Newton's divided difference formula (for unequal intervals)

Unequal Intervals

When the values of the independent variable (Argument) are given at uneven/unequal spaced intervals then the various differences will also be affected by the changes in the values of argument and then the differences involve argument values and denoted as ◊ (divided differences)

x = a b c d e ….

f(x) = f(a) f(b) f(c) f(d) f(e)

$$\underset{b}{\Diamond} f(a) = \frac{f(b) - f(a)}{b - a}$$

$$\Delta_{c} f(b) = \frac{f(c) - f(b)}{c - b} \quad \text{and so on.}$$

$$\Delta_{bc}^{2} f(a) = \frac{\Delta_{c} f(b) - \Delta_{b} f(a)}{c - a}$$

Here in Δ there are two suffixes and Δ^2 there are three suffixes.

We also write

$$\Delta_{b} f(a) = f(a,b) = \frac{f(b) - f(a)}{b - a}$$

$$\underset{bc}{\Diamond^2} f(a) = f(a,b,c) = \frac{f(b,c) - f(a,b)}{c-a}$$

$$\underset{bcd}{\Diamond^3} f(a) = f(a,b,c,d) = \frac{f(b,c,d) - f(a,b,c)}{d-a}$$

$$f(a,b) = \underset{b}{\Diamond} f(a) = \frac{f(b) - f(a)}{b-a}$$

Now

$$= \frac{f(a)}{a-b} + \frac{f(b)}{b-a}$$

$$\underset{bc}{\Diamond^2} f(a) = \underset{c}{\Diamond}[\underset{b}{\Diamond} f(a)]$$

$$= \underset{c}{\Diamond}\left[\frac{f(a)}{a-b} + \frac{f(b)}{b-a}\right]$$

$$= \frac{\dfrac{f(a)}{a-b} + \dfrac{f(b)}{b-a}}{c-a} + \frac{\dfrac{f(c)}{c-b} + \dfrac{f(b)}{b-c}}{a-c}$$

and so on.

Steps

I. Divide f(a) by a-b

II. Where there is a write b in the function and divide the value by b-a.

III. Add the values obtained in steps I and II to get the desired divided difference.

Difference between Δ and ◊

A. Ordinary differences are not affected by the changes in the value of argument.

B. Ordinary differences have numerator(i.e. differenced between successive values of entry or various order differences) whereas divided differences have denominator also.(i.e. differences between two extreme values of argument)

C. The operator and the operand have no suffixes in ordinary differences whereas these suffixes have special significance in divided differences.

The value of the divided differences remain unchanged by interchanging the suffixes of the operator and the operand or divided differences are symmetric functions of their arguments.

Also the nth divided difference of a polynomial of nth degree is constant.

$$\underset{bc..kl}{\Diamond^{n}} f(a) = \frac{\Delta^{n} f(a)}{n!\,h^{n}}$$

h = a + h

c = a + h

l = a + nh.

NEWTONS DIVIDED DIFFERENCE FORMULA

$$f(x) = f(a) + (x-a) \underset{h}{\Diamond} f(a)$$

$$+ (x-a)(x-h) \underset{hc}{\Diamond^2} f(a)$$

$$+ \ldots + (x-a)(x-h)\ldots(x-k) \underset{hc-kl}{\Diamond^n} f(a)$$

$$+ (x-a)(x-h)\ldots(x-h) \underset{bc..lx}{\Diamond^{n+1}} f(a)$$