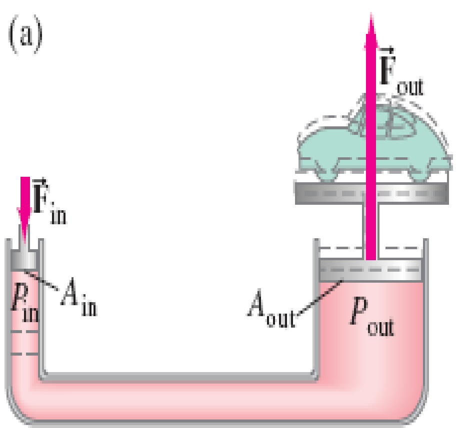
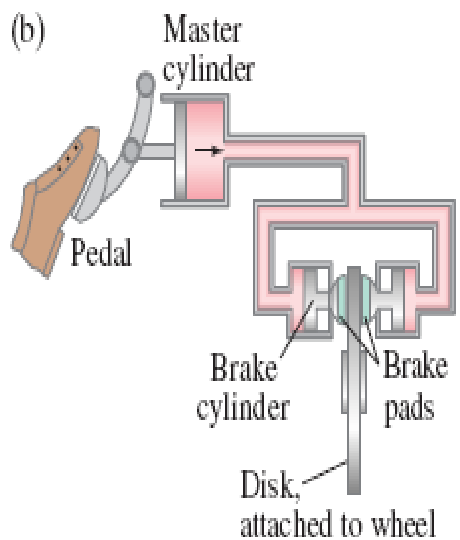
Buoyancy

10.5 Pascal's Principle

- Pascal's Principle - if an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount. Applications: hydraulic lift and brakes





Pout = Pin

And since P = F/a

$$\frac{Fout}{Aout} = \frac{Fin}{Ain}$$

Mechanical Advantage:

$$\frac{Fout}{Fin} = \frac{Aout}{Ain}$$

Problem 10-10

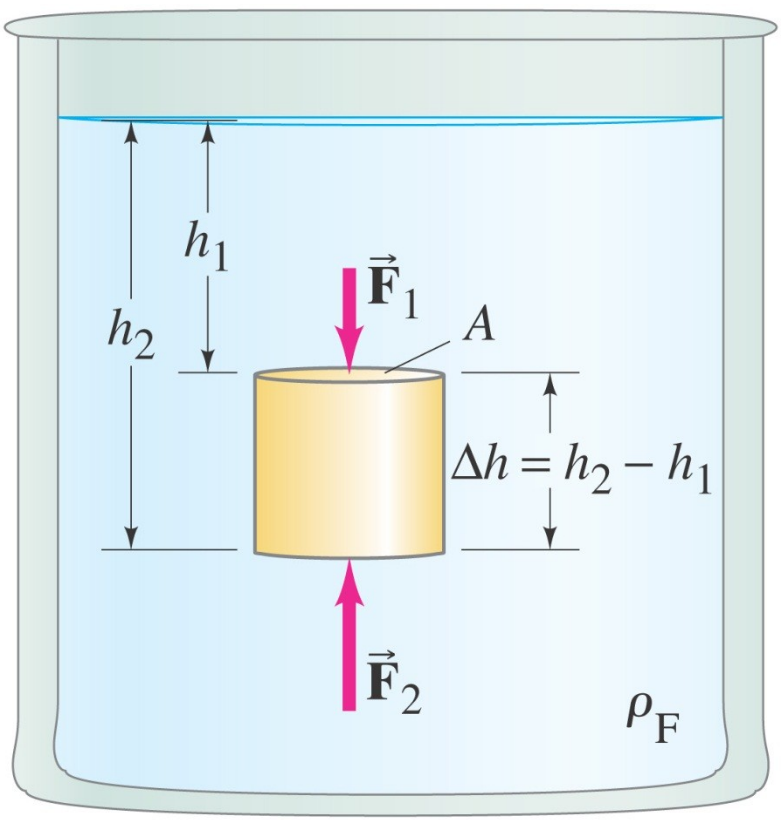
10. (II) In a movie, Tarzan evades his captors by hiding underwater for many minutes while breathing through a long, thin reed. Assuming the maximum pressure difference his lungs can manage and still breathe is calculate the deepest he could have been. (See page 261.)

10. The pressure difference on the lungs is the pressure change from the depth of water

$$\Delta P = \rho g \Delta h \quad \rightarrow \quad \Delta h = \frac{\Delta P}{\rho g} = \frac{(85\,\text{mm-Hg})\left(\dfrac{133\,\text{N/m}^2}{1\,\text{mm-Hg}}\right)}{(1.00\times10^3\,\text{kg/m}^3)(9.80\,\text{m/s}^2)} = 1.154\,\text{m} \approx \boxed{1.2\,\text{m}}$$

10-7 Buoyancy and Archimedes' Principle

This is an object **submerged** in a fluid. There is a net **force** on the object because the **pressures** at the top and bottom of it are different.

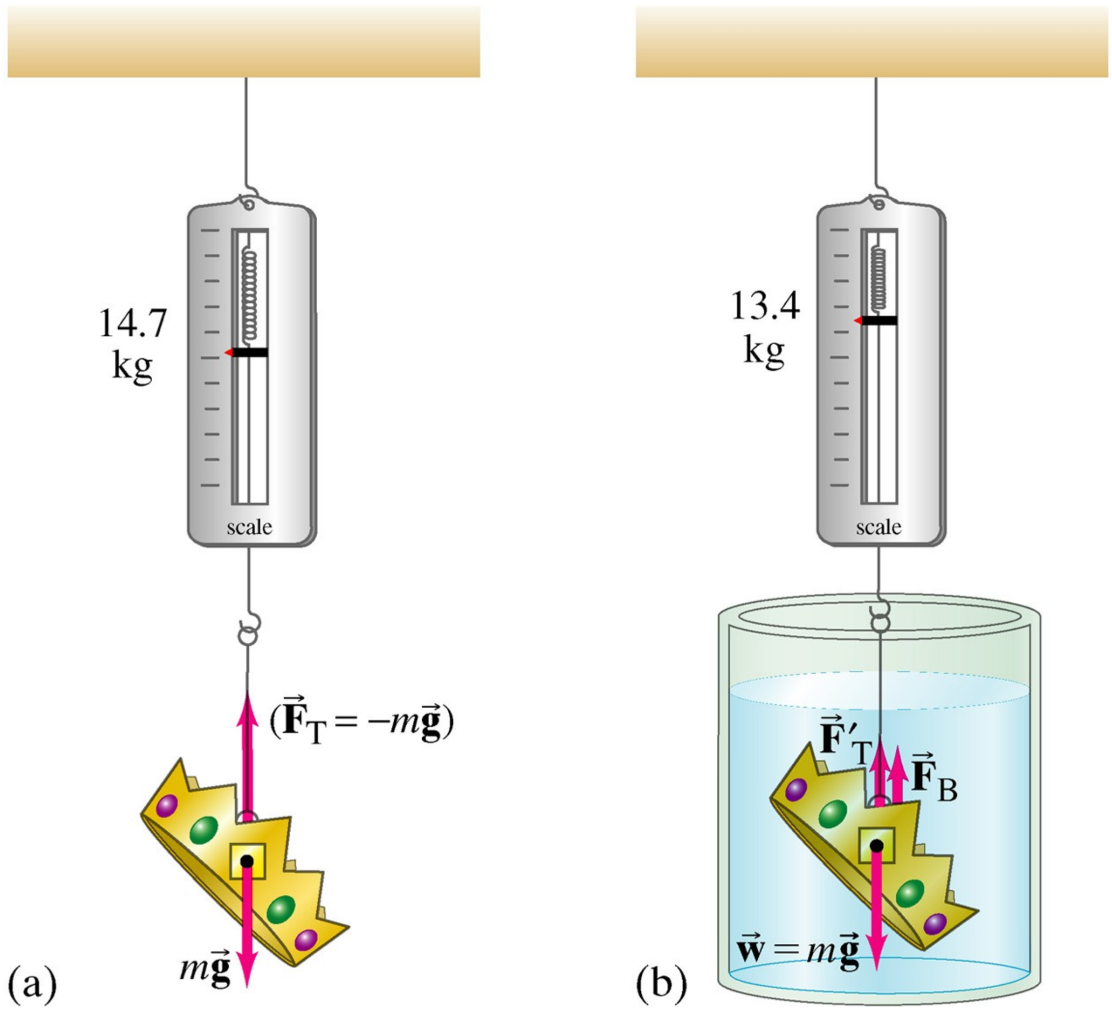


The **buoyant force** is found to be the **upward** force on the **same** volume of water:

$$\begin{aligned} F_B = F_2 - F_1 &= \rho_F g A (h_2 - h_1) \\ &= \rho_F g A \Delta h \\ &= \rho_F V g \\ &= m_F g, \end{aligned}$$

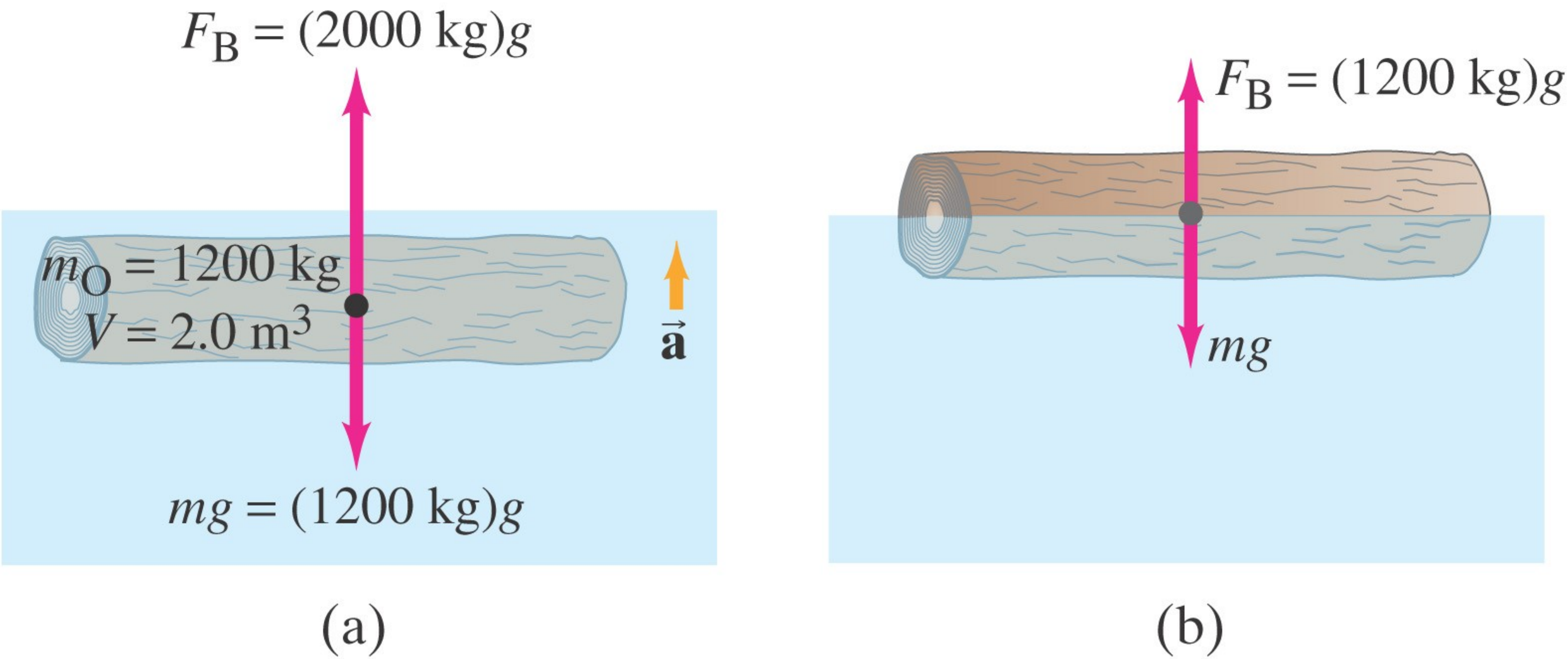
10-7 Buoyancy and Archimedes' Principle

The **net force on** the object is then the difference between the **buoyant force** and the **gravitational** force.



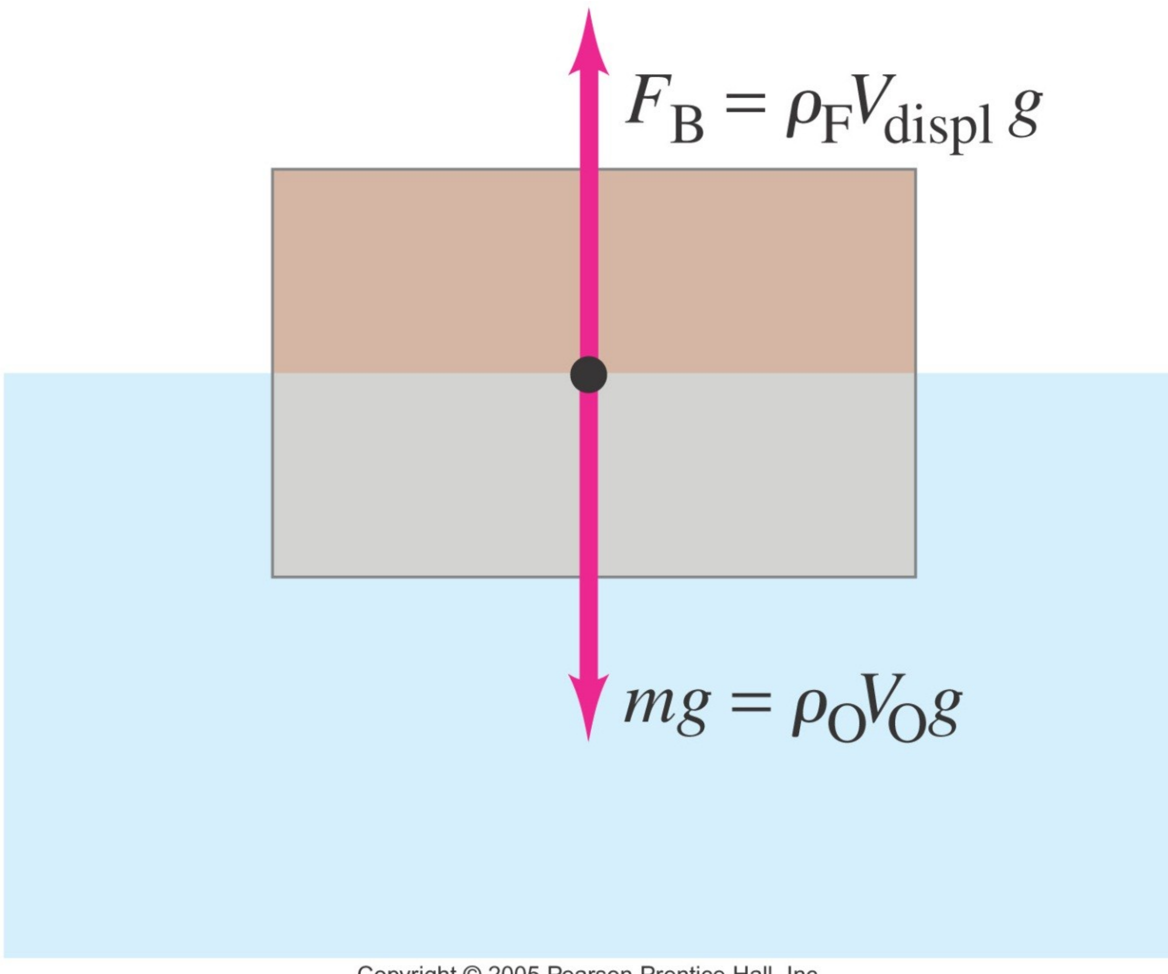
10-7 Buoyancy and Archimedes' Principle

If the object's **density** is **less** than that of water, there will be an **upward** net force on it, and it will rise until it is partially **out** of the water.

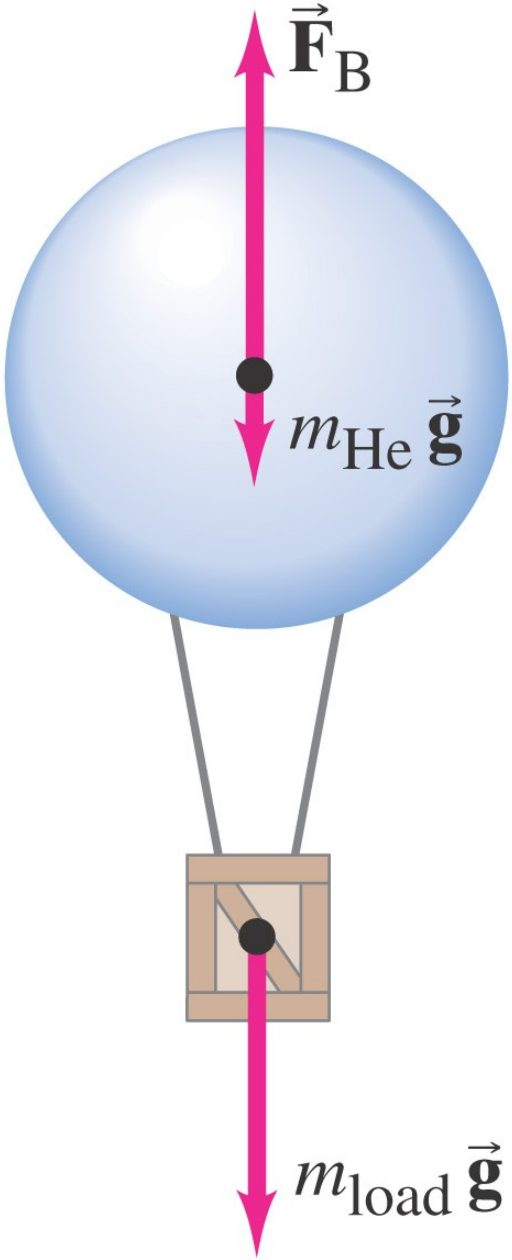


10-7 Buoyancy and Archimedes' Principle

For a **floating** object, the fraction that is **submerged** is given by the ratio of the object's **density** to that of the fluid.



10-7 Buoyancy and Archimedes' Principle



This principle also works in the air; this is why hot-air and helium balloons rise.

22. (I) A geologist finds that a Moon rock whose mass is 9.28 kg has an apparent mass of 6.18 kg when submerged in water. What is the density of the rock?

22. The difference in the actual mass and the apparent mass is the mass of the water displaced by the rock. The mass of the water displaced is the volume of the rock times the density of water, and the volume of the rock is the mass of the rock divided by its density. Combining these relationships yields an expression for the density of the rock.

$$m_{\text{actual}} - m_{\text{apparent}} = \Delta m = \rho_{\text{water}} V_{\text{rock}} = \rho_{\text{water}} \frac{m_{\text{rock}}}{\rho_{\text{rock}}} \rightarrow$$

$$\rho_{\text{rock}} = \rho_{\text{water}} \frac{m_{\text{rock}}}{\Delta m} = \left(1.00\times10^{3}\ \text{kg/m}^{3}\right)\frac{9.28\ \text{kg}}{9.28\ \text{kg} - 6.18\ \text{kg}} = \boxed{2.99\times10^{3}\ \text{kg/m}^{3}}$$

24.(II) A crane lifts the 18,000-kg steel hull of a ship out of the water. Determine (*a*) the tension in the crane's cable when the hull is submerged in the water, and (*b*) the tension when the hull is completely out of the water.

24.(*a*) When the hull is submerged, both the buoyant force and the tension force act upward on the hull, and so their sum is equal to the weight of the hull. The buoyant force is the weight of the water displaced.

$$T + F_{\text{buoyant}} = mg \quad \rightarrow$$

$$T = mg - F_{\text{buoyant}} = m_{\text{hull}}g - \rho_{\text{water}}V_{\text{sub}}g = m_{\text{hull}}g - \rho_{\text{water}}\frac{m_{\text{hull}}}{\rho_{\text{hull}}}g = m_{\text{hull}}g\left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{hull}}}\right)$$

$$= \left(1.8\times10^{4}\,\text{kg}\right)\left(9.80\,\text{m/s}^{2}\right)\left(1 - \frac{1.00\times10^{3}\,\text{kg/m}^{3}}{7.8\times10^{3}\,\text{kg/m}^{3}}\right) = 1.538\times10^{5}\,\text{N} \approx \boxed{1.5\times10^{5}\,\text{N}}$$

24. (*b*)When the hull is completely out of the water, the tension in the crane's cable must be equal to the weight of the hull.

$$T = mg = \left(1.8\times10^{4}\,\text{kg}\right)\left(9.80\,\text{m/s}^{2}\right) = 1.764\times10^{5}\,\text{N} \approx \boxed{1.8\times10^{5}\,\text{N}}$$

$$(\mathrm{SG} = 0.50)$$

34.(III) A 5.25-kg piece of wood floats on water. What minimum mass of lead, hung from the wood by a string, will cause it to sink?

34. For the combination to just barely sink, the total weight of the wood and the lead must be equal to the total buoyant force on the wood and the lead.

$$F_{\text{weight}} = F_{\text{buoyant}} \quad \rightarrow \quad m_{\text{wood}} g + m_{\text{Pb}} g = V_{\text{wood}} \rho_{\text{water}} g + V_{\text{Pb}} \rho_{\text{water}} g \quad \rightarrow$$

$$m_{\text{wood}} + m_{\text{Pb}} = \frac{m_{\text{wood}}}{\rho_{\text{wood}}} \rho_{\text{water}} + \frac{m_{\text{Pb}}}{\rho_{\text{Pb}}} \rho_{\text{water}} \quad \rightarrow \quad m_{\text{Pb}} \left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{Pb}}} \right) = m_{\text{wood}} \left(\frac{\rho_{\text{water}}}{\rho_{\text{wood}}} - 1 \right) \quad \rightarrow$$

$$m_{\text{Pb}} = m_{\text{wood}} \frac{\left(\dfrac{\rho_{\text{water}}}{\rho_{\text{wood}}} - 1 \right)}{\left(1 - \dfrac{\rho_{\text{water}}}{\rho_{\text{Pb}}} \right)} = m_{\text{wood}} \frac{\left(\dfrac{1}{SG_{\text{wood}}} - 1 \right)}{\left(1 - \dfrac{1}{SG_{\text{Pb}}} \right)} = (5.25\,\text{kg}) \frac{\left(\dfrac{1}{0.50} - 1 \right)}{\left(1 - \dfrac{1}{11.3} \right)} = \boxed{5.76\,\text{kg}}$$

10-8 Fluids in Motion; Flow Rate and the Equation of Continuity

We will deal with **laminar** flow.

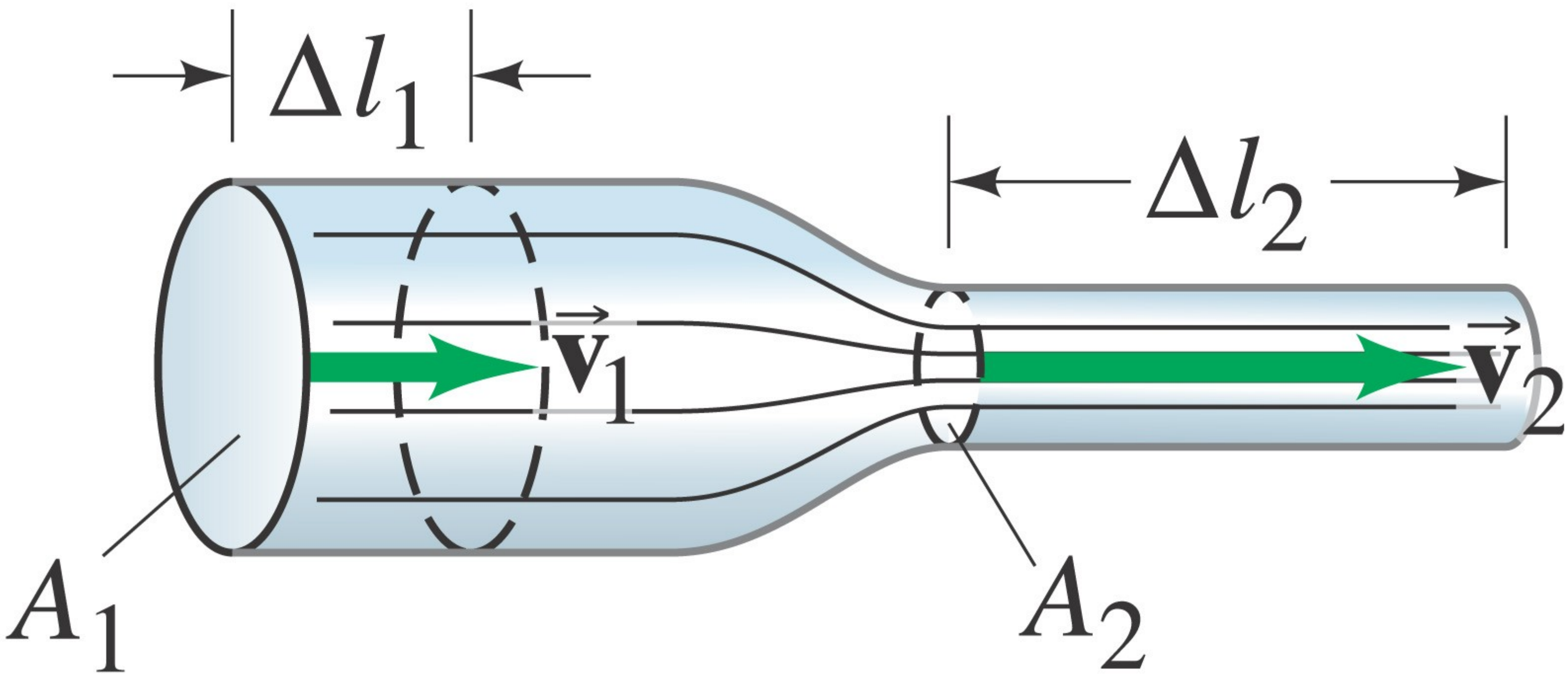
The **mass flow rate** is the mass that passes a given point per unit time. The flow rates at any two points must be **equal**, as long as no fluid is being added or taken away.

This gives us the **equation of continuity**:

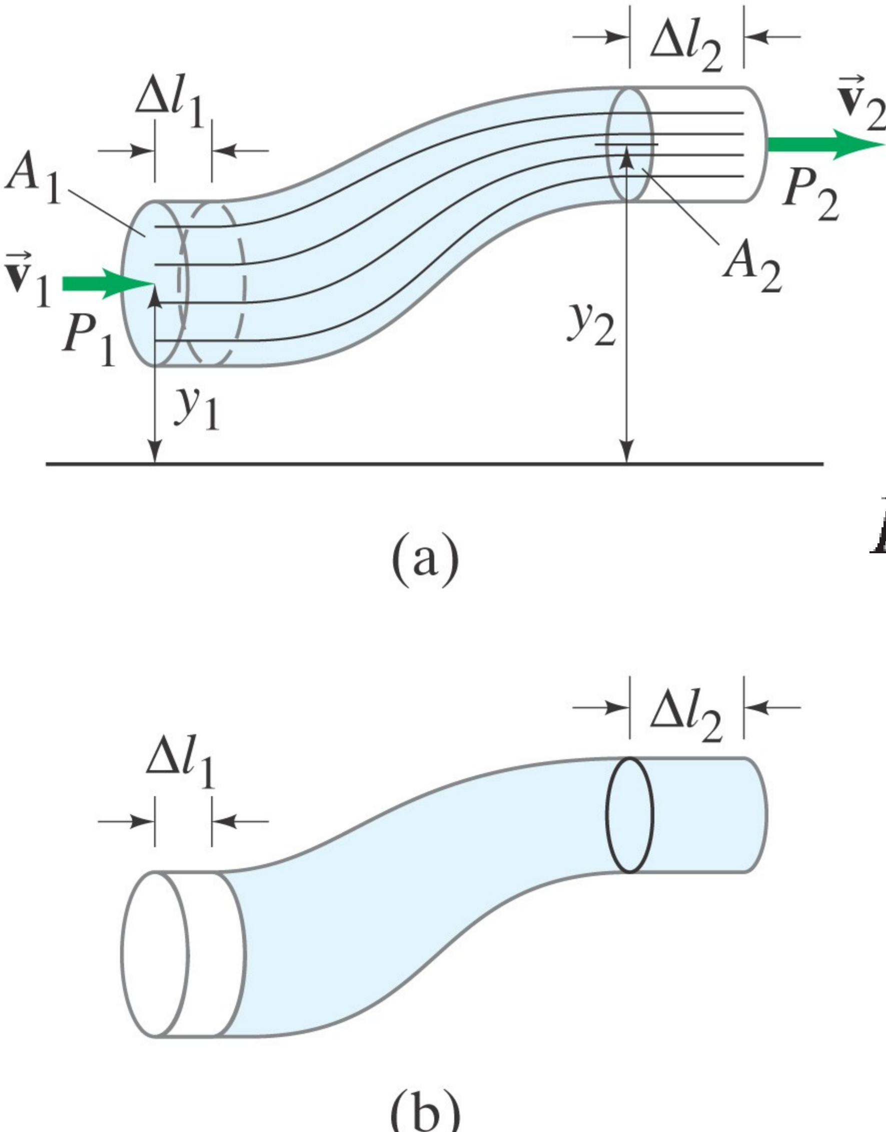
$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \qquad \text{(10-4a)}$$

10-8 Fluids in Motion; Flow Rate and the Equation of Continuity

If the density doesn't change – typical for liquids – this simplifies to $A_1 v_1 = A_2 v_2$.
Where the pipe is **wider**, the flow is **slower**.



10-9 Bernoulli's Equation



A fluid can also change its **height**. By looking at the work done as it moves, we find:

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

This is **Bernoulli's** equation. One thing it tells us is that as the **speed** goes up, the **pressure** goes down.

$$9.2\,\text{m}\times 5.0\,\text{m}\times 4.5\,\text{m}$$

36. (I) A 15-cm-radius air duct is used to replenish the air of a room every 16 min.

$2.0\,\text{cm}^2$.

How fast does air flow in the duct?

36. We apply the equation of continuity at constant density, Eq. 10-4b. Flow rate out of duct = Flow rate into room.

$$A_{\text{duct}}v_{\text{duct}} = \pi r^2 v_{\text{duct}} = \frac{V_{\text{room}}}{t_{\substack{\text{to fill}\\\text{room}}}} \rightarrow v_{\text{duct}} = \frac{V_{\text{room}}}{\pi r^2 t_{\substack{\text{to fill}\\\text{room}}}} = \frac{(9.2\text{ m})(5.0\text{ m})(4.5\text{ m})}{\pi(0.15\text{ m})^2(16\,\text{min})\left(\frac{60\text{ s}}{1\text{ min}}\right)} = \boxed{3.1\,\text{m/s}}$$

- **39.** (II) A $\frac{5}{8}$-inch (inside) diameter garden hose is used to fill a round swimming pool 6.1 m in diameter. How long will it take to fill the pool to a depth of 1.2 m if water issues from the hose at a speed of $0.40\ \text{m/s}$?
- 39. The volume flow rate of water from the hose, multiplied times the time of filling, must equal the volume of the pool.

$$(Av)_{\text{hose}} = \frac{V_{\text{pool}}}{t} \quad \rightarrow \quad t = \frac{V_{\text{pool}}}{A_{\text{hose}} v_{\text{hose}}} = \frac{\pi (3.05\,\text{m})^2 (1.2\,\text{m})}{\pi \left[\frac{1}{2}\left(\frac{5}{8}\right)'' \left(\frac{1\,\text{m}}{39.37''} \right) \right]^2 (0.40\,\text{m/s})} = 4.429 \times 10^5\,\text{s}$$

$$4.429 \times 10^5\,\text{s} \left(\frac{1\,\text{day}}{60 \times 60 \times 24\,\text{s}} \right) = \boxed{5.1\ \text{days}}$$

40. (II) What gauge pressure in the water mains is necessary if a firehose is to spray water to a height of 15 m?

40. Apply Bernoulli's equation with point 1 being the water main, and point 2 being the top of the spray. The velocity of the water will be zero at both points. The pressure at point 2 will be atmospheric pressure. Measure heights from the level of point 1.

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \rightarrow$$

$$P_1 - P_{atm} = \rho g y_2 = \left(1.00\times10^3\ \mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)\left(15\ \mathrm{m}\right) = \boxed{1.5\times10^5\ \mathrm{N/m^2}}$$