



# Digital Image Processing

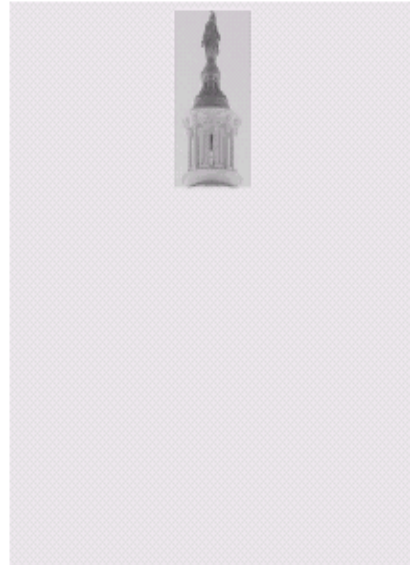
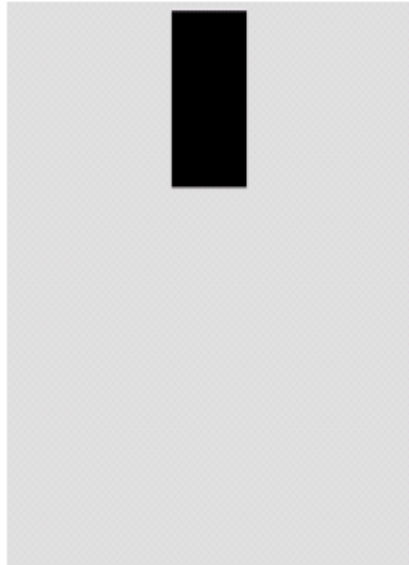
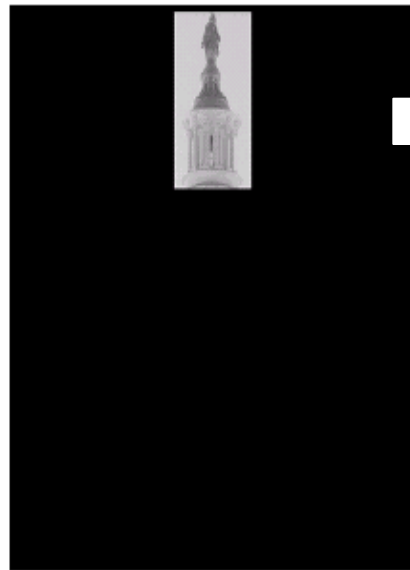
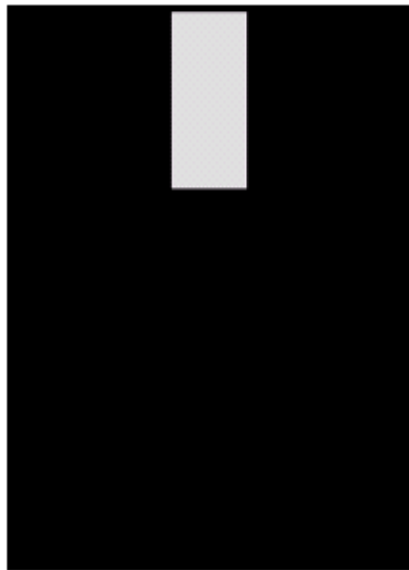
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Image Enhancement Using  
Arithmetic/Logical Operations

# Enhancement Using Arithmetic/Logic Operations

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- AND
- OR
- NOT
- Subtraction
- Addition
- Multiplication
- Division



a	b	c
d	e	f

(a) Original image. (b) AND image mask. (c) Result of the AND operation on images (a) and (b). (d) Original image. (e) OR image mask. (f) Result of operation OR on images (d) and (e).

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- Image subtraction

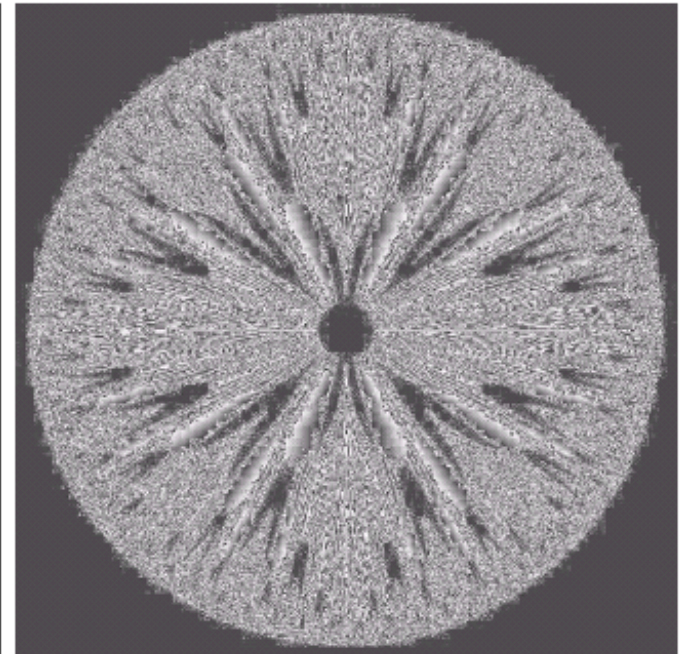
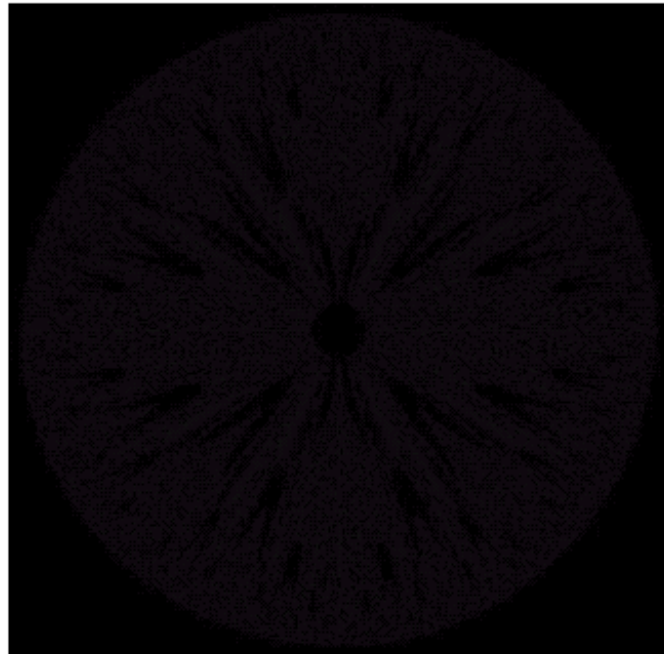
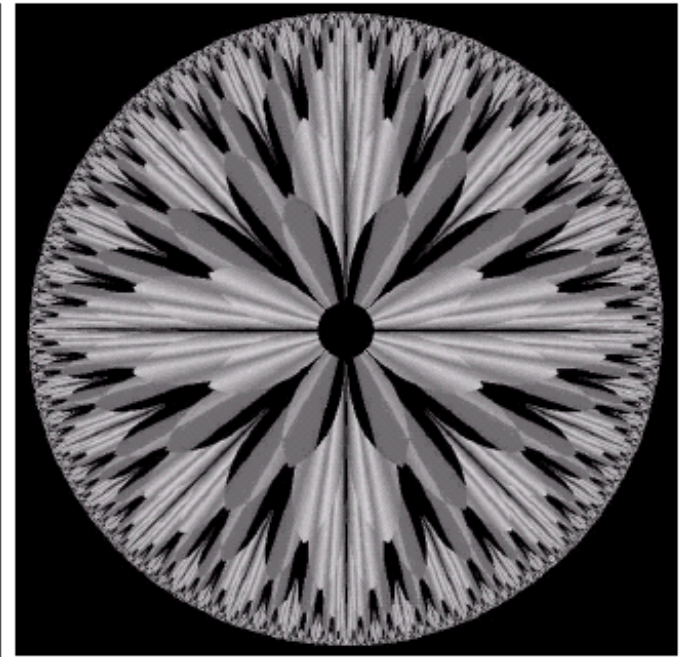
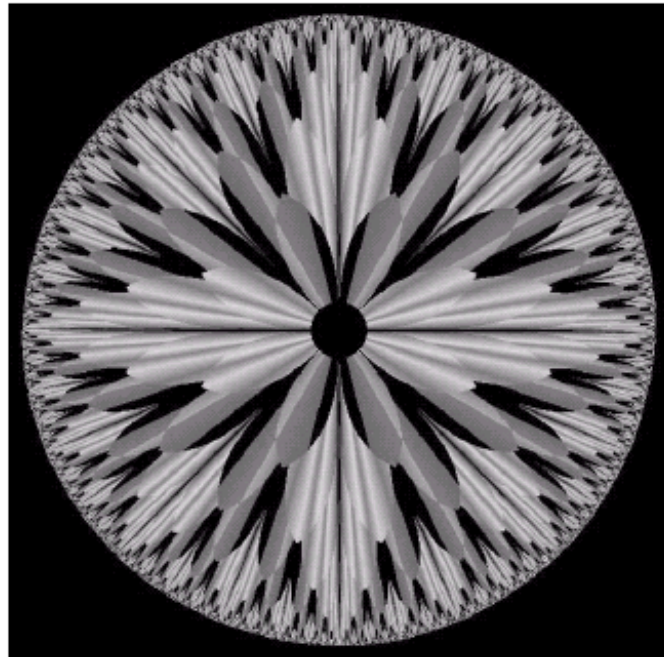
- Enhancement of differences between images

$$g(x, y) = f(x, y) - h(x, y)$$

a	b
c	d

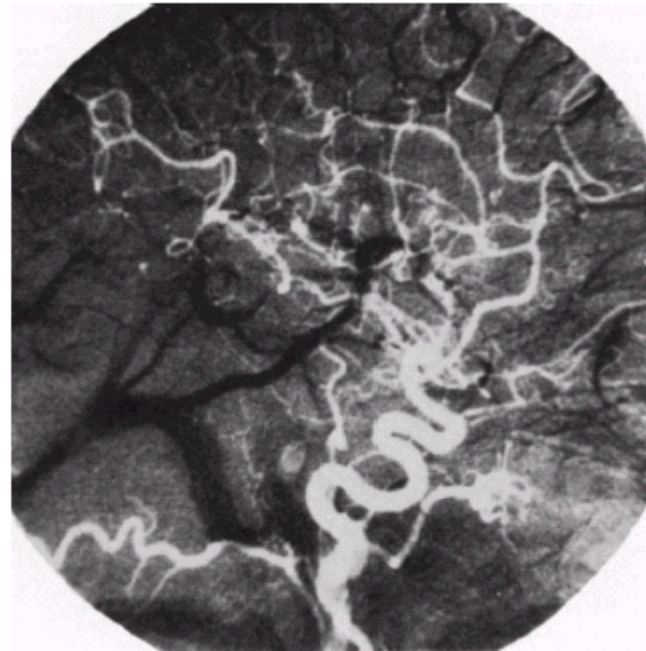
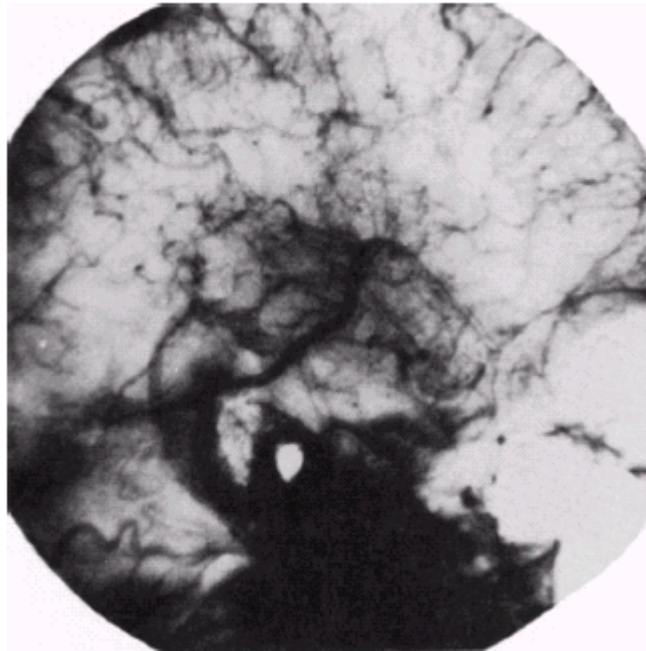
(a) Original fractal image.  
(b) Result of setting the four lower-order bit planes to zero.  
(c) Difference between (a) and (b).  
(d) Histogram-equalized difference image.  
(Original image courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA).

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- Mask mode radiography



a b

**FIGURE 3.29**  
Enhancement by image subtraction.  
(a) Mask image.  
(b) An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out.

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- Image Averaging

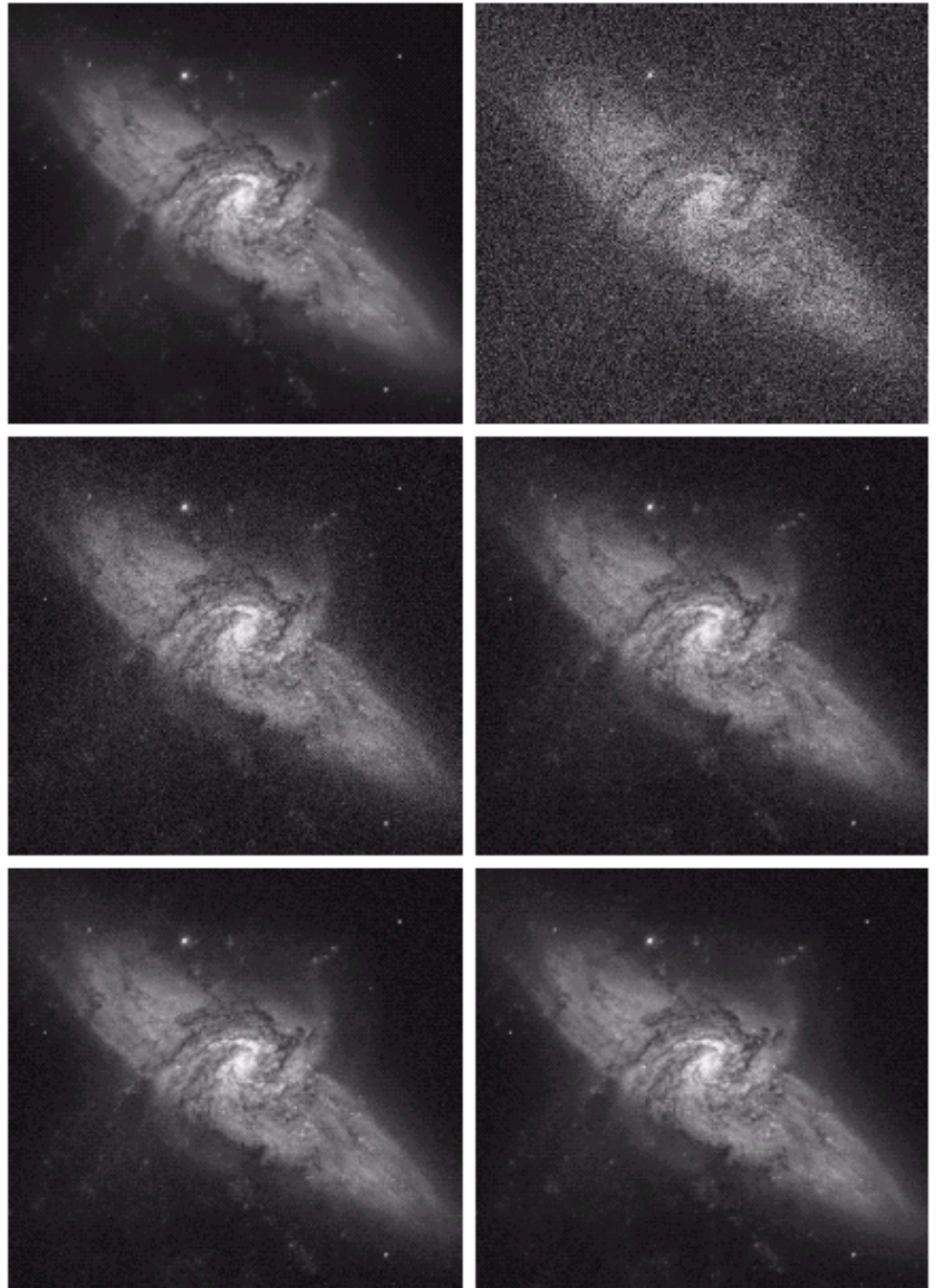
$$g(x, y) = f(x, y) + \eta(x, y)$$

- Averaging K different noisy images

$$\bar{g}(x, y) = \frac{1}{K} \sum_{i=1}^K g_i(x, y)$$

$$E\{\bar{g}(x, y)\} = f(x, y)$$

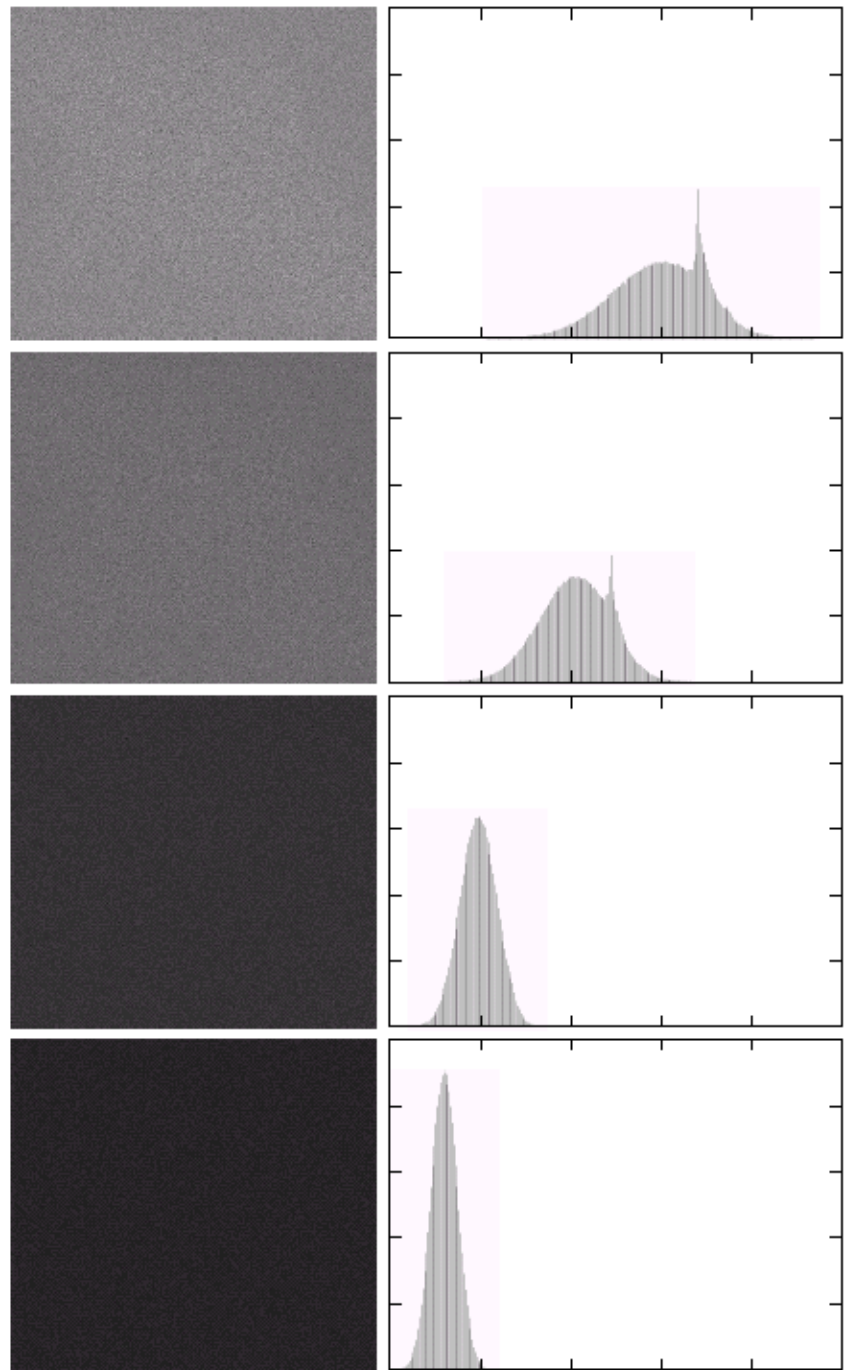
$$\sigma_{\bar{g}(x, y)}^2 = \frac{1}{K} \sigma_{\eta(x, y)}^2 \quad , \quad \sigma_{\bar{g}(x, y)} = \frac{1}{\sqrt{K}} \sigma_{\eta(x, y)}$$



a b  
c d  
e f

**FIGURE 3.30** (a) image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging  $K = 8, 16, 64,$  and  $128$  noisy images. (Original image courtesy of NASA.)





a b

(a) From top to bottom:  
Difference images  
between  
Fig. 3.30(a) and  
the four images in  
Figs. 3.30(c)  
through (f),  
respectively.  
(b) Corresponding  
histograms.

# Basics of Spatial Filtering

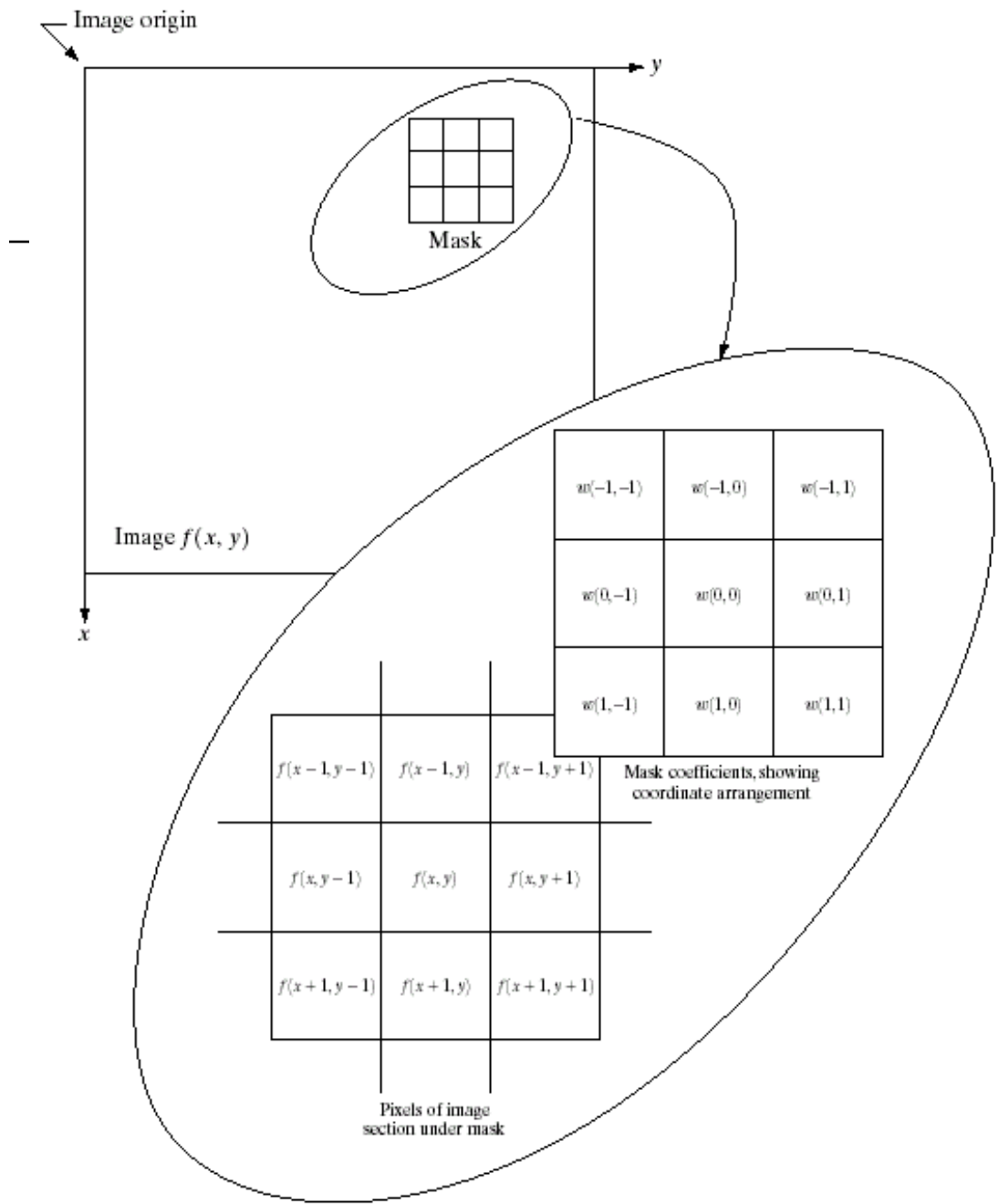
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$$\begin{aligned} R = & w(-1,-1) f(x-1, y-1) + \\ & w(-1,0) f(x-1, y) + \cdots + \\ & w(0,0) f(x, y) + \cdots + \\ & w(1,0) f(x+1, y) + \\ & w(1,1) f(x+1, y+1) \end{aligned}$$

- 
- Image size:  $M \times N$
  - Mask size:  $m \times n$

$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

- $a = (m-1)/2$  and  $b = (n-1)/2$
- $x = 0, 1, 2, \dots, M-1$  and  $y = 0, 1, 2, \dots, N-1$



..... The mechanics of spatial filtering. The magnified drawing shows a  $3 \times 3$  mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

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$$R = w_1 z_1 + w_2 z_2 + \dots + w_9 z_9$$
$$= \sum_{i=1}^9 w_i z_i$$

Another  
representation of  
a general  $3 \times 3$   
spatial filter mask.

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$w_1$	$w_2$	$w_3$
$w_4$	$w_5$	$w_6$
$w_7$	$w_8$	$w_9$



# Smoothing Spatial Filters

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- Smoothing
  - Noise reduction
  - Smoothing of false contours
  - Reduction of irrelevant detail

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$$R = \frac{1}{9} \sum_{i=1}^9 z_i$$

 $\frac{1}{9} \times$ 

1	1	1
1	1	1
1	1	1

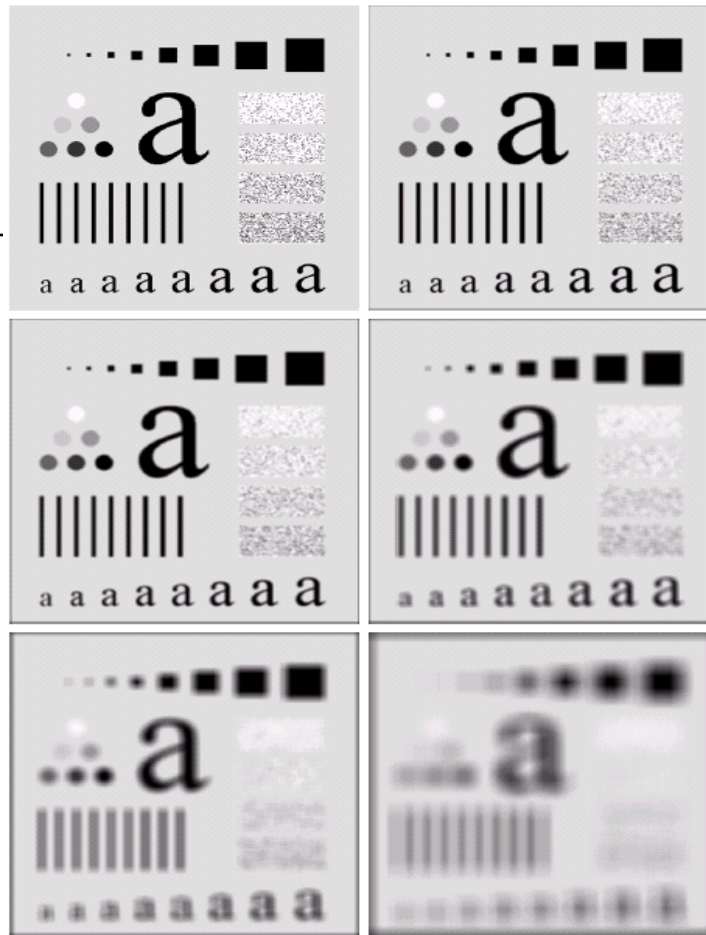
 $\frac{1}{16} \times$ 

1	2	1
2	4	2
1	2	1

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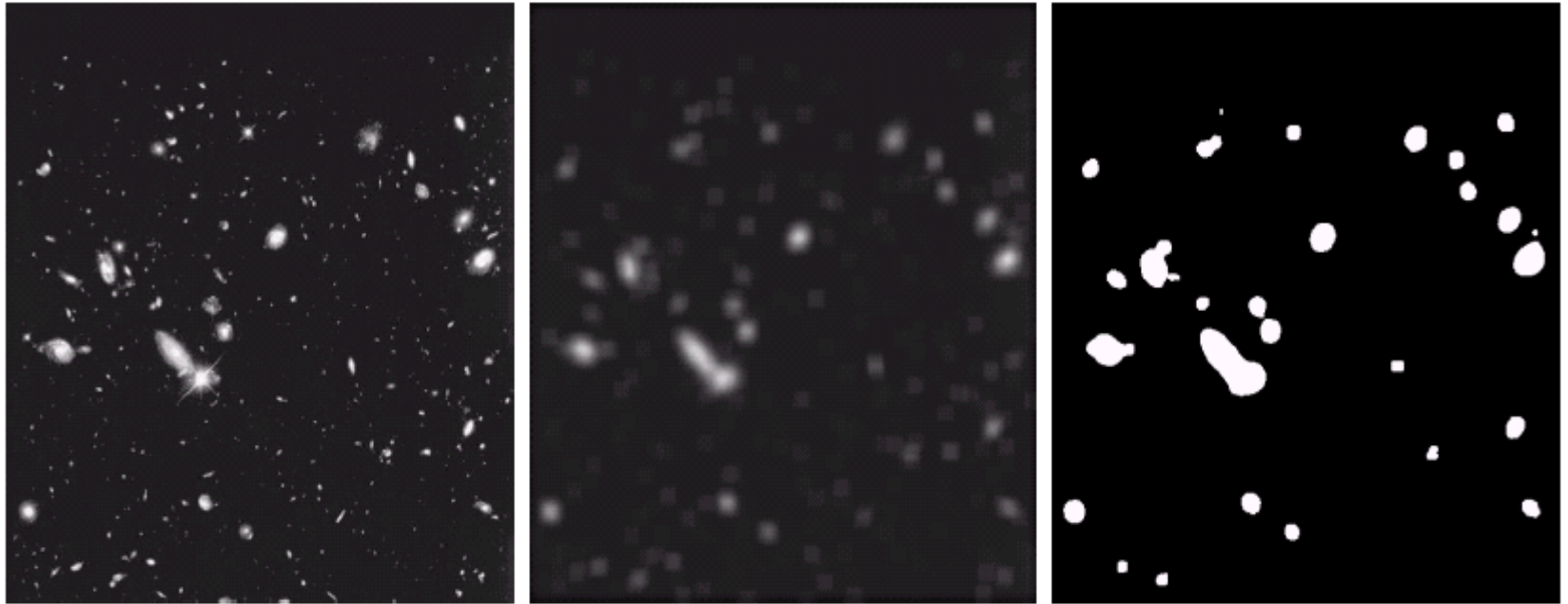
$$g(x, y) = \frac{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)}{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t)}$$





a b  
c d  
e f

Original image, of size  $500 \times 500$  pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes  $n = 3, 5, 9, 15,$  and  $35,$  respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size  $50 \times 120$  pixels.



a b c

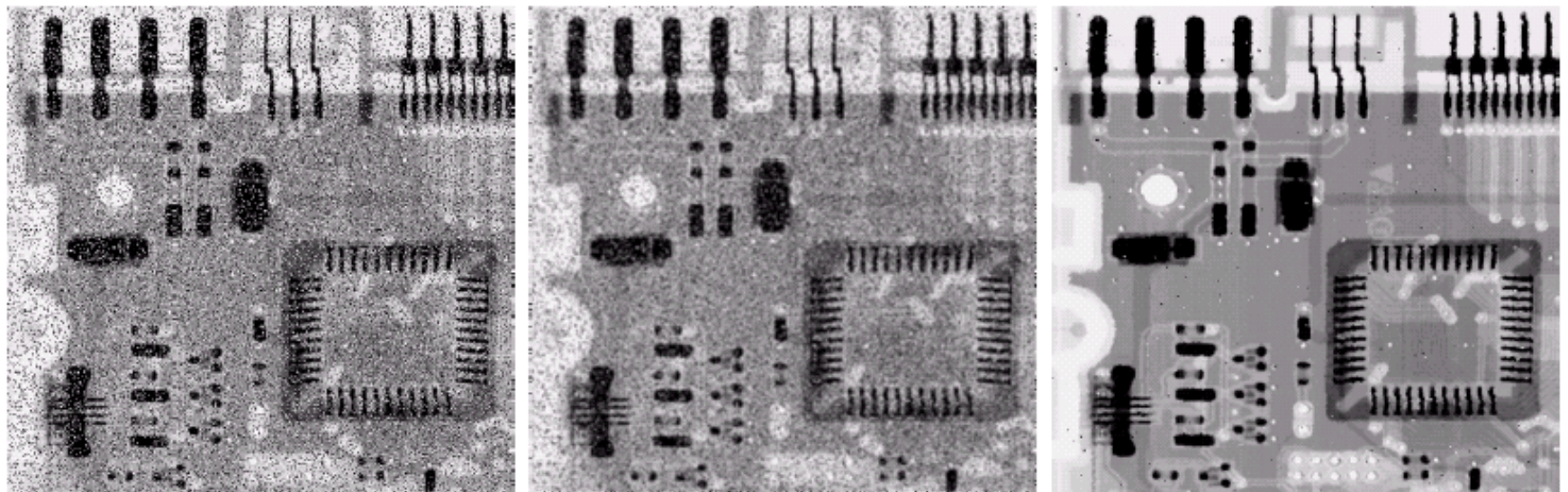
**3.36** (a) Image from the Hubble Space Telescope. (b) Image processed by a  $15 \times 15$  averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)



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## ○ Order-statistic filters

- median filter: Replace the value of a pixel by the median of the gray levels in the neighborhood of that pixel
- Noise-reduction



a b c

**FIGURE 3.37** (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a  $3 \times 3$  averaging mask. (c) Noise reduction with a  $3 \times 3$  median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

# Sharpening Spatial Filters

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## ○ Foundation

- The first-order derivative

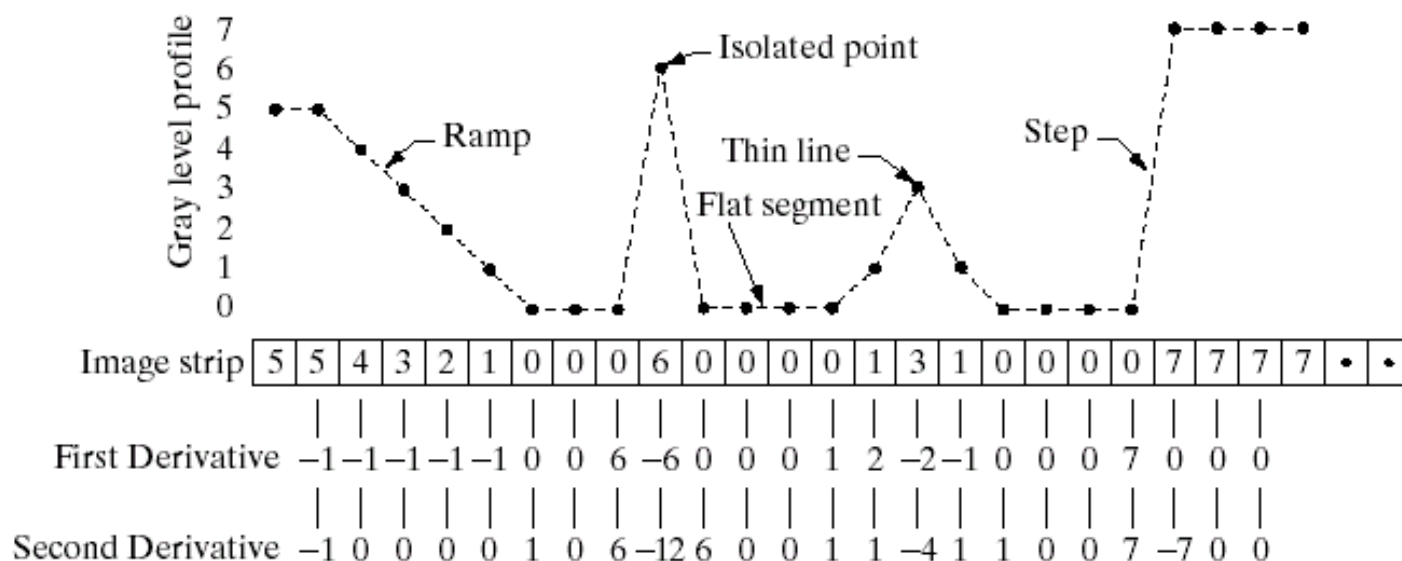
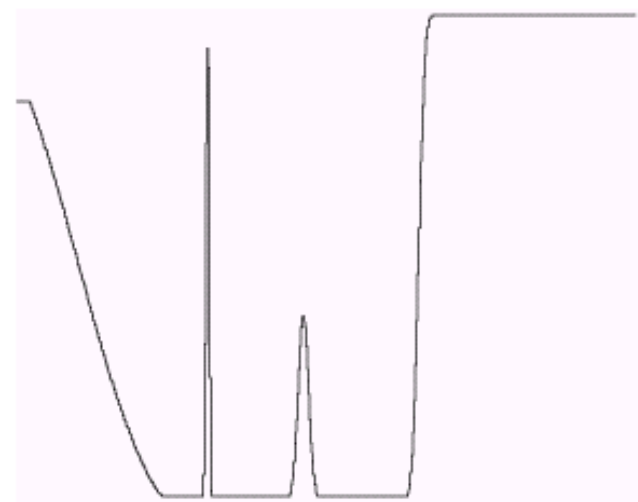
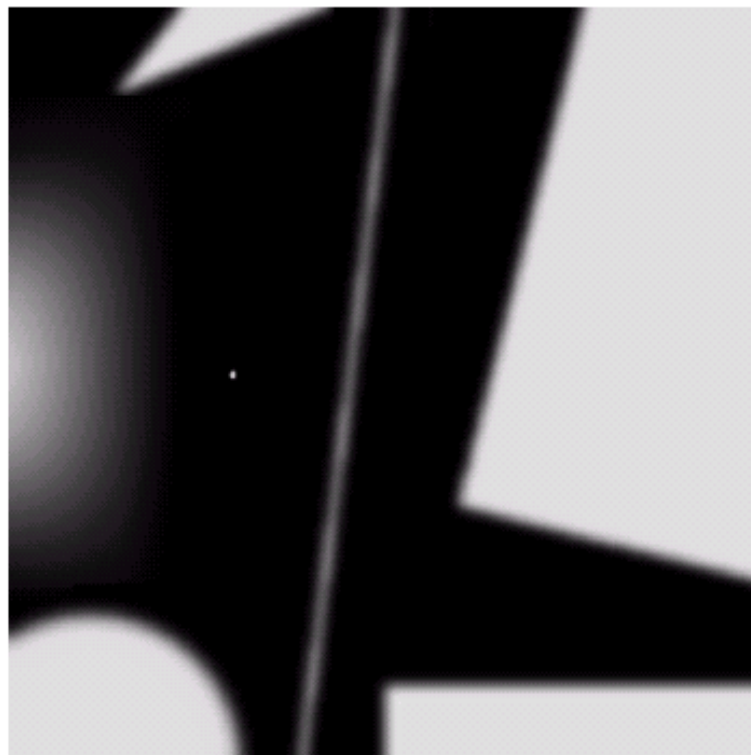
$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- The second-order derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

a b  
c

(a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).



- 
- Use of second derivatives for enhancement-The Laplacian
    - Development of the method

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

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$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient} \\ & \text{of the Laplacian mask} \\ & \text{is negative} \\ f(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient} \\ & \text{of the Laplacian mask} \\ & \text{is positive} \end{cases}$$



0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1
0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1

a	b
c	d

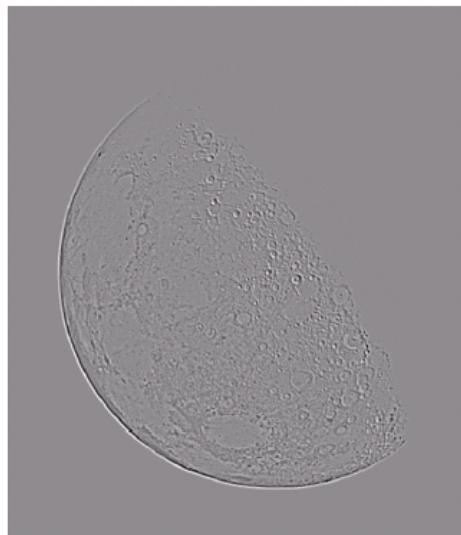
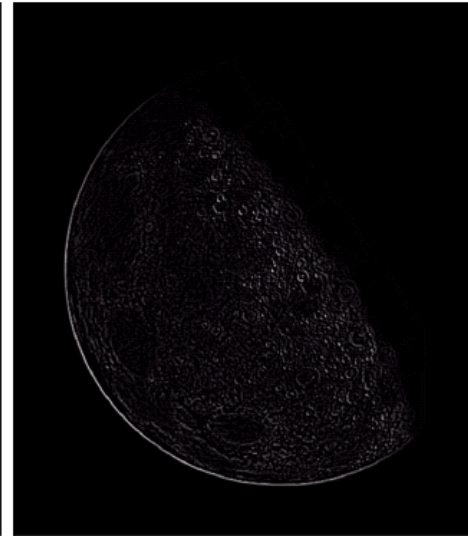
(a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4).

(b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

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a b  
c d

(a) Image of the North Pole of the moon.  
(b) Laplacian-filtered image.  
(c) Laplacian image scaled for display purposes.  
(d) Image enhanced by using Eq. (3.7-5).  
(Original image courtesy of NASA.)



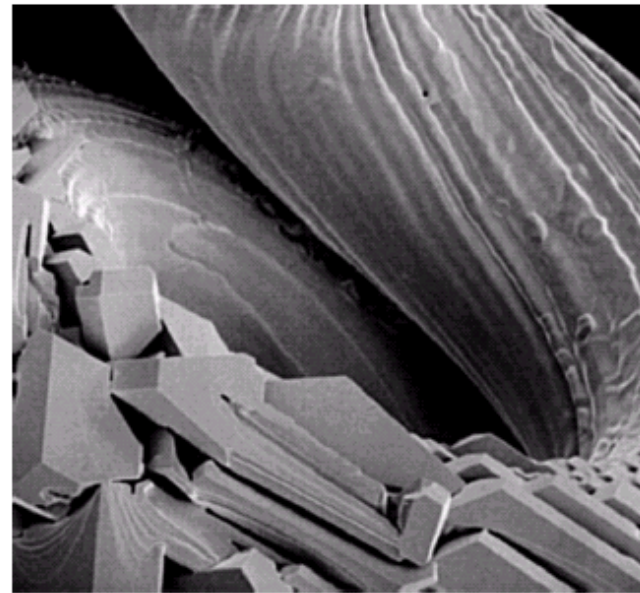
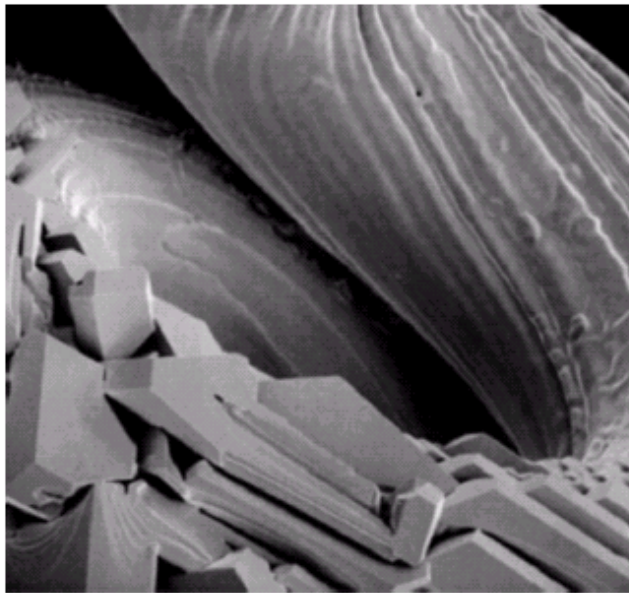
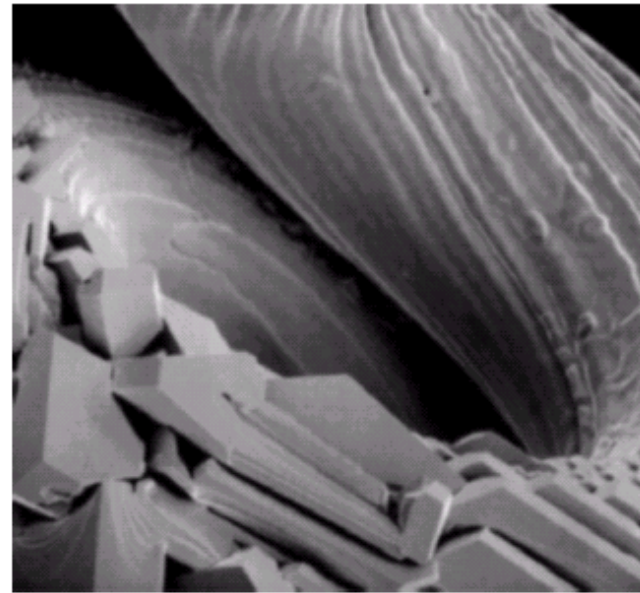
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- Simplifications

$$\begin{aligned}g(x, y) &= f(x, y) - [f(x+1, y) + f(x-1, y) + f(x, y+1) + \\ & f(x, y-1)] + 4f(x, y) \\ &= 5f(x, y) - [f(x+1, y) + f(x-1, y) + f(x, y+1) + \\ & f(x, y-1)]\end{aligned}$$

0	-1	0
-1	5	-1
0	-1	0

-1	-1	-1
-1	9	-1
-1	-1	-1



a b c  
d e

(a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

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- Unsharp masking and high-boost filtering

- Unsharp masking

- Subtract a blurred version of an image from the image itself

$$f_s(x, y) = f(x, y) - \bar{f}(x, y)$$

- $f(x, y)$  : The image,  $\bar{f}(x, y)$  : The blurred image

- 
- High-boost filtering

$$f_{hb}(x, y) = Af(x, y) - \bar{f}(x, y)$$

$$f_{hb}(x, y) = (A-1)f(x, y) + f(x, y) - \bar{f}(x, y)$$

$$f_{hb}(x, y) = (A-1)f(x, y) + f_s(x, y)$$

- 
- Use the Laplacian as the sharpening filtering

$$f_{hb} = \begin{cases} Af(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient} \\ & \text{of the Laplacian mask} \\ & \text{is negative} \\ Af(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient} \\ & \text{of the Laplacian mask} \\ & \text{is positive} \end{cases}$$

---

0	-1	0	-1	-1	-1
-1	$A + 4$	-1	-1	$A + 8$	-1
0	-1	0	-1	-1	-1

a b

The high-boost filtering technique can be implemented with either one of these masks, with  $A \geq 1$ .

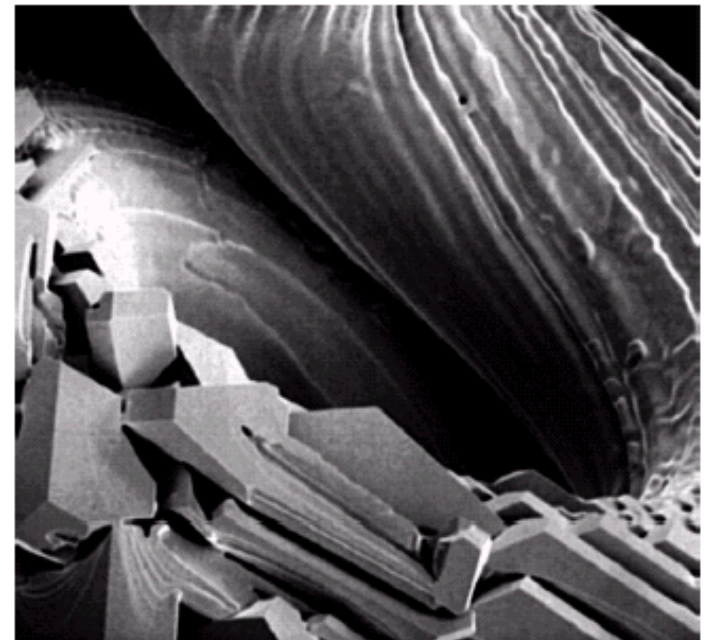
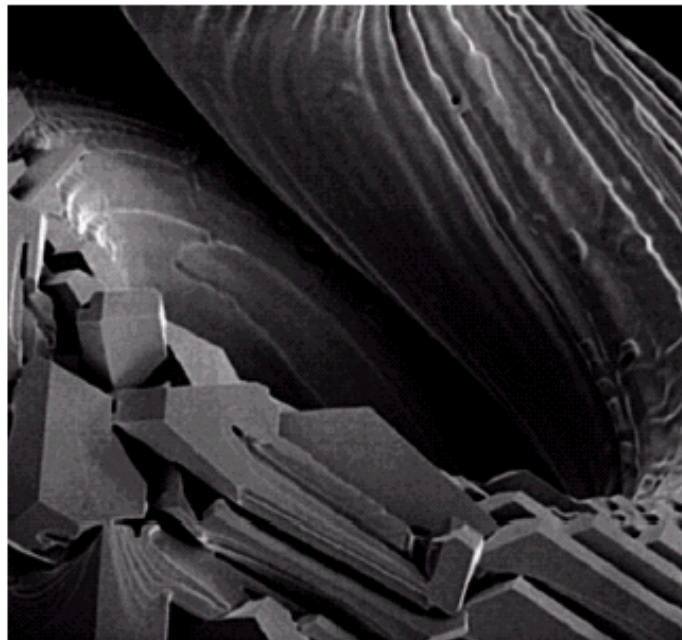
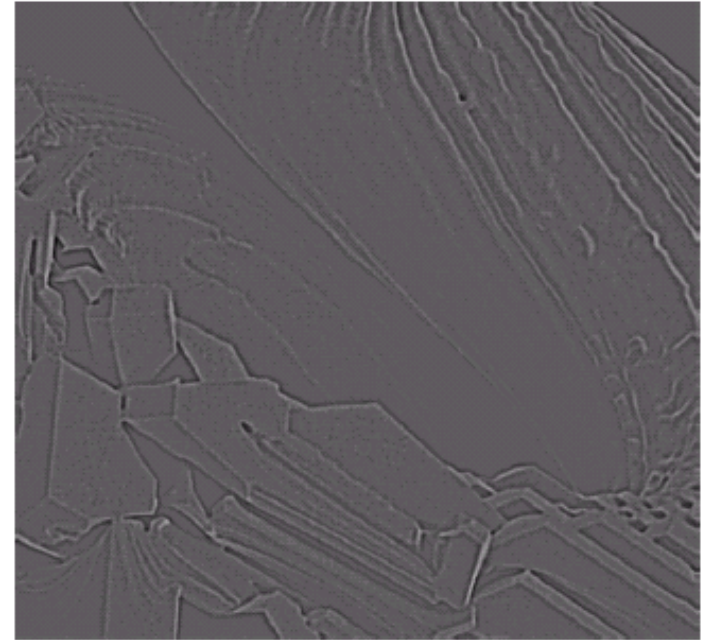
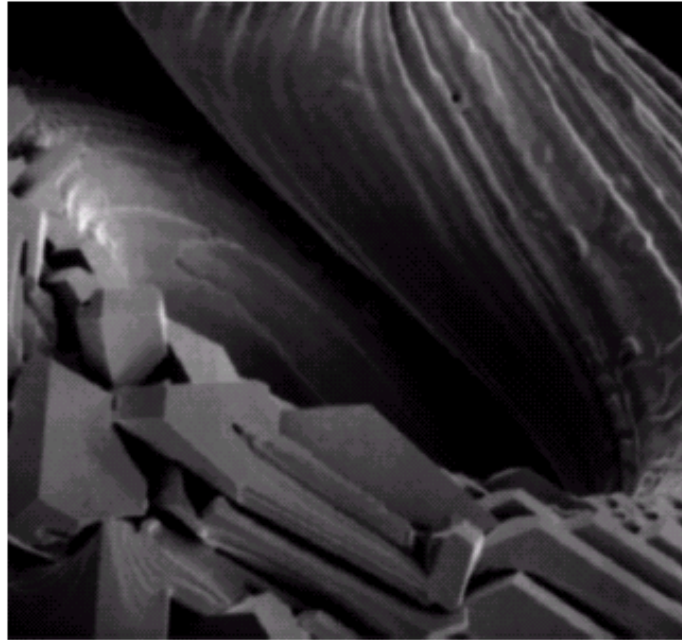


a b  
c d

(a) Same as Fig. 3.41(c), but darker.

(b) Laplacian of (a) computed with the mask in Fig. 3.42(b) using  $A = 0$ .

(c) Laplacian enhanced image using the mask in Fig. 3.42(b) with  $A = 1$ . (d) Same as (c), but using  $A = 1.7$ .



- 
- Use of first derivatives for enhancement—The gradient

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

- 
- The magnitude is rotation invariant (isotropic)

$$\nabla f = \text{mag}(\nabla \mathbf{f}) = \left[ G_x^2 + G_y^2 \right]^{1/2}$$

$$= \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

$$\nabla f \approx |G_x| + |G_y|$$

- 
- Computing using cross differences, Roberts cross-gradient operators

$$G_x = (z_9 - z_5) \quad \text{and} \quad G_y = (z_8 - z_6)$$

$$\nabla f = \left[ (z_9 - z_5)^2 + (z_8 - z_6)^2 \right]^{1/2}$$

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$



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- Sobel operators

- A weight value of 2 is to achieve some smoothing by giving more importance to the center point

$$\nabla f \approx \left| (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3) \right| \\ + \left| (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7) \right|$$

a
b c
d e

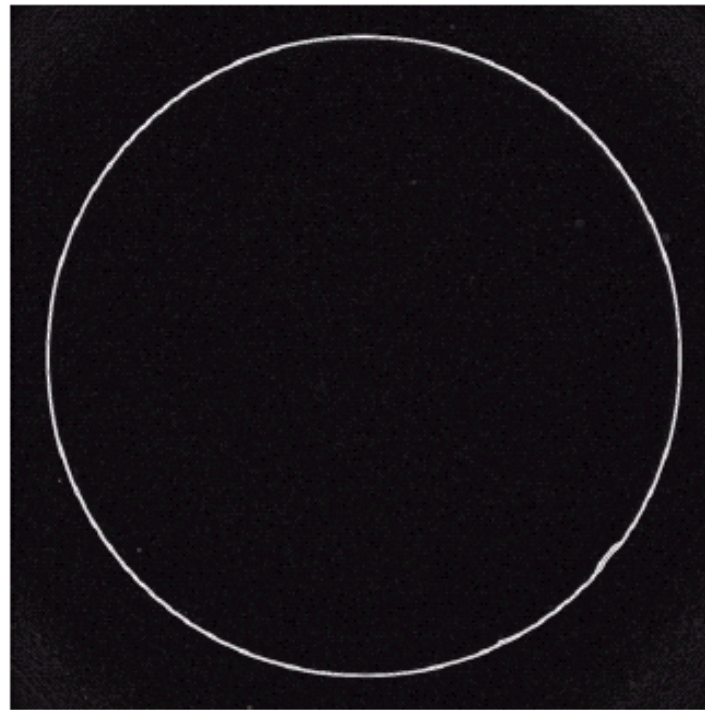
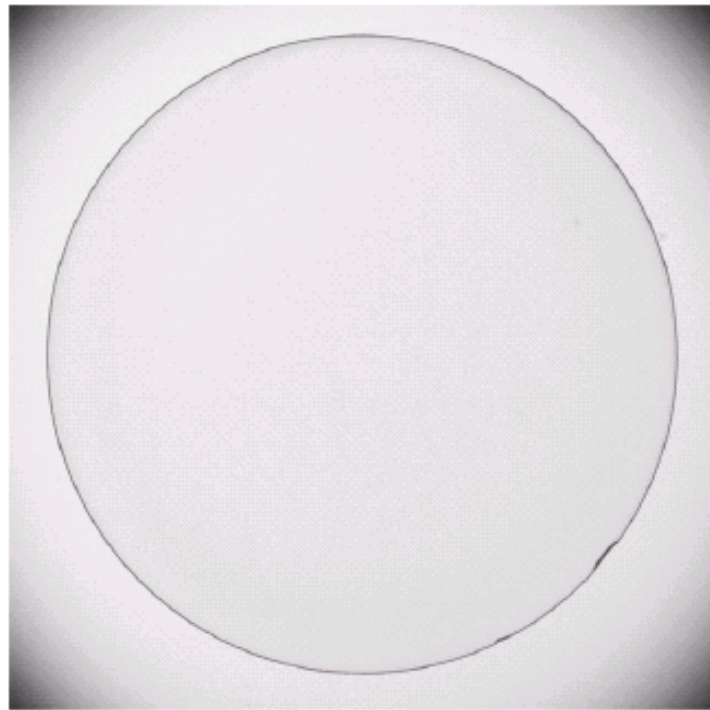
A  $3 \times 3$  region of an image (the  $z$ 's are gray-level values) and masks used to compute the gradient at point labeled  $z_5$ . All masks coefficients sum to zero, as expected of a derivative operator.

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$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$

-1	0	0	-1
0	1	1	0

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1



a b

Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).  
(b) Sobel gradient.  
(Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

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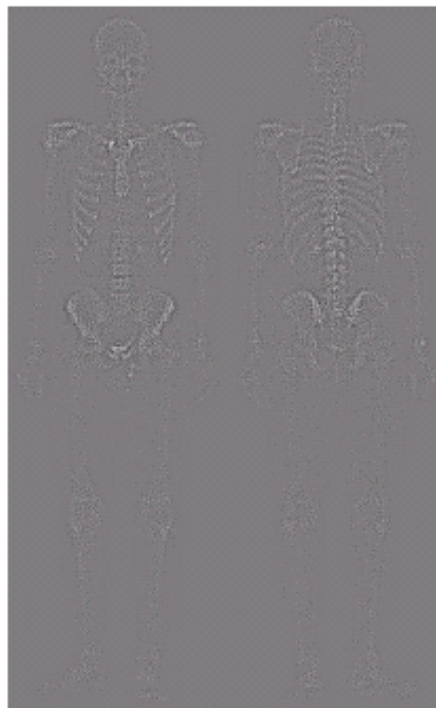
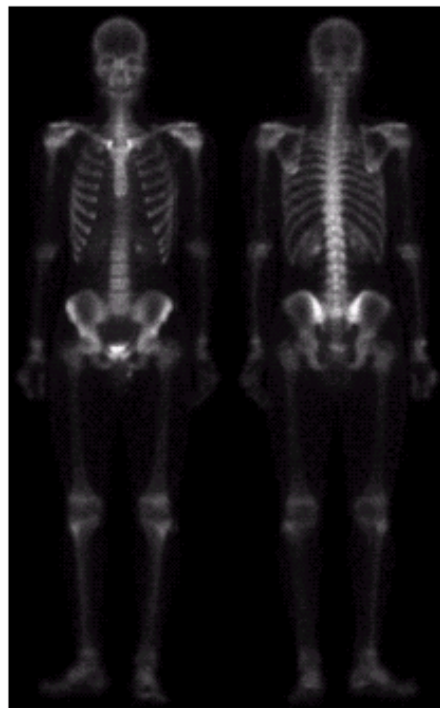
# Combining Spatial Enhancement Methods

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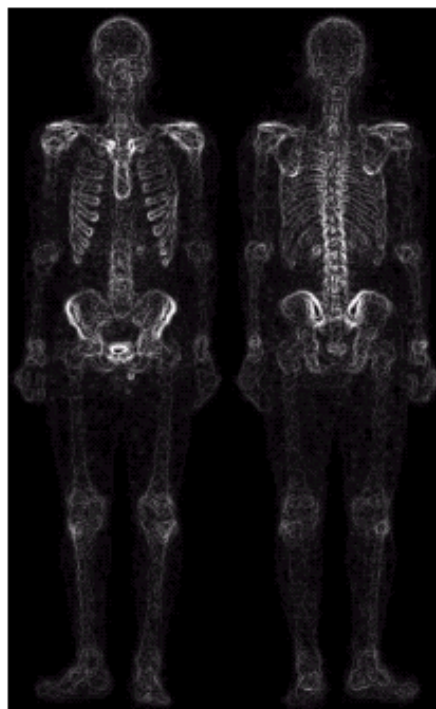
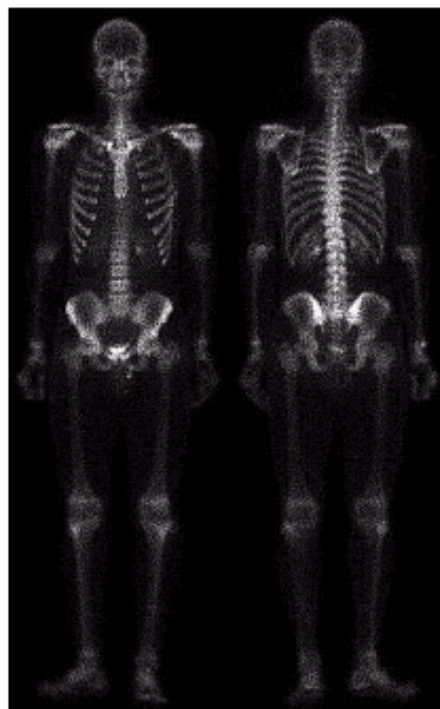
## ○ An example

- Laplacian to highlight fine detail
- Gradient to enhance prominent edges
- Smoothed version of the gradient image used to mask the Laplacian image
- Increase the dynamic range of the gray levels by using a gray-level transformation

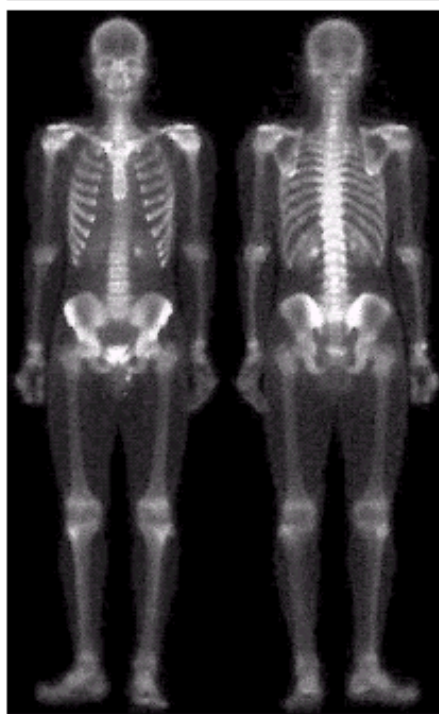
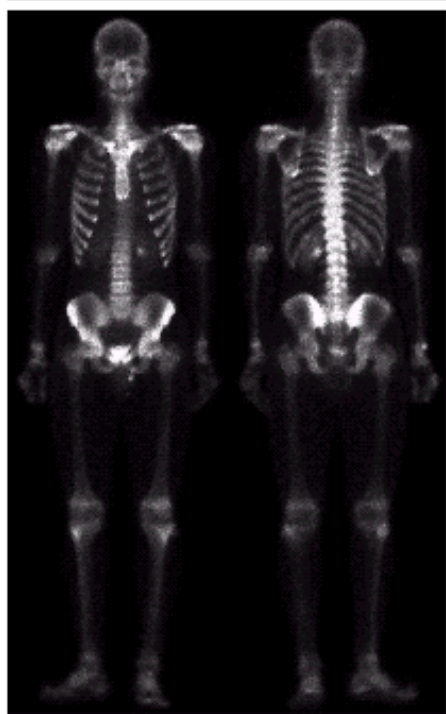
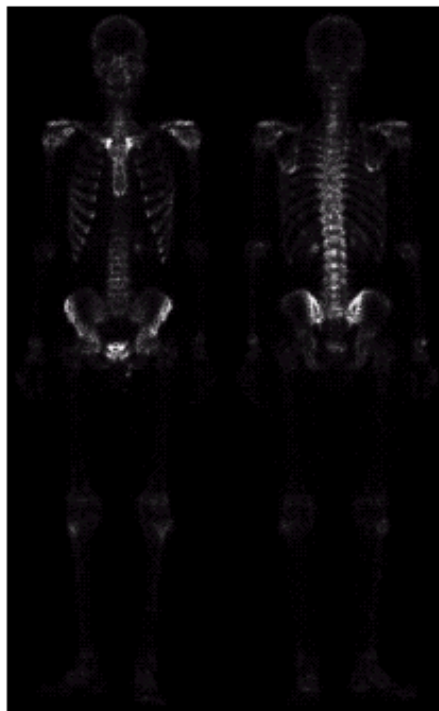
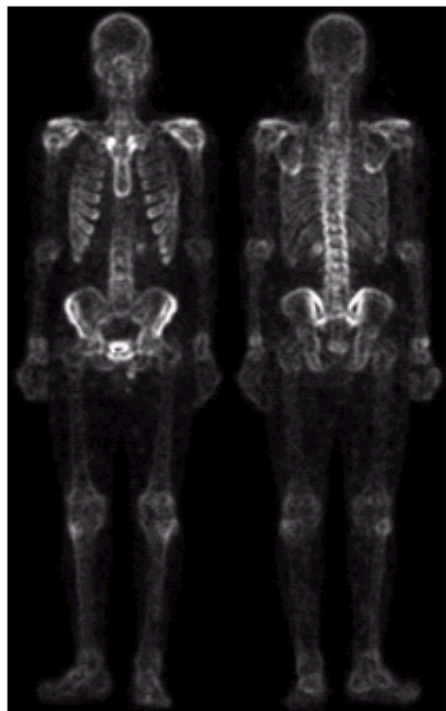




a	b
c	d



- (a) Image of whole body bone scan.  
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).



e	f
g	h

*(Continued)*

(e) Sobel image smoothed with a  $5 \times 5$  averaging filter. (f) Mask image formed by the product of (c) and (e).

(g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

# Example 2

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- **Arithmetic Operations**
- Write a computer program capable of performing the four arithmetic operations between two images. This project is generic, in the sense that it will be used in other projects to follow. In addition to multiplying two images, your multiplication function must be able to handle multiplication of an image by a constant.