Colour
in
Computer Graphics

Outline: This time

- Introduction
- Spectral distributions
- Simple Model for the Visual System
- Simple Model for an Emitter System
- Generating Perceivable Colours
- CIE-RGB Colour Matching Functions

Outline: Next Time

- CIE-RGB Chromaticity Space
- CIE-XYZ Chromaticity Space
- Converting between XYZ and RGB
- Colour Gamuts and Undisplayable Colours
- Summary for Rendering: What to do in practice

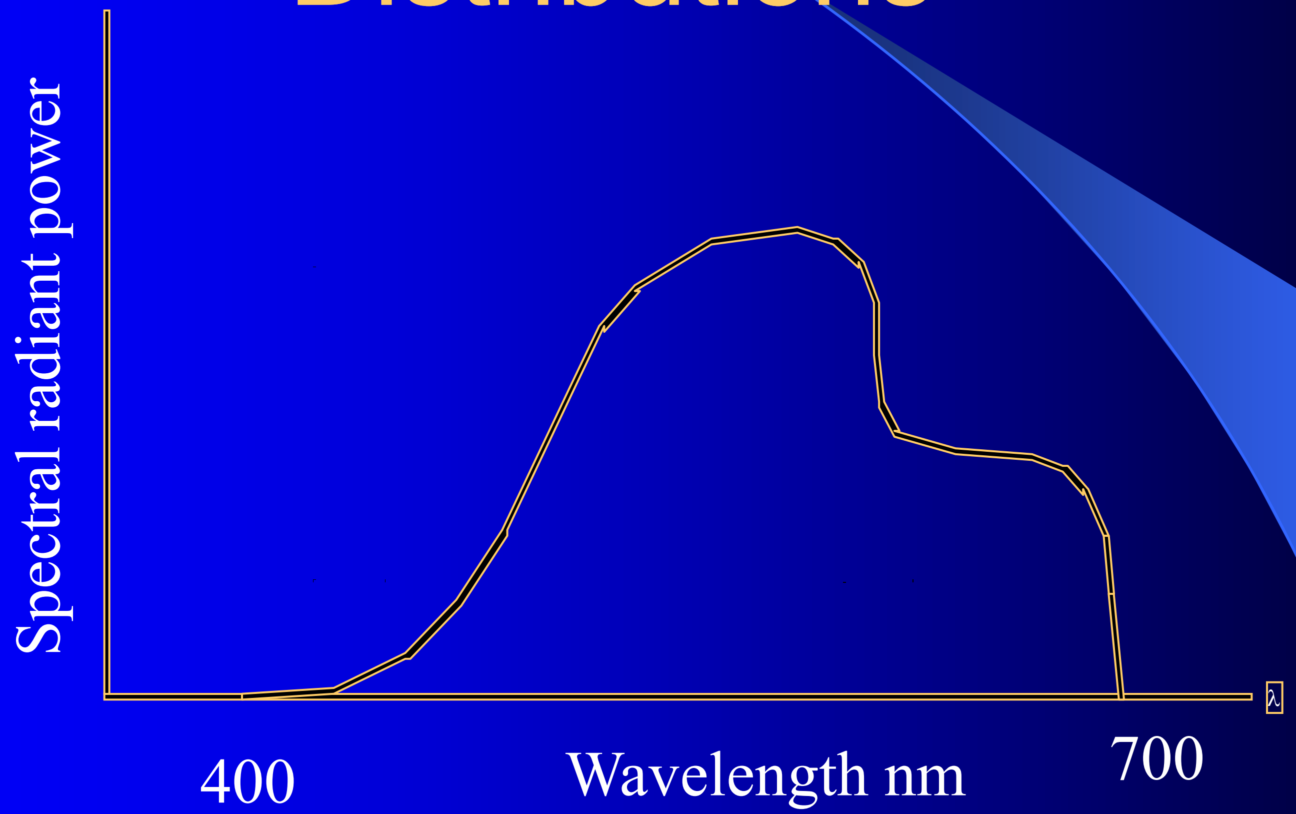
Spectral Distributions

- Radiometry (radiant power, radiance etc)
  - Measurement of light energy
- Photometry (luminance etc)
  - Measurement including response of visual system
- $\Phi(\lambda) = Kn(\lambda)/\lambda$ spectral radiant power distribution
- Generally $C(\lambda)$ defines spectral colour distribution $\lambda \varepsilon [\lambda_a, \lambda_b] = \Lambda$
- In computer graphics C is usually radiance.

Monochromatic Light (pure colour)

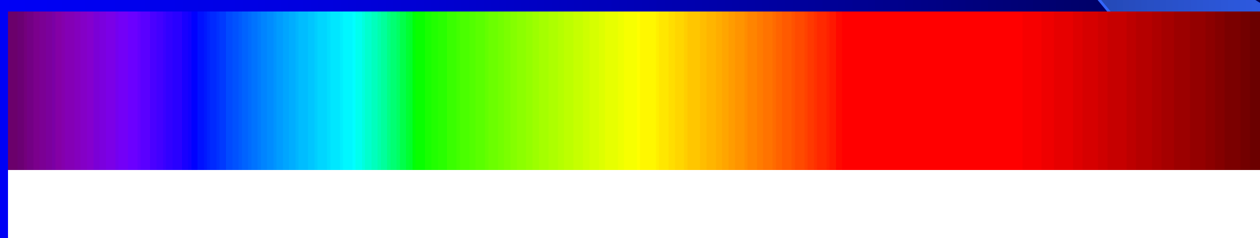
- $\delta(\lambda) = 0, \lambda \neq 0$
- $\int \delta(\lambda)\, d\lambda = 1$
- $\int \delta(t)\, f(x-t) dt = f(x)$
- $C(\lambda) = \delta(\lambda-\lambda_0)$ is spectral distribution for pure colour with wavelength λ_0

Colour as Spectral Distributions



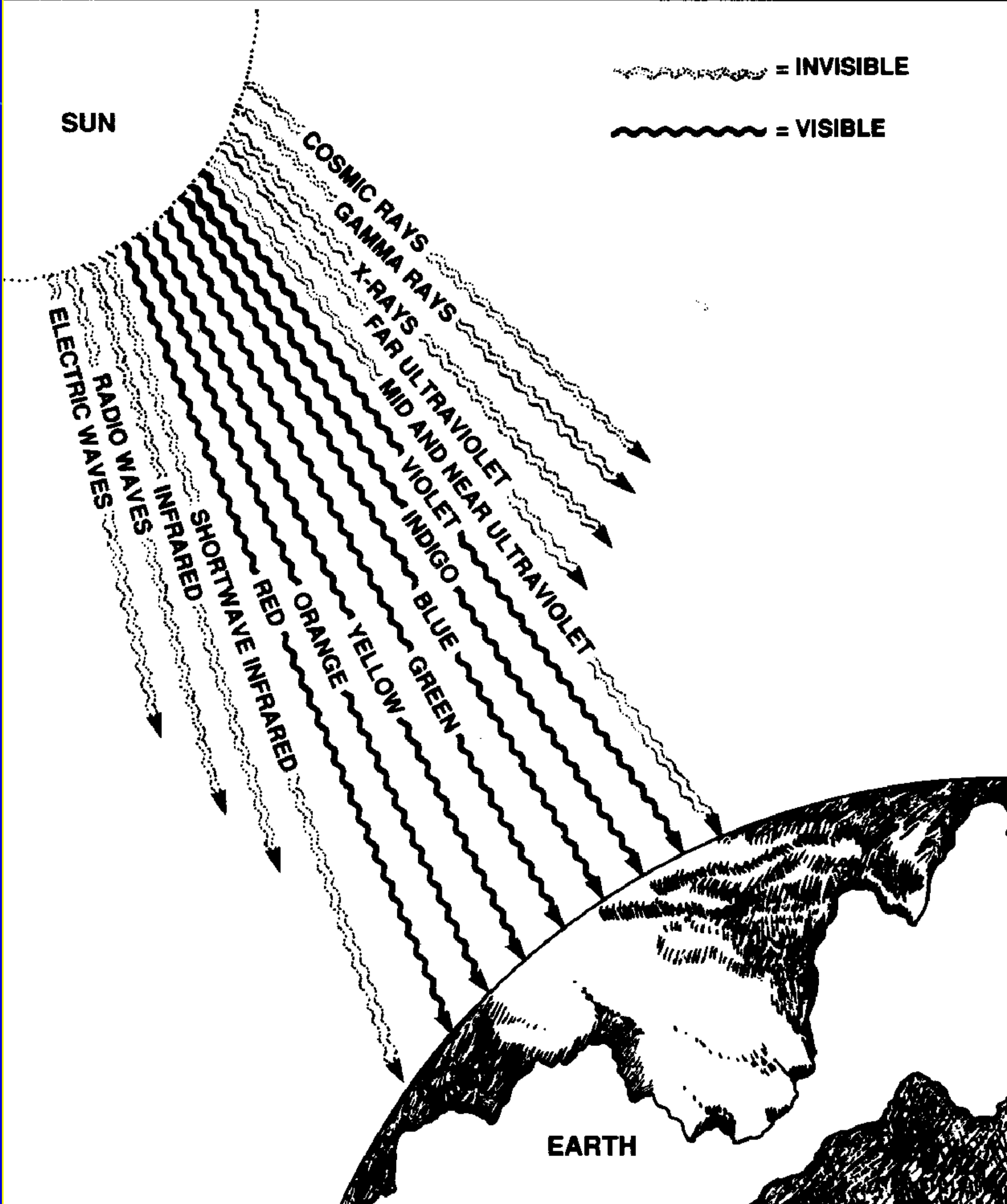
Spectral Energy Distribution

Visible Spectrum



400 700

Schematic Representation of Colour Spectra



Colour Space

- Space of all visible colours equivalent to set of all functions $C : \Lambda \rightarrow R$
  - $C(\lambda) \geq 0$ all λ
  - $C(\lambda) > 0$ some λ.
- (Cardinality of this space is 2^c)

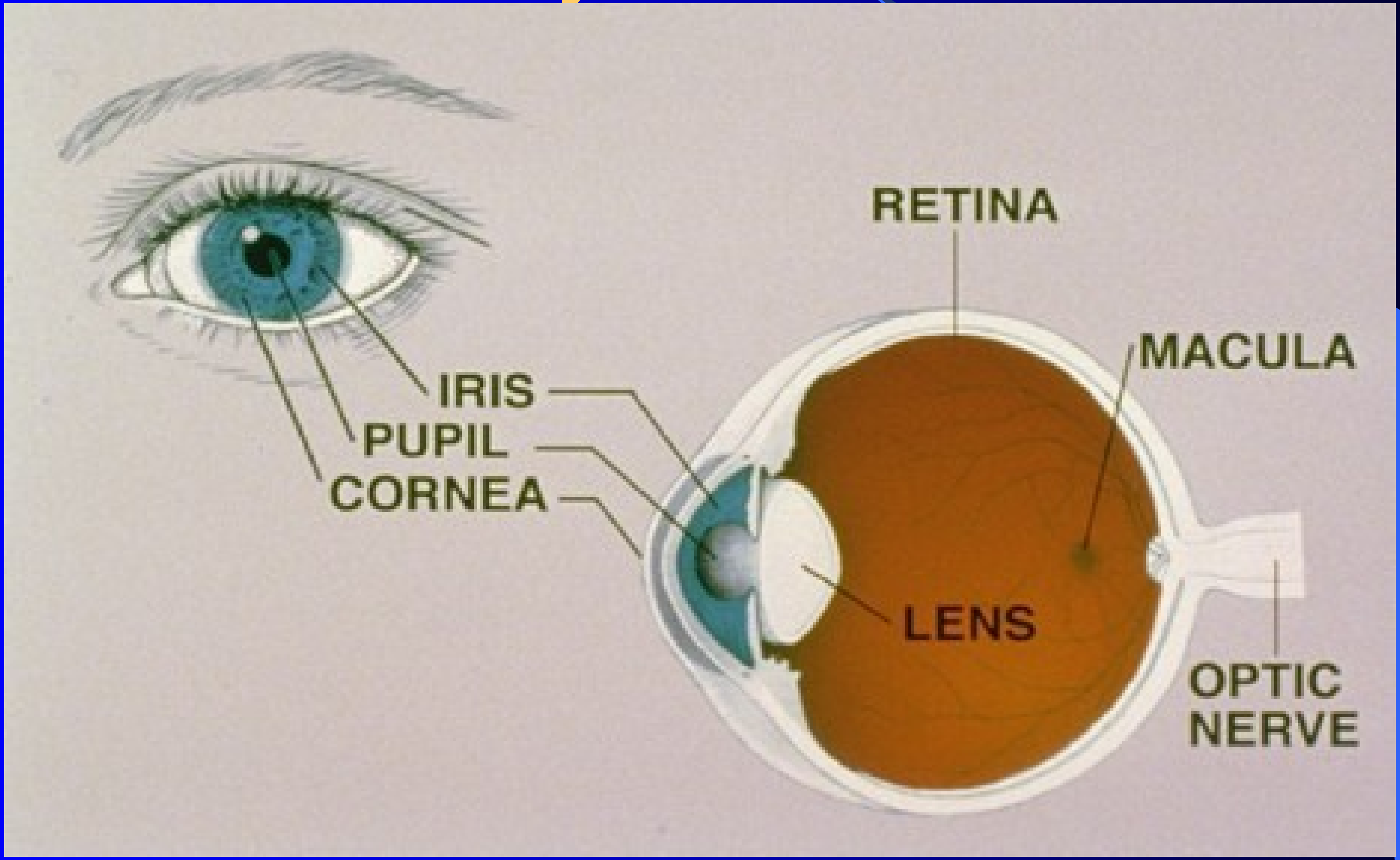
Perception and ‘The Sixth Sense’ movie

- We do not ‘see’ $C(\lambda)$ directly but as filtered through visual system.
- Two different people/animals will ‘see’ $C(\lambda)$ differently.
- Different $C(\lambda)$s can appear exactly the same to one individual (metamer).
- (Ignoring all ‘higher level’ processing, which basically indicates “we see what we expect to see”).

Infinite to Finite

- Colour space is infinite dimensional
- Visual system filters the energy distribution through a finite set of channels
- Constructs a finite signal space (retinal level)
- Through optic nerve to higher order processing (visual cortex ++++).

A Simple Model for the Visual System



Human Eye Schematic

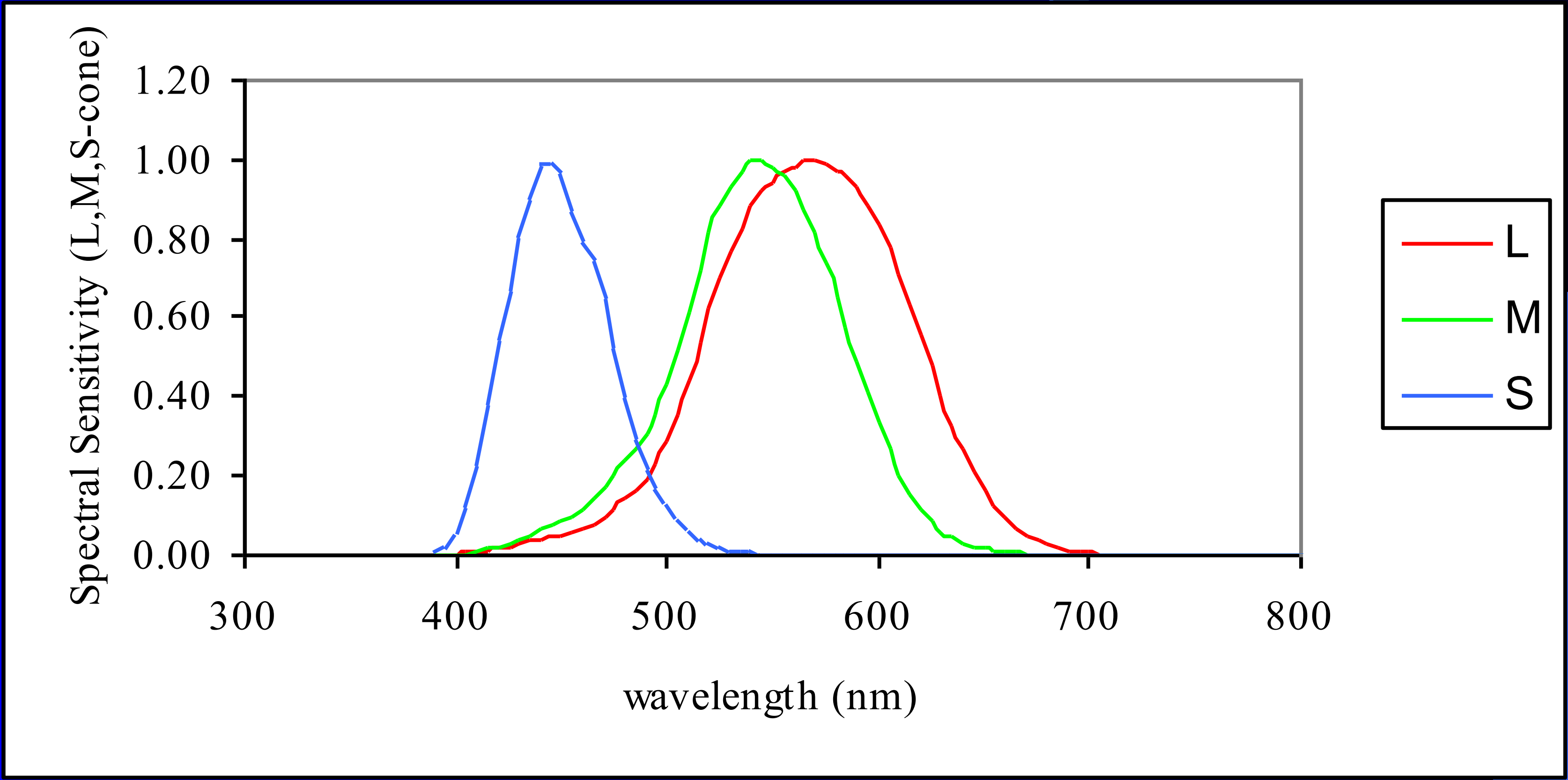
Photosensitive Receptors

- Rods – 130,000,000 night vision + peripheral (scotopic)
- Cones – 5-7,000,000, daylight vision + acuity (one point only)
- Cones
  - L-cones
  - M-cones
  - S-cones

LMS Response Curves

- $l = \int C(\lambda)L(\lambda)d\lambda$
- $m = \int C(\lambda)M(\lambda)d\lambda$
- $s = \int C(\lambda)S(\lambda)d\lambda$
- $C \rightarrow (l,m,s)$ (trichromatic theory)
- $LMS(C) = (l,m,s)$
- $LMS(C_a) = LMS(C_b)$ then C_a, C_b are *metamers*.

2-degree cone normalised response curves



Simple Model for an Emitter System

- Generates chromatic light by mixing streams of energy of light of different spectral distributions
- Finite number (>=3) and independent

Primaries (Basis) for an Emitter

- $C_E(\lambda)=\alpha_1 E_1(\lambda)+ \alpha_2 E_2(\lambda) + \alpha_3 E_3 (\lambda)$
- E_i are the primaries (form a basis)
- α_i are called the *intensities*.
- CIE-RGB Primaries are:
  - $E_R(\lambda) = \delta(\lambda- \lambda_R), \lambda_R = 700nm$
  - $E_G(\lambda) = \delta(\lambda- \lambda_G), \lambda_G = 546.1nm$
  - $E_B(\lambda) = \delta(\lambda- \lambda_B), \lambda_B = 435.8nm$

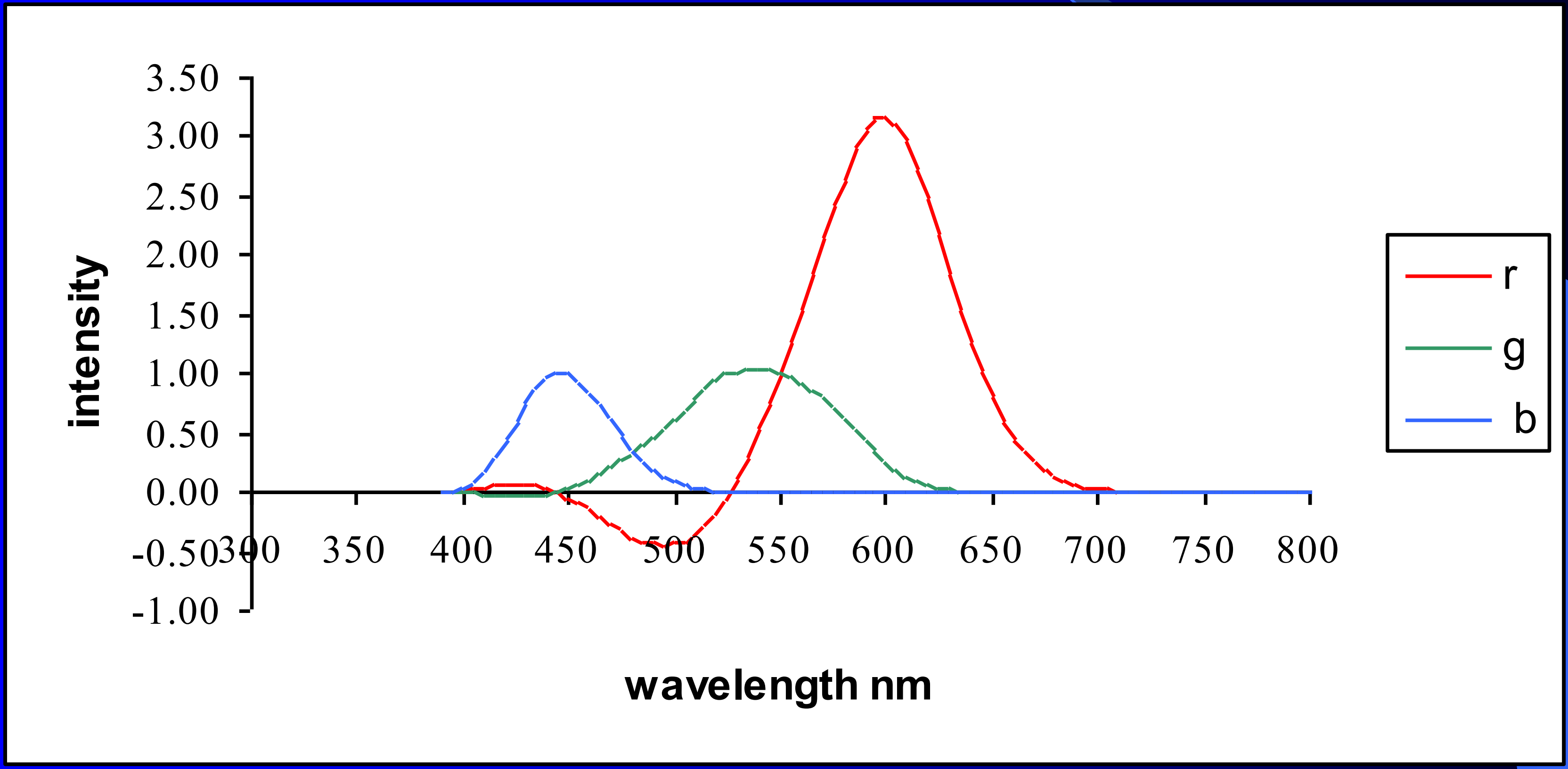
Computing the Intensities

- For a given $C(\lambda)$ problem is to find the intensities α_i such that $C_E(\lambda)$ is metameric to $C(\lambda)$.
- First Method to be shown isn't used, but illustrative of the problem.

Colour Matching Functions

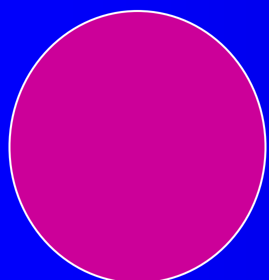
- Previous method relied on knowing L, M, and S response curves accurately.
- Better method based on colour matching functions.
- Define how to get the colour matching functions $\gamma_i(\lambda)$ relative to a given system of primaries.

2-degree RGB Colour Matching Functions

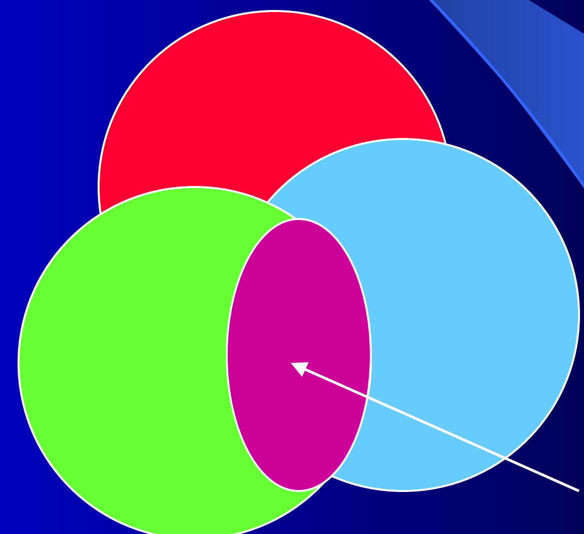


Colour Matching Experiment

Mixing of 3 primaries



Target colour



overlap

Adjust intensities to match the colour

Summary Week 1

- Compute the radiance distribution $C(\lambda)$
- Find out the colour matching functions for the display $\gamma_i(\lambda)$
- Perform the 3 integrals $\int \gamma_i(\lambda)C(\lambda)d\,\lambda$ to get the intensities for the metamer for that colour on the display.
- ….
- Except that's not how it is done …
- ….to be continued….

Outline: Week 2

- CIE-RGB Chromaticity Space
- CIE-XYZ Chromaticity Space
- Converting between XYZ and RGB
- Colour Gamuts and Undisplayable Colours
- Summary for Rendering: What to do in practice

CIE-RGB Chromaticity Space

- Consider CIE-RGB primaries:
  - For each $C(\lambda)$ there is a point $(\alpha_R, \alpha_G, \alpha_B)$:
    - $C(\lambda) \approx \alpha_R E_R(\lambda) + \alpha_G E_G(\lambda) + \alpha_B E_B(\lambda)$
  - Considering all such possible points
    - $(\alpha_R, \alpha_G, \alpha_B)$
  - Results in 3D RGB colour space
  - Hard to visualise in 3D
  - so we'll find a 2D representation instead.

CIE-RGB Chromaticity Space

- Consider 1st only monochromatic colours:
  - $C(\lambda) = \delta(\lambda - \lambda_0)$
- Let the CIE-RGB matching functions be
  - $r(\lambda), g(\lambda), b(\lambda)$
- Then, eg,
  - $\alpha_R(\lambda_0) = \int \delta(\lambda - \lambda_0)\, r(\lambda) d\lambda = r(\lambda_0)$
- Generally
  - $(\alpha_R(\lambda_0), \alpha_G(\lambda_0), \alpha_B(\lambda_0)) = (r(\lambda_0), g(\lambda_0), b(\lambda_0))$

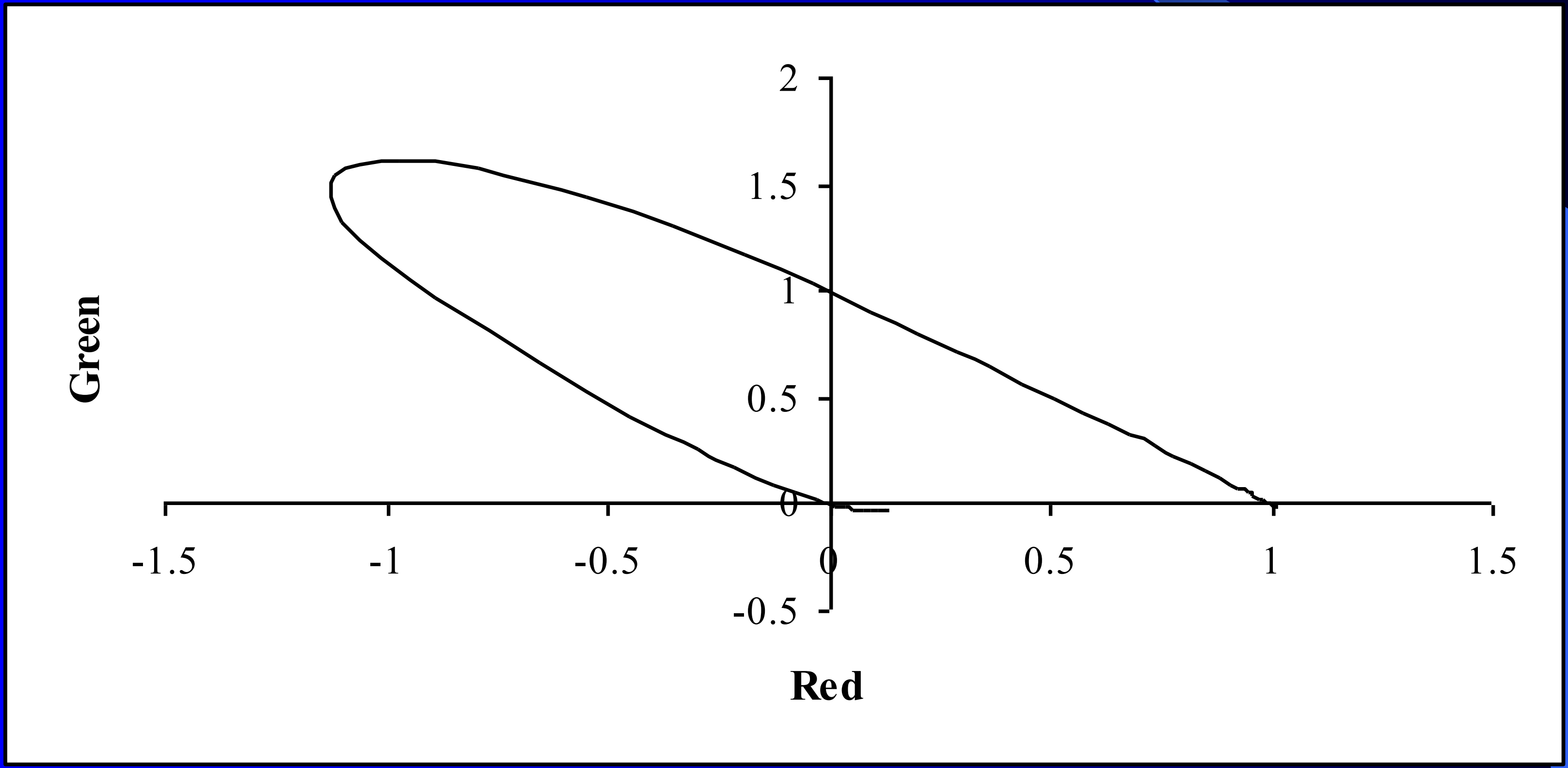
CIE-RGB Chromaticity Space

- As λ_0 varies over all wavelengths
  - $(r(\lambda_0), g(\lambda_0), b(\lambda_0))$ sweeps out a 3D curve.
- This curve gives the metamer intensities for all monochromatic colours.
- To visualise this curve, conventionally project onto the plane
  - $\alpha_R + \alpha_G + \alpha_B = 1$

CIE-RGB Chromaticity Space

- It is easy to show that projection of $(\alpha_R, \alpha_G, \alpha_B)$ onto $\alpha_R + \alpha_G + \alpha_B = 1$ is:
  - $(\alpha_R/D, \alpha_G/D, \alpha_B/D)$,
    - $D = \alpha_R + \alpha_G + \alpha_B$
- Show that interior and boundary of the curve correspond to visible colours.
- CIE-RGB chromaticity space.

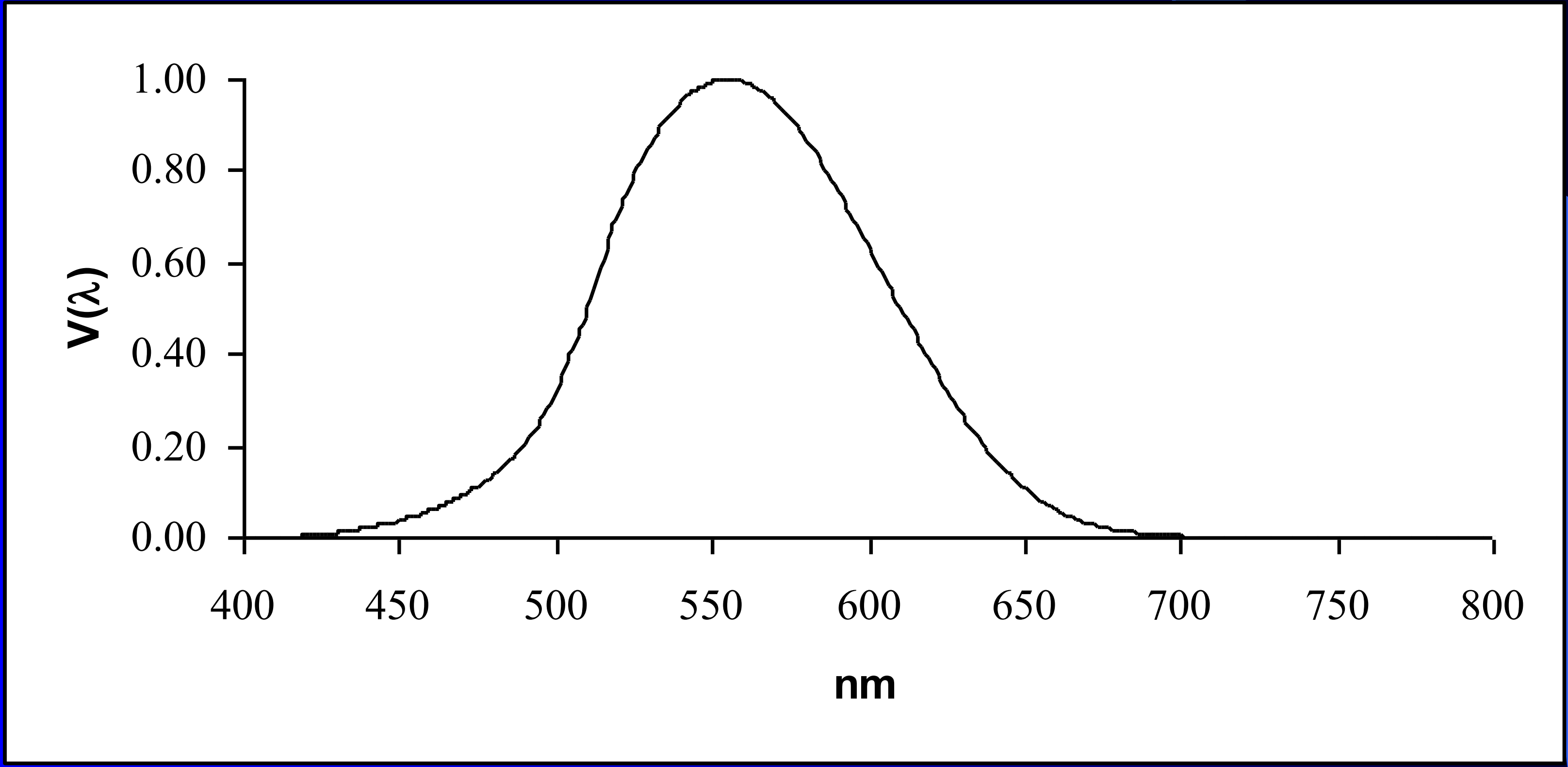
CIE-RGB Chromaticity Diagram



CIE-RGB Chromaticity

- Define:
  - $V(\lambda) = \beta_1 L(\lambda) + \beta_2 M(\lambda) + \beta_3 S(\lambda)$
- Specific constants β_i results in
  - Spectral luminous efficiency curve
- Overall response of visual system to $C(\lambda)$
  - $L(C) = K \int C(\lambda)\, V(\lambda) d\lambda$
- For K=680 lumens/watt, and C as radiance, L is called the luminance (candelas per square metre)

Spectral Luminous Efficiency Function



CIE-RGB Chromaticity

- Since
  - $C(\lambda) \approx \alpha_R E_R(\lambda) + \alpha_G E_G(\lambda) + \alpha_B E_B(\lambda)$
- Then
  - $L(C) =$
    $\alpha_R \int E_R(\lambda) V(\lambda) d\lambda$
    $+ \alpha_G \int E_G(\lambda) V(\lambda) d\lambda$
    $+ \alpha_B \int E_B(\lambda) V(\lambda) d\lambda$
- Or
  - $L(C) = \alpha_R l_R + \alpha_G l_G + \alpha_B l_B$

Luminance and Chrominance

- $L(C) = \alpha_R l_R + \alpha_G l_G + \alpha_B l_B$
  - and l_R l_G l_B are constants
- Consider set of all $(\alpha_R, \alpha_G, \alpha_B)$ satisfying this equation…
  - a plane of constant luminance in RGB space
- Only one point on plane corresponds to colour C
  - so what is varying?
- Chrominance
  - The part of a colour (hue) abstracting away the luminance
- Colour = chrominance + luminance (independent)

Luminance and Chrominance

- Consider plane of constant luminance
  - $\alpha_R\, l_R + \alpha_G\, l_G + \alpha_B\, l_B = L$
- Let $\alpha^* = (\alpha^*_R, \alpha^*_G, \alpha^*_B)$ be a point on this plane.
  - $(t\alpha^*_R, t\alpha^*_G, t\alpha^*_B)$, $t>0$ is a line from 0 through α^*
- Luminance is increasing (tL) but projection on $\alpha_R + \alpha_G + \alpha_B = 1$ is the same.
- Projection on $\alpha_R + \alpha_G + \alpha_B = 1$ is a way of providing 2D coord system for chrominance.

(Change of Basis)

- E and F are two different primaries
  - $C(\lambda) \approx \alpha_1 E_1(\lambda) + \alpha_2 E_2(\lambda) + \alpha_3 E_3(\lambda)$
  - $C(\lambda) \approx \beta_1 F_1(\lambda) + \beta_2 F_2(\lambda) + \beta_3 F_3(\lambda)$
- Let A be the matrix that expresses F in terms of E
  - $F(\lambda) = AE(\lambda)$
- Then
  - $\alpha = \beta A$
  - $\gamma_{Ej}(\lambda) = \sum_i \gamma_{Fi}(\lambda)\, \alpha_{ij}$ (CMFs)

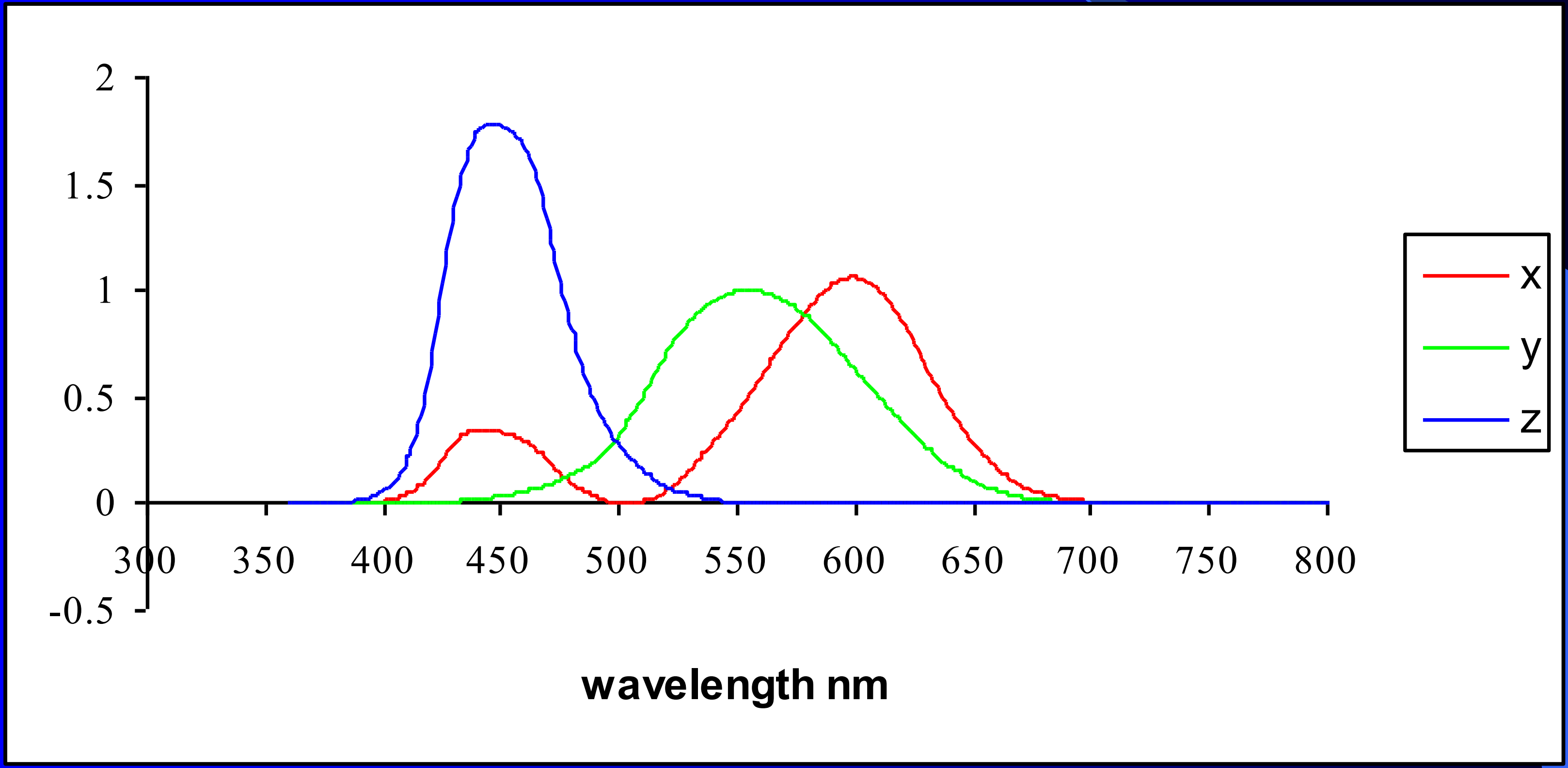
CIE-XYZ Chromaticity Space

- CIE-RGB representation not ideal
  - Colours outside 1st quadrant not achievable
  - Negative CMF function ranges
- CIE derived a different XYZ basis with better mathamatical behaviour
  - $X(\lambda)$, $Y(\lambda)$, $Z(\lambda)$ basis functions (imaginary primaries)
  - X, Z have zero luminance
  - CMF for Y is specral luminous efficiency function V
- Known matrix A for transformation to CIE-RGB

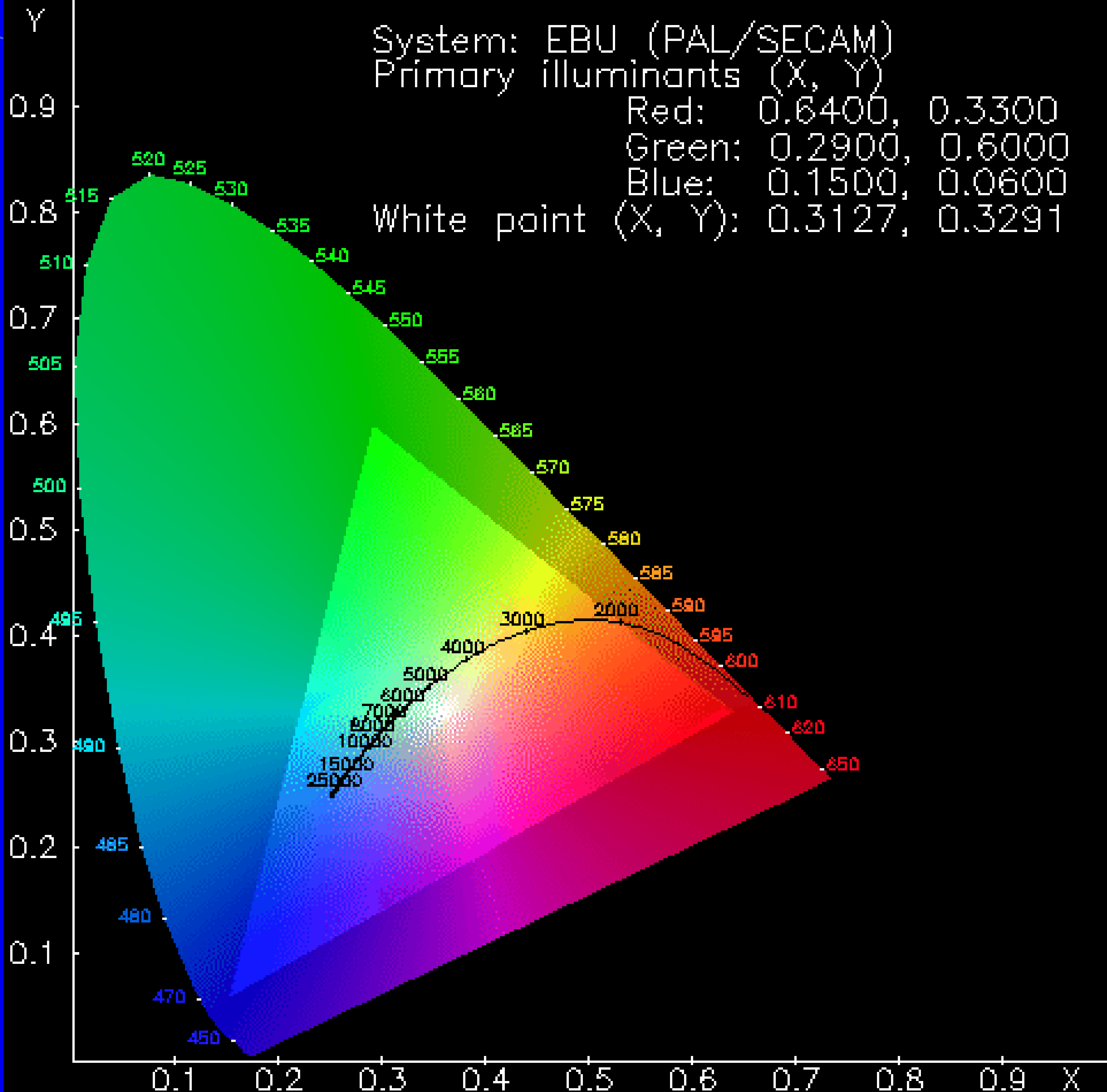
CIE-XYZ Chromaticity Space

- $C(\lambda) \approx X.\ X(\lambda) + Y.\ Y(\lambda) + Z.\ Z(\lambda)$
  - $X = \int C(\lambda)x(\lambda)d\lambda$
  - $Y = \int C(\lambda)y(\lambda)d\lambda$ [luminance]
  - $Z = \int C(\lambda)z(\lambda)d\lambda$
  - x,y,z are the CMFs

2-deg XYZ Colour Matching Functions



CIE-XYZ Chromaticity Diagram



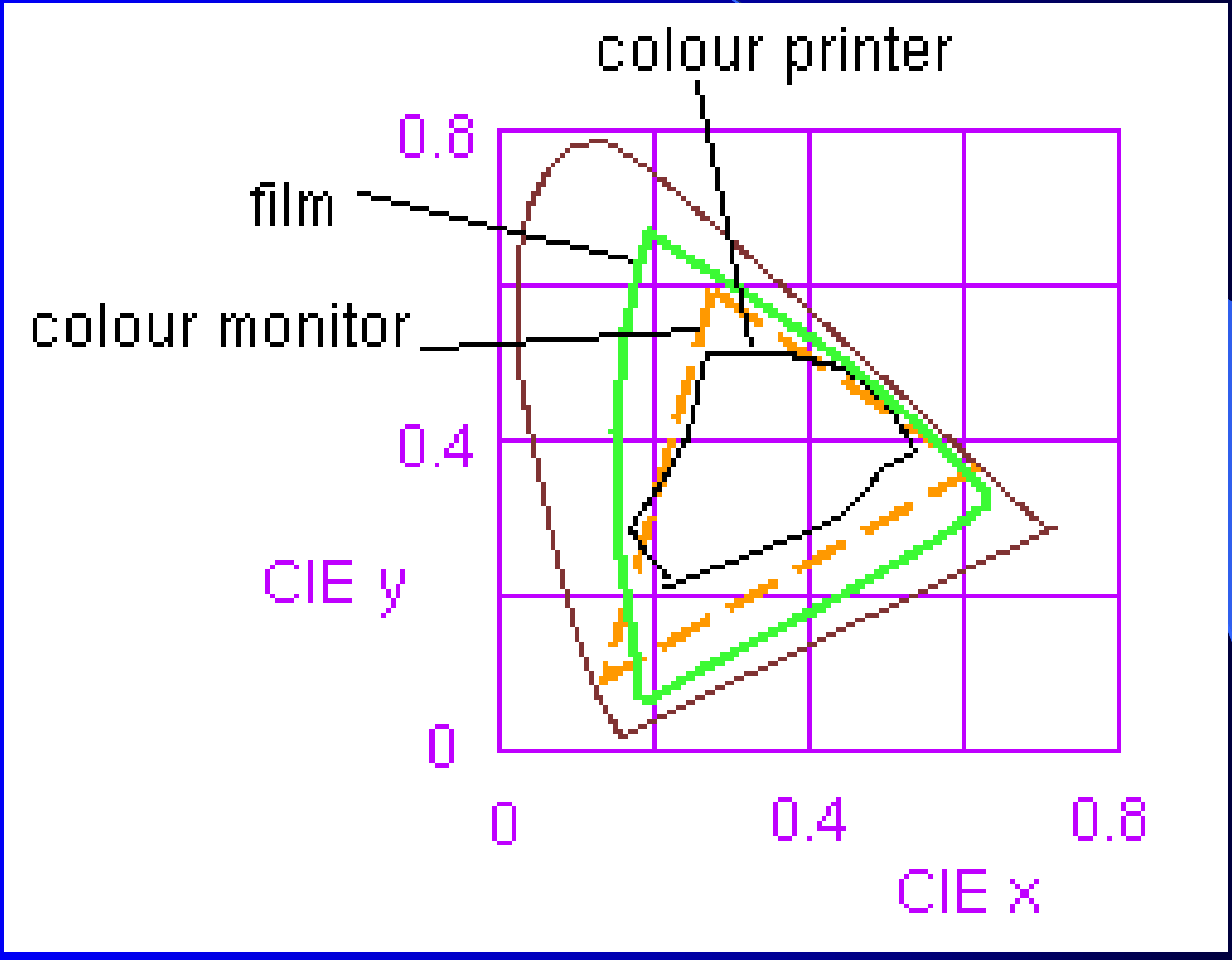
Converting Between XYZ and RGB

- System has primaries $R(\lambda)$, $G(\lambda)$, $B(\lambda)$
- How to convert between a colour expressed in RGB and vice versa?
- Derivation…

Colour Gamuts and Undisplayable Colours

- Display has RGB primaries, with corresponding XYZ colours C_R, C_G, C_B
- Chromaticities c_R, c_G, c_B will form triangle on CIE-XYZ diagram
- All points in the triangle are displayable colours
  - forming the colour gamut

Some Colour Gamuts



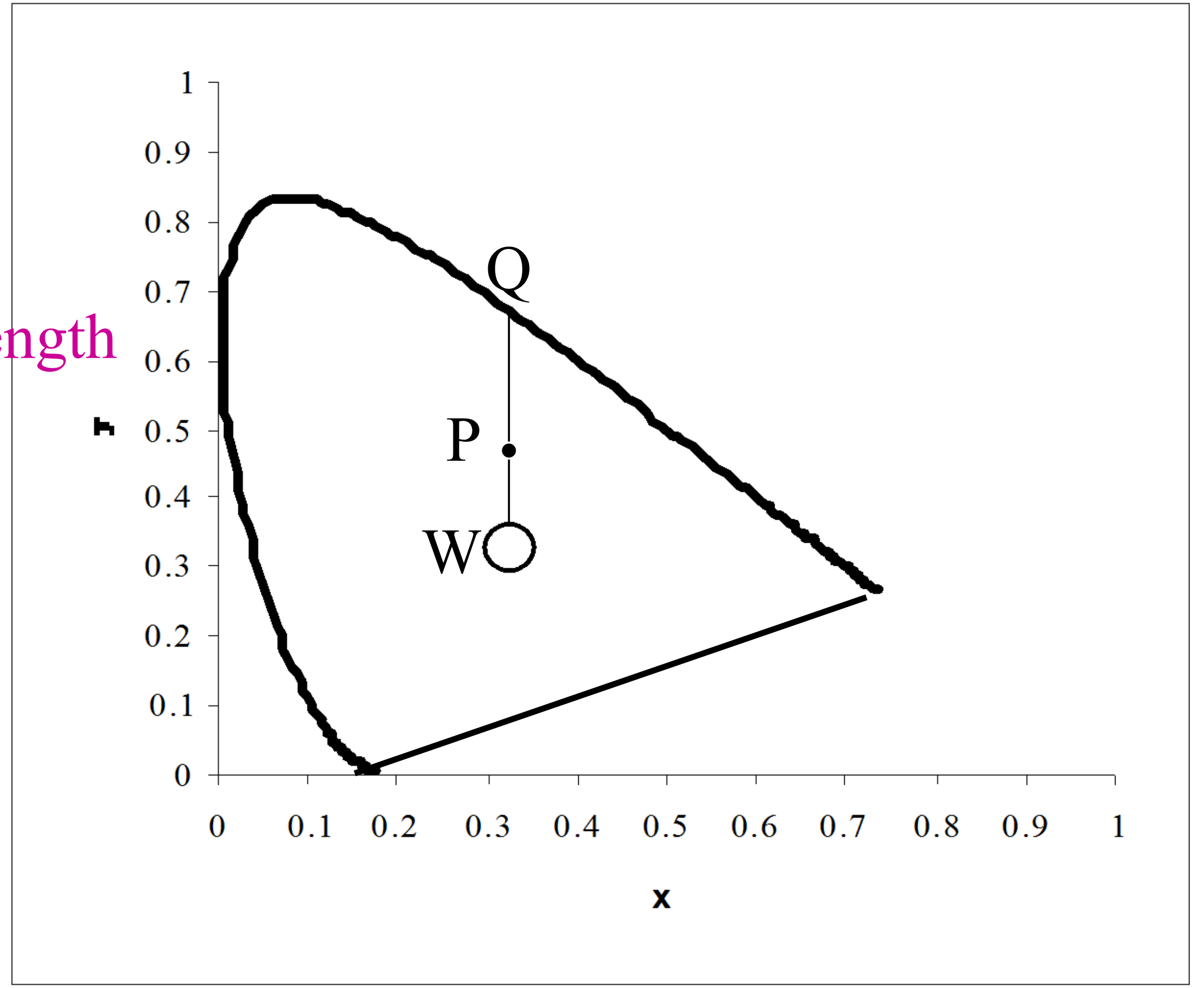
Undisplayable Colours

- Suppose XYZ colour computed, but not displayable?
- Terminology
  - Dominant wavelength
  - Saturation

XYZ with White Point

For colour at P

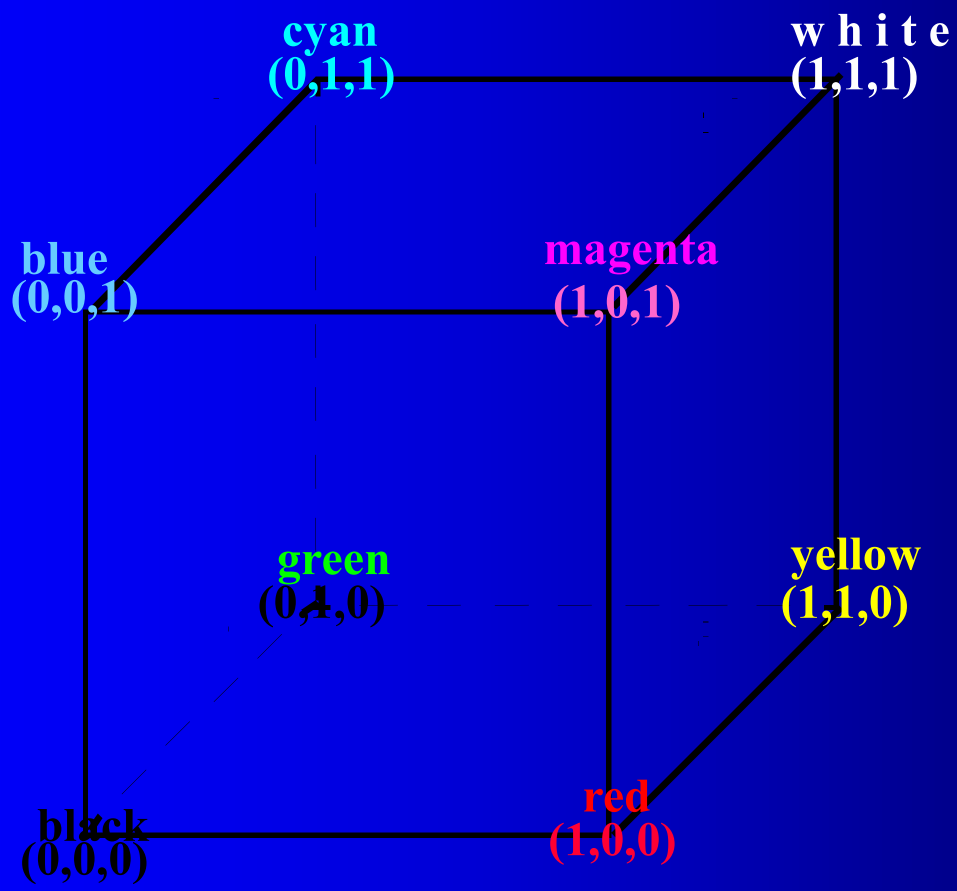
- Q dominant wavelength
- WP/WQ saturation



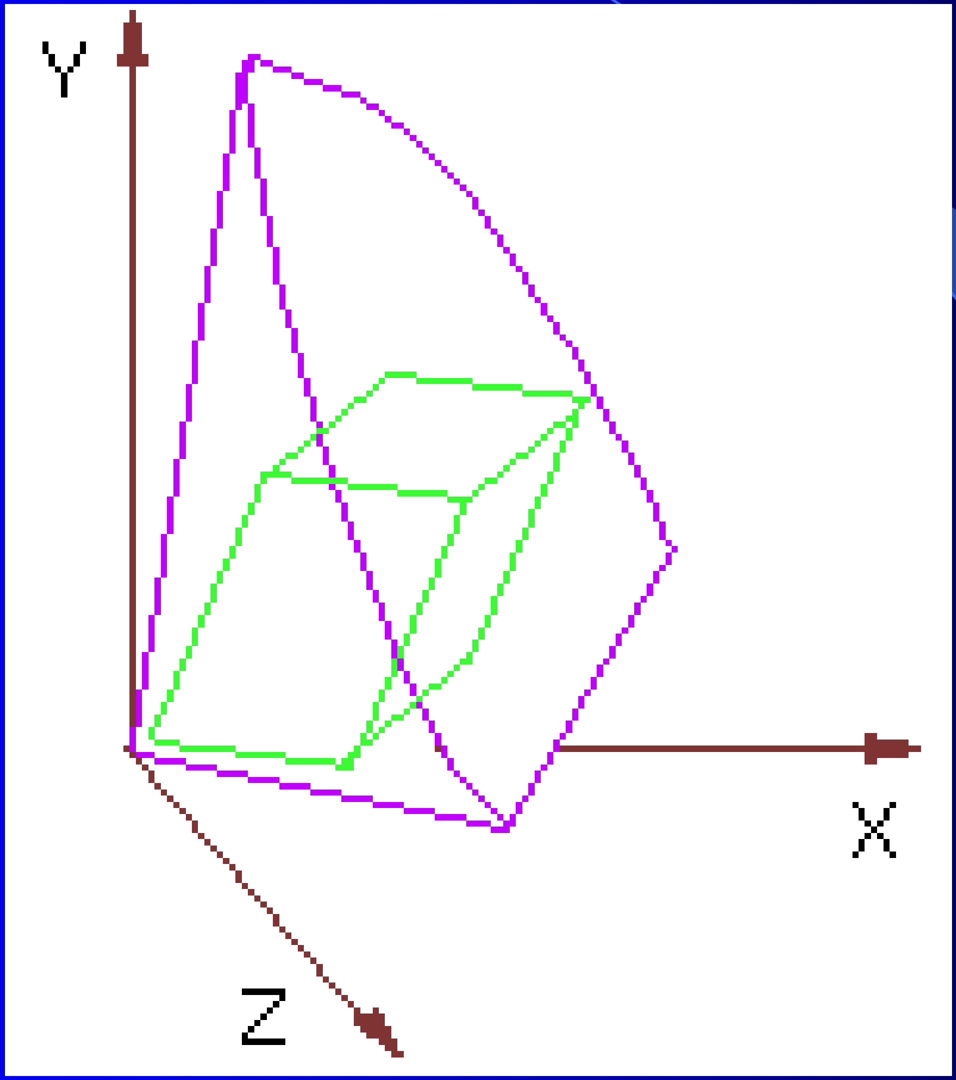
Colour might not be displayable

- Falls outside of the triangle (its chromaticity not displayable on this device)
  - Might desaturate it, move it along line QW until inside gamut (so dominant wavelength invariant)
- Colour with luminance outside of displayable range.
  - Clip vector through the origin to the RGB cube (chrominance invariant)

RGB Colour Cube



RGB Cube Mapped to XYZ Space



Summary for Rendering

- Incorrect to use RGB throughout!!!
  - Different displays will produce different results
  - RGB is not the appropriate measure of light energy (neither radiometric nor photometric).
  - But depends on application
    - Most applications of CG do not require ‘correct’ colours…
    - …but colours that are appropriate for the application.

For Rendering

- Algorithm should compute $C(\lambda)$ for surfaces
  - means computing at a sufficient number of wavelengths to estimate C (not 'RGB').
- Transform into XYZ space
  - $X = \int C(\lambda)x(\lambda)d\lambda$
  - $Y = \int C(\lambda)y(\lambda)d\lambda$
  - $Z = \int C(\lambda)z(\lambda)d\lambda$
- Map to RGB space, with clipping and gamma correction.