4. Fluid Kinematics

- 4.1. Velocity Field
- 4.2. Continuity Equation

Fluid Kinematics

What is fluid kinematics?

- Fluid kinematics is the study on fluid motion in space and time without considering the force which causes the fluid motion.

- According to the continuum hypothesis the local velocity of fluid is the velocity of an infinitesimally small fluid particle/element at a given instant t. It is generally a continuous function in space and time.

• How small an how large should be a fluid particle/element in frame of the continuum concept?

• The characteristic length of the fluid system of interest >> The characteristic length of a fluid particle/element >> The characteristic spacing between the molecules contained in the volume of the fluid particle/element :

$$L >> d >> \lambda; \lambda/L = Kn \ (Knudsen\ No.)$$

For air at sea-level conditions, $15\ ^{\circ}C\ and\ 10.133 \times 10^{4}\ Pa$

- 3×10^{7} molecules in a volume of $(10^{-3}\ mm)^{3}$

- $\lambda = 10^{-6}\ mm$ (λ : mean free path)

The continuum concept is valid!

4.1. Velocity Field

- Eulerian Flow Description
- Lagrangian Flow Description
- Streamline
- Pathline
- Streakline

4.1.1. In the Eulerian Method

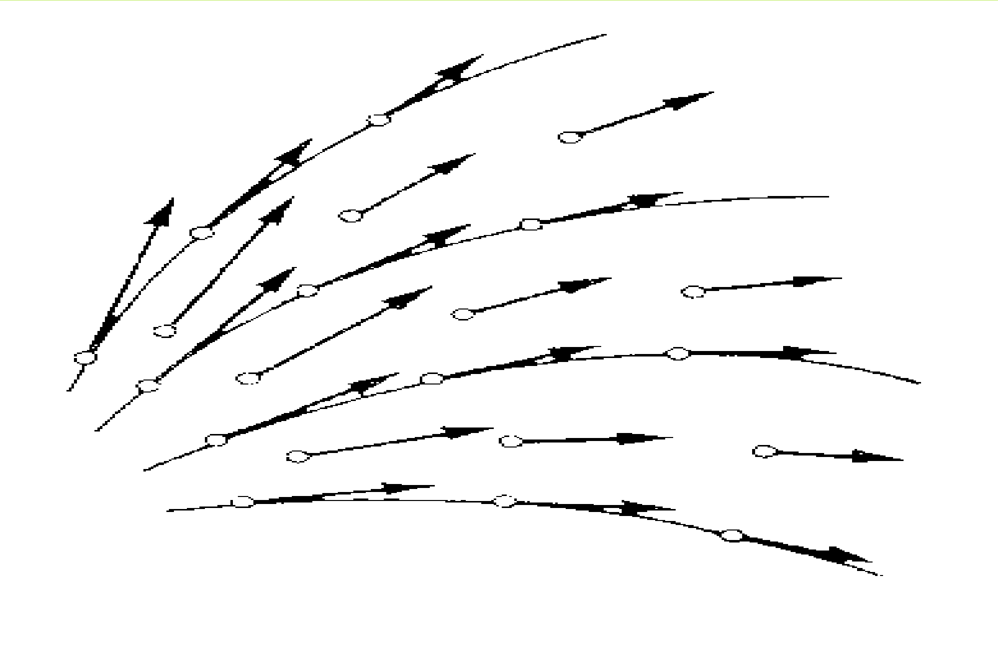
- The flow quantities, like $\vec{u}$, p, ρ, T, are described as a function of space and time without referring to any individual identity of the fluid particle :

4.1.2. Streamline

- A line in the fluid whose tangent is parallel to at a given instant t.
- The family of streamlines at time t are solutions of

$$\frac{dx}{u_x(\vec{r},t)} = \frac{dy}{u_y(\vec{r},t)} = \frac{dz}{u_z(\vec{r},t)}$$

- Where $\vec{u}_x,\ \vec{u}_y,\ and\ \ \vec{u}_z$ are velocity components in the respective direction



▲ **Fig. 4.1**

- Steady flow : the streamlines are fixed in space for all time.
- Unsteady flow : the streamlines are changing from instant to instant.

4.1.3. Flow Dimensionality

- Most of the real flow are 3-dimensional and unsteady : $\vec{u}(x, y, z, t)$
- For many situations simplifications can be made :

  2-dimensional unsteady and steady flow

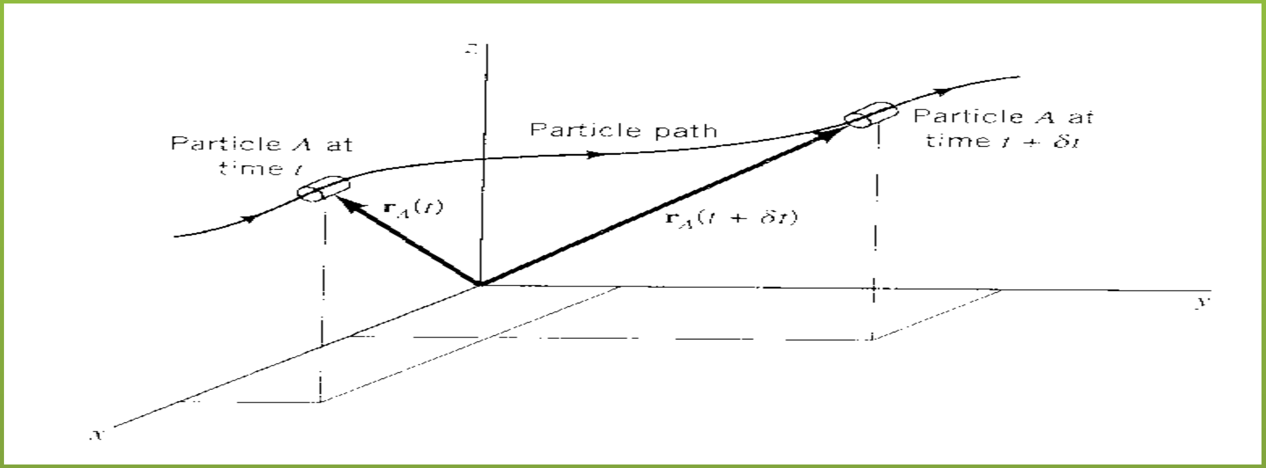
  $\vec{u}(x, y, t)$; $\vec{u}(x, y)$

  1-dimensional unsteady and steady flow

  $\vec{u}(x, t)$; $\vec{u}(x)$

4.1.4. In the Lagrangian Method

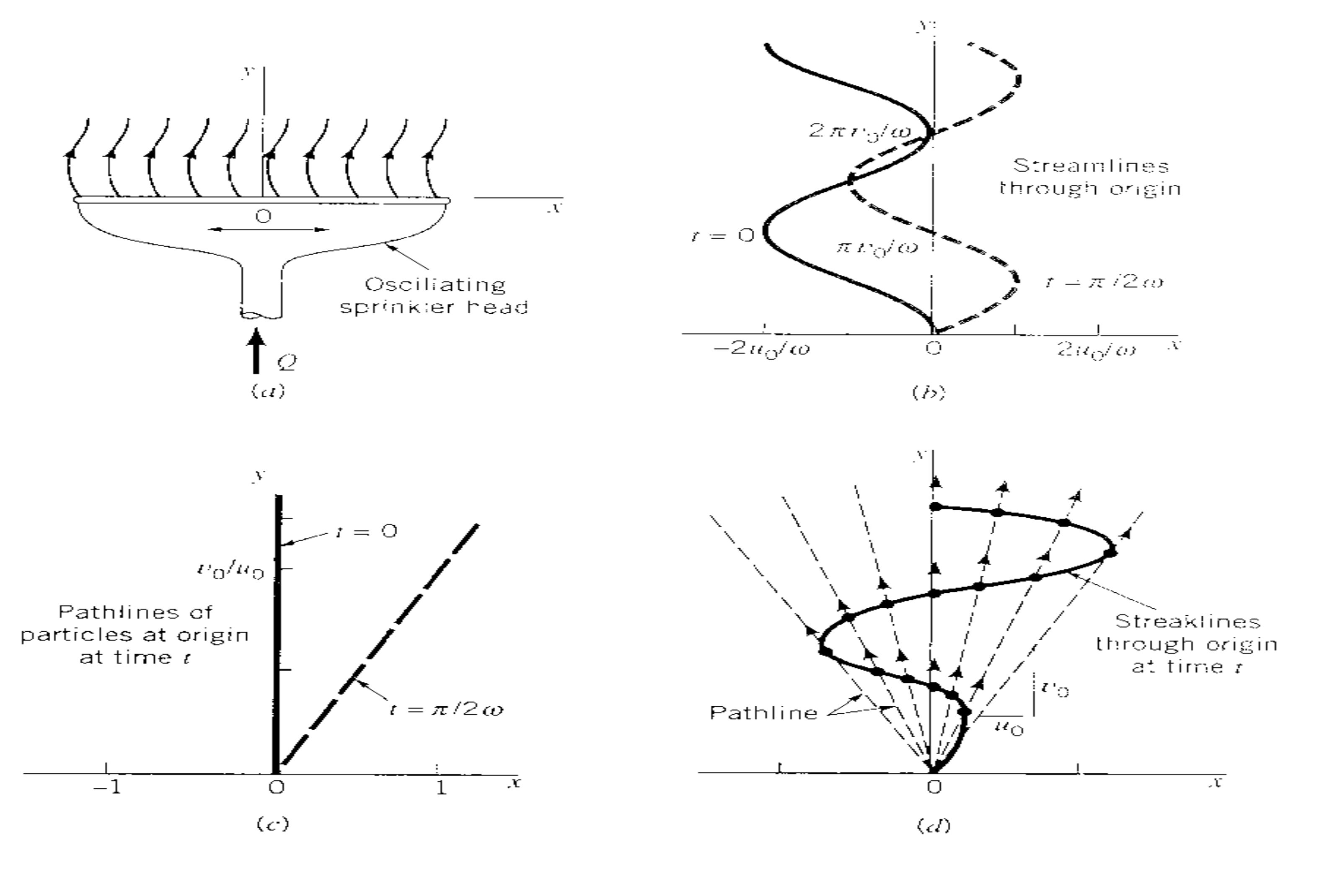
- The flow quantities are described for each individually identifiable fluid particle moving through flow field of interest. The position of the individual fluid particle is a function of time :



▲ **Fig. 4.2**

4.1.5. Pathline

- A line traced by an individual fluid particle $\vec{r}(t)$
- For a steady flow the pathlines are identical with the streamlines.



y
x
O
Oscillating
sprinkler head
Q
(a)
y
$2\pi v_0/\omega$
Streamlines
through origin
$t = 0$
$\pi v_0/\omega$
$t = \pi/2\omega$
$-2u_0/\omega$
O
$2u_0/\omega$
x
(b)
y
$t = 0$
v_0/u_0
Pathlines of
particles at origin
at time t
$t = \pi/2\omega$
-1
O
1
x
(c)
y
Streaklines
through origin
at time t
v_0
Pathline
u_0
O
x
(d)

▲ Fig. 4.3

4.1.6. Streakline

- A streakline consists of all fluid particles in a flow that have previously passed through a common point. Such a line can be produced by continuously injecting marked fluid (smoke in air, or dye in water) at a given location.

- For steady flow : The streamline, the pathline, and the streakline are the same.

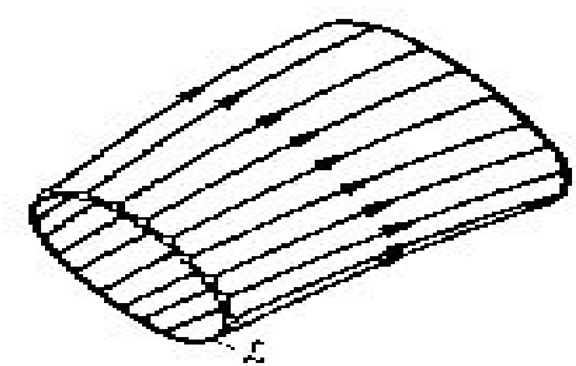
4.2. Stream-tube and Continuity Equation

- Stream-tube

- Continuity Equation of a Steady Flow

4.2.1. Stream-tube

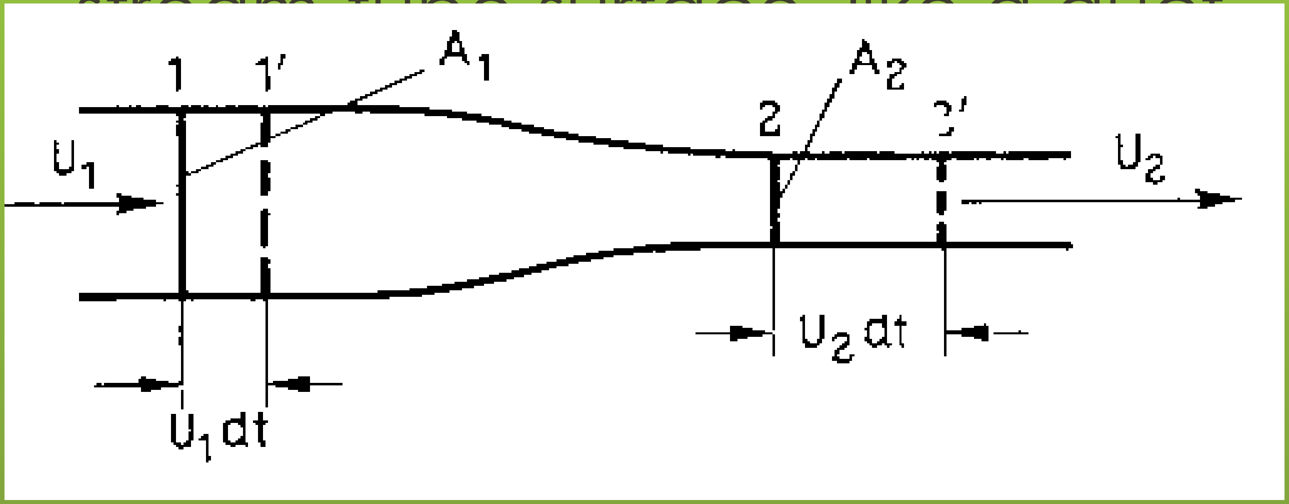
- is the surface formed instantaneously by all the streamlines that pass through a given closed curve in the fluid.



▲ **Fig. 4.4**

4.2.2. Continuity Equation of a Steady Flow

- For a steady flow the stream-tube formed by a closed curved fixed in space is also fixed in space, and no fluid can penetrate through the



▲ Fig. 4.5

Considering a stream-tube of cylindrical cross sections $A_1 \ and \ A_2$ with velocities $u_1 \ and \ u_2$ perpendicular to the cross sections $A_1 \ and \ A_2$ and densities $\rho_1 \ and \ \rho_2$ at the respective cross sections $A_1 \ and \ A_2$ and assuming the velocities and densities are constant across the whole cross section $A_1 \ and \ A_2$, a fluid mass closed between cross section 1 and 2 at an instant t will be moved after a time interval dt by $u_1 \cdot dt \ and \ u_2 \cdot dt$ to the cross section 1' and 2' respectively.

Because the closed mass between 1 and 2 must be the same between 1' and 2', and the mass between 1' and 2 for a steady flow can not change from t and t+dt, the mass between 1 and 1' moved in dt, $\rho_1 A_1 u_1 dt$ must be the same as the mass between 2 and 2' moved in the same time dt, $\rho_2 A_2 u_2 dt$:

- Therefore the continuity equation of steady flow :

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2 \qquad \textbf{(4.1)}$$

Interpretation : The mass flow rate $\dot{m} = \rho A u = const.$

through a steady stream-tube or a duct.

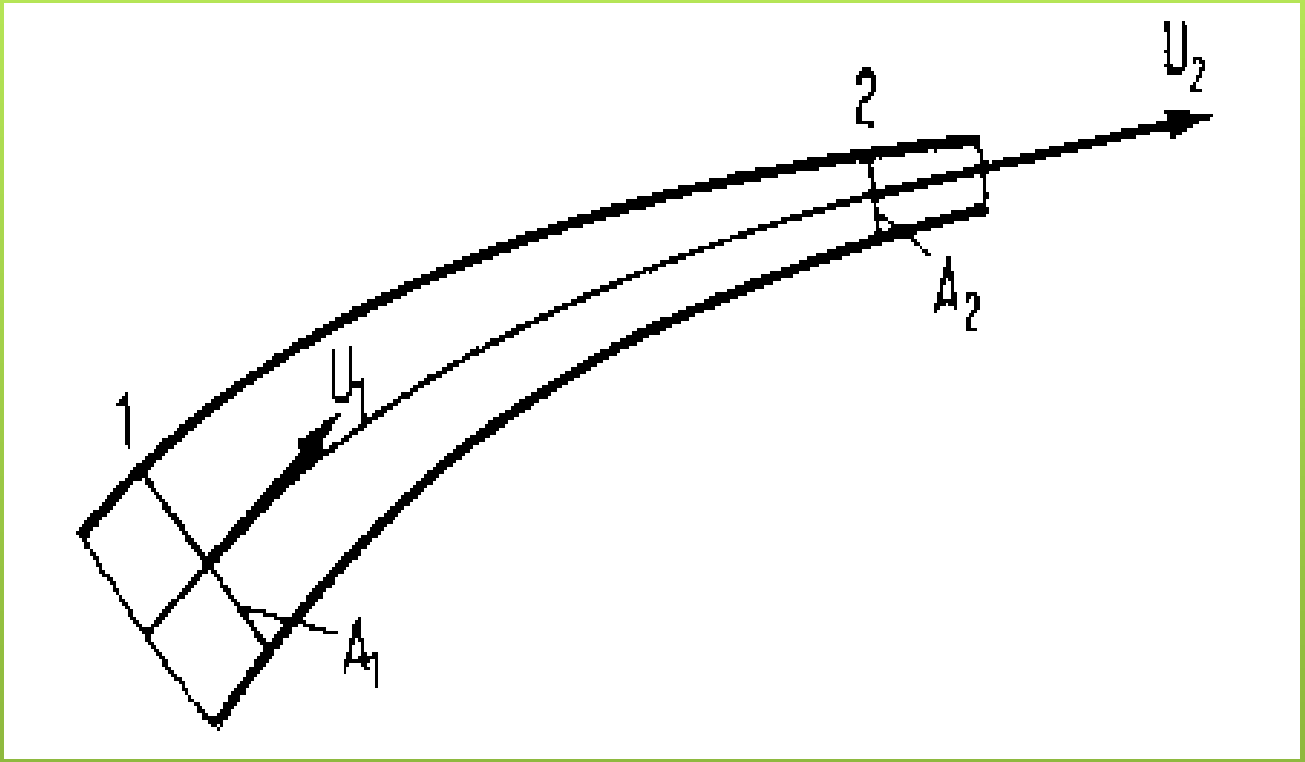
- For incompressible fluid with $\rho_1 = \rho_2$:

$$A_1 u_1 = A_2 u_2 \qquad \textbf{(4.2)}$$

Interpretation : The volume flow rate $\dot{V} = A u = const.$

- From the continuity equation for incompressible fluid :

$\dfrac{u_1}{u_2} = \dfrac{A_2}{A_1}$ for a stream-tube.



▲ Fig. 4.6