

Binary Sign Representations

Sign-magnitude: The left bit is the sign (0 for + numbers and 1 for – numbers).

All bits to right are the number magnitude

Left bit is the sign bit

Advantages to sign-magnitude:

- Simple to implement.

- Useful for floating point representation.

Disadvantage of sign-magnitude:

If $m = 00000000$, then $m = 11111111$; there are two zeros in this method as well.

In an n -bit representation, there are no extra bits! If adding 2 n -bit numbers results in $n+1$ bits, the left most bit is discarded! Thus: Let $n = 0000\ 0000$. Then $m = n + 1$, or $m = 1111\ 1111 + 1 = (1)\ 0000\ 0000 = 0000\ 0000$. The 1 is discarded, since in a computer, there are no extra columns. There are only 8-bits, so the (9th-column) 1 is “thrown away.”

Therefore, the 2's complement of 0 is 0.

Finding Two's Complements: Examples

To convert a negative decimal number to 2's complement binary: Convert the decimal number to a positive binary number.

Take the 1's complement of that binary number and add 1.

- Converting negative numbers (still using a single 8-bit byte length):

– 50: 50 = 0011 0010; 1's C. = 1100 1101; 2's C. = 1100 1110.

– 127: 127 = 0111 1111; 1's C. = 1000 0000; 2's C. = 1000 0001.

– 1: 1 = 0000 0001; 1's C. = 1111 1110; 2's
C. = 1111 1111.

But: Positive decimal numbers are converted
simply to positive binary numbers as before
(no 2's complement).

Example: +67 (using method of successive div.)
→ 0100 0011.

Two's Complement Binary to Decimal

Converting the 2's complement to decimal is also simple. Simply do the following: Check the sign bit (left-most bit).

If the sign bit is 0 (positive number), simply convert the number directly to a positive decimal number as we learned previously.

If the sign bit is 1, the number is a 2's complement negative number. To convert this number to decimal: Take the 2's complement of the negative binary number. Convert the resulting + number to decimal and add a negative sign.

Two's Complement Binary to Decimal (2)

Binary 2's complement-to-decimal examples, negative numbers:

$$1111\ 1111 \rightarrow 0000\ 0000 + 1 = 0000\ 0001 = 1 \rightarrow -1.$$

$$1010\ 0011 \rightarrow 0101\ 1100 + 1 = 0101\ 1101 = 93 \rightarrow -93.$$

$$1000\ 1111 \rightarrow 0111\ 0000 + 1 = 0111\ 0001 = 113 \rightarrow -113.$$

$$1000\ 0010 \rightarrow 0111\ 1101 + 1 = 0111\ 1110 = 126 \rightarrow -126.$$

But for a positive binary number:

$$0000\ 0001 \rightarrow \text{Not a negative number} \rightarrow 1.$$

$$0000\ 1111 \rightarrow \text{Not a negative number} \rightarrow 15.$$

$$0110\ 1100 \rightarrow \text{Not a negative number} \rightarrow 108.$$

$$0111\ 1111 \rightarrow \text{Not a negative number} \rightarrow 127.$$

To subtract binary number b from a , simply take the 2's complement of b , and add to a .

That is:

$$a - b = a + (\text{2's comp. of } b) = a + (b + 1) = a + b + 1.$$

To add a positive and negative number, simply perform the addition and the minus sign (i.e., the left-most bit in the number) will take care of itself (assuming the result is within the range of representation).

Two's Complement Math

Subtract 0111 0101 from 0111 1100:

The 2's complement of 0111 0101 is $1000\ 1010 + 1 = 1000\ 1011$.

Adding:

$$\begin{array}{r} 0111\ 1100 \quad \text{Check: } 124 \\ + \underline{1000\ 1011} \quad \quad \quad - \underline{117} \\ (1)0000\ 0111\ 007 \quad \quad \quad 007 \end{array}$$

Add 1100 0001 + 0110 1110:

Note that the 2's complement of 1100 0001 is 0011 1111, so the first number is equivalent to -63 decimal.

Adding:

$$\begin{array}{r} 1100\ 0001 \quad \text{Check: } -63 \\ + \underline{0110\ 1110} \quad \quad \quad + \underline{110} \\ (1)0010\ 1111 \quad \quad \quad +47 \end{array}$$

Subtract 1101 1101 from 0101 1100 (note we are subtracting a negative number):

Adding:	0101 1100	Check:	92
	+ 0010 0011		<u>-(-35)</u>
	0111 1111		+127

Add 1000 0001 + 0111 0010

0111 0010	check:	114
+ 1000 0001		<u>+(-127)</u>
1111 0011		-13

Check: 2's C of 1111 0011 = 0000 1101 = 13, so the number = -13.