

# Chapter 7. Cluster Analysis

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1. What is Cluster Analysis?
2. Types of Data in Cluster Analysis
3. A Categorization of Major Clustering Methods
4. Partitioning Methods
5. Hierarchical Methods
6. Density-Based Methods
7. Grid-Based Methods
8. Model-Based Methods
9. Clustering High-Dimensional Data
10. Constraint-Based Clustering
11. Outlier Analysis
12. Summary

# What is Cluster Analysis?

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- Cluster: a collection of data objects
  - Similar to one another within the same cluster
  - Dissimilar to the objects in other clusters
- Cluster analysis
  - Finding similarities between data according to the characteristics found in the data and grouping similar data objects into clusters
- **Unsupervised learning**: no predefined classes
- Typical applications
  - As a **stand-alone tool** to get insight into data distribution
  - As a **preprocessing step** for other algorithms

# Clustering: Rich Applications and Multidisciplinary Efforts

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- Pattern Recognition
- Spatial Data Analysis
  - Create thematic maps in GIS by clustering feature spaces
  - Detect spatial clusters or for other spatial mining tasks
- Image Processing
- Economic Science (especially market research)
- WWW
  - Document classification
  - Cluster Weblog data to discover groups of similar access patterns

# Examples of Clustering Applications

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- Marketing: Help marketers discover distinct groups in their customer bases, and then use this knowledge to develop targeted marketing programs
- Land use: Identification of areas of similar land use in an earth observation database
- Insurance: Identifying groups of motor insurance policy holders with a high average claim cost
- City-planning: Identifying groups of houses according to their house type, value, and geographical location
- Earth-quake studies: Observed earth quake epicenters should be clustered along continent faults

# Quality: What Is Good Clustering?

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- A good clustering method will produce high quality clusters with
  - high intra-class similarity
  - low inter-class similarity
- The quality of a clustering result depends on both the similarity measure used by the method and its implementation
- The quality of a clustering method is also measured by its ability to discover some or all of the hidden patterns

# Measure the Quality of Clustering

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- **Dissimilarity/Similarity metric**: Similarity is expressed in terms of a distance function, typically metric:  $d(i, j)$
- There is a separate “quality” function that measures the “goodness” of a cluster.
- The definitions of **distance functions** are usually very different for interval-scaled, boolean, categorical, ordinal ratio, and vector variables.
- Weights should be associated with different variables based on applications and data semantics.
- It is hard to define “similar enough” or “good enough”
  - the answer is typically highly subjective.


# Requirements of Clustering in Data Mining

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- Scalability
- Ability to deal with different types of attributes
- Ability to handle dynamic data
- Discovery of clusters with arbitrary shape
- Minimal requirements for domain knowledge to determine input parameters
- Able to deal with noise and outliers
- Insensitive to order of input records
- High dimensionality
- Incorporation of user-specified constraints
- Interpretability and usability

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# Type of data in clustering analysis

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- Interval-scaled variables
- Binary variables
- Nominal, ordinal, and ratio variables
- Variables of mixed types

# Interval-scaled variables

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- Standardize data

- Calculate the mean absolute deviation:

$$s_f = \frac{1}{n} (|x_{1f} - m_f| + |x_{2f} - m_f| + \dots + |x_{nf} - m_f|)$$

where  $m_f = \frac{1}{n}(x_{1f} + x_{2f} + \dots + x_{nf})$ .

- Calculate the standardized measurement (*z-score*)

$$z_{if} = \frac{x_{if} - m_f}{s_f}$$

- Using mean absolute deviation is more robust than using standard deviation

# Similarity and Dissimilarity Between Objects

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- Distances are normally used to measure the similarity or dissimilarity between two data objects
- Some popular ones include: *Minkowski distance*:

$$d(i, j) = \sqrt[q]{(|x_{i1} - x_{j1}|^q + |x_{i2} - x_{j2}|^q + \dots + |x_{ip} - x_{jp}|^q)}$$

where  $i = (x_{i1}, x_{i2}, \dots, x_{ip})$  and  $j = (x_{j1}, x_{j2}, \dots, x_{jp})$  are two  $p$ -dimensional data objects, and  $q$  is a positive integer

- If  $q = 1$ ,  $d$  is Manhattan distance

$$d(i, j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \dots + |x_{ip} - x_{jp}|$$

# Similarity and Dissimilarity Between Objects (Cont.)

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- If  $q = 2$ ,  $d$  is Euclidean distance:

$$d(i,j) = \sqrt{(|x_{i_1} - x_{j_1}|^2 + |x_{i_2} - x_{j_2}|^2 + \dots + |x_{i_p} - x_{j_p}|^2)}$$

- Properties

- $d(i,j) \geq 0$
- $d(i,i) = 0$
- $d(i,j) = d(j,i)$
- $d(i,j) \leq d(i,k) + d(k,j)$

- Also, one can use weighted distance, parametric Pearson product moment correlation, or other dissimilarity measures

# Binary Variables

- A contingency table for binary data

		Object $j$		
		1	0	<i>sum</i>
Object $i$	1	$a$	$b$	$a+b$
	0	$c$	$d$	$c+d$
<i>sum</i>		$a+c$	$b+d$	$p$

- Distance measure for symmetric binary variables:

$$d(i, j) = \frac{b+c}{a+b+c+d}$$

- Distance measure for asymmetric binary variables:

$$d(i, j) = \frac{b+c}{a+b+c}$$

- Jaccard coefficient (*similarity* measure for *asymmetric* binary variables):

$$sim_{Jaccard}(i, j) = \frac{a}{a+b+c}$$

# Dissimilarity between Binary Variables

- Example

Name	Gender	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	M	Y	N	P	N	N	N
Mary	F	Y	N	P	N	P	N
Jim	M	Y	P	N	N	N	N

- gender is a symmetric attribute
- the remaining attributes are asymmetric binary
- let the values Y and P be set to 1, and the value N be set to 0
- Suppose distance is computed based on the asymmetric variables

$$d(\mathit{jack}, \mathit{mary}) = \frac{0 + 1}{2 + 0 + 1} = 0.33$$

$$d(\mathit{jack}, \mathit{jim}) = \frac{1 + 1}{1 + 1 + 1} = 0.67$$

$$d(\mathit{jim}, \mathit{mary}) = \frac{1 + 2}{1 + 1 + 2} = 0.75$$

# Categorical Variables

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- A generalization of the binary variable in that it can take more than 2 states, e.g., red, yellow, blue, green
- Method 1 (sym.): Simple matching
  - $m$ : # of matches,  $p$ : total # of variables

$$d(i, j) = \frac{p - m}{p}$$

- Method 2 (asym.): use a large number of binary variables
  - creating a new binary variable for each of the  $M$  nominal states



# Ordinal Variables

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- An ordinal variable can be discrete or continuous
- Order is important, e.g., rank
- Can be treated like interval-scaled
  - replace  $x_{if}$  by their rank  $r_{if} \in \{1, \dots, M_f\}$
  - map the range of each variable onto  $[0, 1]$  by replacing  $i$ -th object in the  $f$ -th variable by

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

- compute the dissimilarity using methods for interval-scaled variables

# Ratio-Scaled Variables

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- Ratio-scaled variable: a positive measurement on a nonlinear scale, approximately at exponential scale, such as  $Ae^{Bt}$  or  $Ae^{-Bt}$
- Methods:
  - treat them like interval-scaled variables—*not a good choice!* (why?—the scale can be distorted)
  - apply logarithmic transformation
$$y_{if} = \log(x_{if})$$
  - treat them as continuous ordinal data treat their rank as interval-scaled

# Variables of Mixed Types

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- A database may contain all the six types of variables
  - symmetric binary, asymmetric binary, nominal, ordinal, interval and ratio

- One may use a weighted formula to combine their effects

$$d(i, j) = \frac{\sum_{f=1}^p \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^p \delta_{ij}^{(f)}}$$

- $f$  is binary or nominal:

$d_{ij}^{(f)} = 0$  if  $x_{if} = x_{jf}$ , or  $d_{ij}^{(f)} = 1$  otherwise

- $f$  is interval-based: use the normalized distance
- $f$  is ordinal or ratio-scaled

- compute ranks  $r_{if}$  and

- and treat  $z_{if}$  as interval-scaled

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

# Vector Objects

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- Vector objects: keywords in documents, gene features in micro-arrays, etc.
- Broad applications: information retrieval, biologic taxonomy, etc.
- Cosine measure:  $\cos(\theta) = s(\vec{X}, \vec{Y}) = \frac{\vec{X}^t \cdot \vec{Y}}{|\vec{X}| |\vec{Y}|}$ ,

$\vec{X}^t$  is a transposition of vector  $\vec{X}$ ,  $|\vec{X}|$  is the Euclidean normal of vector  $\vec{X}$ ,

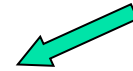
- A variant: Tanimoto coefficient

$$s(\vec{X}, \vec{Y}) = \frac{\vec{X}^t \cdot \vec{Y}}{\vec{X}^t \cdot \vec{X} + \vec{Y}^t \cdot \vec{Y} - \vec{X}^t \cdot \vec{Y}}$$

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# Major Clustering Approaches (I)

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- Partitioning approach:
  - Construct various partitions and then evaluate them by some criterion, e.g., minimizing the sum of square errors
  - Typical methods: k-means, k-medoids, CLARANS
- Hierarchical approach:
  - Create a hierarchical decomposition of the set of data (or objects) using some criterion
  - Typical methods: Diana, Agnes, BIRCH, ROCK, CHAMELEON
- Density-based approach:
  - Based on connectivity and density functions
  - Typical methods: DBSCAN, OPTICS, DenClue

# Major Clustering Approaches (II)

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- Grid-based approach:
  - based on a multiple-level granularity structure
  - Typical methods: STING, WaveCluster, CLIQUE
- Model-based:
  - A model is hypothesized for each of the clusters and tries to find the best fit of that model to each other
  - Typical methods: EM, SOM, COBWEB
- Frequent pattern-based:
  - Based on the analysis of frequent patterns
  - Typical methods: pCluster
- User-guided or constraint-based:
  - Clustering by considering user-specified or application-specific constraints
  - Typical methods: COD (obstacles), constrained clustering

# Typical Alternatives to Calculate the Distance between Clusters

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- **Single link:** smallest distance between an element in one cluster and an element in the other, i.e.,  $\text{dis}(K_i, K_j) = \min(t_{ip}, t_{jq})$
- **Complete link:** largest distance between an element in one cluster and an element in the other, i.e.,  $\text{dis}(K_i, K_j) = \max(t_{ip}, t_{jq})$
- **Average:** avg distance between an element in one cluster and an element in the other, i.e.,  $\text{dis}(K_i, K_j) = \text{avg}(t_{ip}, t_{jq})$
- **Centroid:** distance between the centroids of two clusters, i.e.,  $\text{dis}(K_i, K_j) = \text{dis}(C_i, C_j)$
- **Medoid:** distance between the medoids of two clusters, i.e.,  $\text{dis}(K_i, K_j) = \text{dis}(M_i, M_j)$ 
  - Medoid: one chosen, centrally located object in the cluster



# Centroid, Radius and Diameter of a Cluster (for numerical data sets)

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- Centroid: the “middle” of a cluster

$$C_m = \frac{\sum_{i=1}^N (t_{ip})}{N}$$

- Radius: square root of average distance from any point of the cluster to its centroid

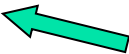
$$R_m = \sqrt{\frac{\sum_{i=1}^N (t_{ip} - c_m)^2}{N}}$$

- Diameter: square root of average mean squared distance between all pairs of points in the cluster

$$D_m = \sqrt{\frac{\sum_{i=1}^N \sum_{i=1}^N (t_{ip} - t_{iq})^2}{N(N-1)}}$$

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# Partitioning Algorithms: Basic Concept

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- Partitioning method: Construct a partition of a database  $D$  of  $n$  objects into a set of  $k$  clusters, s.t., some objective is minimized. E.g., min sum of squared distance in k-means

$$\sum_{m=1}^k \sum_{t_{mi} \in K_m} (C_m - t_{mi})^2$$

- Given a  $k$ , find a partition of  $k$  clusters that optimizes the chosen partitioning criterion
  - Global optimal: exhaustively enumerate all partitions
  - Heuristic methods: *k-means* and *k-medoids* algorithms
  - *k-means* (MacQueen' 67): Each cluster is represented by the center of the cluster
  - *k-medoids* or PAM (Partition around medoids) (Kaufman & Rousseeuw' 87): Each cluster is represented by one of the objects in the cluster

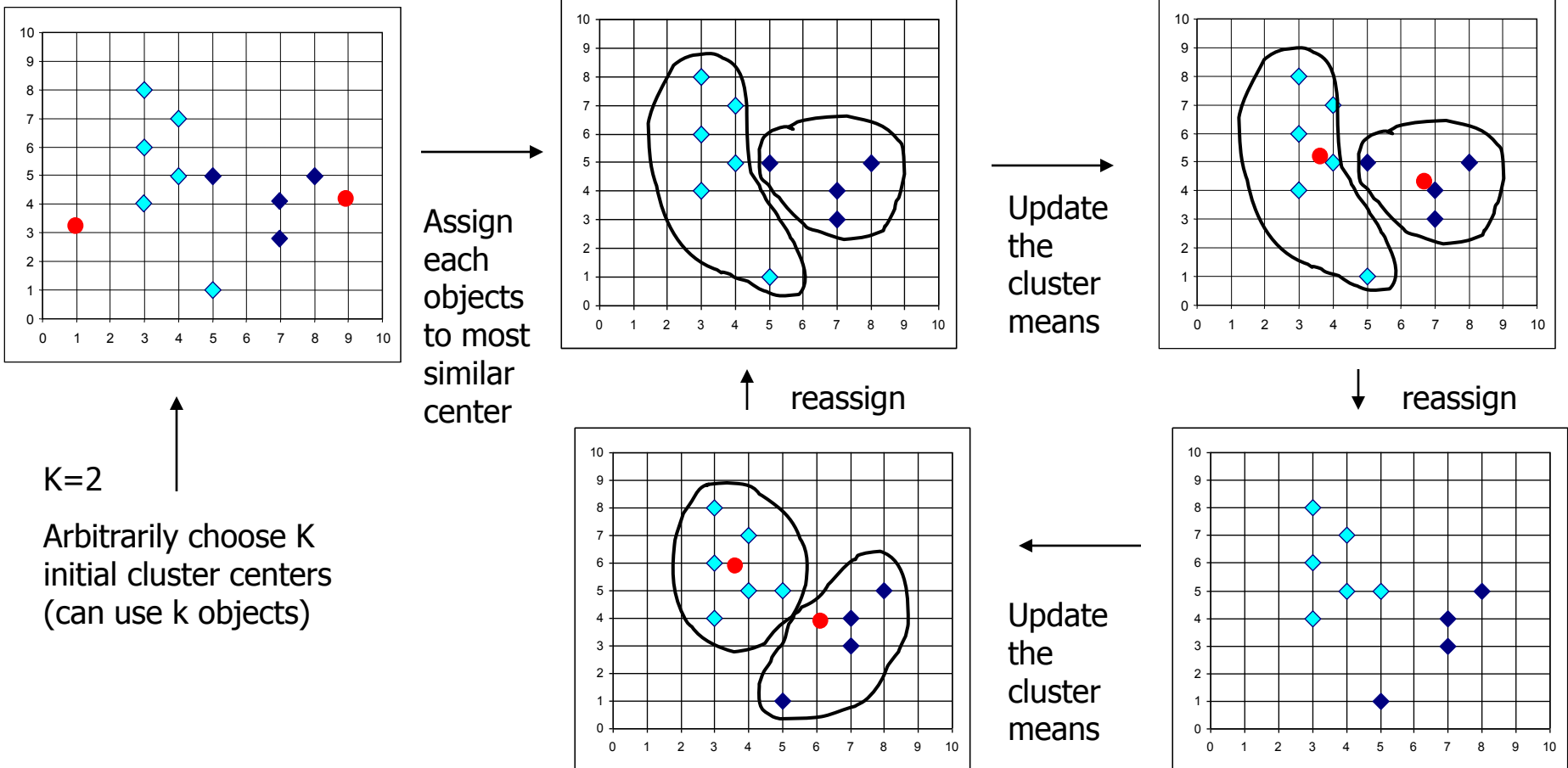
# The *K-Means* Clustering Method

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- Given  $k$ , the *k-means* algorithm is implemented in four steps:
  - Partition objects into  $k$  nonempty subsets
  - Compute seed points as the centroids of the clusters of the current partition (the centroid is the center, i.e., *mean point*, of the cluster)
  - Assign each object to the cluster with the nearest seed point
  - Go back to Step 2, stop when no more new assignment

# The *K-Means* Clustering Method

## ■ Example



# Comments on the *K-Means* Method

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- Strength: *Relatively efficient*:  $O(tkn)$ , where  $n$  is # objects,  $k$  is # clusters, and  $t$  is # iterations. Normally,  $k, t \ll n$ .
  - Comparing: PAM:  $O(k(n-k)^2)$ , CLARA:  $O(ks^2 + k(n-k))$
- Comment: Often terminates at a *local optimum*. The *global optimum* may be found using techniques such as: *deterministic annealing* and *genetic algorithms*
- Weakness
  - Applicable only when *mean* is defined, then what about categorical data?
  - Need to specify  $k$ , the *number* of clusters, in advance
  - Unable to handle noisy data and *outliers*
  - Not suitable to discover clusters with *non-convex shapes*

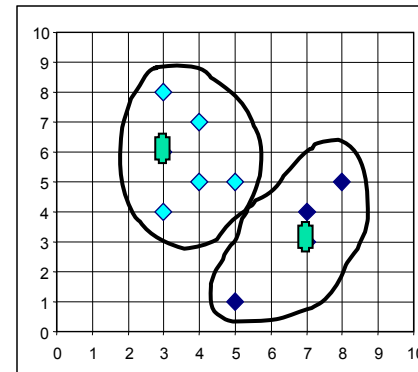
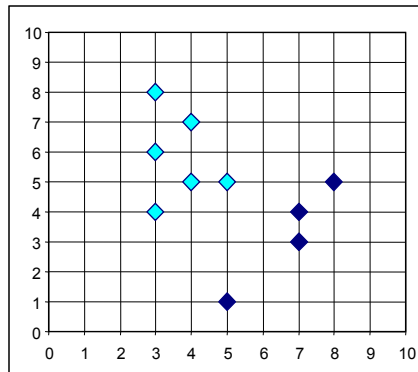
# Variations of the *K-Means* Method

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- A few variants of the *k-means* which differ in
  - Selection of the initial *k* means
  - Dissimilarity calculations
  - Strategies to calculate cluster means
- Handling categorical data: *k-modes* (Huang' 98)
  - Replacing means of clusters with modes
  - Using new dissimilarity measures to deal with categorical objects
  - Using a frequency-based method to update modes of clusters
  - A mixture of categorical and numerical data: *k-prototype* method

# What Is the Problem of the K-Means Method?

- The k-means algorithm is sensitive to outliers !
  - Since an object with an extremely large value may substantially distort the distribution of the data.
- K-Medoids: Instead of taking the **mean** value of the object in a cluster as a reference point, **medoids** can be used, which is the **most centrally located** object in a cluster.





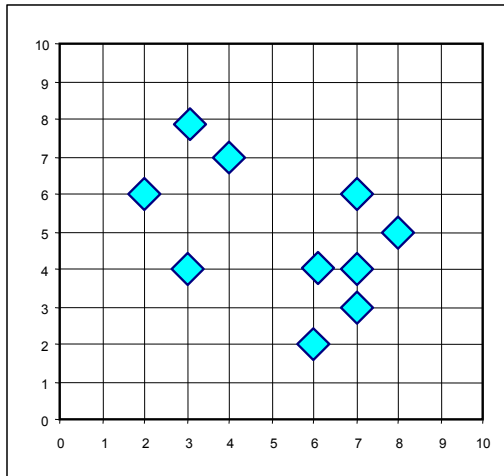
# The *K-Medoids* Clustering Method

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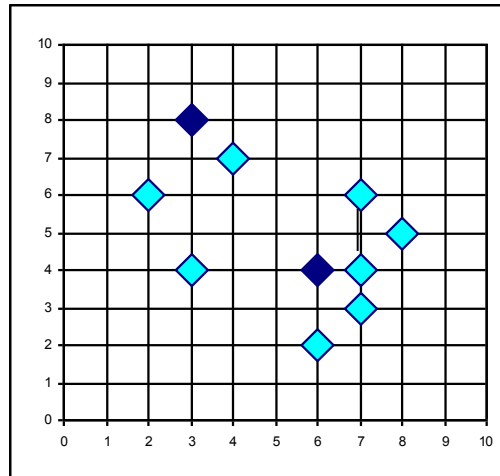
- Find *representative* objects, called medoids, in clusters
- *PAM* (Partitioning Around Medoids, 1987)
  - starts from an initial set of medoids and iteratively replaces one of the medoids by one of the non-medoids if it improves the total distance of the resulting clustering
  - *PAM* works effectively for small data sets, but does not scale well for large data sets
- *CLARA* (Kaufmann & Rousseeuw, 1990)
- *CLARANS* (Ng & Han, 1994): Randomized sampling
- Focusing + spatial data structure (Ester et al., 1995)

# A Typical K-Medoids Algorithm (PAM)

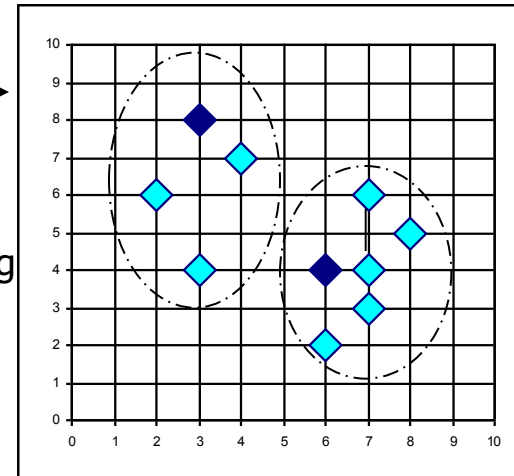
Total Cost = 20



Arbitrary  
choose  $k$   
object as  
initial  
medoids



Assign  
each  
remaining  
object to  
nearest  
medoids

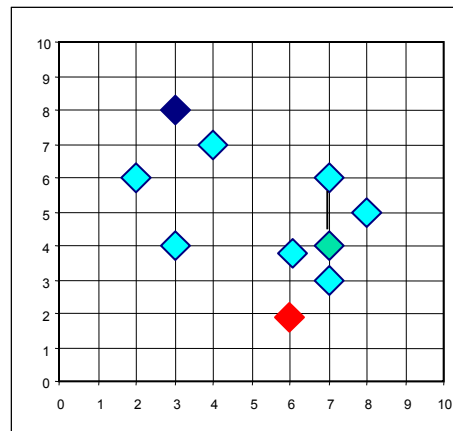


$K=2$

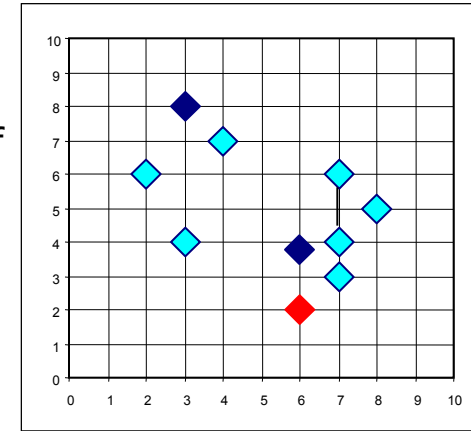
**Do loop  
Until no  
change**

Swapping  $O$   
and  $O_{\text{random}}$   
If quality is  
improved.

Total Cost = 26



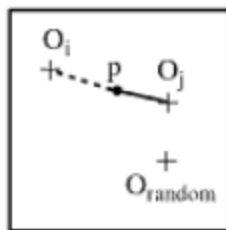
Compute  
total cost of  
swapping



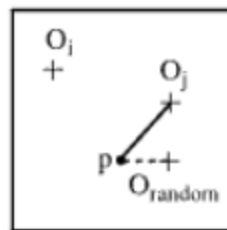
Randomly select a  
nonmedoid object,  $O_{\text{random}}$

# Swapping cost

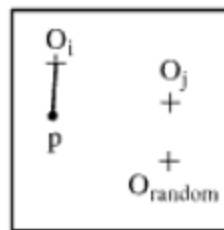
- **Case 1:**  $p$  currently belongs to representative object,  $o_j$ . If  $o_j$  is replaced by  $o_{random}$  as a representative object and  $p$  is closest to one of the other representative objects,  $o_i$ ,  $i \neq j$ , then  $p$  is reassigned to  $o_i$ .
- **Case 2:**  $p$  currently belongs to representative object,  $o_j$ . If  $o_j$  is replaced by  $o_{random}$  as a representative object and  $p$  is closest to  $o_{random}$ , then  $p$  is reassigned to  $o_{random}$ .
- **Case 3:**  $p$  currently belongs to representative object,  $o_i$ ,  $i \neq j$ . If  $o_j$  is replaced by  $o_{random}$  as a representative object and  $p$  is still closest to  $o_i$ , then the assignment does not change.
- **Case 4:**  $p$  currently belongs to representative object,  $o_i$ ,  $i \neq j$ . If  $o_j$  is replaced by  $o_{random}$  as a representative object and  $p$  is closest to  $o_{random}$ , then  $p$  is reassigned to  $o_{random}$ .



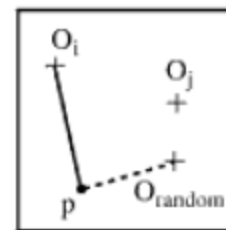
1. Reassigned to  $O_i$



2. Reassigned to  $O_{random}$



3. No change



4. Reassigned to  $O_{random}$

- data object
- + cluster center
- before swapping
- after swapping

# What Is the Problem with PAM?

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- PAM is more robust than k-means in the presence of noise and outliers because a medoid is less influenced by outliers or other extreme values than a mean
- Pam works efficiently for small data sets but does not **scale well** for large data sets.
  - $O(k(n-k)^2)$  for each iteration

where  $n$  is # of data,  $k$  is # of clusters

→ Sampling based method,

CLARA(Clustering LARge Applications)

# CLARA (Clustering Large Applications) (1990)

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- *CLARA* (Kaufmann and Rousseeuw in 1990)
  - Built in statistical analysis packages, such as S+
- It draws *multiple samples* of the data set, applies *PAM* on each sample, and gives the best clustering as the output
- Strength: deals with larger data sets than *PAM*
- Weakness:
  - Efficiency depends on the sample size
  - A good clustering based on samples will not necessarily represent a good clustering of the whole data set if the sample is biased




# *CLARANS* (“Randomized” CLARA) (1994)

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- *CLARANS* (A Clustering Algorithm based on Randomized Search) (Ng and Han’ 94)
- *CLARANS* draws sample of neighbors dynamically
- The clustering process can be presented as searching a graph where every node is a potential solution, that is, a set of  $k$  medoids
- If the local optimum is found, *CLARANS* starts with new randomly selected node in search for a new local optimum
- It is more efficient and scalable than both *PAM* and *CLARA*
- Focusing techniques and spatial access structures may further improve its performance (Ester et al.’ 95)

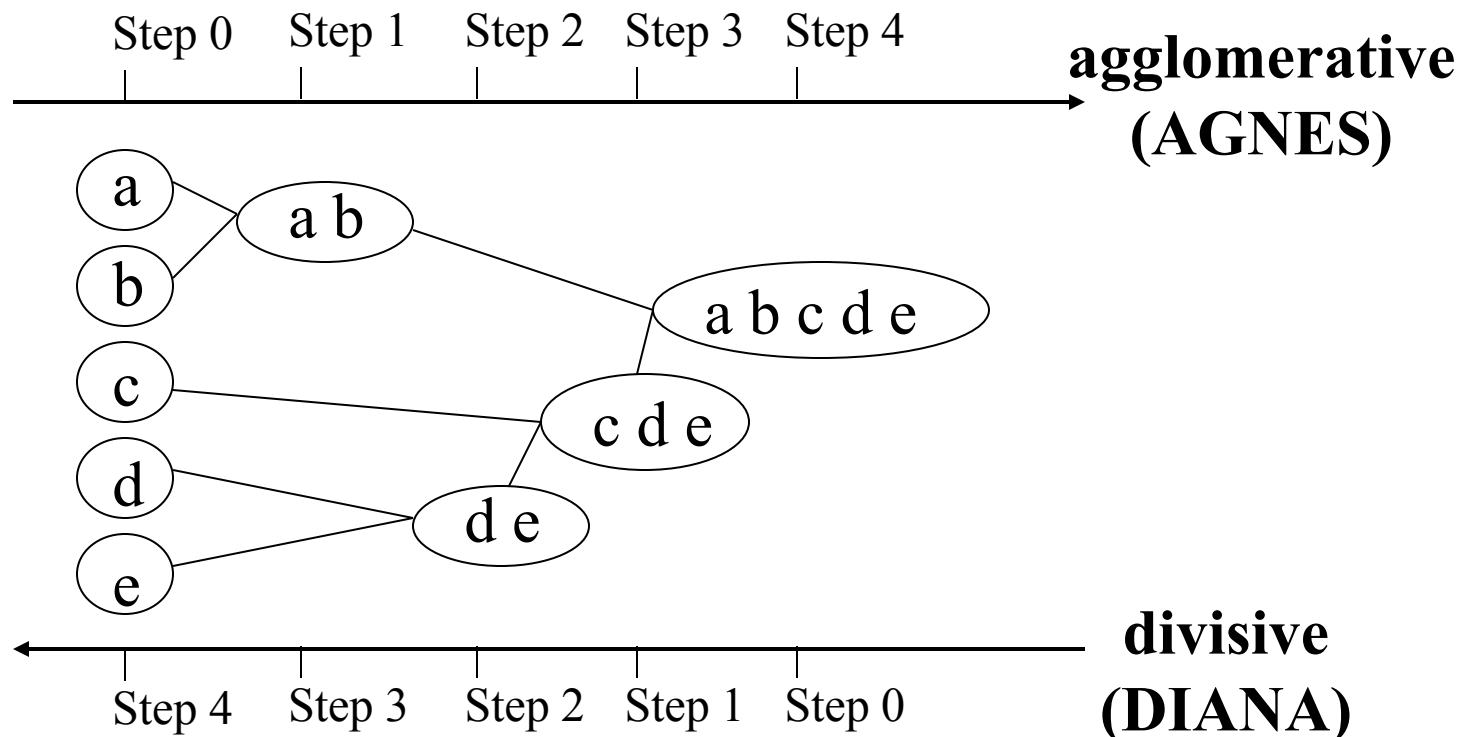
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# Hierarchical Clustering

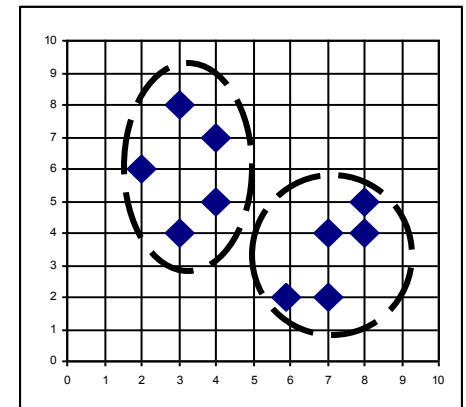
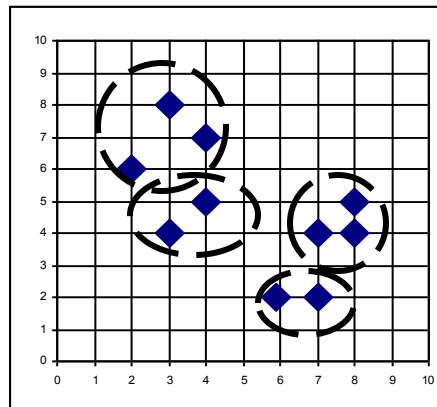
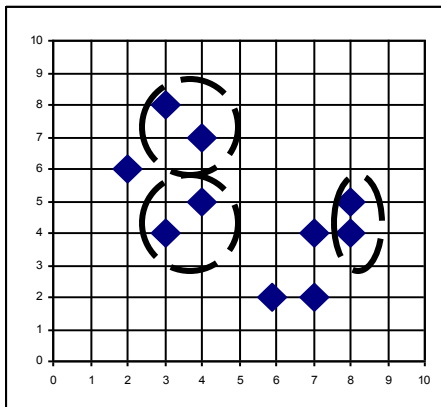
- Use distance matrix. This method does not require the number of clusters  $k$  as an input, but needs a termination condition





# AGNES (Agglomerative Nesting)

- Introduced in Kaufmann and Rousseeuw (1990)
- Implemented in statistical analysis packages, e.g., Splus
- Use the Single-Link method and the dissimilarity matrix.
- Merge nodes that have the least dissimilarity
- Go on in a non-descending fashion
- Eventually all nodes belong to the same cluster

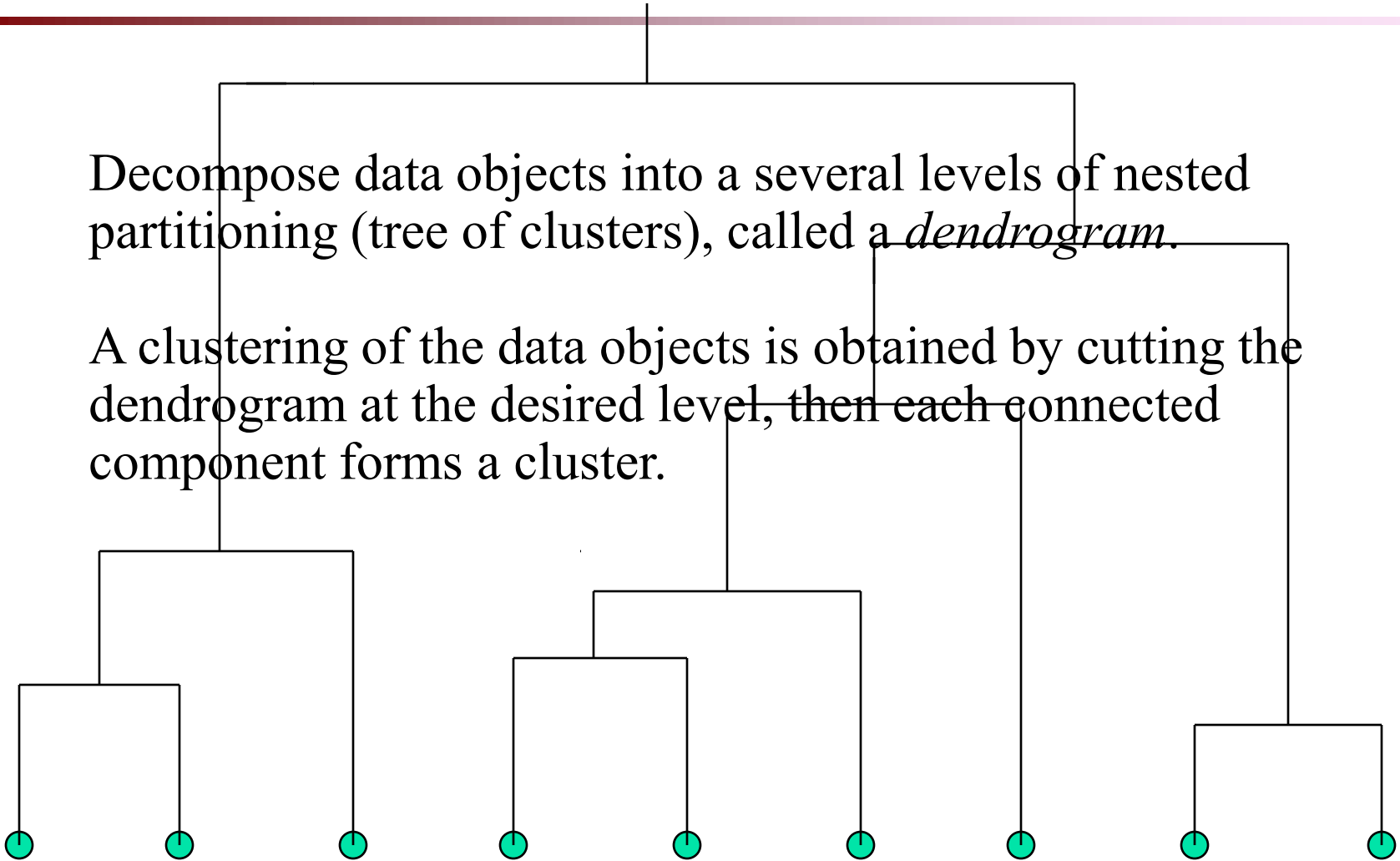


# *Dendrogram: Shows How the Clusters are Merged*

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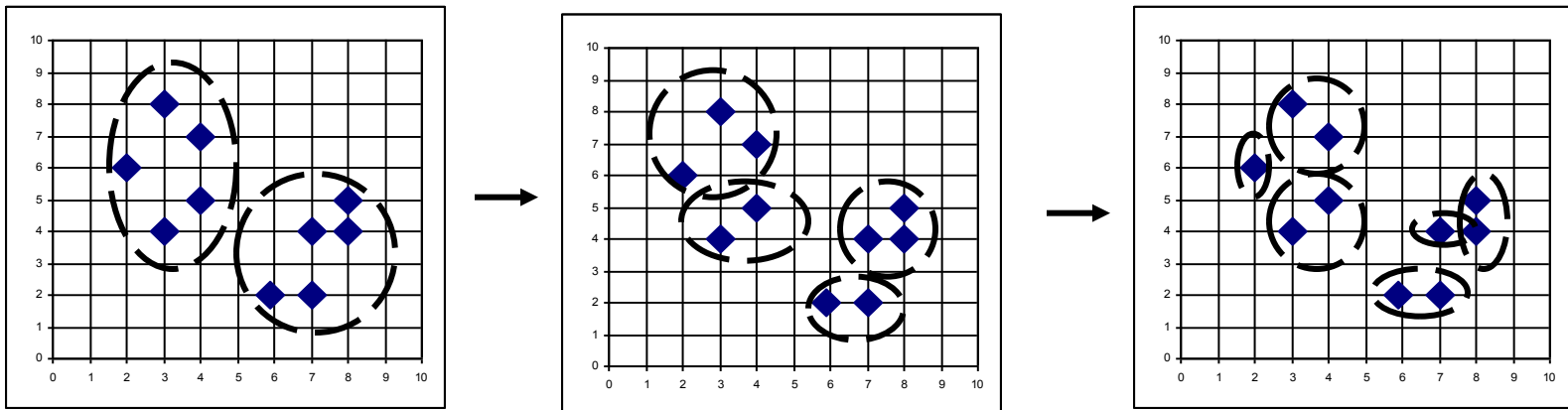
Decompose data objects into a several levels of nested partitioning (tree of clusters), called a *dendrogram*.

A clustering of the data objects is obtained by cutting the dendrogram at the desired level, then each connected component forms a cluster.



# DIANA (Divisive Analysis)

- Introduced in Kaufmann and Rousseeuw (1990)
- Implemented in statistical analysis packages, e.g., Splus
- Inverse order of AGNES
- Eventually each node forms a cluster on its own



# Recent Hierarchical Clustering Methods

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- Major weakness of agglomerative clustering methods
  - do not scale well: time complexity of at least  $O(n^2)$ , where  $n$  is the number of total objects
  - can never undo what was done previously
- Integration of hierarchical with distance-based clustering
  - BIRCH (1996): uses CF-tree and incrementally adjusts the quality of sub-clusters
  - ROCK (1999): clustering categorical data by neighbor and link analysis
  - CHAMELEON (1999): hierarchical clustering using dynamic modeling

# BIRCH (1996)

---

- Birch: Balanced Iterative Reducing and Clustering using Hierarchies (Zhang, Ramakrishnan & Livny, SIGMOD'96)
- Incrementally construct a CF (Clustering Feature) tree, a hierarchical data structure for multiphase clustering
  - Phase 1: scan DB to build an initial in-memory CF tree (a multi-level compression of the data that tries to preserve the inherent clustering structure of the data)
  - Phase 2: use an arbitrary clustering algorithm to cluster the leaf nodes of the CF-tree
- *Scales linearly*: finds a good clustering with a single scan and improves the quality with a few additional scans
- *Weakness*: handles only numeric data, and sensitive to the order of the data record.

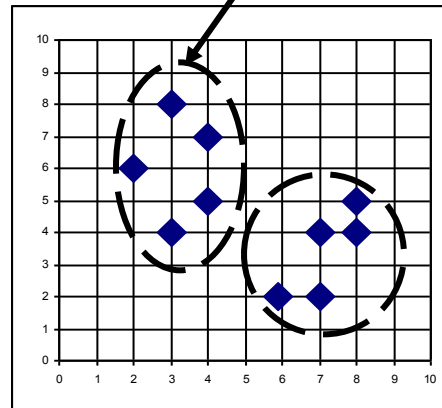
# Clustering Feature Vector in BIRCH

**Clustering Feature:**  $CF = (N, LS, SS)$

**$N$ :** Number of data points

**$LS$  (linear sum):**  $\sum_{i=1}^N X_i$

**$SS$  (square sum):**  $\sum_{i=1}^N X_i^2$



$CF = (5, (16,30), (54,190))$

(3,4)

(2,6)

(4,5)

(4,7)

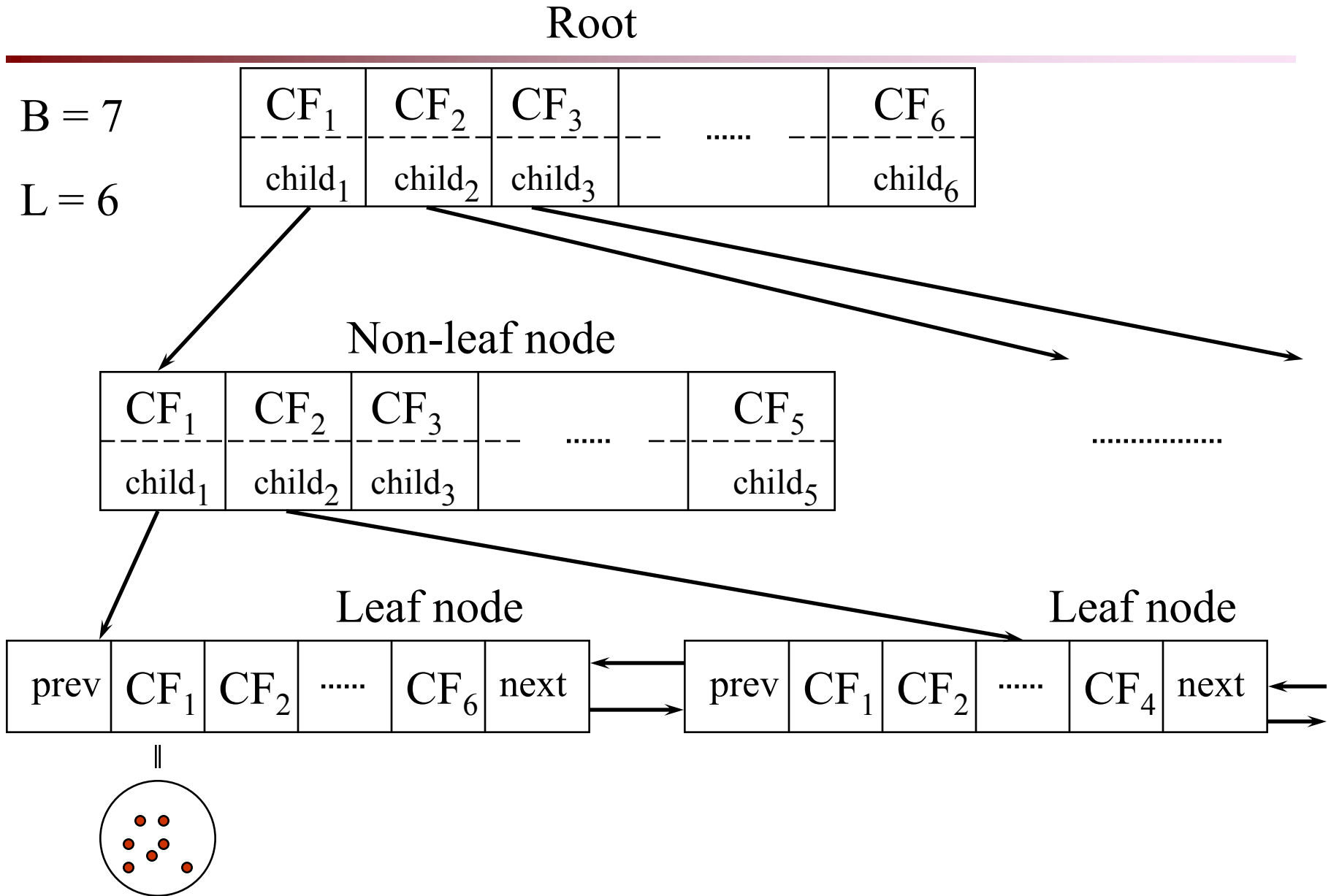
(3,8)

# CF-Tree in BIRCH

---

- Clustering feature:
  - summary of the statistics for a given subcluster: the 0-th, 1st and 2nd moments of the subcluster from the statistical point of view.
  - registers crucial measurements for computing cluster and utilizes storage efficiently
- A CF tree is a height-balanced tree that stores the clustering features for a hierarchical clustering
  - A nonleaf node in a tree has descendants or “children”
  - The nonleaf nodes store sums of the CFs of their children
- A CF tree has two parameters
  - Branching factor: specify the maximum number of children.
  - threshold: max diameter of sub-clusters stored at the leaf nodes
- If memory not enough, rebuild the tree from leaf node by adjusting threshold

# The CF Tree Structure







# Clustering Categorical Data: The ROCK Algorithm

---

- ROCK: RObust Clustering using linkS
  - S. Guha, R. Rastogi & K. Shim, ICDE' 99
- Major ideas
  - Use links to measure similarity/proximity
  - Not distance-based
  - Computational complexity:  $O(n^2 + nm_m m_a + n^2 \log n)$
- Algorithm: sampling-based clustering
  - Draw random sample
  - Cluster with links
  - Label data in disk
- Experiments
  - Congressional voting, mushroom data



# Similarity Measure in ROCK

- Traditional measures for categorical data may not work well, e.g., Jaccard coefficient
- Example: Two groups (clusters) of transactions
  - $C_1$ .  $\langle a, b, c, d, e \rangle$ :  $\{a, b, c\}$ ,  $\{a, b, d\}$ ,  $\{a, b, e\}$ ,  $\{a, c, d\}$ ,  $\{a, c, e\}$ ,  $\{a, d, e\}$ ,  $\{b, c, d\}$ ,  $\{b, c, e\}$ ,  $\{b, d, e\}$ ,  $\{c, d, e\}$
  - $C_2$ .  $\langle a, b, f, g \rangle$ :  $\{a, b, f\}$ ,  $\{a, b, g\}$ ,  $\{a, f, g\}$ ,  $\{b, f, g\}$
- Jaccard co-efficient may lead to wrong clustering result
  - $C_1$ : 0.2 ( $\{a, b, c\}$ ,  $\{b, d, e\}$ ) to 0.5 ( $\{a, b, c\}$ ,  $\{a, b, d\}$ )
  - $C_1$  &  $C_2$ : could be as high as 0.5 ( $\{a, b, c\}$ ,  $\{a, b, f\}$ )
- Jaccard co-efficient-based similarity function:

$$Sim(T_1, T_2) = \frac{|T_1 \cap T_2|}{|T_1 \cup T_2|}$$

- Ex. Let  $T_1 = \{a, b, c\}$ ,  $T_2 = \{c, d, e\}$

$$Sim(T_1, T_2) = \frac{|\{c\}|}{|\{a, b, c, d, e\}|} = \frac{1}{5} = 0.2$$

# Link Measure in ROCK



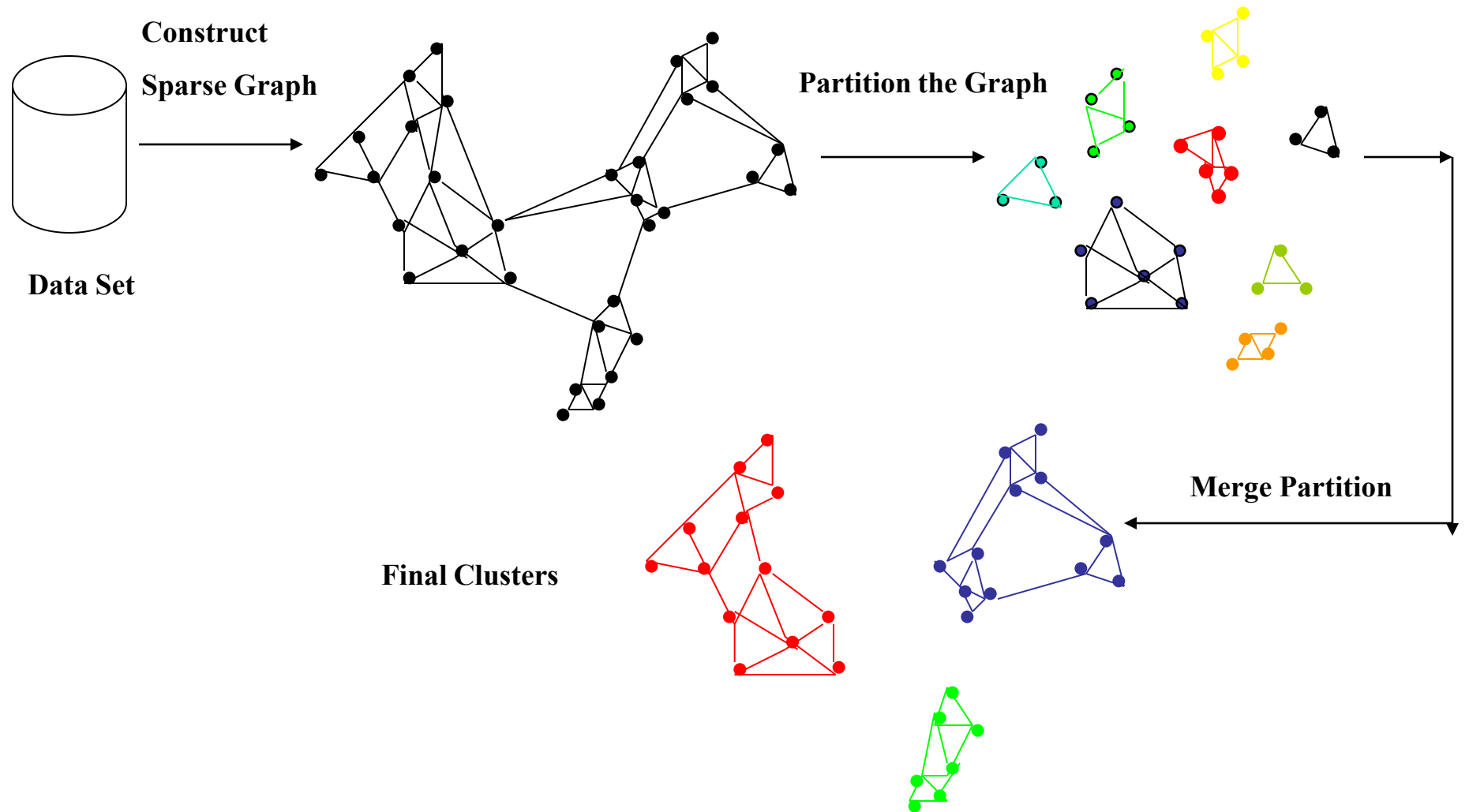
- Links: # of common neighbors
  - $C_1 \langle a, b, c, d, e \rangle$ :  $\{\underline{a, b, c}\}$ ,  $\{a, b, d\}$ ,  $\{a, b, e\}$ ,  $\{a, c, d\}$ ,  $\{a, c, e\}$ ,  $\{a, d, e\}$ ,  $\{b, c, d\}$ ,  $\{b, c, e\}$ ,  $\{b, d, e\}$ ,  $\{\underline{c, d, e}\}$
  - $C_2 \langle a, b, f, g \rangle$ :  $\{\underline{a, b, f}\}$ ,  $\{a, b, g\}$ ,  $\{a, f, g\}$ ,  $\{b, f, g\}$
- Let  $T_1 = \{a, b, c\}$ ,  $T_2 = \{c, d, e\}$ ,  $T_3 = \{a, b, f\}$ 
  - $\text{link}(T_1, T_2) = 4$ , *since they have 4 common neighbors*
    - $\{a, c, d\}$ ,  $\{a, c, e\}$ ,  $\{b, c, d\}$ ,  $\{b, c, e\}$
  - $\text{link}(T_1, T_3) = 3$ , *since they have 3 common neighbors*
    - $\{a, b, d\}$ ,  $\{a, b, e\}$ ,  $\{a, b, g\}$
- Thus link is a better measure than Jaccard coefficient

# CHAMELEON: Hierarchical Clustering Using Dynamic Modeling (1999)

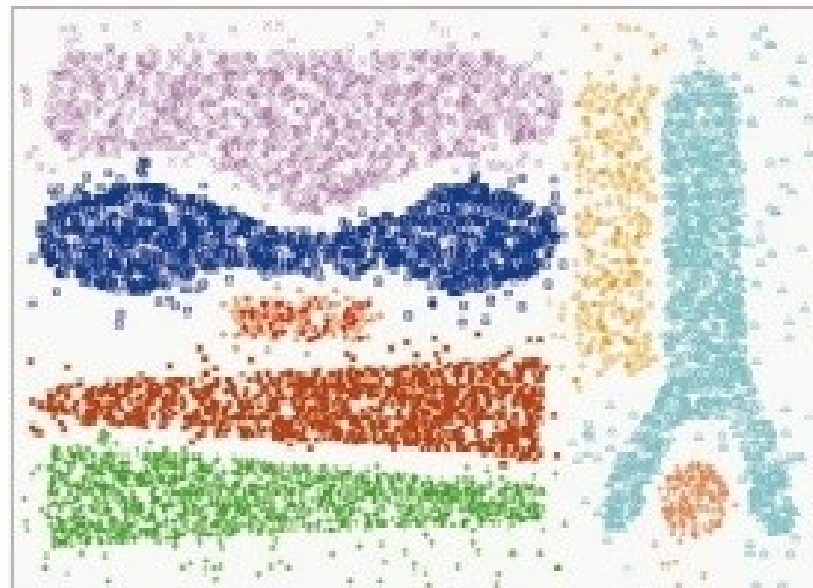
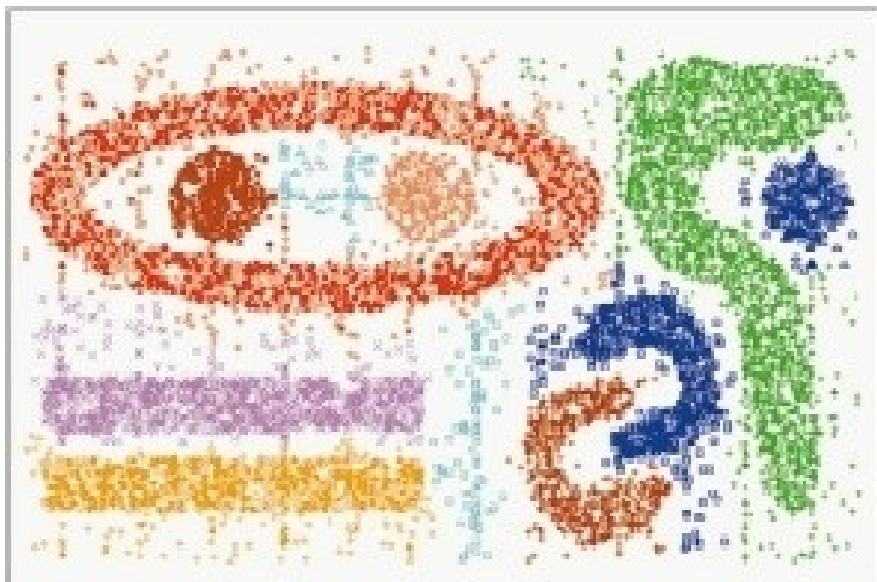
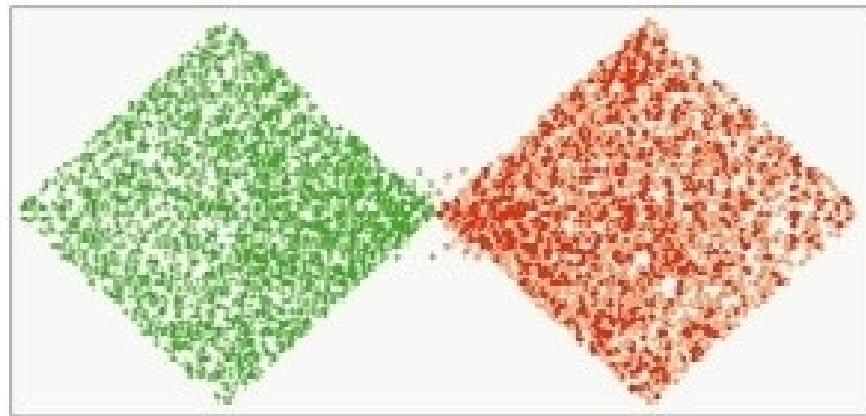
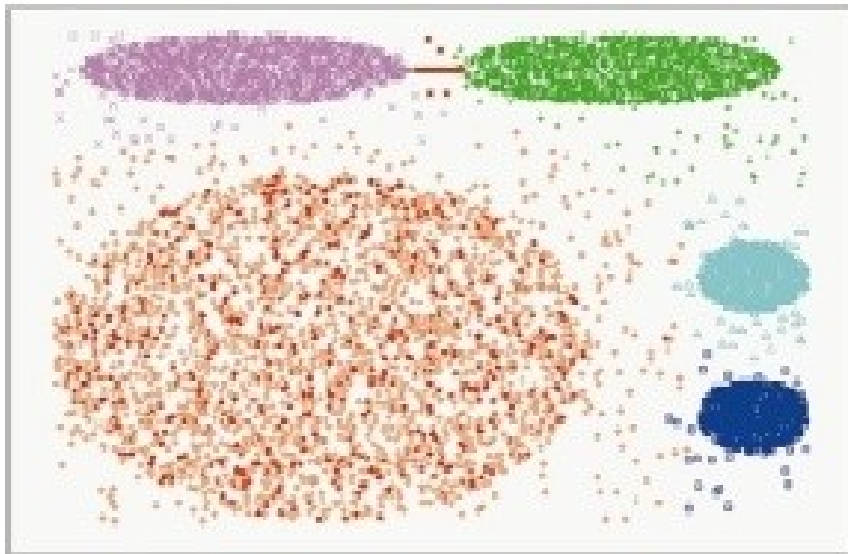


- CHAMELEON: by G. Karypis, E.H. Han, and V. Kumar'99
- Measures the similarity based on a dynamic model
  - Two clusters are merged only if the *interconnectivity* and *closeness (proximity)* between two clusters are high *relative to* the internal interconnectivity of the clusters and closeness of items within the clusters
  - **Cure** ignores information about **interconnectivity** of the objects, **Rock** ignores information about the **closeness** of two clusters
- A two-phase algorithm
  1. Use a graph partitioning algorithm: cluster objects into a large number of relatively small sub-clusters
  2. Use an agglomerative hierarchical clustering algorithm: find the genuine clusters by repeatedly combining these sub-clusters

# Overall Framework of CHAMELEON

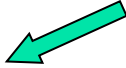


# CHAMELEON (Clustering Complex Objects)



# Chapter 7. Cluster Analysis

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1. What is Cluster Analysis?
2. Types of Data in Cluster Analysis
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4. Partitioning Methods
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7. Grid-Based Methods
8. Model-Based Methods
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12. Summary

# Density-Based Clustering Methods

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- Clustering based on density (local cluster criterion), such as density-connected points
- Major features:
  - Discover clusters of arbitrary shape
  - Handle noise
  - One scan
  - Need density parameters as termination condition
- Several interesting studies:
  - DBSCAN: Ester, et al. (KDD'96)
  - OPTICS: Ankerst, et al (SIGMOD'99).
  - DENCLUE: Hinneburg & D. Keim (KDD'98)
  - CLIQUE: Agrawal, et al. (SIGMOD'98) (more grid-based)

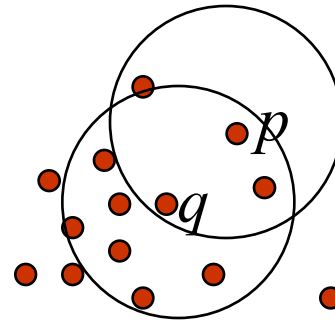


# Density-Based Clustering: Basic Concepts

- Two parameters:
  - *Eps*: Maximum radius of the neighbourhood
  - *MinPts*: Minimum number of points in an Eps-neighbourhood of that point
- $N_{Eps}(p)$ :  $\{q \text{ belongs to } D \mid \text{dist}(p,q) \leq Eps\}$
- **Directly density-reachable**: A point  $p$  is directly density-reachable from a point  $q$  w.r.t.  $Eps$ ,  $MinPts$  if

- $p$  belongs to  $N_{Eps}(q)$
- core point condition:

$$|N_{Eps}(q)| \geq MinPts$$



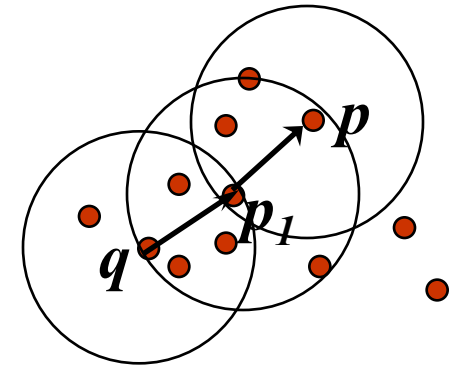
$MinPts = 5$

$Eps = 1 \text{ cm}$

# Density-Reachable and Density-Connected

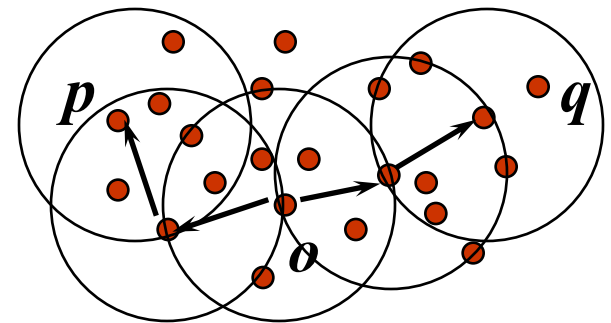
- Density-reachable:

- A point  $p$  is **density-reachable** from a point  $q$  w.r.t.  $Eps$ ,  $MinPts$  if there is a chain of points  $p_1, \dots, p_n$ ,  $p_1 = q$ ,  $p_n = p$  such that  $p_{i+1}$  is directly density-reachable from  $p_i$



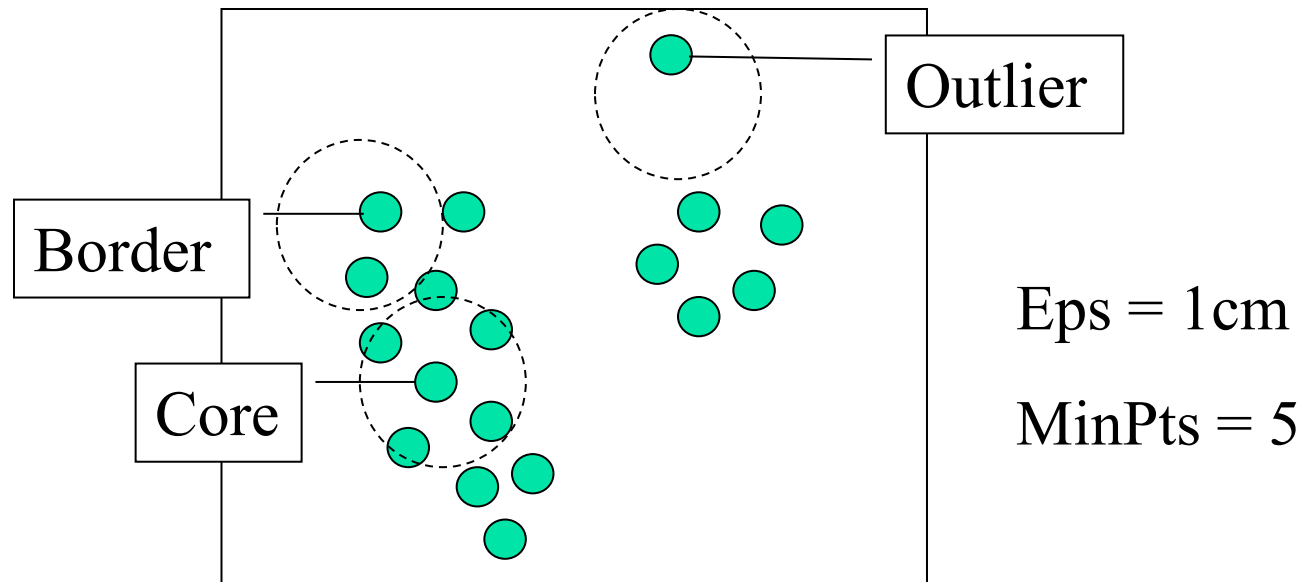
- Density-connected

- A point  $p$  is **density-connected** to a point  $q$  w.r.t.  $Eps$ ,  $MinPts$  if there is a point  $o$  such that both,  $p$  and  $q$  are density-reachable from  $o$  w.r.t.  $Eps$  and  $MinPts$



# DBSCAN: Density Based Spatial Clustering of Applications with Noise

- Relies on a *density-based* notion of cluster: A *cluster* is defined as a maximal set of density-connected points
- Discovers clusters of arbitrary shape in spatial databases with noise



# DBSCAN: The Algorithm

---

- Arbitrary select a point  $p$
- Retrieve all points density-reachable from  $p$  w.r.t.  $Eps$  and  $MinPts$ .
- If  $p$  is a core point, a cluster is formed.
- If  $p$  is a border point, no points are density-reachable from  $p$  and DBSCAN visits the next point of the database.
- Continue the process until all of the points have been processed.

# DBSCAN: Sensitive to Parameters

Figure 8. DBScan results for DS1 with MinPts at 4 and Eps at (a) 0.5 and (b) 0.4.

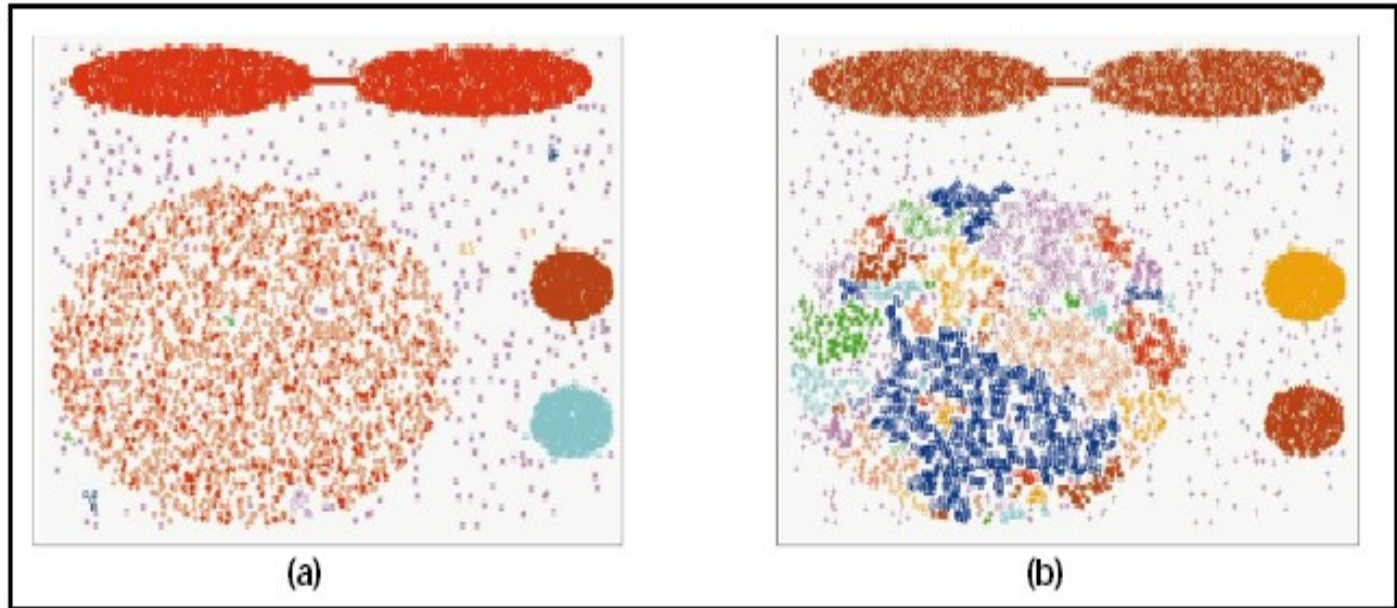
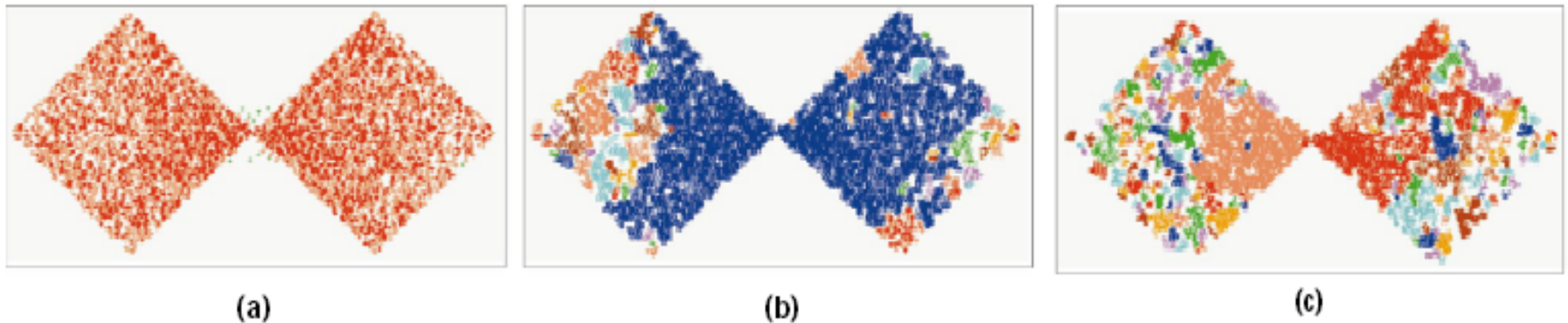


Figure 9. DBScan results for DS2 with MinPts at 4 and Eps at (a) 5.0, (b) 3.5, and (c) 3.0.



# OPTICS: A Cluster-Ordering Method (1999)

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- OPTICS: Ordering Points To Identify the Clustering Structure
  - Ankerst, Breunig, Kriegel, and Sander (SIGMOD'99)
  - Produces a special order of the database wrt its density-based clustering structure
  - This cluster-ordering contains info equiv to the density-based clusterings corresponding to a broad range of parameter settings
  - Good for both automatic and interactive cluster analysis, including finding intrinsic clustering structure
  - Can be represented graphically or using visualization techniques

# OPTICS: Some Extension from DBSCAN



- Index-based:
  - $k$  = number of dimensions
  - $N = 20$
  - $p = 75\%$
  - $M = N(1-p) = 5$

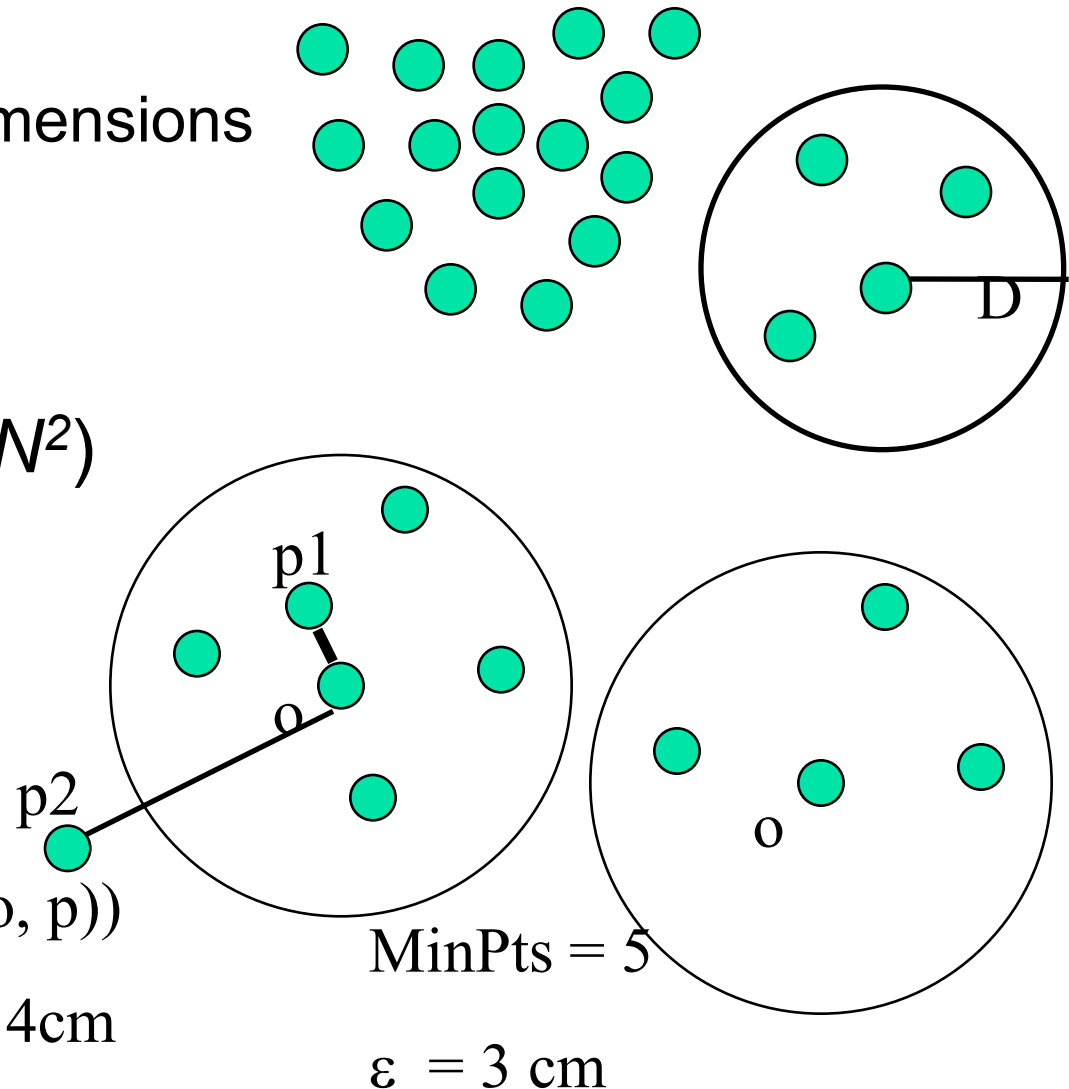
■ Complexity:  $O(kN^2)$

- Core Distance

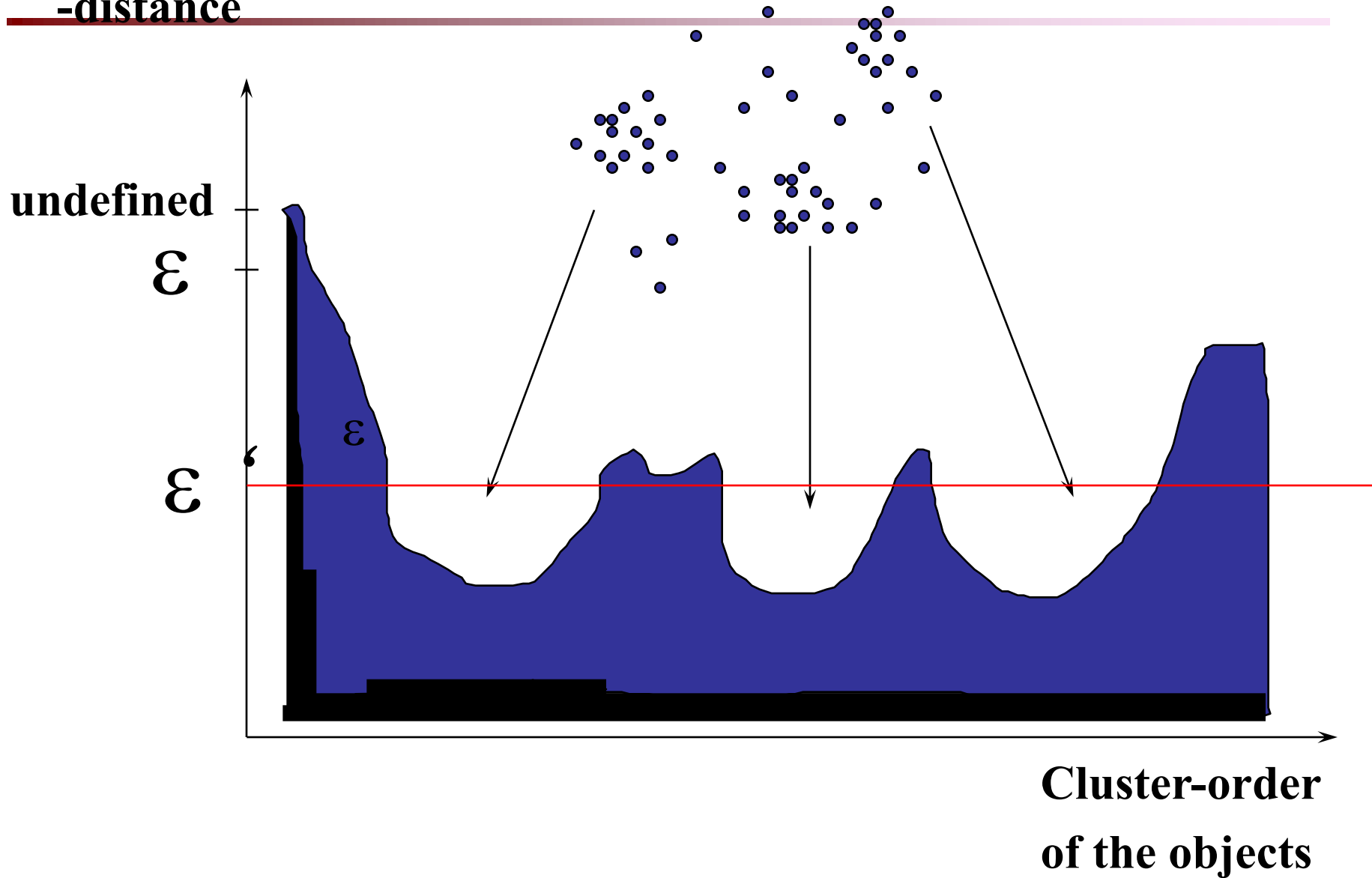
- Reachability Distance

$\text{Max}(\text{core-distance}(o), d(o, p))$

$r(p1, o) = 2.8\text{cm}$ .  $r(p2, o) = 4\text{cm}$

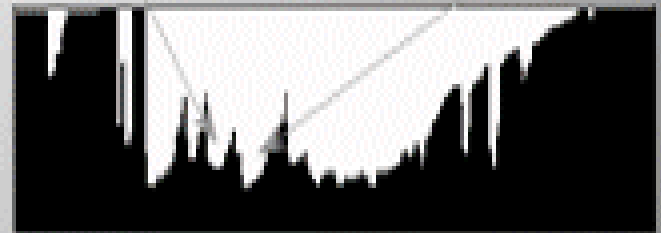
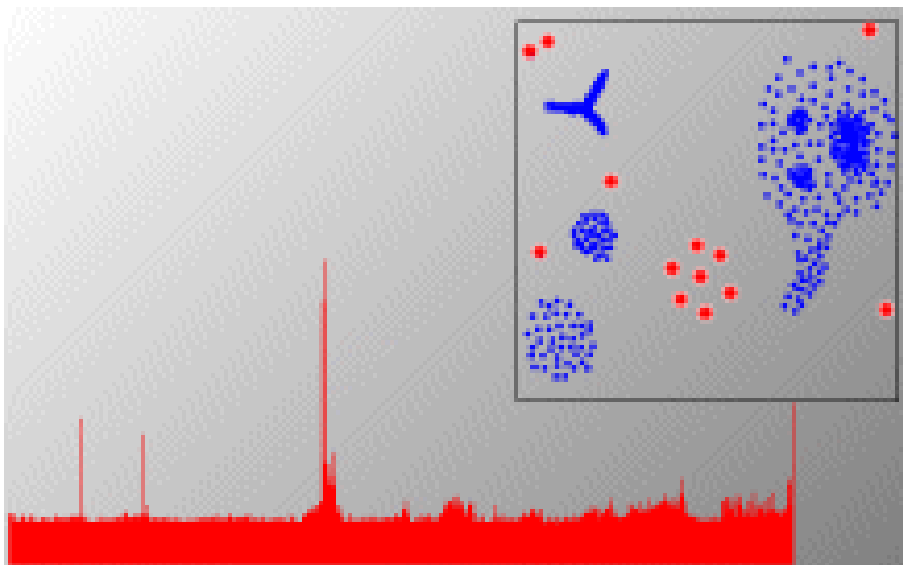
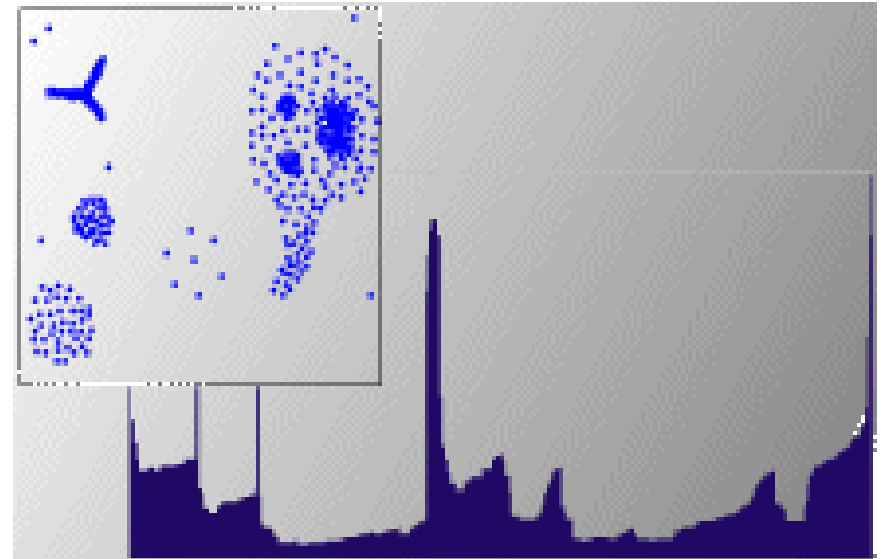
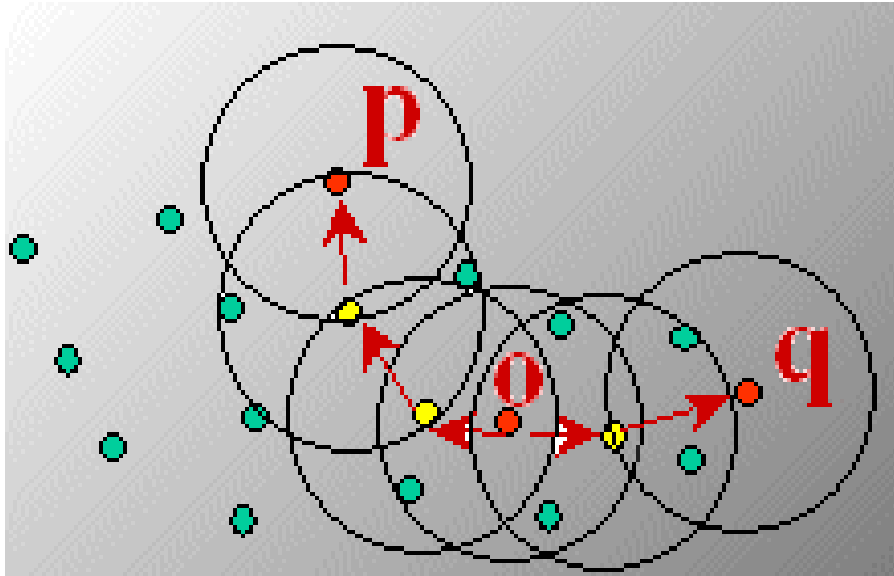


# Reachability -distance





# Density-Based Clustering: OPTICS & Its Applications



# DENCLUE: Using Statistical Density Functions



- DENSity-based CLUstEring by Hinneburg & Keim (KDD'98)

- Using statistical density functions:  $f_{Gaussian}(x, y) = e^{-\frac{d(x, y)^2}{2\sigma^2}}$

$$f_{Gaussian}^D(x) = \sum_{i=1}^N e^{-\frac{d(x, x_i)^2}{2\sigma^2}}$$

- Major features

$$\nabla f_{Gaussian}^D(x, x_i) = \sum_{i=1}^N (x_i - x) \cdot e^{-\frac{d(x, x_i)^2}{2\sigma^2}}$$

- Solid mathematical foundation
- Good for data sets with large amounts of noise
- Allows a compact mathematical description of arbitrarily shaped clusters in high-dimensional data sets
- Significant faster than existing algorithm (e.g., DBSCAN)
- But needs a large number of parameters



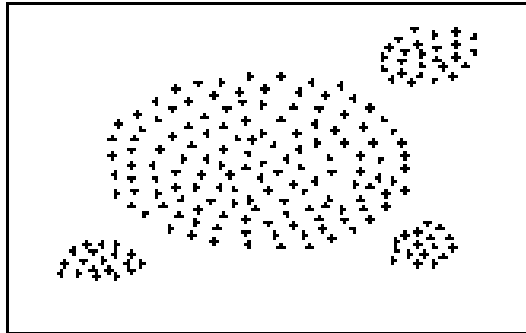
# Denclue: Technical Essence

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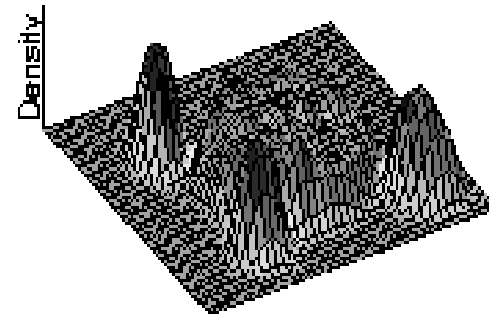
- Uses grid cells but only keeps information about grid cells that do actually contain data points and manages these cells in a tree-based access structure
- Influence function: describes the impact of a data point within its neighborhood
- Overall density of the data space can be calculated as the sum of the influence function of all data points
- Clusters can be determined mathematically by identifying density attractors
- Density attractors are local maximal of the overall density function



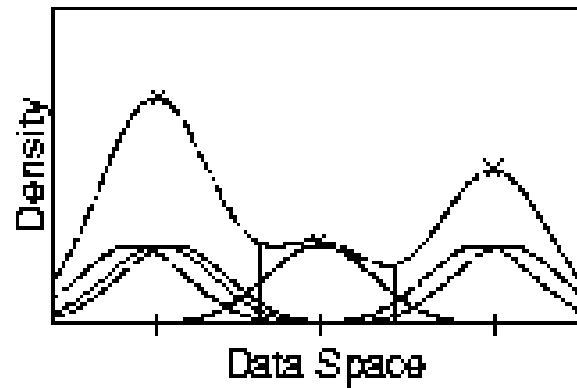
# Density Attractor



(a) Data Set



(c) Gaussian





# Center-Defined and Arbitrary

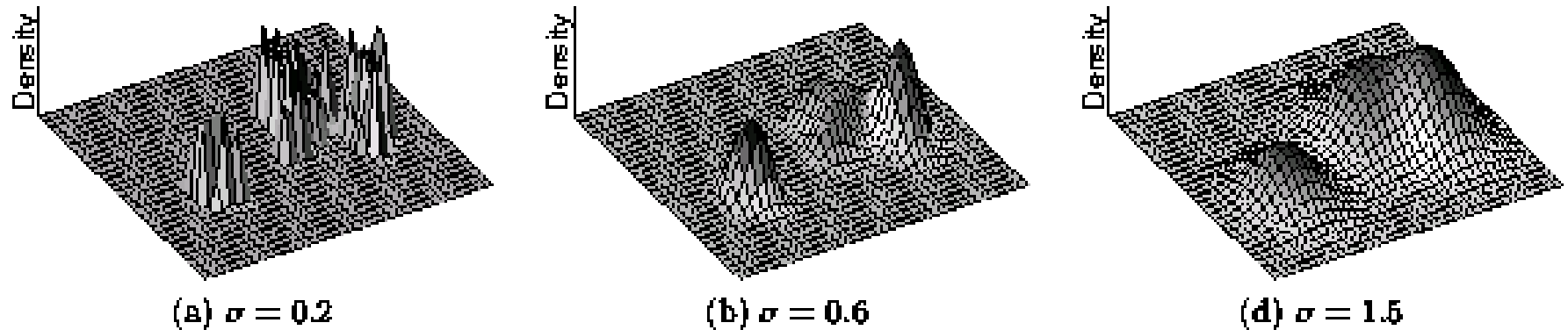


Figure 3: Example of Center-Defined Clusters for different  $\sigma$

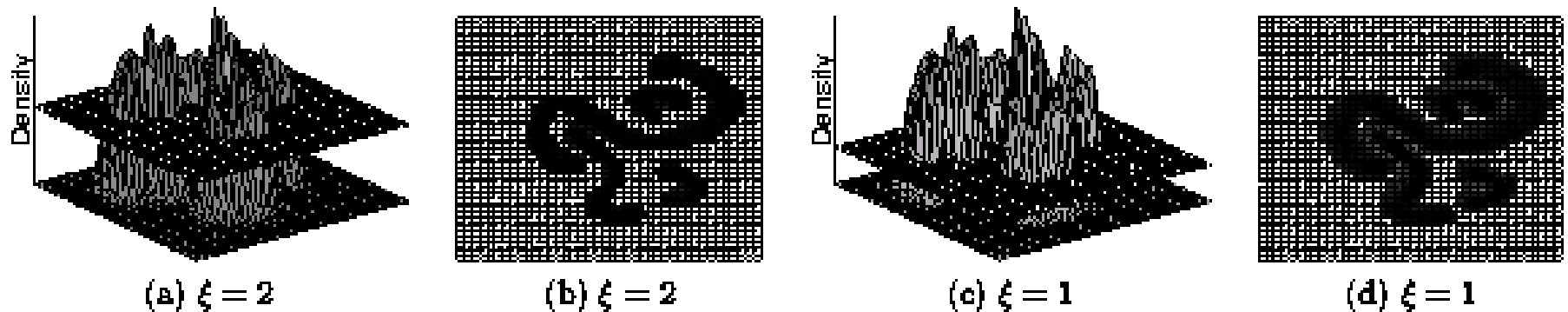
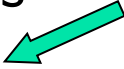


Figure 4: Example of Arbitrary-Shape Clusters for different  $\xi$

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# Grid-Based Clustering Method

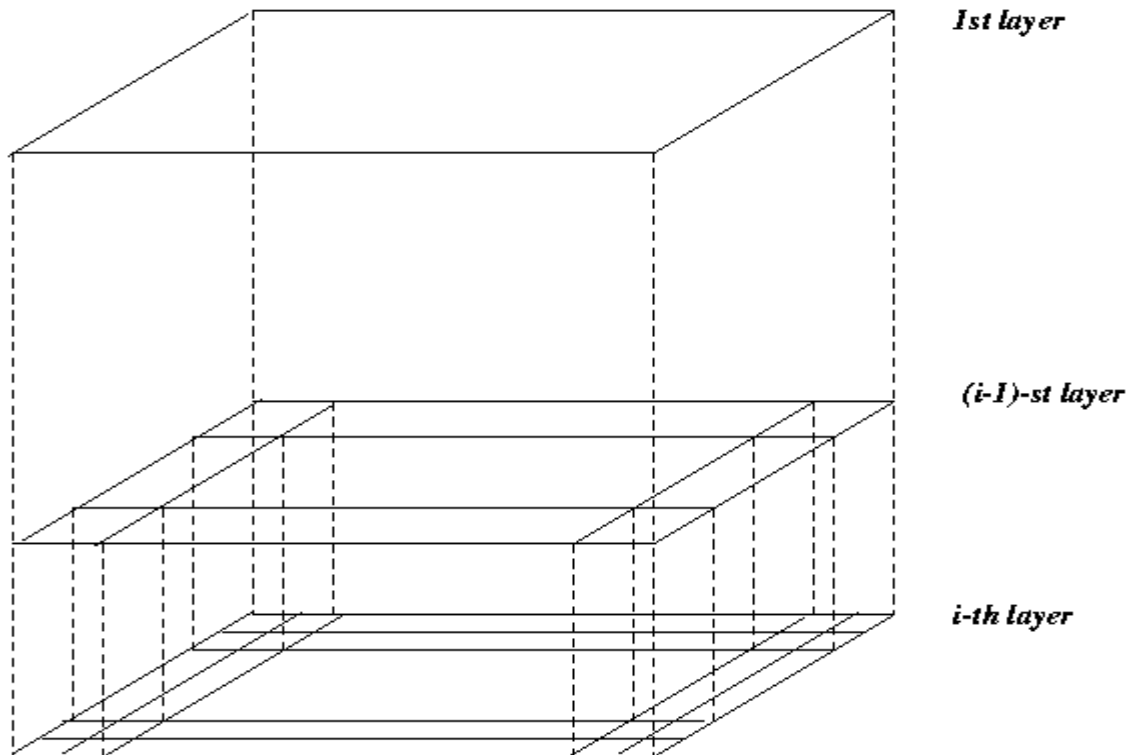
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- Using multi-resolution grid data structure
- Several interesting methods
  - **STING** (a S**T**atistical **I**Nformation Grid approach) by Wang, Yang and Muntz (1997)
  - **WaveCluster** by Sheikholeslami, Chatterjee, and Zhang (VLDB' 98)
    - A multi-resolution clustering approach using wavelet method
  - **CLIQUE**: Agrawal, et al. (SIGMOD'98)
    - On high-dimensional data (thus put in the section of clustering high-dimensional data)



# STING: A Statistical Information Grid Approach

- Wang, Yang and Muntz (VLDB' 97)
- The spatial area is divided into rectangular cells
- There are several levels of cells corresponding to different levels of resolution







# The STING Clustering Method

---

- Each cell at a high level is partitioned into a number of smaller cells in the next lower level
- Statistical info of each cell is calculated and stored beforehand and is used to answer queries
- Parameters of higher level cells can be easily calculated from parameters of lower level cell
  - *count, mean, s, min, max*
  - type of distribution—normal, *uniform*, etc.
- Use a top-down approach to answer spatial data queries
- Start from a pre-selected layer—typically with a small number of cells
- For each cell in the current level compute the confidence interval

# Comments on STING



- Remove the irrelevant cells from further consideration
- When finish examining the current layer, proceed to the next lower level
- Repeat this process until the bottom layer is reached
- Advantages:
  - Query-independent, easy to parallelize, incremental update
  - $O(K)$ , where  $K$  is the number of grid cells at the lowest level
- Disadvantages:
  - All the cluster boundaries are either horizontal or vertical, and no diagonal boundary is detected



# WaveCluster: Clustering by Wavelet Analysis (1998)

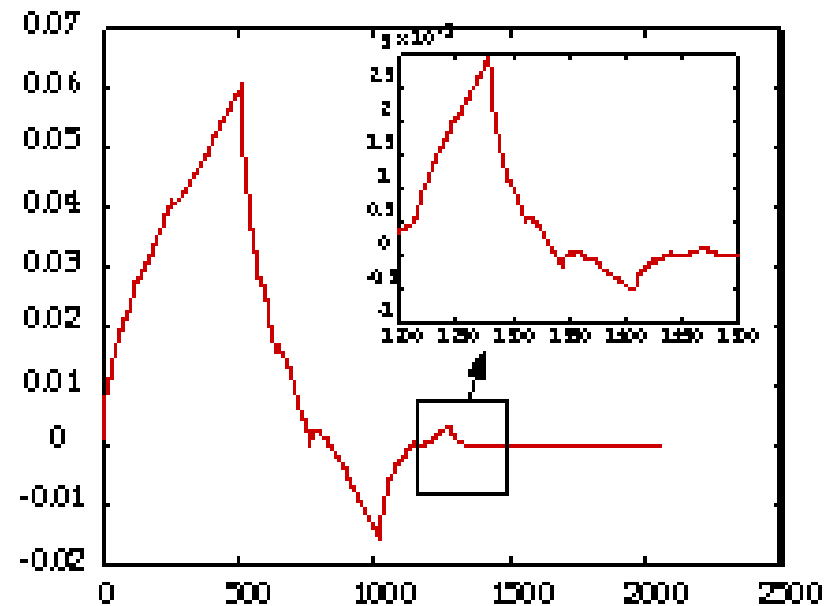
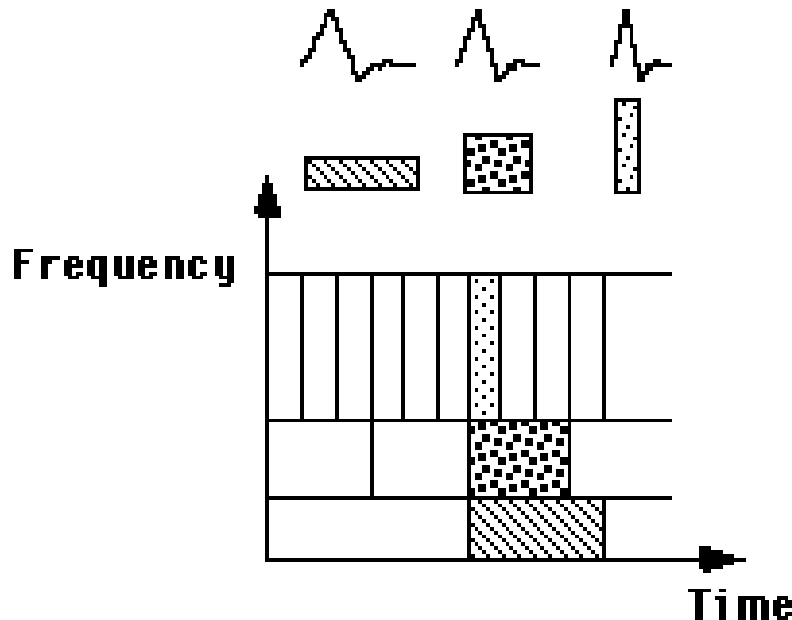
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- Sheikholeslami, Chatterjee, and Zhang (VLDB' 98)
- A multi-resolution clustering approach which applies wavelet transform to the feature space
- How to apply wavelet transform to find clusters
  - Summarizes the data by imposing a multidimensional grid structure onto data space
  - These multidimensional spatial data objects are represented in a n-dimensional feature space
  - Apply wavelet transform on feature space to find the dense regions in the feature space
  - Apply wavelet transform multiple times which result in clusters at different scales from fine to coarse



# Wavelet Transform

- Wavelet transform: A signal processing technique that decomposes a signal into different frequency sub-band (can be applied to n-dimensional signals)
- Data are transformed to preserve relative distance between objects at different levels of resolution
- Allows natural clusters to become more distinguishable





# The WaveCluster Algorithm

---

- Input parameters
  - # of grid cells for each dimension
  - the wavelet, and the # of applications of wavelet transform
- Why is wavelet transformation useful for clustering?
  - Use hat-shape filters to emphasize region where points cluster, but simultaneously suppress weaker information in their boundary
  - Effective removal of outliers, multi-resolution, cost effective
- Major features:
  - Complexity  $O(N)$
  - Detect arbitrary shaped clusters at different scales
  - Not sensitive to noise, not sensitive to input order
  - Only applicable to low dimensional data
- Both grid-based and density-based

# Quantization & Transformation

- First, quantize data into m-D gric structure, then wavelet transform
  - a) scale 1: high resolution
  - b) scale 2: medium resolution
  - c) scale 3: low resolution

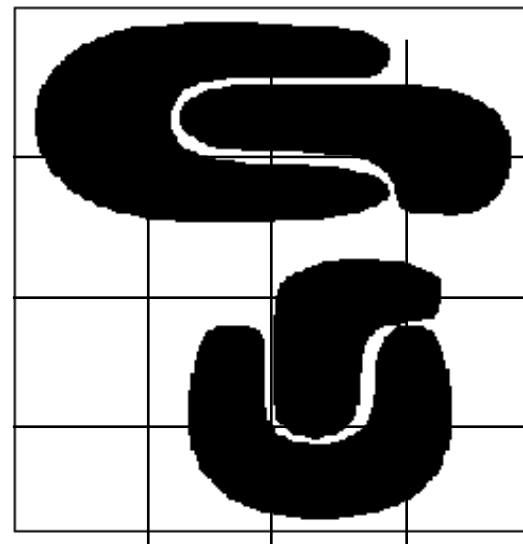
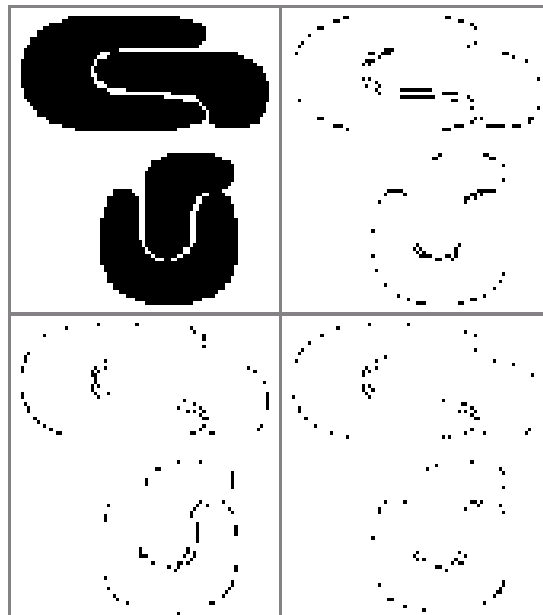


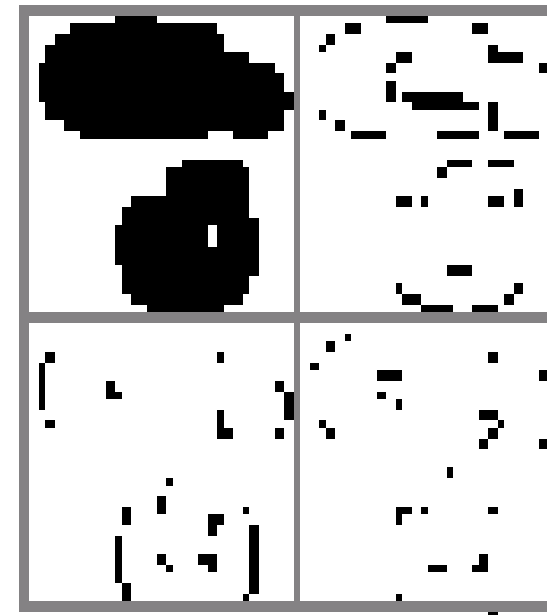
Figure 1: A sample 2-dimensional feature space.



a)




b)



c)

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---

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# Model-Based Clustering

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- What is model-based clustering?
  - Attempt to optimize the fit between the given data and some mathematical model
  - Based on the assumption: Data are generated by a mixture of underlying probability distribution
- Typical methods
  - Statistical approach
    - EM (Expectation maximization), AutoClass
  - Machine learning approach
    - COBWEB, CLASSIT
  - Neural network approach
    - SOM (Self-Organizing Feature Map)



# EM — Expectation Maximization

---

- EM — A popular iterative refinement algorithm
- An extension to k-means
  - Assign each object to a cluster according to a weight (prob. distribution)
  - New means are computed based on weighted measures
- General idea
  - Starts with an initial estimate of the parameter vector
  - Iteratively rescores the patterns against the mixture density produced by the parameter vector
  - The rescored patterns are used to update the parameter updates
  - Patterns belonging to the same cluster, if they are placed by their scores in a particular component
- Algorithm converges fast but may not be in global optima

# The EM (Expectation Maximization) Algorithm

---

- Initially, randomly assign  $k$  cluster centers
- Iteratively refine the clusters based on two steps
  - **Expectation step:** assign each data point  $X_i$  to cluster  $C_k$  with the following probability

$$P(X_i \in C_k) = p(C_k | X_i) = \frac{p(C_k)p(X_i | C_k)}{p(X_i)},$$

- **Maximization step:**
  - Estimation of model parameters

$$m_k = \frac{1}{N} \sum_{i=1}^N \frac{X_i P(X_i \in C_k)}{\sum_j P(X_i \in C_j)},$$

# Conceptual Clustering

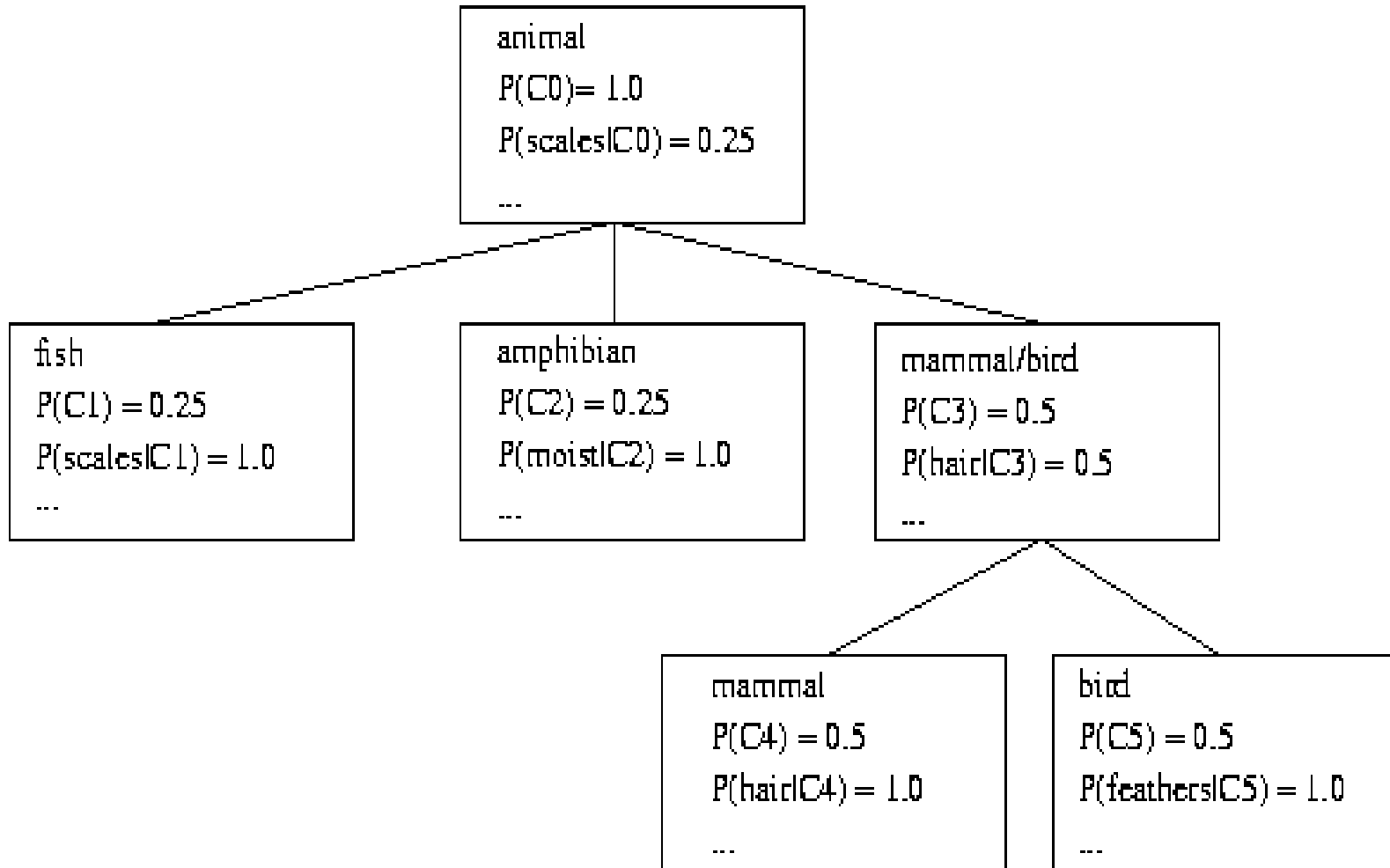


- Conceptual clustering
  - A form of clustering in machine learning
  - Produces a classification scheme for a set of unlabeled objects
  - Finds characteristic description for each concept (class)
- COBWEB (Fisher' 87)
  - A popular a simple method of incremental conceptual learning
  - Creates a hierarchical clustering in the form of a **classification tree**
  - Each node refers to a concept and contains a probabilistic description of that concept



# COBWEB Clustering Method

## A classification tree



# More on Conceptual Clustering



- Limitations of COBWEB
  - The assumption that the attributes are independent of each other is often too strong because correlation may exist
  - Not suitable for clustering large database data – skewed tree and expensive probability distributions
- CLASSIT
  - an extension of COBWEB for incremental clustering of continuous data
  - suffers similar problems as COBWEB
- AutoClass (Cheeseman and Stutz, 1996)
  - Uses Bayesian statistical analysis to estimate the number of clusters
  - Popular in industry

# Neural Network Approach



- Neural network approaches
  - Represent each cluster as an exemplar, acting as a “prototype” of the cluster
  - New objects are distributed to the cluster whose exemplar is the most similar according to some distance measure
- Typical methods
  - SOM (Soft-Organizing feature Map)
  - Competitive learning
    - Involves a hierarchical architecture of several units (neurons)
    - Neurons compete in a “winner-takes-all” fashion for the object currently being presented

# Self-Organizing Feature Map (SOM)

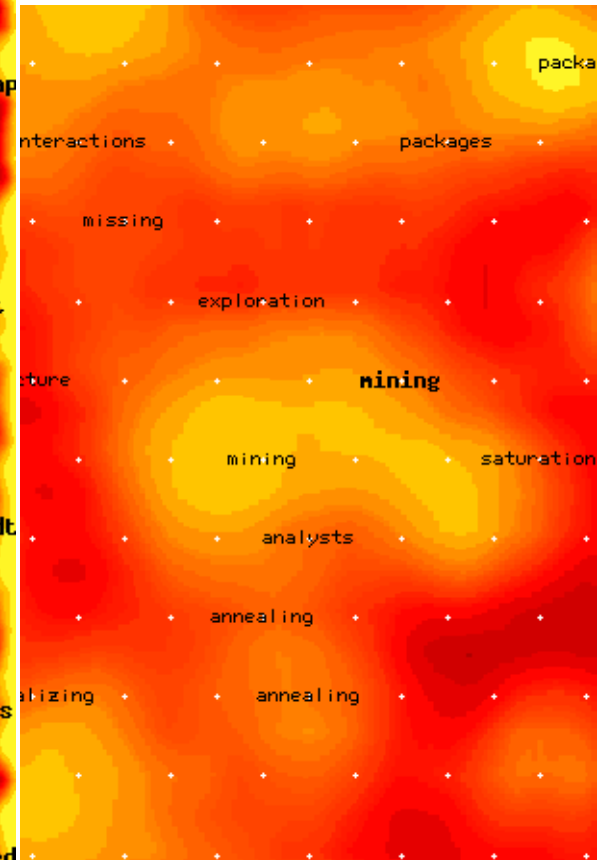
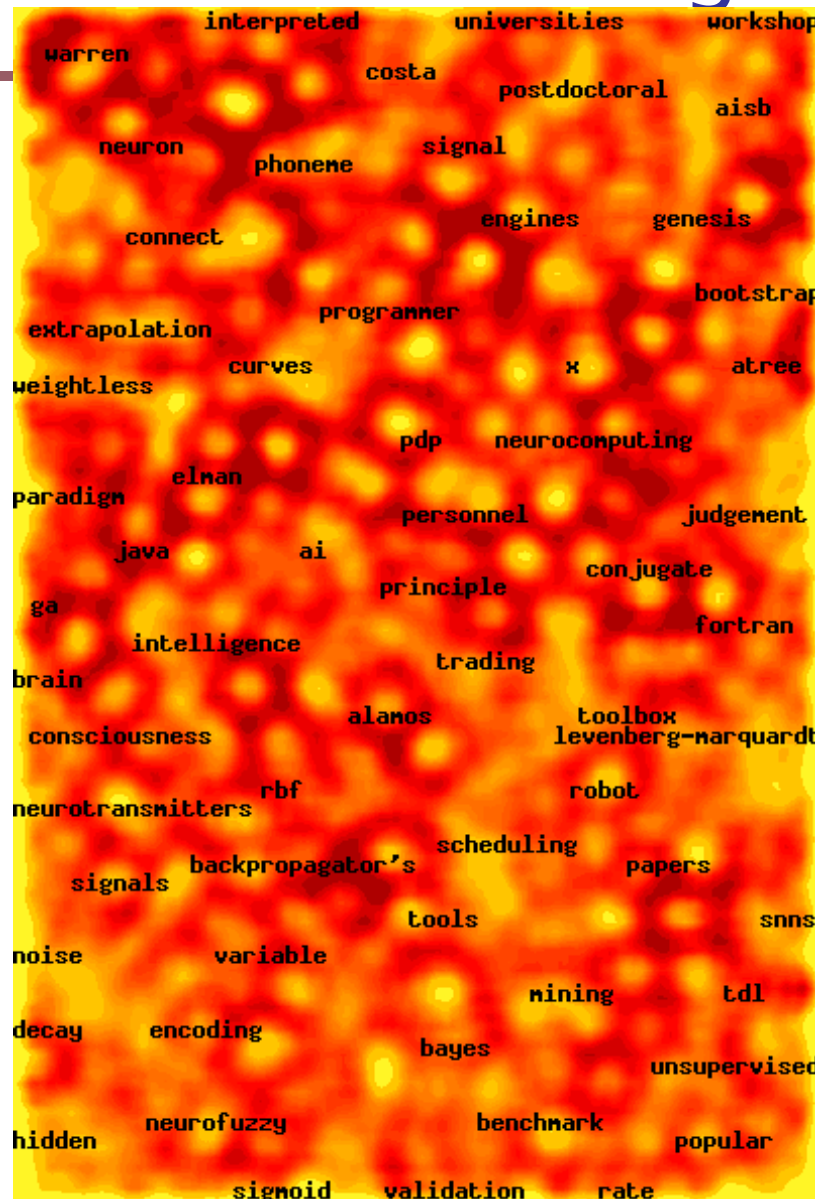


- SOMs, also called topological ordered maps, or Kohonen Self-Organizing Feature Map (KSOMs)
- It maps all the points in a high-dimensional source space into a 2 to 3-d target space, s.t., the distance and proximity relationship (i.e., topology) are preserved as much as possible
- Similar to k-means: cluster centers tend to lie in a low-dimensional manifold in the feature space
- Clustering is performed by having several units competing for the current object
  - The unit whose weight vector is closest to the current object wins
  - The winner and its neighbors learn by having their weights adjusted
- SOMs are believed to resemble processing that can occur in the brain
- Useful for visualizing high-dimensional data in 2- or 3-D space



# Web Document Clustering Using SOM

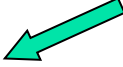
- The result of SOM clustering of 12088 Web articles
- The picture on the right: drilling down on the keyword “mining”
- Based on websom.hut.fi Web page





# Chapter 6. Cluster Analysis

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# Clustering High-Dimensional Data

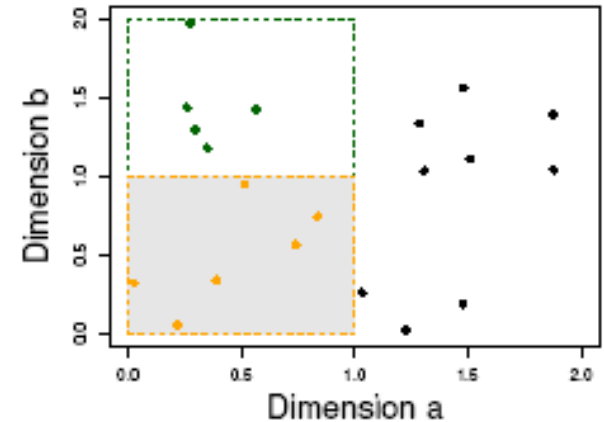
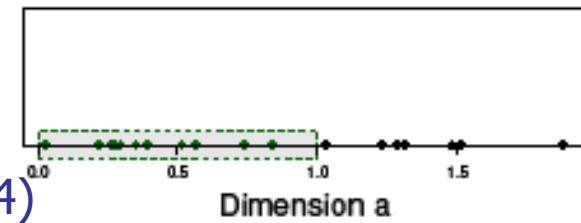
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- Clustering high-dimensional data
  - Many applications: text documents, DNA micro-array data
  - Major challenges:
    - Many irrelevant dimensions may mask clusters
    - Distance measure becomes meaningless—due to equi-distance
    - Clusters may exist only in some subspaces
- Methods
  - Feature transformation: only effective if most dimensions are relevant
    - PCA & SVD useful only when features are highly correlated/redundant
  - Feature selection: wrapper or filter approaches
    - useful to find a subspace where the data have nice clusters
  - Subspace-clustering: find clusters in all the possible subspaces
    - CLIQUE, ProClus, and frequent pattern-based clustering

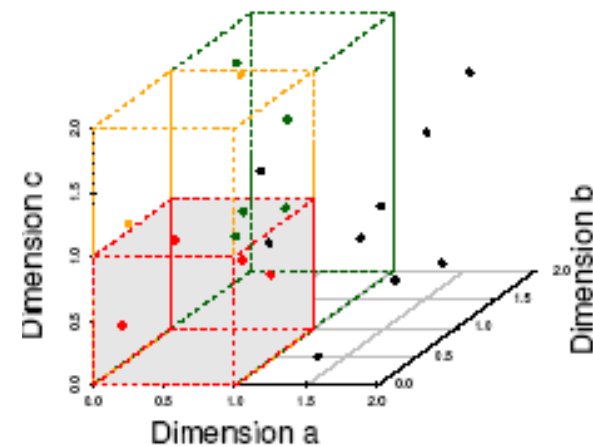
# The Curse of Dimensionality

(graphs adapted from Parsons et al. KDD Explorations 2004)

- Data in only one dimension is relatively packed
- Adding a dimension “stretch” the points across that dimension, making them further apart
- Adding more dimensions will make the points further apart—high dimensional data is extremely sparse
- Distance measure becomes meaningless—due to equi-distance



(b) 6 Objects in One Unit Bin

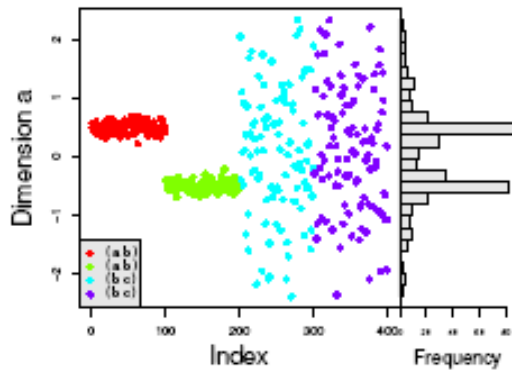
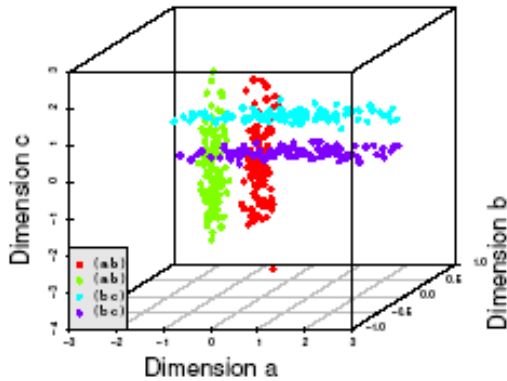


(c) 4 Objects in One Unit Bin

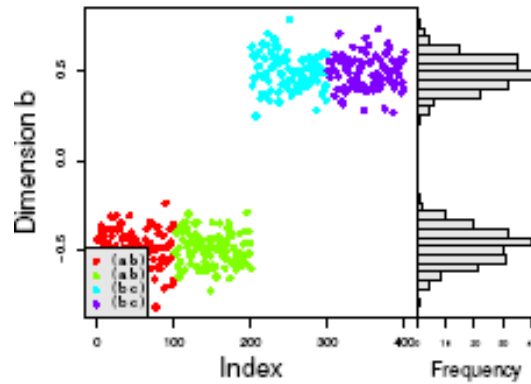
# Why Subspace Clustering?

(adapted from Parsons et al. SIGKDD Explorations 2004)

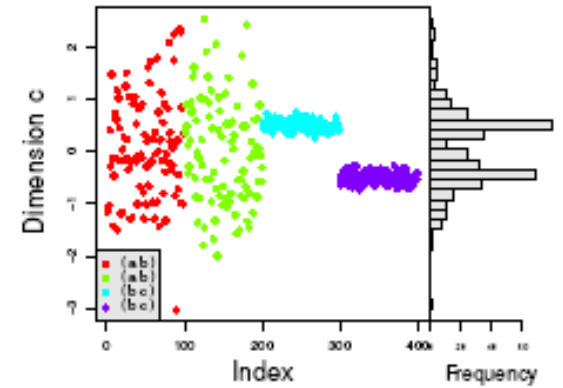
- Clusters may exist only in some subspaces
- Subspace-clustering: find clusters in all the subspaces



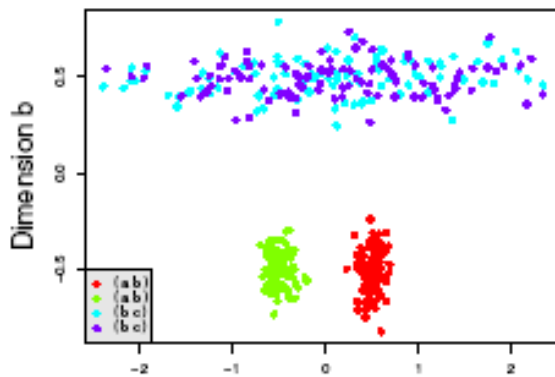
(a) Dimension *a*



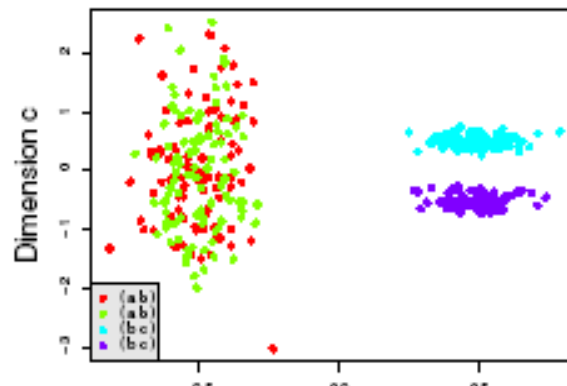
(b) Dimension *b*



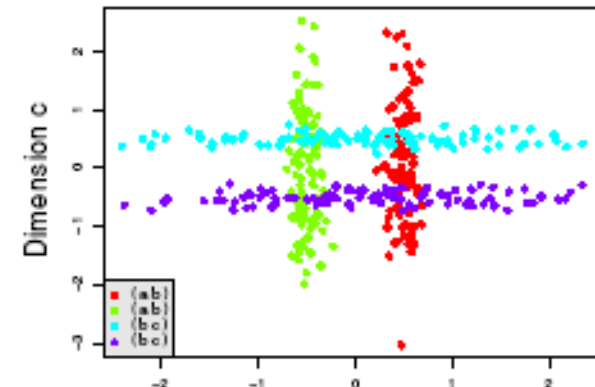
(c) Dimension *c*



(a) Dims *a* & *b*



(b) Dims *b* & *c*



(c) Dims *a* & *c*

# CLIQUE (Clustering In QUES)

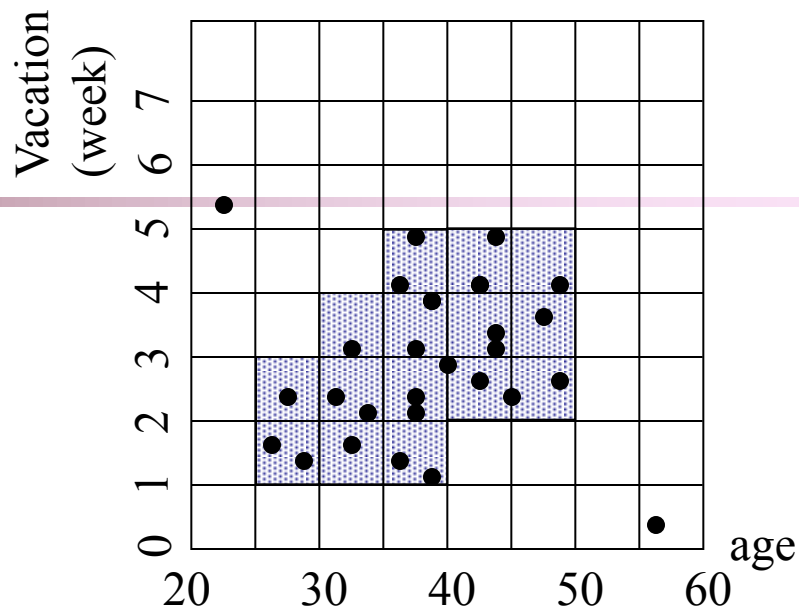
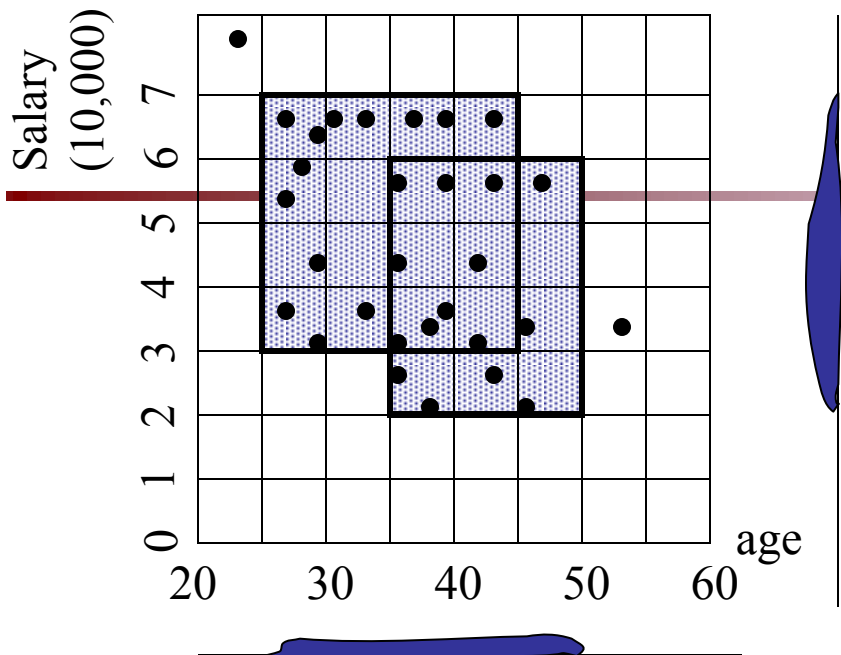
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- Agrawal, Gehrke, Gunopulos, Raghavan (SIGMOD' 98)
- Automatically identifying subspaces of a high dimensional data space that allow better clustering than original space
- CLIQUE can be considered as both density-based and grid-based
  - It partitions each dimension into the same number of equal length interval
  - It partitions an m-dimensional data space into non-overlapping rectangular units
  - A unit is dense if the fraction of total data points contained in the unit exceeds the input model parameter
  - A cluster is a maximal set of connected dense units within a subspace

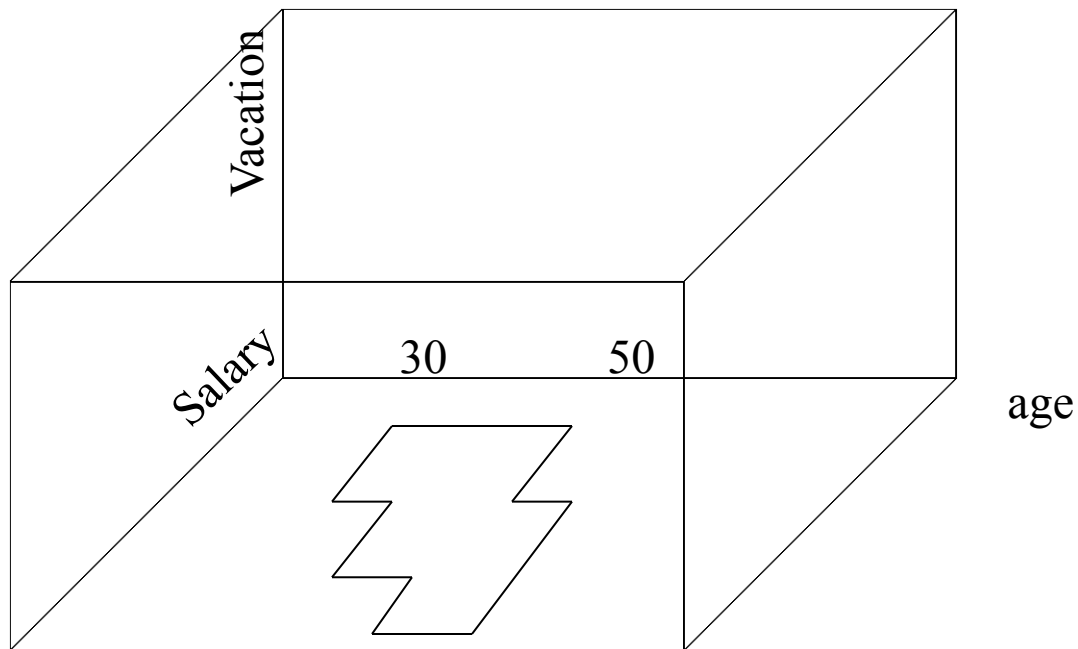
# CLIQUE: The Major Steps

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- Partition the data space and find the number of points that lie inside each cell of the partition.
- Identify the subspaces that contain clusters using the Apriori principle
- Identify clusters
  - Determine dense units in all subspaces of interests
  - Determine connected dense units in all subspaces of interests.
- Generate minimal description for the clusters
  - Determine maximal regions that cover a cluster of connected dense units for each cluster
  - Determination of minimal cover for each cluster



$\tau = 3$



# Strength and Weakness of *CLIQUE*

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## ■ Strength

- *automatically* finds subspaces of the highest dimensionality such that high density clusters exist in those subspaces
- *insensitive* to the order of records in input and does not presume some canonical data distribution
- scales *linearly* with the size of input and has good scalability as the number of dimensions in the data increases

## ■ Weakness

- The accuracy of the clustering result may be degraded at the expense of simplicity of the method



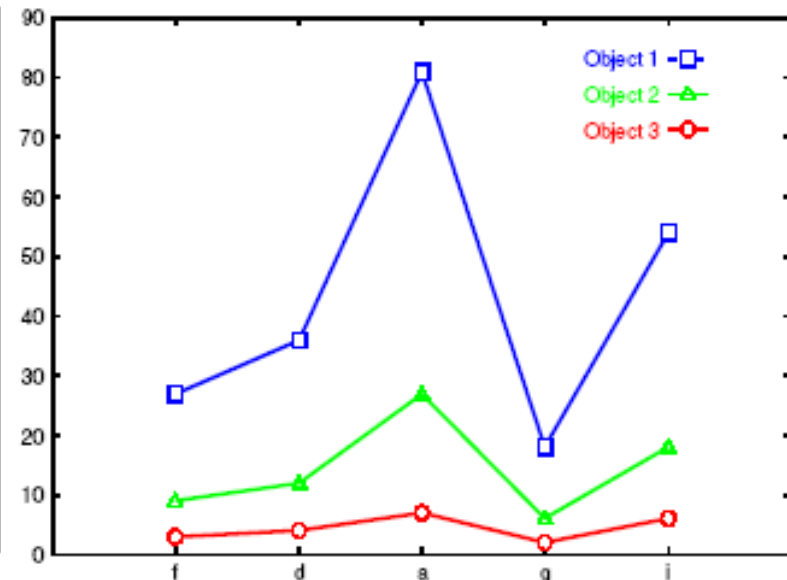
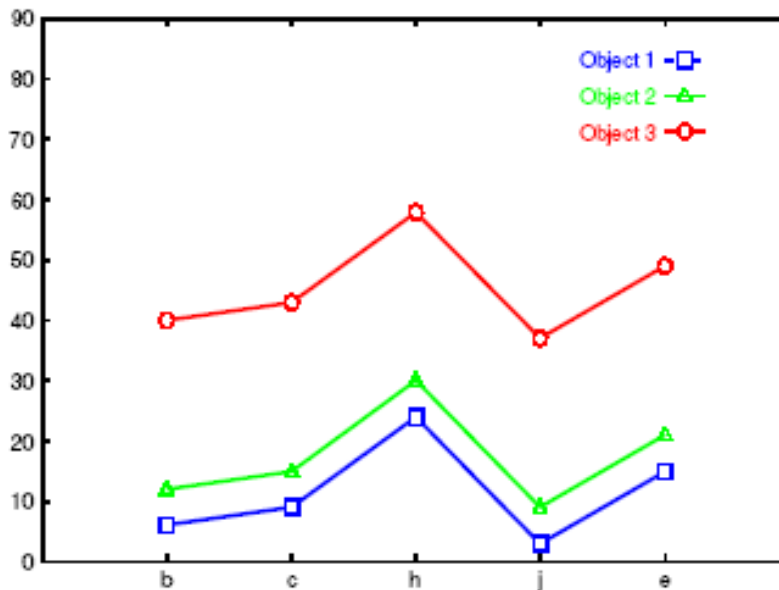
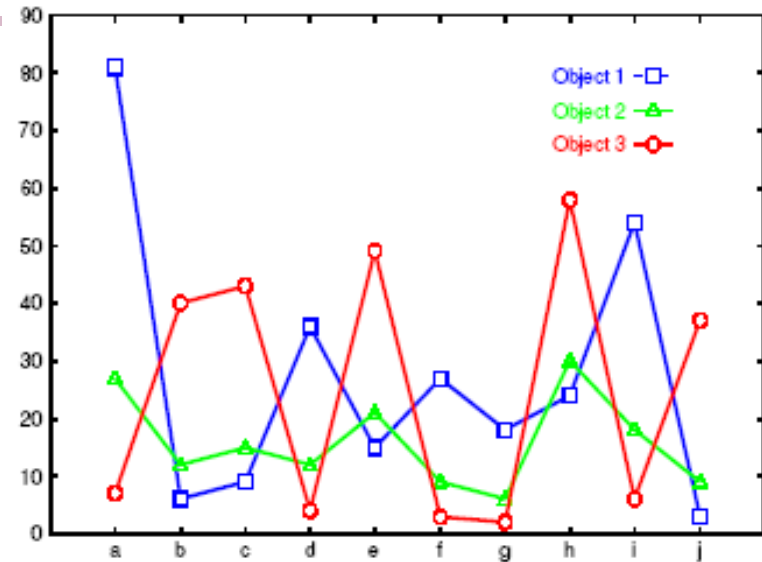
# Frequent Pattern-Based Approach



- Clustering high-dimensional space (e.g., clustering text documents, microarray data)
  - Projected subspace-clustering: which dimensions to be projected on?
    - CLIQUE, ProClus
  - Feature extraction: costly and may not be effective?
  - Using frequent patterns as “features”
    - “Frequent” are inherent features
    - Mining freq. patterns may not be so expensive
- Typical methods
  - Frequent-term-based document clustering
  - Clustering by pattern similarity in micro-array data (pClustering)

# Clustering by Pattern Similarity ( $p$ -Clustering)

- Right: The micro-array “raw” data shows 3 genes and their values in a multi-dimensional space
  - Difficult to find their patterns
- Bottom: Some subsets of dimensions form nice **shift** and **scaling** patterns

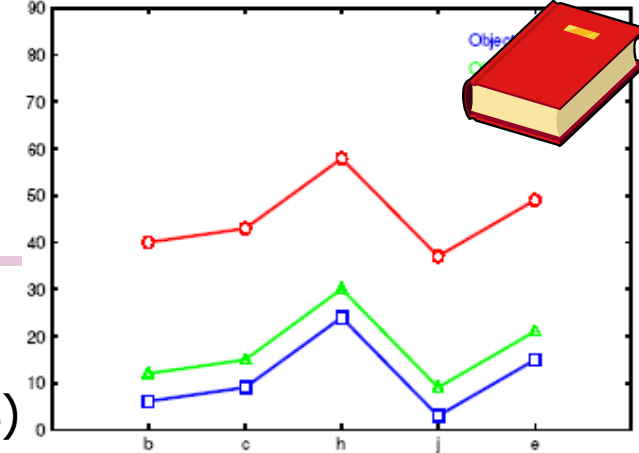


# Why $p$ -Clustering?

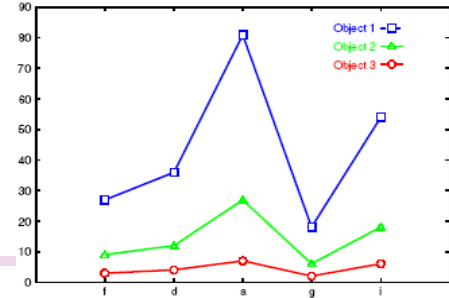
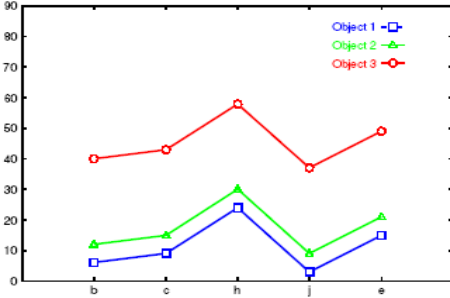
- Microarray data analysis may need to
  - Clustering on thousands of dimensions (attributes)
  - Discovery of both **shift** and **scaling** patterns
- Clustering with Euclidean distance measure? — cannot find shift patterns
- Clustering on derived attribute  $A_{ij} = a_i - a_j$ ? — introduces  $N(N-1)$  dimensions
- Bi-cluster using transformed mean-squared residue score matrix (I, J)

$$H(IJ) = \frac{1}{|I||J|} \sum_{i \in I, j \in J} (d_{ij} - d_{iJ} - d_{Ij} + d_{IJ})^2$$

- Where
  - $d_{ij} = \frac{1}{|J|} \sum_{j \in J} d_{ij}$
  - $d_{Ij} = \frac{1}{|I|} \sum_{i \in I} d_{ij}$
  - $d_{IJ} = \frac{1}{|I||J|} \sum_{i \in I, j \in J} d_{ij}$
- A submatrix is a  $\delta$ -cluster if  $H(I, J) \leq \delta$  for some  $\delta > 0$
- Problems with bi-cluster
  - No downward closure property,
  - Due to averaging, it may contain outliers but still within  $\delta$ -threshold



# $p$ -Clustering: Clustering by Pattern Similarity



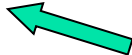
- Given object  $x, y$  in  $O$  and features  $a, b$  in  $T$ ,  $p$ Cluster is a 2 by 2 matrix

$$pScore\left(\begin{bmatrix} d_{xa} & d_{xb} \\ d_{ya} & d_{yb} \end{bmatrix}\right) = |(d_{xa} - d_{xb}) - (d_{ya} - d_{yb})|$$

- A pair  $(O, T)$  is in  $\delta$ - $p$ Cluster if for any 2 by 2 matrix  $X$  in  $(O, T)$ ,  $pScore(X) \leq \delta$  for some  $\delta > 0$
- Properties of  $\delta$ - $p$ Cluster
  - Downward closure
  - Clusters are more homogeneous than bi-cluster (thus the name: pair-wise Cluster)
- Pattern-growth algorithm has been developed for efficient mining
- For scaling patterns, one can observe, taking logarithmic on  $\frac{d_{xa} / d_{ya}}{d_{xb} / d_{yb}} < \delta$  will lead to the  $p$ Score form

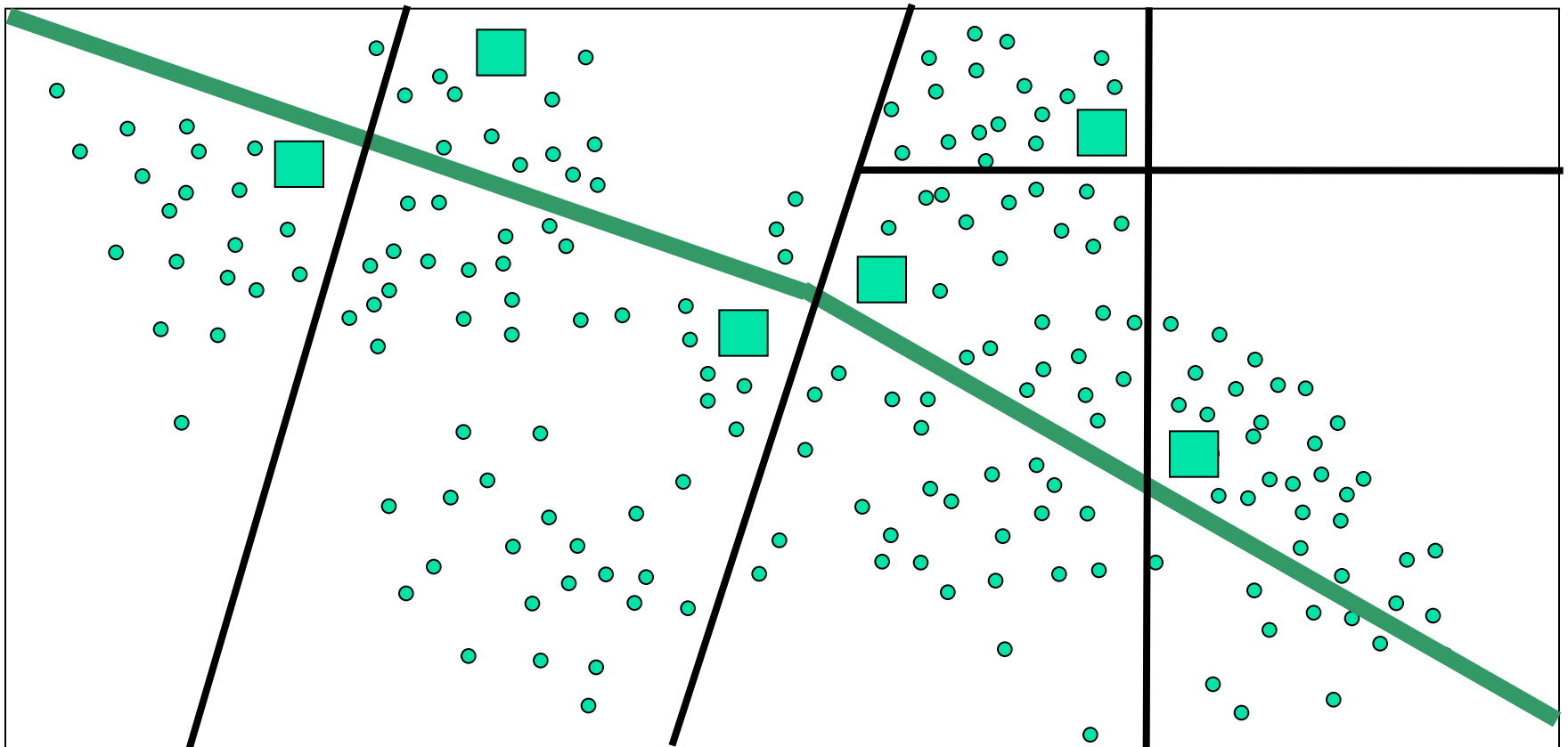
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# Why Constraint-Based Cluster Analysis?

- Need user feedback: Users know their applications the best
- Less parameters but more user-desired constraints, e.g., an ATM allocation problem: obstacle & desired clusters





# A Classification of Constraints in Cluster Analysis

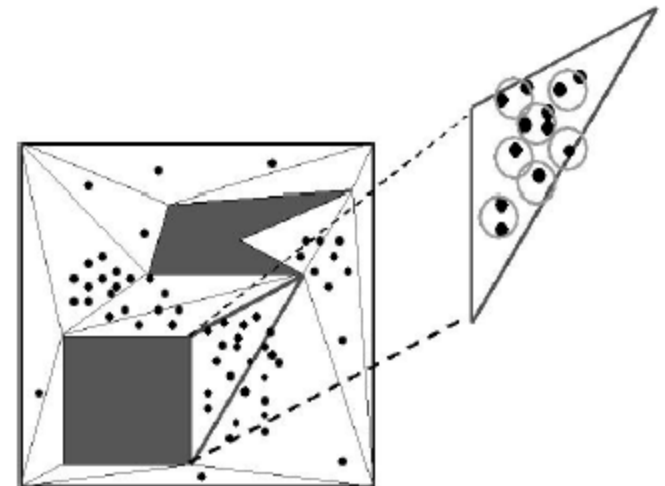
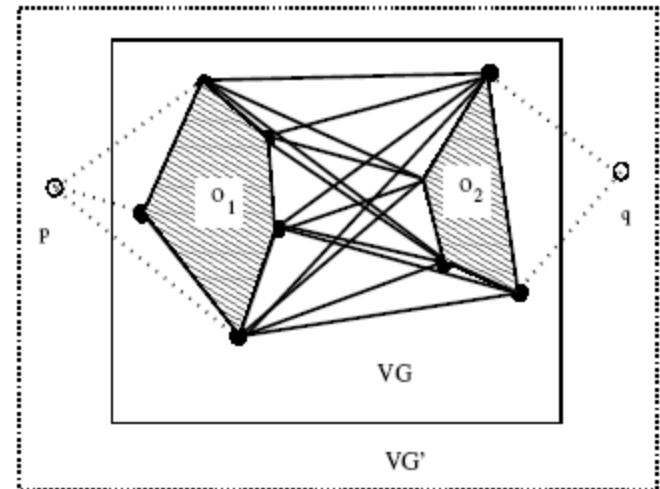
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- Clustering in applications: desirable to have user-guided (i.e., constrained) cluster analysis
- Different constraints in cluster analysis:
  - Constraints on individual objects (do selection first)
    - Cluster on houses worth over \$300K
  - Constraints on distance or similarity functions
    - Weighted functions, obstacles (e.g., rivers, lakes)
  - Constraints on the selection of clustering parameters
    - # of clusters, MinPts, etc.
  - User-specified constraints
    - Contain at least 500 valued customers and 5000 ordinary ones
  - Semi-supervised: giving small training sets as “constraints” or hints

# Clustering With Obstacle Objects



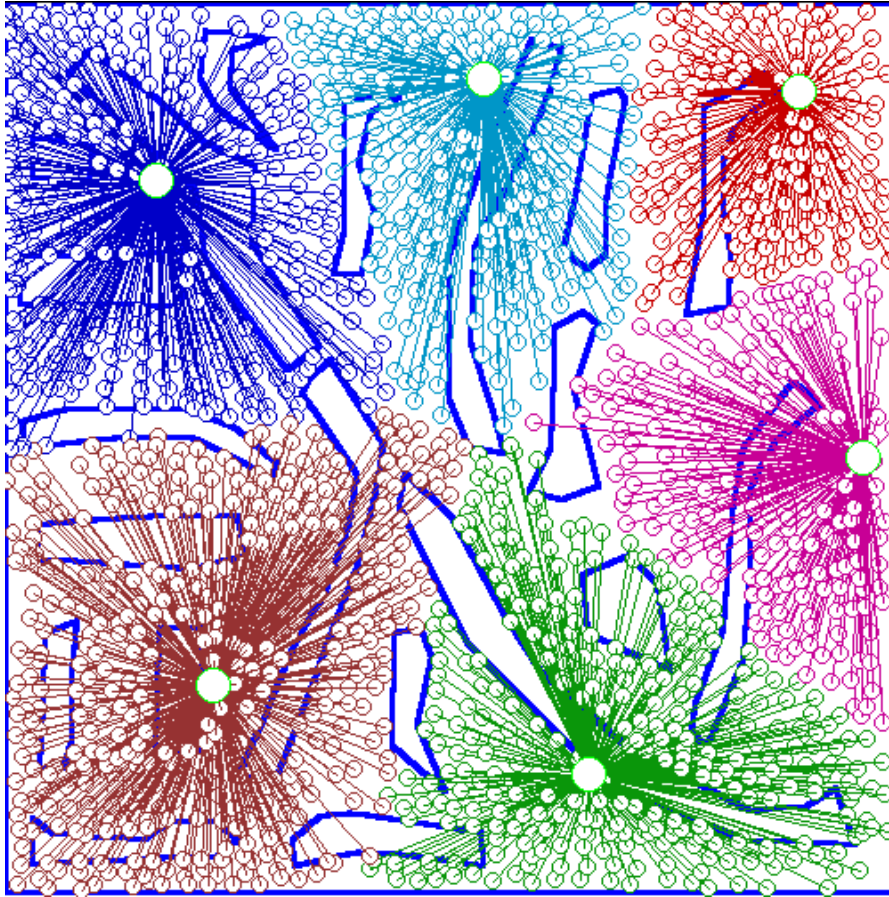
- K-medoids is more preferable since k-means may locate the ATM center in the middle of a lake
- Visibility graph and shortest path
- Triangulation and micro-clustering
- Two kinds of join indices (shortest-paths) worth pre-computation
  - VV index: indices for any pair of obstacle vertices
  - MV index: indices for any pair of micro-cluster and obstacle indices







# An Example: Clustering With Obstacle Objects



*Not* Taking obstacles into account



Taking obstacles into account

# Clustering with User-Specified Constraints

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- Example: Locating  $k$  delivery centers, each serving at least  $m$  valued customers and  $n$  ordinary ones
- Proposed approach
  - Find an initial “solution” by partitioning the data set into  $k$  groups and satisfying user-constraints
  - Iteratively refine the solution by micro-clustering relocation (e.g., moving  $\delta$   $\mu$ -clusters from cluster  $C_i$  to  $C_j$ ) and “deadlock” handling (break the microclusters when necessary)
  - Efficiency is improved by micro-clustering
- How to handle more complicated constraints?
  - E.g., having approximately same number of valued customers in each cluster?! — Can you solve it?

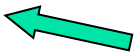
# Semi-supervised clustering

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- Must-link
- Cannot-link

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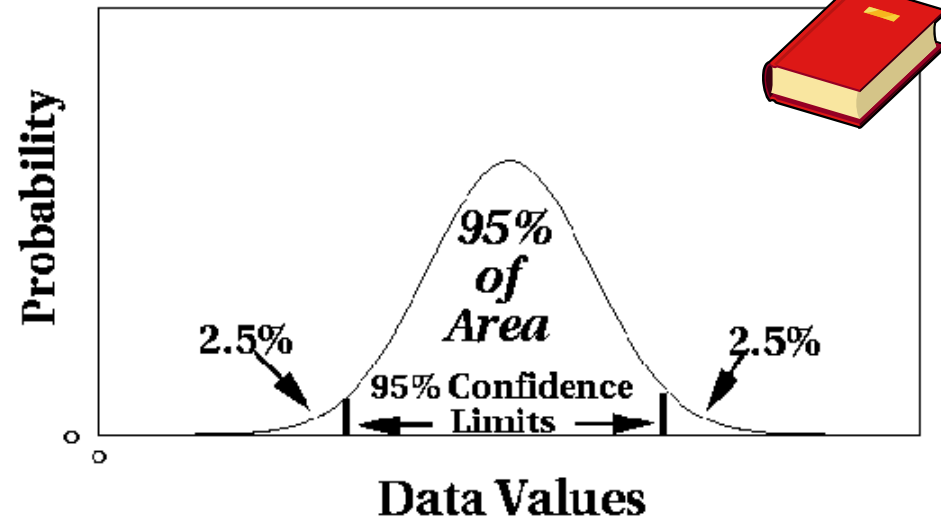
# What Is Outlier Discovery?

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- What are outliers?
  - The set of objects are considerably dissimilar from the remainder of the data
  - Example: Sports: Michael Jordon, Wayne Gretzky, ...
- Problem: Define and find outliers in large data sets
- Applications:
  - Credit card fraud detection
  - Telecom fraud detection
  - Customer segmentation
  - Medical analysis

# Outlier Discovery: Statistical Approaches

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- ✧ Assume a model underlying distribution that generates data set (e.g. normal distribution)
- Use discordancy tests depending on
  - data distribution
  - distribution parameter (e.g., mean, variance)
  - number of expected outliers
- Drawbacks
  - most tests are for single attribute
  - In many cases, data distribution may not be known

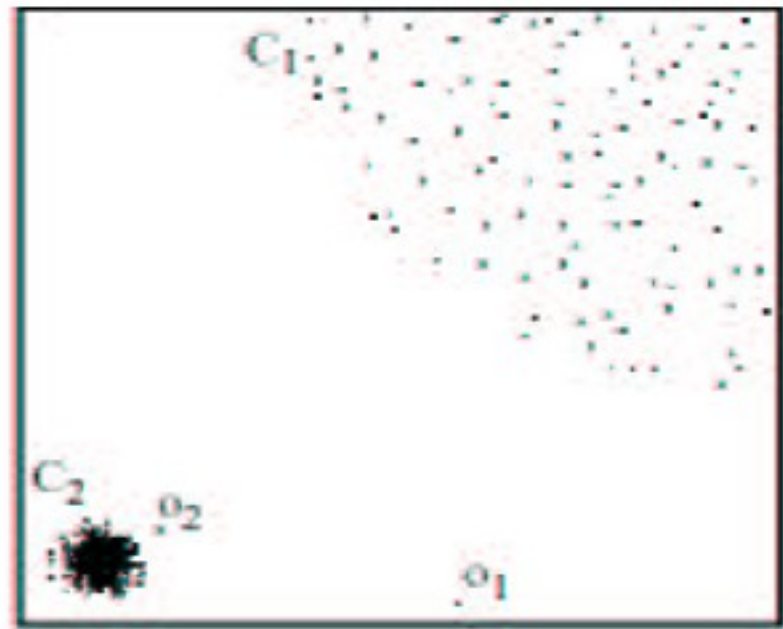
# Outlier Discovery: Distance-Based Approach



- Introduced to counter the main limitations imposed by statistical methods
  - We need multi-dimensional analysis without knowing data distribution
- Distance-based outlier: A  $DB(p, D)$ -outlier is an object  $O$  in a dataset  $T$  such that at least a fraction  $p$  of the objects in  $T$  lies at a distance greater than  $D$  from  $O$
- Algorithms for mining distance-based outliers
  - Index-based algorithm
  - Nested-loop algorithm
  - Cell-based algorithm

# Density-Based Local Outlier Detection

- Distance-based outlier detection is based on global distance distribution
- It encounters difficulties to identify outliers if data is not uniformly distributed
- Ex.  $C_1$  contains 400 loosely distributed points,  $C_2$  has 100 tightly condensed points, 2 outlier points  $o_1, o_2$
- Distance-based method cannot identify  $o_2$  as an outlier
- Need the concept of local outlier



- Local outlier factor (LOF)
  - Assume outlier is not crisp
  - Each point has a LOF





# Outlier Discovery: Deviation-Based Approach

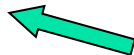
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- Identifies outliers by examining the main characteristics of objects in a group
- Objects that “deviate” from this description are considered outliers
- Sequential exception technique
  - simulates the way in which humans can distinguish unusual objects from among a series of supposedly like objects
- OLAP data cube technique
  - uses data cubes to identify regions of anomalies in large multidimensional data

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# Summary

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- **Cluster analysis** groups objects based on their **similarity** and has wide applications
- Measure of similarity can be computed for **various types of data**
- Clustering algorithms can be **categorized** into partitioning methods, hierarchical methods, density-based methods, grid-based methods, and model-based methods
- **Outlier detection** and analysis are very useful for fraud detection, etc. and can be performed by statistical, distance-based or deviation-based approaches
- There are still lots of research issues on cluster analysis

# Problems and Challenges

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- Considerable progress has been made in scalable clustering methods
  - Partitioning: k-means, k-medoids, CLARANS
  - Hierarchical: BIRCH, ROCK, CHAMELEON
  - Density-based: DBSCAN, OPTICS, DenClue
  - Grid-based: STING, WaveCluster, CLIQUE
  - Model-based: EM, Cobweb, SOM
  - Frequent pattern-based: pCluster
  - Constraint-based: COD, constrained-clustering
- Current clustering techniques do not address all the requirements adequately, still an active area of research

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