# Inverse Fourier Transform

# Fourier Transform and Its Inverse

- Writing the exponential function in (4) as a product of exponential functions, we have

**(5)** $$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(v) e^{-iwv} \, dv \right] e^{iwx} \, dw.$$

The expression in brackets is a function of *w*, is denoted by $\hat{f}(w)$ and is called the **Fourier transform** of *f*; writing *v* = *x*, we have

**(6)** $$\hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx} \, dx.$$

With this, (5) becomes

**(7)**
$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(w) e^{iwx}\, dw$$

and is called the **inverse Fourier transform** of $\hat{f}(w)$

- Another notation for the Fourier transform is

  so that $\hat{f} = \mathcal{F}(f), \qquad f = \mathcal{F}^{-1}(\hat{f}).$

- The process of obtaining the Fourier transform

  $\mathcal{F}(f) = \hat{f}$

  from a given $f$ is also called the **Fourier transform** or the *Fourier transform method.*

# Table of Fourier Transforms

| | $f(x)$ | $\hat{f}(\omega)$ |
|---|---|---|
| 1. | $\dfrac{1}{x^2+a^2} \quad (a>0)$ | $\dfrac{\pi}{a}e^{-a\lvert\omega\rvert}$ |
| 2. | $H(x)e^{-ax} \quad (\operatorname{Re} a>0)$ | $\dfrac{1}{a+i\omega}$ |
| 3. | $H(-x)e^{ax} \quad (\operatorname{Re} a>0)$ | $\dfrac{1}{a-i\omega}$ |
| 4. | $e^{-a\lvert x\rvert} \quad (a>0)$ | $\dfrac{2a}{\omega^2+a^2}$ |
| 5. | $e^{-x^2}$ | $\sqrt{\pi}e^{-\omega^2/4}$ |
| 6. | $\dfrac{1}{2a\sqrt{\pi}}e^{-x^2/(2a)^2} \quad (a>0)$ | $e^{-a^2\omega^2}$ |
| 7. | $\dfrac{1}{\sqrt{\lvert x\rvert}}$ | $\sqrt{\dfrac{2\pi}{\lvert\omega\rvert}}$ |
| 8. | $e^{-a\lvert x\rvert/\sqrt{2}}\sin\left(\dfrac{a}{\sqrt{2}}\lvert x\rvert+\dfrac{\pi}{4}\right) \quad (a>0)$ | $\dfrac{2a^3}{\omega^4+a^4}$ |
| 9. | $H(x+a)-H(x-a)$ | $\dfrac{2\sin\omega a}{\omega}$ |