its volume?

412 in^3