# An Introduction to Control Charts

Basic Conceptions

What is a control chart?

- The control chart is a graph used to study how a process changes over time. Data are plotted in time order.
- A control chart always has a central line for the average, an upper line for the upper control limit and a lower line for the lower control limit.
- Lines are determined from historical data. By comparing current data to these lines, you can draw conclusions about whether the process variation is consistent (in control) or is unpredictable (out of control, affected by special causes of variation).
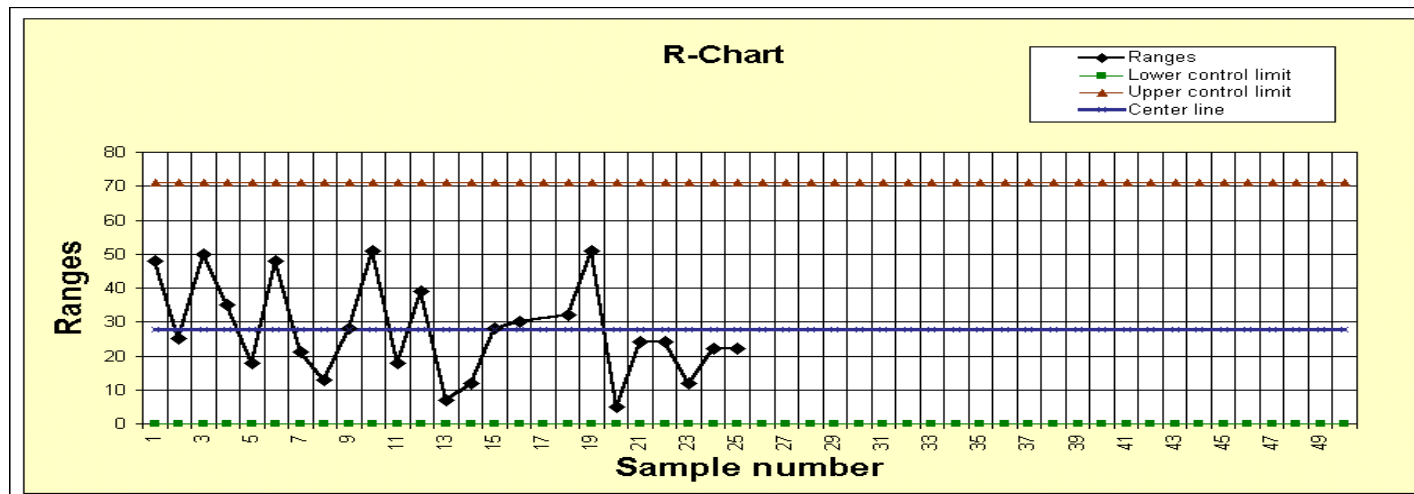
Basic Conceptions

When to use a control chart?

- Controlling ongoing processes by finding and correcting problems as they occur.

- Predicting the expected range of outcomes from a process.

- Determining whether a process is stable (in statistical control).

- Analyzing patterns of process variation from special causes (non-routine events) or common causes (built into the process).

- Determining whether the quality improvement project should aim to prevent specific problems or to make fundamental changes to the process.

Basic Conceptions

Control Chart Basic Procedure

- ❑ Choose the appropriate control chart for the data.
- ❑ Determine the appropriate time period for collecting and plotting data.
- ❑ Collect data, construct the chart and analyze the data.
- ❑ Look for "out-of-control signals" on the control chart. When one is identified, mark it on the chart and investigate the cause. Document how you investigated, what you learned, the cause and how it was corrected.
- ❑ Continue to plot data as they are generated. As each new data point is plotted, check for new out-of-control signals.

(http://www.asq.org/learn-about-quality/data-collection-analysis-tools/overview/control-chart.html)

Basic Principles

- ## Basic components of control charts
  - ❑ A centerline, usually the mathematical average of all the samples plotted;
  - ❑ Lower and upper control limits defining the constraints of common cause variations;
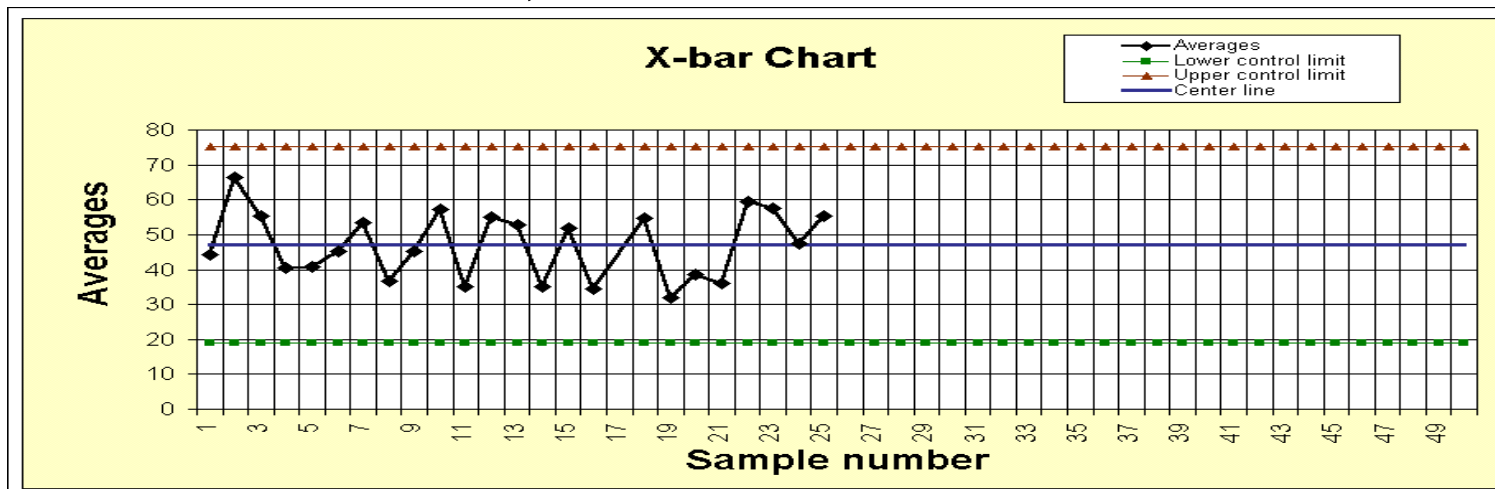
Basic Principles

- ## General model for a control chart
  - ❑ UCL = μ + kσ
  - ❑ CL = μ
  - ❑ LCL = μ – kσ

where μ is the mean of the variable, and σ is the standard deviation of the variable. UCL=upper control limit; LCL = lower control limit; CL = center line. where k is the distance of the control limits from the center line, expressed in terms of standard deviation units. When k is set to 3, we speak of 3-sigma control charts. Historically, k = 3 has become an accepted standard in industry.

(Proposed by Walter A. Shewhart in1920's)

Basic Principles of Control Charts

Types of the control charts

- Variables control charts
    - Variable data are measured on a continuous scale. For example: time, weight, distance or temperature can be measured in fractions or decimals.
    - Applied to data with continuous distribution

- Attributes control charts
    - Attribute data are counted and cannot have fractions or decimals. Attribute data arise when you are determining only the presence or absence of something: success or failure, accept or reject, correct or not correct. For example, a report can have four errors or five errors, but it cannot have four and a half errors.
    - Applied to data following discrete distribution

Basic Principles of Control Charts

- **Variables control charts**
  - X-bar and R chart (also called averages and range chart)
  - X-bar and s chart
  - moving average–moving range chart (also called MA–MR chart)
  - target charts (also called difference charts, deviation charts and nominal charts)
  - CUSUM (cumulative sum chart)
  - EWMA (exponentially weighted moving average chart)
  - multivariate chart

http://www.statgraphics.com/control_charts.htm#variables

Basic Principles of Control Charts

- **Attributes control charts**
  - p chart (proportion chart)
  - n_p chart
  - c chart (count chart)
  - u chart

R-Chart

- Always look at the Range chart first. The control limits on the X-bar chart are derived from the average range, so if the Range chart is out of control, then the control limits on the X-bar chart are meaningless.

- Look for out of control points. If there are any, then the special causes must be eliminated..

- There should be more than five distinct values plotted, and no one value should appear more than 25% of the time. If there are values repeated too often, then you have inadequate resolution of your measurements, which will adversely affect your control limit calculations. In this case, you'll have to look at how you measure the variable, and try to measure it more precisely.

- Once the effect of the out of control points from the Range chart is removed, look at the X-bar Chart.

$$UCL = \overline{R}D_4$$

$$LCL = \overline{R}D_3$$

Example: R Control Chart

In the manufacturing of a certain machine part, the percentage of aluminum in the finished part is especially critical. For each production day, the aluminum percentage of five parts is measured. The table below consists of the average aluminum percentage of ten consecutive production days, along with the minimum and maximum sample values (aluminum percentage) for each day. The sum of the 10 samples means (below) is 258.8.

Day	1	2	3	4	5	6	7	8	9	10
Sample Mean	25.2	26.0	25.2	25.2	26.0	25.6	26.0	26.0	24.6	29.0
Maximum Value	26.6	27.6	27.7	27.4	27.6	27.4	27.5	27.9	26.8	31.6
Minimum Value	23.5	24.4	24.6	23.2	23.3	23.3	24.1	23.8	23.5	27.4
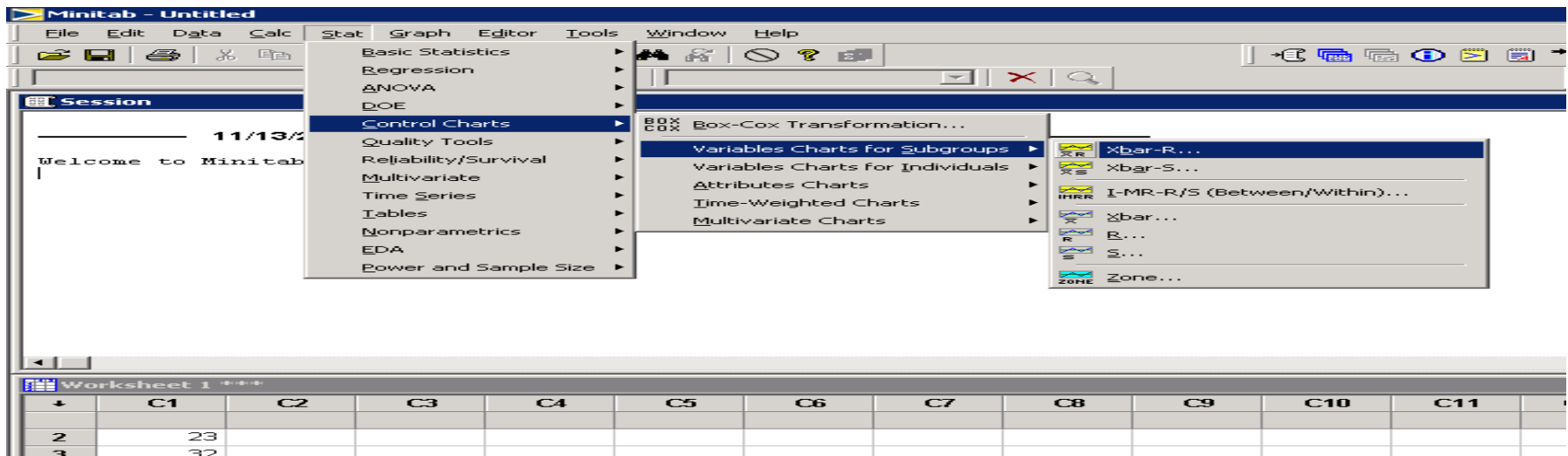
X-bar Chart

$$UCL = \bar{\bar{x}} + A_3 \bar{s} (x and S chart)$$

$$LCL = \bar{\bar{x}} - A_3 \bar{s} (x and S chart)$$

$$A_2 \bar{R} \, for \, \bar{x} and R chart$$

- The X-bar chart monitors the process location over time, based on the average of a series of observations, called a subgroup.
- **X-bar / Range charts are used when you can rationally collect measurements in groups (subgroups) of between two and ten observations**. Each subgroup represents a "snapshot" of the process at a given point in time. The charts' x-axes are time based, so that the charts show a history of the process. For this reason, data should be time-ordered; that is, entered in the sequence from which it was generated. If this is not the case, then trends or shifts in the process may not be detected, but instead attributed to random (common cause) variation.
- **For subgroup sizes greater than ten, use X-bar / Sigma charts**, since the range statistic is a poor estimator of process sigma for large subgroups. In fact, the subgroup sigma is always a better estimate of subgroup variation than subgroup range. The popularity of the Range chart is only due to its ease of calculation, dating to its use before the advent of computers.
- For **subgroup sizes equal to one**, an Individual-X / Moving Range chart can be used, as well as EWMA or CuSum charts.
- X-bar Charts are efficient at detecting relatively large shifts in the process average, typically shifts of +-1.5 sigma or larger. The larger the subgroup, the more sensitive the chart will be to shifts, providing a Rational Subgroup can be formed. For more sensitivity to smaller process shifts, use an EWMA or CuSum chart.

S Chart

- The sample standard deviations are plotted in order to control the variability of a variable.

- For sample size (n>10), the S-chart is more efficient than R-chart.

- For situations where sample size exceeds 10, the X-bar chart and the S-chart should be used.

$$s_i = \sqrt{\frac{\sum (x_i - \overline{x})^2}{n-1}}$$

$$\overline{s} = \frac{\sum s_i}{k}$$

$$UCL = \overline{s}B_4$$

$$LCL = \overline{s}B_3$$

S**2 Chart

- In this chart, the sample variances are plotted in order to control the variability of a variable.

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}$$

(http://www.statsoft.com/textbook/stquacon.html)

Moving Average (MA)/Range Chart

- Moving Average / Range Charts are a set of control charts for variables data.

- The Moving Average chart monitors the process location over time, based on the average of the current subgroup and one or more prior subgroups. The Range chart monitors the process variation over time.

- Moving Average Charts are generally used for detecting **small shifts** in the process mean. **They will detect shifts of .5 sigma to 2 sigma much faster. They are, however, slower in detecting large shifts in the process mean.**

- Always look at the Range chart first. The control limits on the Moving Average chart are derived from the average range, so if the Range chart is out of control, then the control limits on the Moving Average chart are meaningless.

(http://www.statsoft.com/textbook/stquacon.html)

Cumulative Sum (CUSUM) Chart

- The CUSUM chart reacts more sensitively than the X-bar chart to a shifting of the mean value in the range of 0.5-2s; therefore, it is sited for monitoring processes with a high degree of imprecision.

- If one plots the cumulative sum of deviations of successive sample means from a target specification, even minor, permanent shifts in the process mean will eventually lead to a sizable cumulative sum of deviations. Thus, this chart is particularly well-suited for detecting such small permanent shifts that may go undetected when using the X-bar chart.

(http://www.statsoft.com/textbook/stquacon.html)

Exponentially-weighted Moving Average (EWMA) Chart

- The idea of moving averages of successive (adjacent) samples can be generalized. In principle, in order to detect a trend we need to weight successive samples to form a moving average; however, instead of a simple arithmetic moving average, we could compute a geometric moving average. It is also called Geometric Moving Average chart, see Montgomery, 1985, 1991).

- EWMA Charts are generally used for detecting small shifts in the process mean. They will detect shifts of .5 sigma to 2 sigma much faster. They are, however, slower in detecting large shifts in the process mean. In addition, typical run tests cannot be used because of the inherent dependence of data points.

- EWMA Charts may also be preferred when the subgroups are of size n=1.

$$EWMA_{(t+1)} = \lambda Y_t + (1-\lambda)EWMA_t$$

$$UCL = EWMA_1 + ks\sqrt{\frac{\lambda}{2-\lambda}}$$

$$LCL = EWMA_1 - ks\sqrt{\frac{\lambda}{2-\lambda}}$$

where λ is the weighting factor. The factor k is chosen generally to be 2 or 3.

Attributes Control Charts

- An example of a common quality characteristic classification would be designating units as "conforming units" or "nonconforming units". Another quality characteristic criteria would be sorting units into "non defective" and "defective" categories. Quality characteristics of that type are called attributes.

- Examples of quality characteristics that are attributes are the number of failures in a production run, the proportion of malfunctioning wafers in a lot, the number of people eating in the cafeteria on a given day, etc.

- Control charts dealing with the number of defects or nonconformities are called c charts (for count).

- Control charts dealing with the proportion or fraction of defective product are called p chart (for proportion).

P-Chart

- To evaluate process stability when counting the fraction defective.
- It is used when the sample size varies: the total number of circuit boards, meals, or bills delivered varies from one sampling period to the next.

Repeated samples of 150 coffee cans are inspected to determine whether a can is out of round or whether it contains leaks due to improper construction. Such a can is said to be nonconforming. Following is the data.

Sample	1	2	3	4	5	6	7	8	9	10
Nonconforming#	19	10	4	6	8	9	3	1	0	4

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_j}}$$

$$LCL = \max[\,0, \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_j}}\,]$$

n$_p$-Chart

- Evaluating process stability when counting the fraction defective.
- The n$_p$ chart is useful when it's easy to count the number of defective items and the sample size is always the same. Examples might include: the number of defective circuit boards, meals in a restaurant, teller interactions in a bank, invoices, or bills.
- A fully capable process delivers zero defects. Although this may be difficult to achieve, it should still be our goal. Once we resolve the out-of-control point, we could use the quality problem solving process to begin to eliminate the common causes of defective paychecks. What are the most common types of paycheck errors? Why do they occur? What are the root causes of these paycheck errors?

$$\overline{p} = \frac{\sum_{j=1}^{m}(count)_j}{m \bullet n}$$

$$\overline{np} = \frac{\sum_{j=1}^{m}(count)_j}{m}$$

$$UCL_{np} = n\overline{p} + 3\sqrt{n\overline{p}\left(1 - \overline{p}\right)}$$

$$LCL_{np} = MAX\left[0, n\overline{p} - 3\sqrt{n\overline{p}\left(1 - \overline{p}\right)}\right]$$

C-Chart

- Determining stability of "counted" data (e.g., errors per widget, inquiries per month, etc.)
- The c chart will help evaluate process stability when there can be more than one defect per unit. Examples might include: the number of defective elements on a circuit board, the number of defects in a dining experience--order wrong, food too cold, check wrong, or the number of defects in bank statement, invoice, or bill.
- This chart is especially useful when you want to know how many defects there are not just how many defective items there are.
- The c chart is useful when it's easy to count the number of defects and the sample size is always the same.

An automobile assembly worker is interested in monitoring and controlling the # of minor paint blemishes appearing on the outside door panel on the driver's side of a certain make of automobile. The following data were obtained, using a sample of 25 door panel.

Sample	1	2	3	4	5	6	7	-----	-----	25
# of Paint Blemishes	19	10	4	6	8	9	3	------	-----	4

$$\bar{c} = \frac{\sum_{j=1}^{m}(count)_j}{m}$$

.com/qiwizard/cchart.html)

$$UCL = \bar{c} + 3\sqrt{\bar{c}} \qquad LCL = MAX\left[0, \bar{c} - 3\sqrt{\bar{c}}\right]$$

U-Chart

- Determining stability of "counted" data (e.g., errors per widget, inquiries per month, etc.) when the sample size varies.
- The u chart will help evaluate process stability when there can be more than one defect per unit.
- This chart is especially useful when you want to know how many defects there are not just how many defective items there are. It's one thing to know how many defective circuit boards, meals, statements, invoices, or bills there are; it is another thing to know how many defects were found in these defective items.
- It is used when the sample size varies: the number of circuit boards, meals, or bills delivered each day varies.

$$u_j = \frac{(count)_j}{n_j}$$

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_j}}$$

$$\bar{u} = \frac{\sum_{j=1}^{m}(count)_j}{m}$$

$$LCL = MAX\left[0, \bar{u} - 3\sqrt{\frac{\bar{u}}{n_j}}\right]$$

(http://www.qimacros.com/qiwizard/uchart.html)