LRFD-Steel Design

Chapter 5

5.1 INTRODUCTION

Beams: Structural members that support transverse loads and are therefore subjected primarily to flexure, or bending.

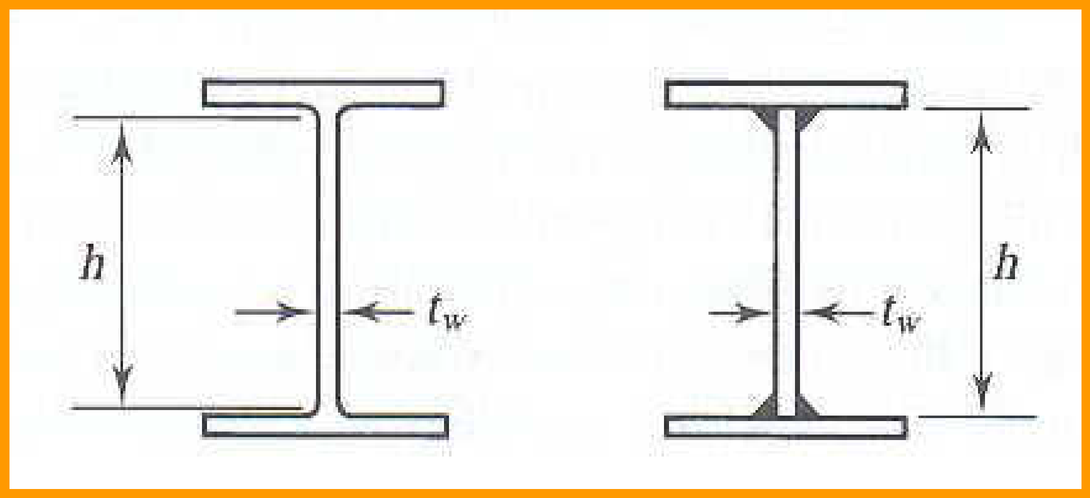
- structural member is considered to be a beam if it is loaded so as to cause bending
- Commonly used cross-sectional shapes include the W-, S-, and M-shapes. Channel shapes are sometimes used.
- Doubly symmetric shapes such as the standard rolled W-, M-, and S-shapes are the most efficient.
- AISC Specification distinguishes beams from plate girders on the basis of the width-thickness ratio of the web.

- Both a hot-rolled shape and a built up shape along with the dimensions to be used for the width-thickness ratios.

- If

$$\frac{h}{t_w} \leq 5.70\sqrt{\frac{E}{F_y}}$$

then the member is to be treated as a beam, regardless of whether it is a rolled shape or is built-up.



- If

$$\frac{h}{t_w} > 5.70\sqrt{\frac{E}{F_y}}$$

then the member is considered to be a plate girder.

- For beams, the basic relationship between load effects and strength is

$$M_u \leq \phi_b M_n$$

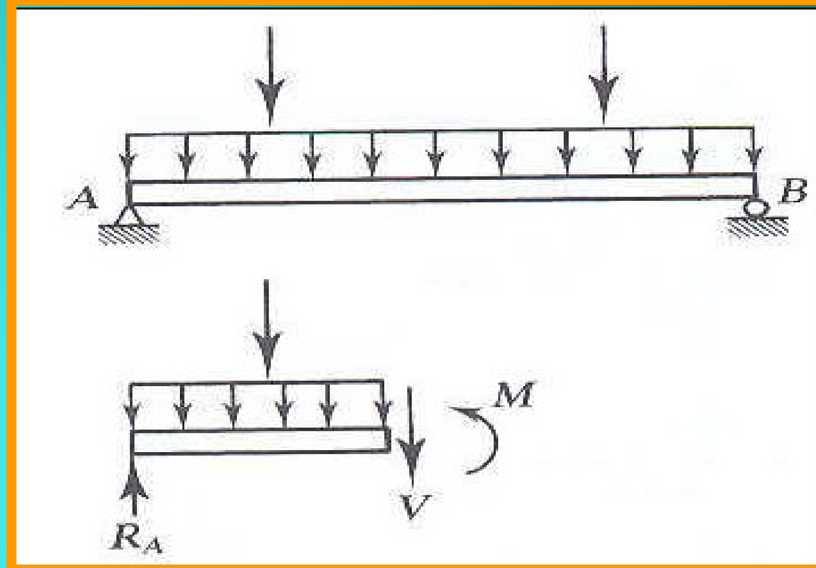
where

M_u = controlling combination of factored load moments
ϕ_b = resistance factor for beams = 0.90
M_n = nominal moment strength

The design strength, $\phi_b M_n$, is sometimes called the *design moment*.

5.2 BENDING STRESS AND THE PLASTIC MOMENT:

- Consider the beam which is oriented so that bending is about the major principal axis.

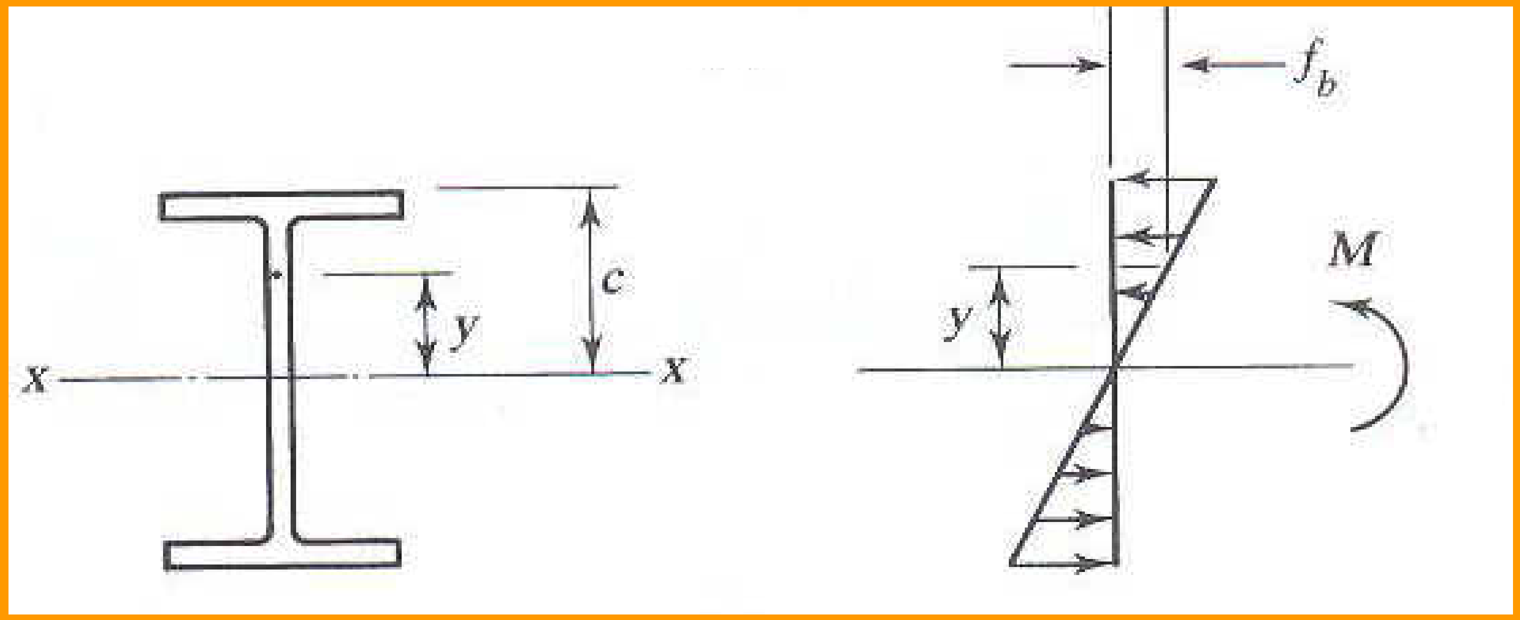


- The stress at any point can be found from the flexure formula:

$$f_b = \frac{My}{I_x}$$

- Where M is the bending moment at the cross section under consideration, y is the perpendicular distance

- For maximum stress, Equation takes the form:



$$f_{\max} = \frac{Mc}{I_x} = \frac{M}{I_x/c} = \frac{M}{S_x}$$

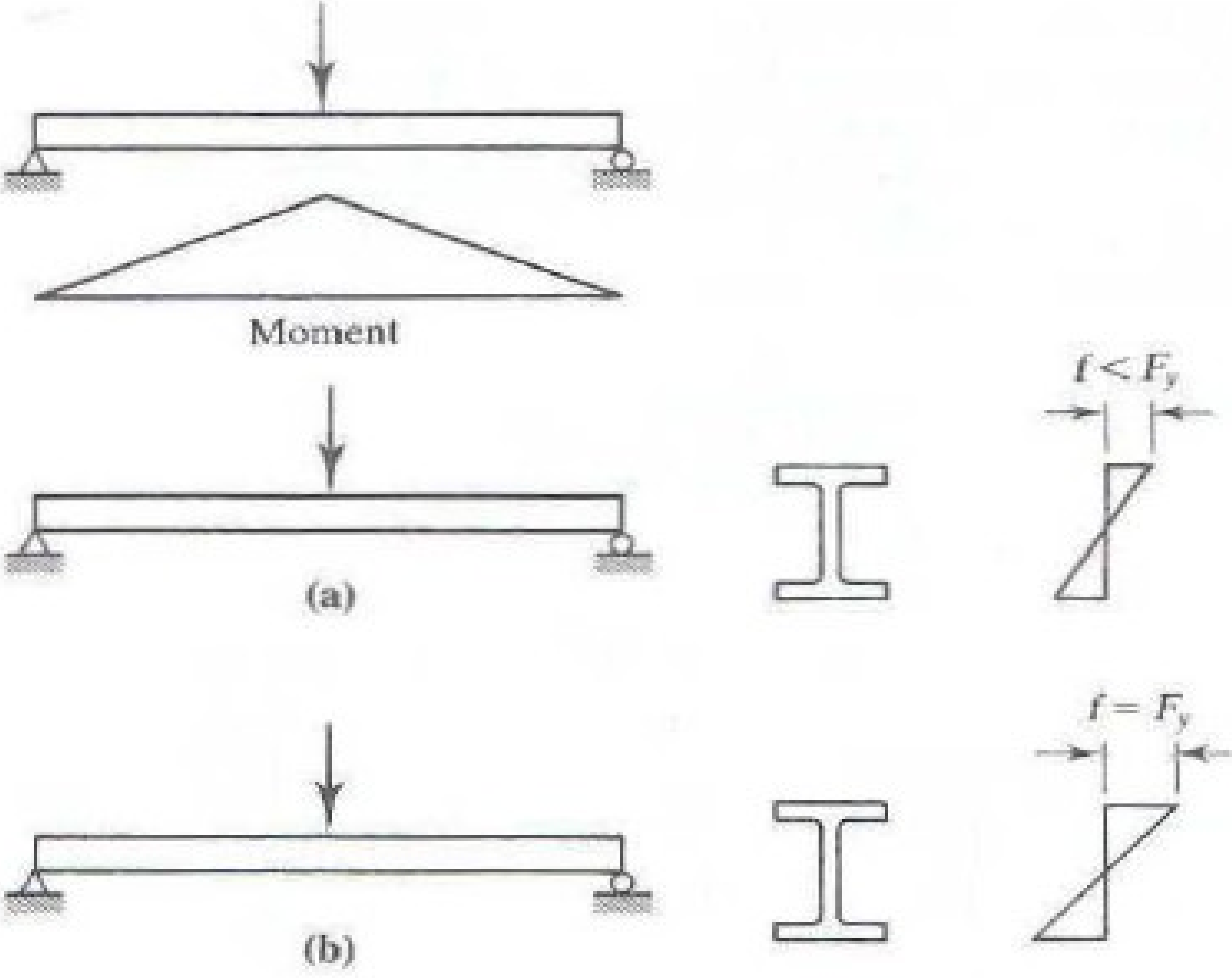
  - where c is the perpendicular distance from the neutral axis to the extreme fiber, and S_x is the elastic section modulus of the cross section

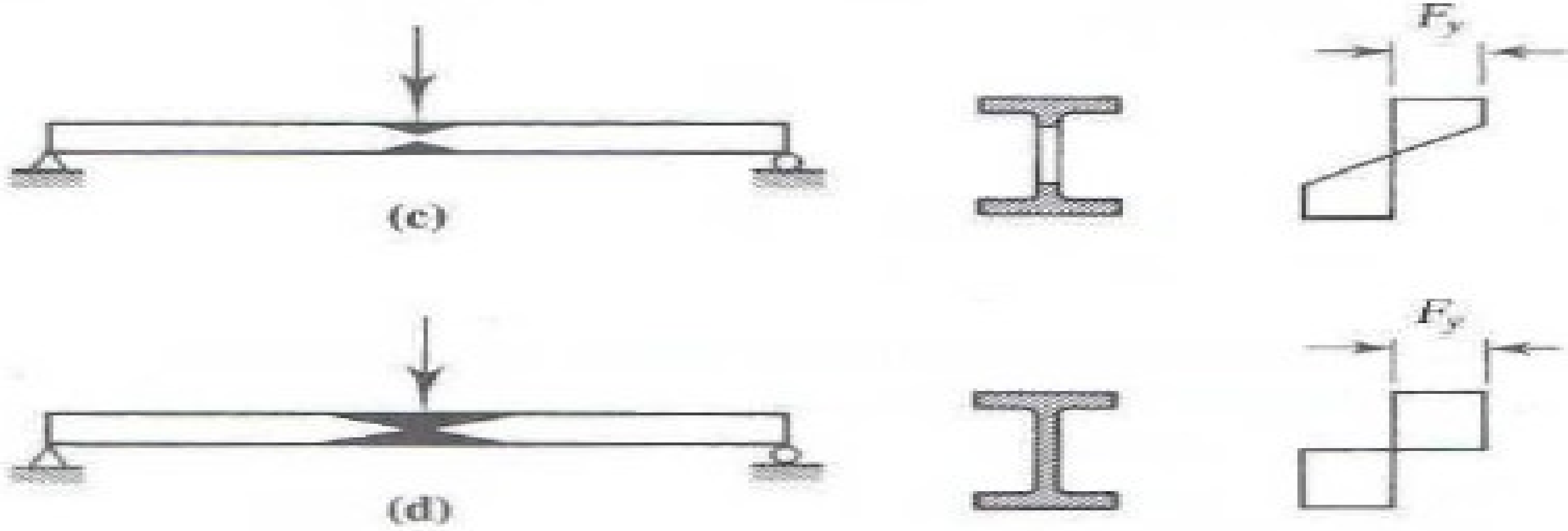
- Equations are valid as long as the loads are small enough that the material remains within its linear elastic range. For structural steel, this means that the stress f_{max} must not exceed Fy and that the bending moment must not exceed

$$M_y = F_y * S_x$$

Where My is the bending moment that brings the beam to the point of yielding.

Once yielding begins, the distribution of stress on the cross section will no longer be linear, and yielding will progress from the extreme fiber toward the neutral axis.

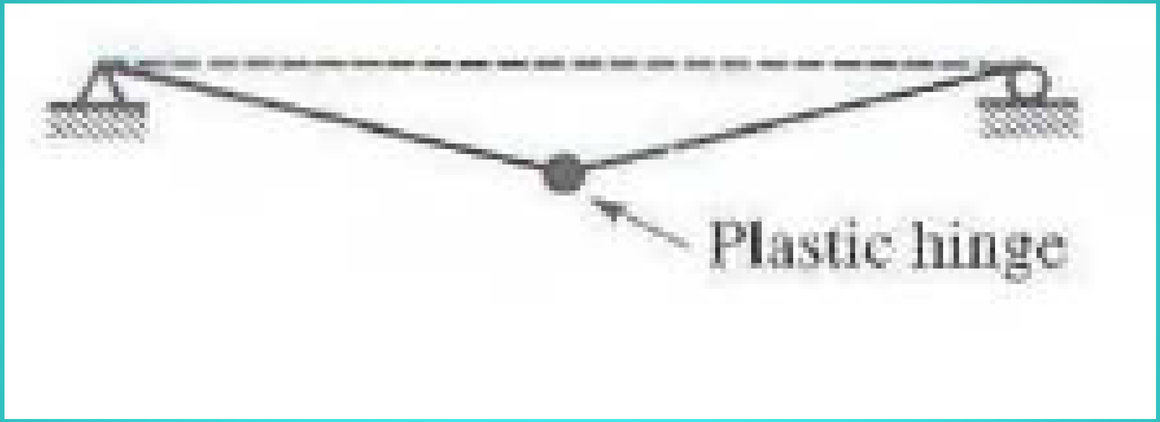

Moment
$f < F_y$
(a)
$f = F_y$
(b)



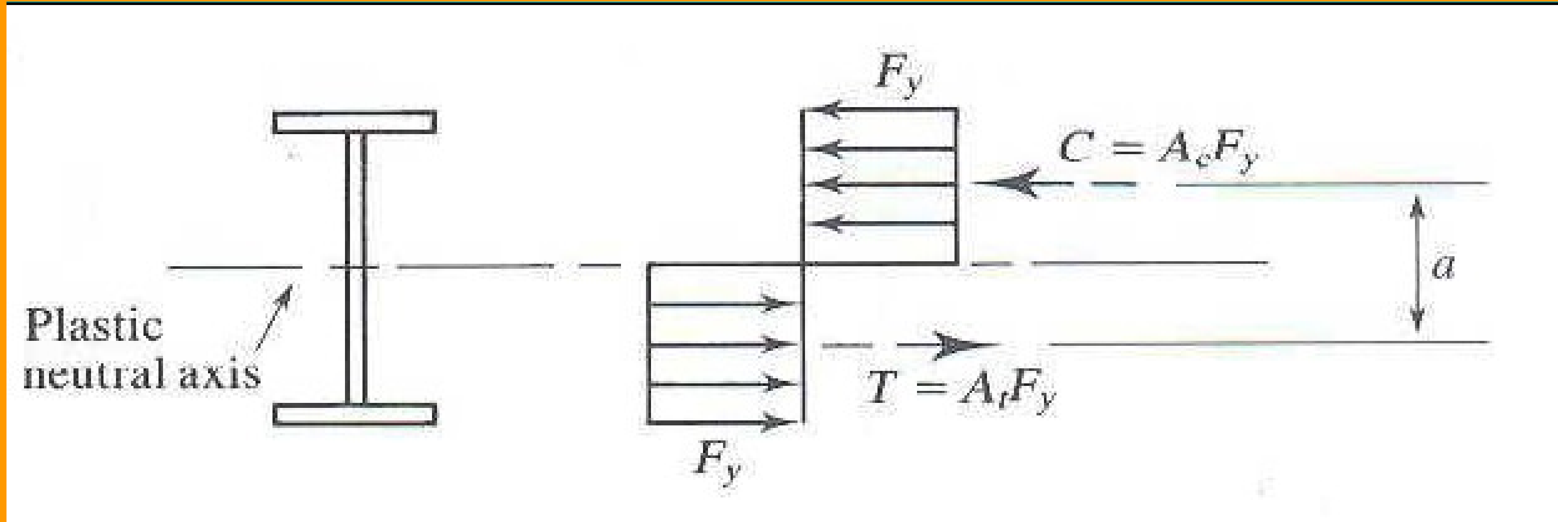
– The additional moment required to bring the beam from stage b to stage d is, on the average, approximately 12% of the yield moment for W-shapes.

– When stage d has been reached, any further increase in the load will cause collapse, since all elements have reached the yield value of the stress-strain carve and unrestricted plastic flow will occur)

- A plastic hinge is said to have formed at the center of the beam.
- At this moment the beam consider in an unstable mechanism.
- The mechanism motion will be as shown



- Structural analysis based on a consideration of collapse mechanism is called plastic analysis.



- The plastic moment capacity, which is the moment required to form the plastic hinge, can easily be computed from a consideration of the corresponding stress distribution, From equilibrium of forces:

$$C = T$$

$$A_c F_y = A_t F_y$$

$$A_c = A_t$$

◆The plastic moment, M_p is the resisting couple formed by the two equal and opposite forces, or

$$M_p = F_y(A_c)a = F_y(A_t)a = F_y\left(\frac{A}{2}\right)a = F_y Z$$

where

A = total cross-sectional area

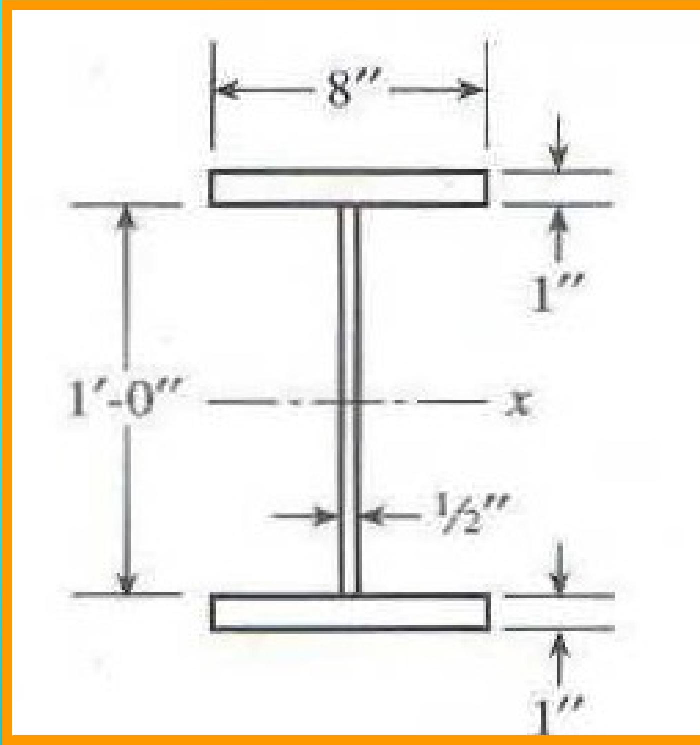
a = distance between the centroids of the two half-areas

$Z = \left(\frac{A}{2}\right)a$ = plastic section modulus

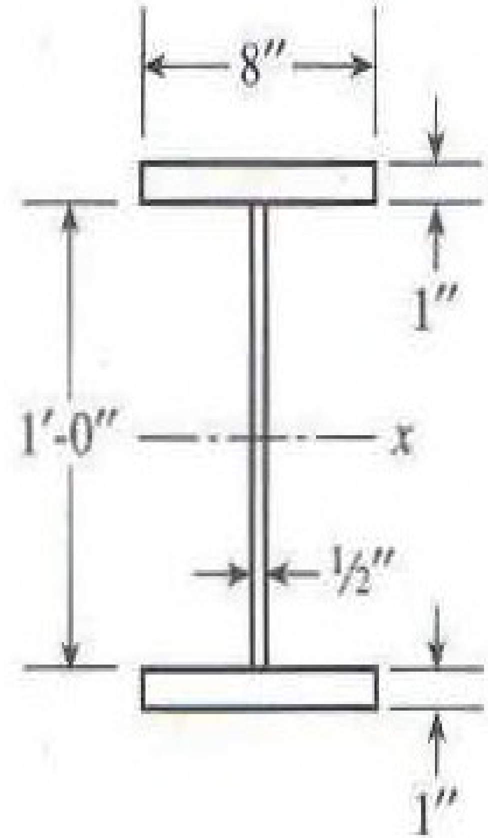
Example 5.1: For the built-up shape, determine (a) the elastic section modulus S and the yield moment My and (b) the plastic section modulus Z and the plastic moment Mp-Bending is about the x-axis, and the steel is A572 Grade 50.

Solution

Because of symmetry, the elastic neutral axis is located at mid-depth of the cross section. The moment of inertia of the cross section can be found by using the parallel axis theorem, and the results of the calculations are summarized in the next table.



Component	$\bar{I}$	A	d	$\bar{I} + Ad^2$
Flange	0.6667	8	6.5	338.7
Flange	0.6667	8	6.5	338.7
Web	72	—	—	72.0
Sum				749.4



The elastic section modulus is

$$S = \frac{I}{c} = \frac{749.4}{1 + (12/2)} = \frac{749.4}{7} = 107 \text{ in.}^3$$

and the yield moment is

$$M_y = F_y S = 50(107) = 5350 \text{ in.-kips} = 446 \text{ ft-kips}$$

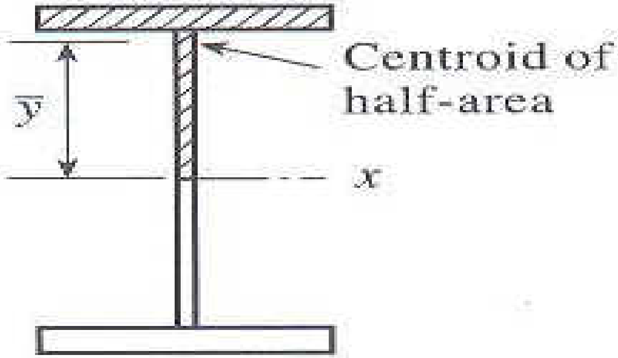
$S = 107$ in.^{3} and $M_y = 446$ ft-kips. **Answer (A)**

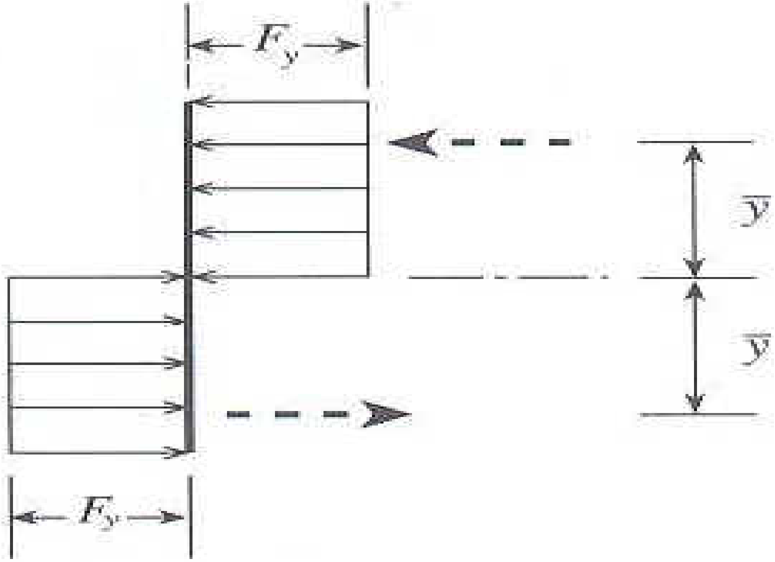
Example 5.1(cont.):

Because this shape is symmetrical about the x-axis, this axis divides the cross section into equal areas and is therefore the plastic neutral axis. The centroid of the top half-area can be found by the principle of moments. Taking moments about the x-axis (the neutral axis of the entire cross section) and tabulating the computations in the next Table, we get

Component	A	y	Ay
Flange	8	6.5	52
Web	3	3	9
Sum	11		61

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{61}{11} = 5.545 \text{ in.}$$





$$a = 2\bar{y} = 2(5.545) = 11.09 \text{ in.}$$

and that the plastic section modulus is

$$\left(\frac{A}{2}\right)a = 11(11.09) = 122 \text{ in.}^3$$

The plastic moment is

$$M_p = F_yZ = 50(122) = 6100 \text{ in.-kips} = 508 \text{ ft-kips}$$

$Z = 122 \text{ in.}^3$ and $M_p = 508$ ft-kips. **answer**

Example 5.2 :

Compute the plastic moment, M_p, for a W10 × 60 of A992 steel.

Solution

From the dimensions and properties tables in Part 1 of the *Manual*,

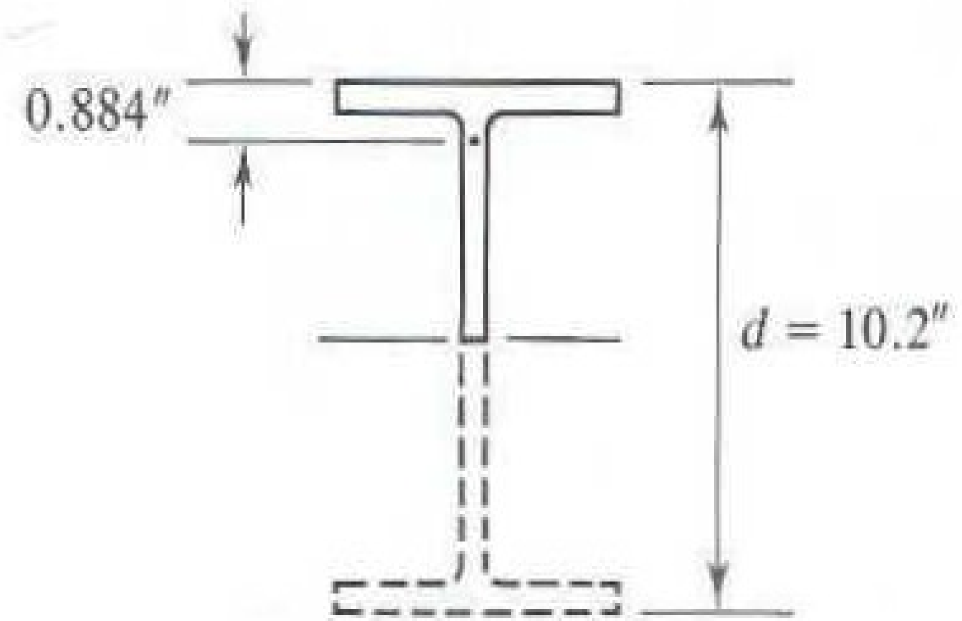
$$A = 17.6 \text{ in.}^2$$

$$\frac{A}{2} = \frac{17.6}{2} = 8.8 \text{ in.}^2$$

The centroid of the half-area can be found in the tables for WT-shapes, which are cut from W-shapes. The relevant shape here is the WT5 × 30, and the distance from the outside face of the flange to the centroid is 0.884 inch, as shown in Figure 5.8.

$$a = d - 2(0.884) = 10.2 - 2(0.884) = 8.432 \text{ in.}$$

$$Z = \left(\frac{A}{2}\right)a = 8.8(8.432) = 74.20 \text{ in.}^3$$



This result compares favorably with the value of 74.6 given in the dimensions and properties tables (the difference results from rounding of the tabular values).

$M_p = F_y Z = 50(74.20) = 3710$ in.-kips $= 309$ ft-kips.

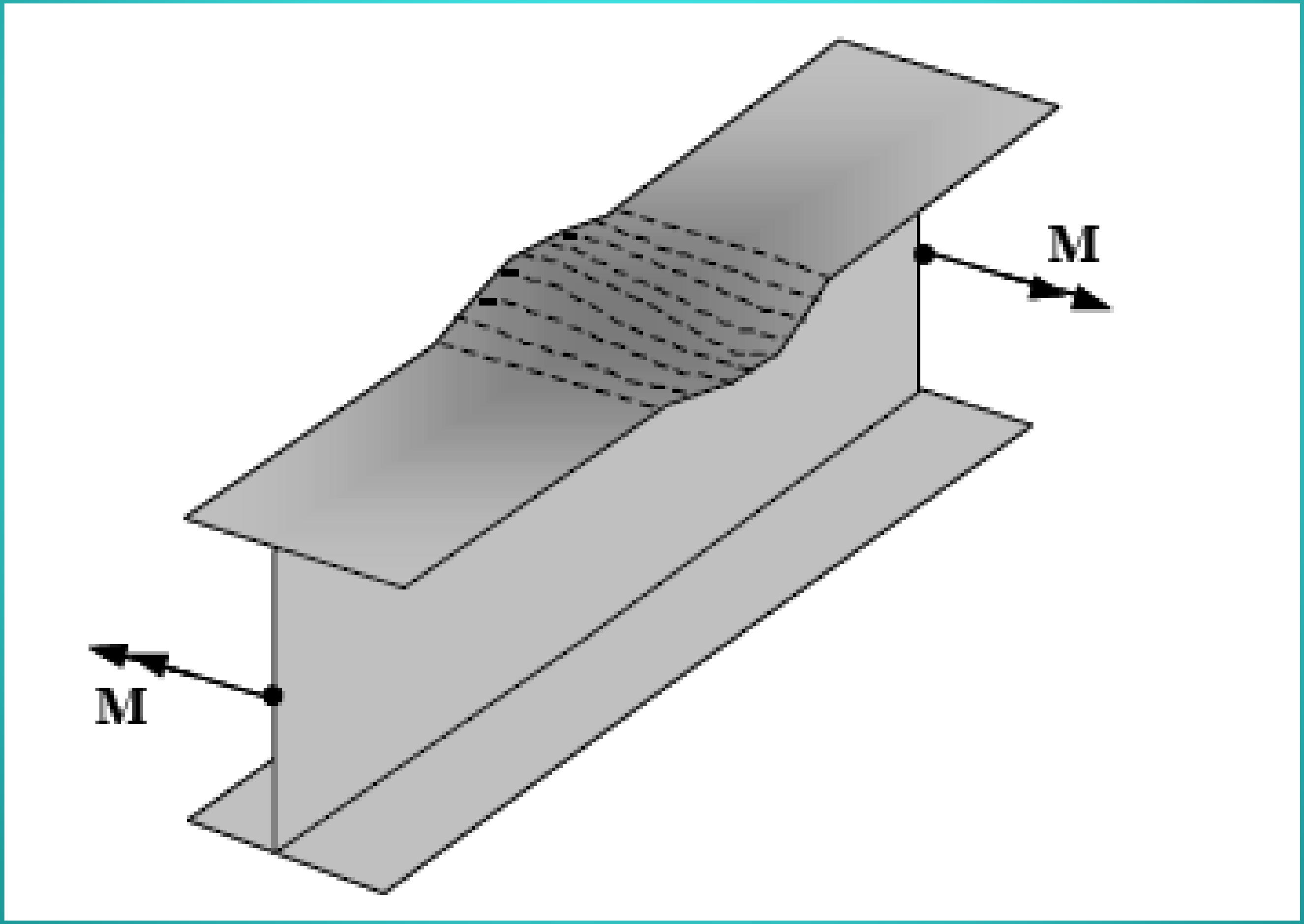
5.3 STABILITY:

–If a beam can be counted on to remain stable up to the fully plastic condition, the nominal moment strength can be taken as:

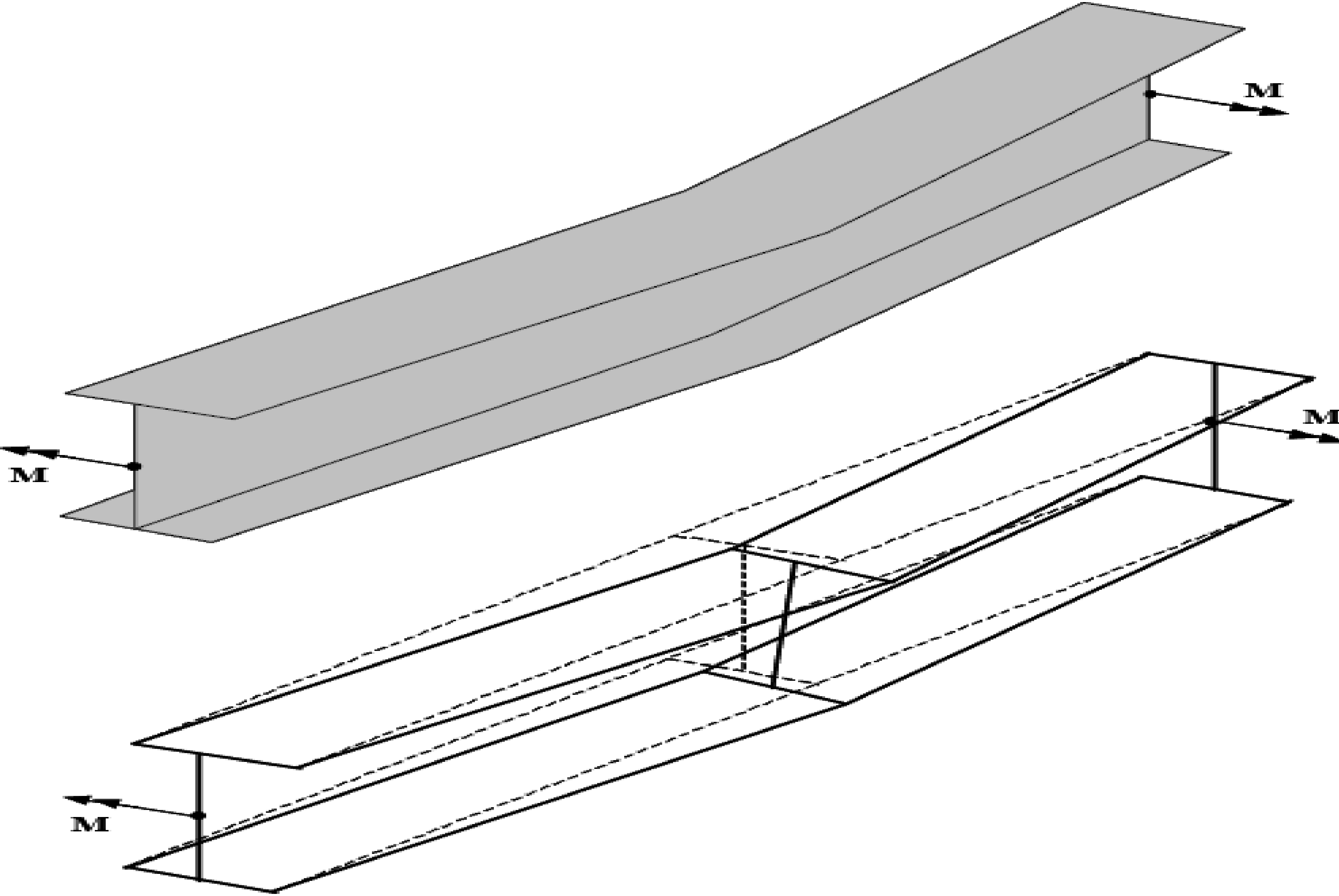
$$\boxed{M_n = M_p}$$

–When a beam bend, the compression region is analogous to a column, and in a manner similar to a column, it will buckle if the member is slender enough. Unlike a column however, the compression portion of the cross section is restrained by the tension portion. and the outward deflection (flexural buckling) is accompanied by twisting (torsion).

This form of instability is called lateral-tensional buckling (LTB). Lateral tensional buckling can be prevented by bracing the beam against twisting at sufficiently close intervals



Local buckling of flange due to compressive stress (σ)



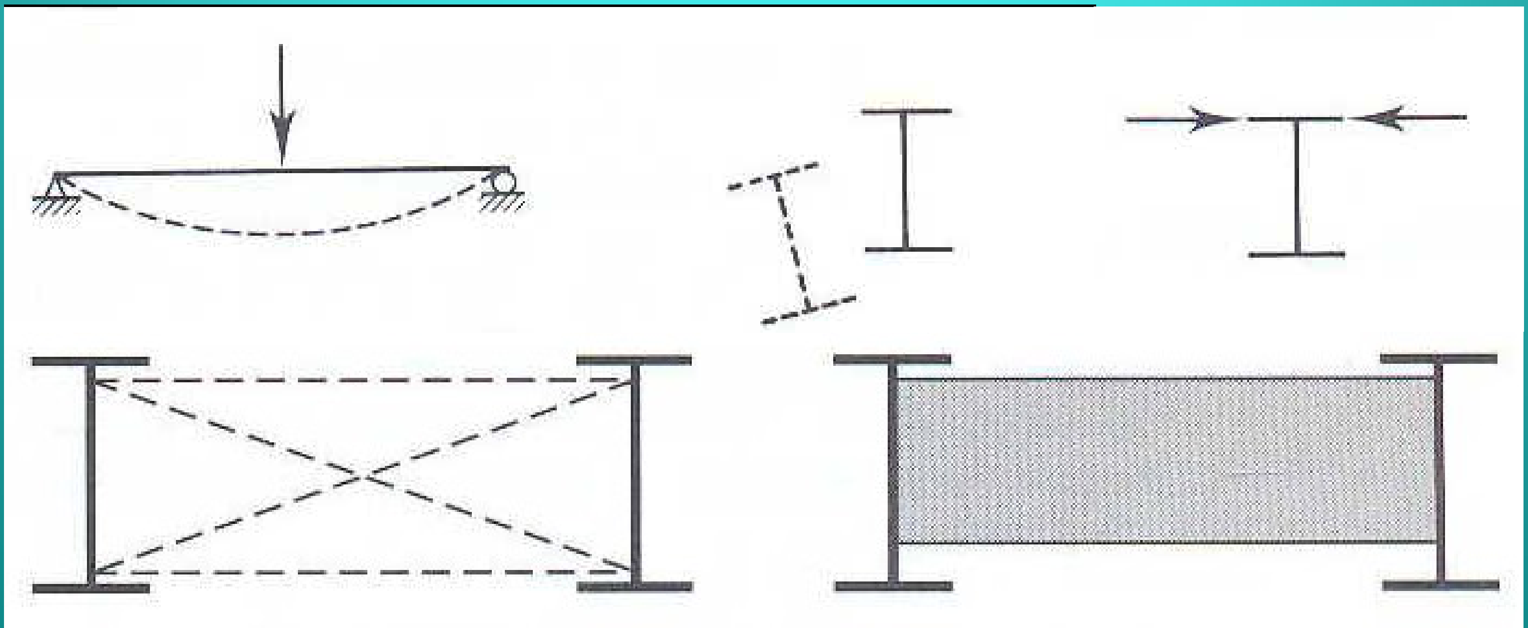
Lateral-torsional buckling of a wide-flange beam subjected to constant moment.

This can be accomplished with either of two types of stability bracing:

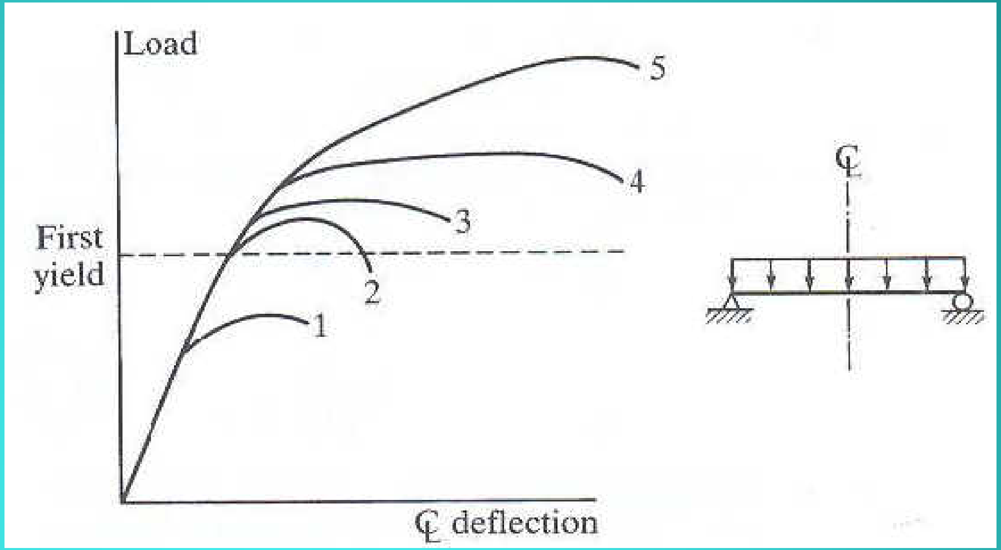
Lateral bracing: which prevents lateral translation. should be applied as close to the compression flange as possible.

Tensional bracing :prevents twist directly.

The moment strength depends in part on the unbraced length, which is the distance between points of bracing.



The next Figure illustrates the effects of local and lateral-tensional buckling.



–This graph of load versus central deflection.

◆ Curve 1 is the load-deflection curve of a beam that becomes unstable and loses its load-carrying capacity before first yield

◆Curves 2 and 3 correspond to beams that can be loaded past first yield but not far enough for the formation of a plastic hinge and the resulting plastic collapse

Curve 4 is for the case of uniform moment over the full length of the beam.

curve 5 is for a beam with a variable bending moment

Safe designs can be achieved with beams corresponding to any of these curves, but curves 1 and 2 represent inefficient use of material.

5.4 CLASSIFICATION OF SHAPES

The analytical equations for local buckling of steel plates with various edge conditions and the results from experimental investigations have been used to develop limiting slenderness ratios for the individual plate elements of the cross-sections.

Steel sections are classified as compact, non-compact, or slender depending upon the slenderness (λ) ratio of the individual plates of the cross-section.

1- Compact section if all elements of cross-section have $\lambda \leq \lambda_p$

2- Non-compact sections if any one element of the cross-section has $\lambda_p \leq \lambda \leq \lambda_r$

3- Slender section if any element of the cross-section has $\lambda_r \leq \lambda$

Where: λ is the width-thickness ratio, λ_p is the upper limit for compact category and λ_r is the upper limit for noncompact category

It is important to note that:

A- If $\lambda \leq \lambda_p$, then the individual plate element can develop and sustain σ_y for large values of ε before local buckling occurs.

B- If $\lambda_p \leq \lambda \leq \lambda_r$, then the individual plate element can develop σ_y but cannot sustain it before local buckling occurs.

C- If $\lambda_r \leq \lambda$, then elastic local buckling of the individual plate element occurs.

Thus, slender sections cannot develop M_p due to elastic local buckling. Non-compact sections can develop M_y but not M_p before local buckling occurs. Only compact sections can develop the plastic moment M_p.

All rolled wide-flange shapes are compact with the following exceptions, which are non-compact.

W40x174, W14x99, W14x90, W12x65, W10x12, W8x10, W6x15 (made from A992)

The definition of λ and the values for λ_p and λ_r for the individual elements of various cross-sections are given in Table B5.1 and shown graphically on page 16.1-183. For example,

Table B5.1, values for λp and λr for various cross-sections

Section	**Plate element**	λ	λ_p	λ_r
Wide-flange	Flange	$b_f/2t_f$	$0.38\sqrt{E/F_y}$	$0.38\sqrt{E/F_L}$
	Web	h/t_w	$3.76\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$
Channel	Flange	b_f/t_f	$0.38\sqrt{E/F_y}$	$0.38\sqrt{E/F_L}$
	Web	h/t_w	$3.76\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$
Square or Rect. Box	Flange	$(b-3t)/t$	$1.12\sqrt{E/F_y}$	$1.40\sqrt{E/F_y}$
	Web	$(b-3t)/t$	$3.76\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$

5.5 BENDING STRENGTH OF COMPACT SHAPES:

(Uniform bending moment)

Beam can fail by reaching M_p and becoming fully plastic, or it can fail by:

- Lateral-torsional buckling. (LTB)
- Flange local buckling (FLB).
- Web local buckling (WLB).

If the maximum bending stress is less than the proportional limit when buckling occurs, the failure is said to be elastic. Otherwise, it is inelastic.

compact shapes, defined as those whose webs are continuously connected to the flanges and that satisfy the following:

$$\frac{b_f}{2t_f} \le 0.38\sqrt{\frac{E}{F_y}} \quad \text{and} \quad \frac{h}{t_w} \le 3.76\sqrt{\frac{E}{F_y}}$$

The web criterion is met by all standard I and C shapes listed in the manual, so only the flange ratio need to be checked.

If the beam is compact and has continuous lateral support, or if the unbraced length is very short , the nominal moment strength, M_n is the full plastic moment capacity of the shape, M_p

–For members with inadequate lateral support, the moment resistance is limited by the LTB strength, either inelastic or elastic.

The first category, laterally supported compact beams is the simplest case

The nominal strength as

$$M_n = M_p \qquad \text{(AISC Equation F1-1)}$$

where

$$M_p = F_y Z \leq 1.5 M_y$$

The limit of $1.5M_y$ for M_p is to prevent excessive working-load deformations and is satisfied when

$$F_y Z \leq 1.5 F_y S \quad \text{or} \quad \frac{Z}{S} \leq 1.5$$

For channels and I- and H-shapes bent about the *strong* axis, Z/S will *always* be ≤ 1.5. (For I- and H-shapes bent about the *weak* axis, however, Z/S will *never* be ≤ 1.5.)

–Example 5.3:

The beam shown in Figure 5.11 is a W16 × 31 of A992 steel. It supports a reinforced concrete floor slab that provides continuous lateral support of the compression flange. The service dead load is 450 lb/ft. This load is *superimposed* on the beam; it does not include the weight of the beam itself. The service live load is 550 lb/ft. Does this beam have adequate moment strength?

The total service dead load, including the weight of the beam, is

$$w_D = 450 + 31 = 481 \text{ lb/ft}$$

For a simply supported, uniformly loaded beam, the maximum bending moment occurs at midspan and is equal to

$$M_{\max} = \frac{1}{8} wL^2$$

where w is the load in units of force per unit length, and L is the span length. Then

$$M_D = \frac{1}{8} w_D L^2 = \frac{0.481(30)^2}{8} = 54.11 \text{ ft-kips}$$

$$M_L = \frac{0.550(30)^2}{8} = 61.88 \text{ ft-kips}$$

The dead load is less than 8 times the live load, so load combination 2 controls:

$$M_u = 1.2M_D + 1.6M_L = 1.2(54.11) + 1.6(61.88) = 164 \text{ ft-kips}$$

Alternatively, the loads can be factored at the outset:

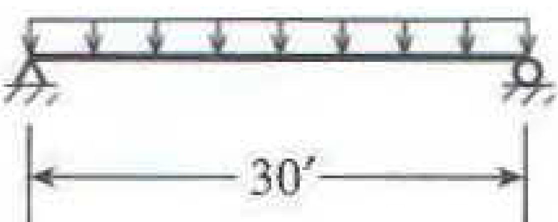
$$w_u = 1.2w_D + 1.6w_L = 1.2(0.481) + 1.6(0.550) = 1.457 \text{ kips/ft}$$

$$M_u = \frac{1}{8}w_u L^2 = \frac{1.457(30)^2}{8} = 164 \text{ ft-kips}$$

Check for compactness:

$$\frac{b_f}{2t_f} = 6.28 \qquad \text{(from Part 1 of the } \textit{Manual}\text{)}$$

$$0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15 > 6.28 \qquad \therefore \text{ the flange is compact.}$$



$w_D = 450$ lb/ft
$w_L = 550$ lb/ft

$$\frac{h}{t_w} < 3.76\sqrt{\frac{E}{F_y}} \qquad \text{(for all shapes in the } \textit{Manual}\text{)}$$

$\therefore$ a W16 × 31 is compact for $F_y = 50$ ksi.

Because the beam is compact and laterally supported,

$$M_n = M_p = F_y Z_x = 50(54.0) = 2700 \text{ in.-kips} = 225.0 \text{ ft-kips}$$

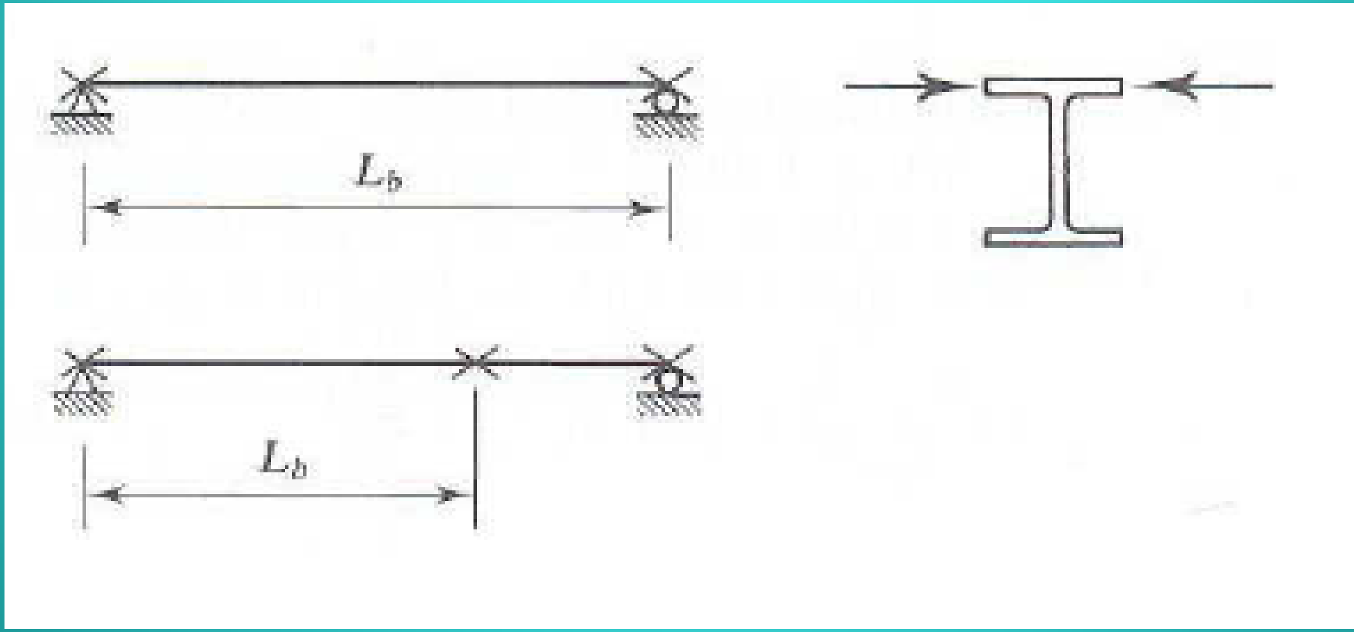
Check for $M_p \leq 1.5M_y$:

$$\frac{Z_x}{S_x} = \frac{54.0}{47.2} = 1.14 < 1.5 \qquad \text{(OK)}$$

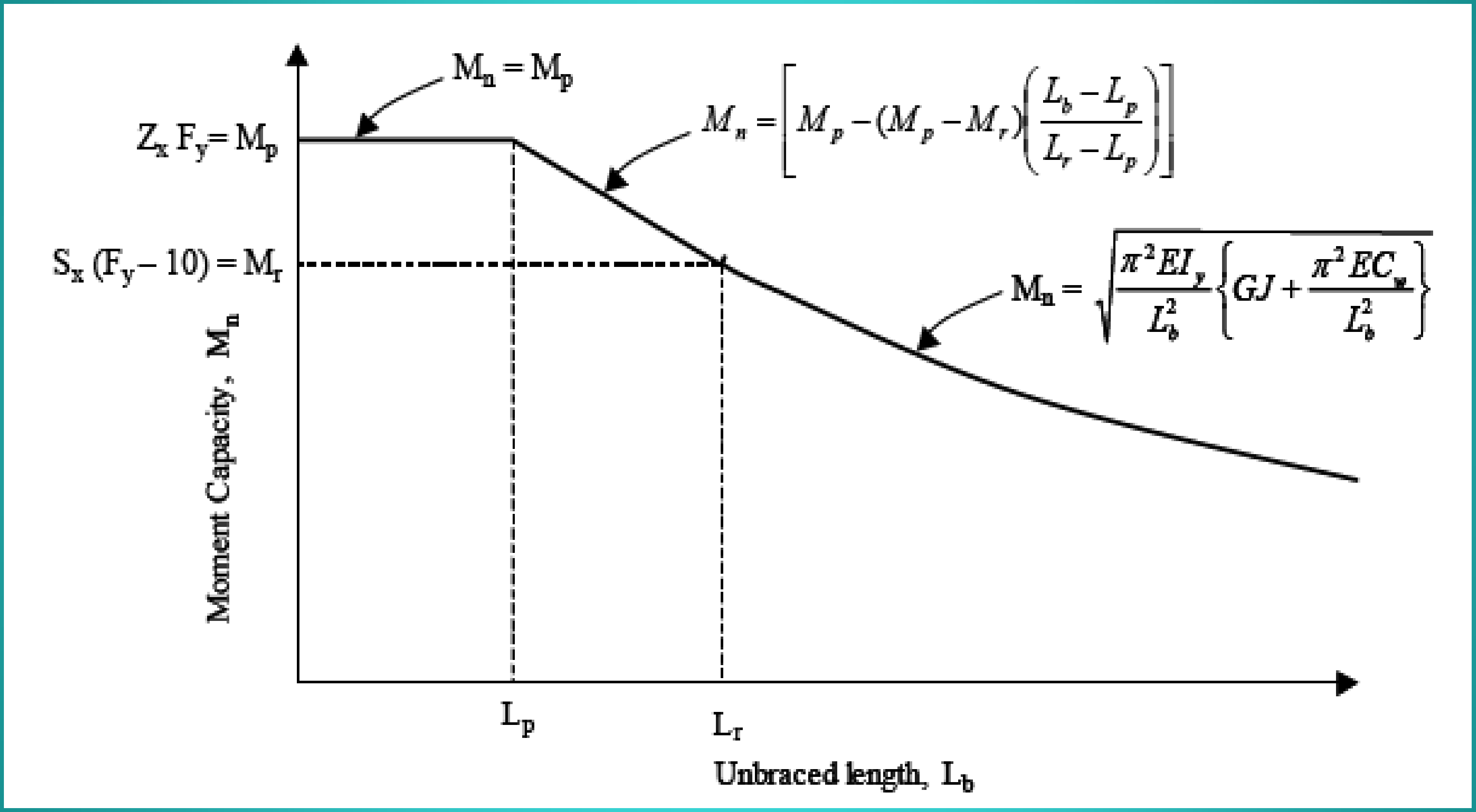
$$\phi_b M_n = 0.90(225.0) = 203 \text{ ft-kips} > 164 \text{ ft-kips} \qquad \text{(OK)}$$

The design moment is greater than the factored load moment, so the W16×31 is satisfactory. ▩

- The moment strength of compact shapes is a function of the unbraced length, L_b, defined as the distance between points of lateral support, or bracing.
- We will indicate points of lateral support with an X as shown in the Figure:



The relationship between the nominal strength, M_n, and the unbraced length, L_b, is shown in the following Figure:



If the unbraced length is less than L_p, the beam is considered to have full lateral support and $M_n = M_p$.

If L_b is greater than L_p then lateral torsional buckling will occur and the moment capacity of the beam will be reduced below the plastic strength M_p as shown in Figure.

The lateral-torsional buckling moment ($M_n = M_{cr}$) is a function of the laterally unbraced length Lb and can be calculated using the eq.:

$$M_n = M_{cr} = \frac{\pi}{L_b}\sqrt{E \times I_y \times G \times J + \left(\frac{\pi \times E}{L_b}\right)^2 \times I_y \times C_w}$$

Where, M_n = moment capacity, L_b = laterally unsupported length. M_{cr} = critical lateral-torsional buckling moment., E = 29000 ksi;, G = 11,200 ksi., I_y = moment of inertia about minor or y-axis (in4), J = torsional constant (in^4) from the AISC manual and C_w = warping constant (in^6) from the AISC manual.

This equation is valid for ELASTIC lateral torsional buckling only. That is it will work only as long as the cross-section is elastic and no portion of the cross-section has yielded.

As soon as any portion of the cross-section reaches the F_y , the elastic lateral torsional buckling equation cannot be used, and the moment corresponding to first yield is:

$$M_r = S_x (F_y - 10).$$

As shown in the figure, the boundary between elastic and inelastic behavior will be an unbraced length of L_r, which is the value of unbraced length that corresponds to a lateral-torsional buckling moment.

$$L_r = \frac{r_y X_1}{(F_y - 10)} \sqrt{1 + \sqrt{1 + X_2 (F_y - 10)^2}}$$

Where:

$$X_1 = \frac{\pi}{S_x} \sqrt{\frac{EGJA}{2}}$$

$$X_2 = \frac{4C_w}{I_y} \left(\frac{S_x}{GJ} \right)^2$$

Inelastic behavior of beam is more complicated than elastic behavior, and empirical formulas are often used.

$$M_n = M_p - (M_p - M_r) \left(\frac{L_b - L_p}{L_r - L_p} \right)$$

where

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}}$$

Moment Capacity of beams subjected to non-uniform B.M.

The case with uniform bending moment is worst for lateral torsional buckling.

For cases with non-uniform B.M, the lateral torsional buckling moment is greater than that for the case with uniform moment.

The AISC specification says that:

The lateral torsional buckling moment for non-uniform B.M case = C_b x lateral torsional buckling moment for uniform moment case.

C_b is always greater than 1.0 for non-uniform bending moment.

C_b is equal to 1.0 for uniform bending moment.

Sometimes, if you cannot calculate or figure out C_b, then it can be conservatively assumed as 1.0.

$$C_b = \frac{12.5\, M_{max}}{2.5\, M_{max} + 3\, M_A + 4\, M_B + 3\, M_c}$$

where,

M_{max} = magnitude of maximum bending moment in L_b

M_A = magnitude of bending moment at quarter point of L_b

M_B = magnitude of bending moment at half point of L_b

M_C = magnitude of bending moment at three-quarter point of L_b

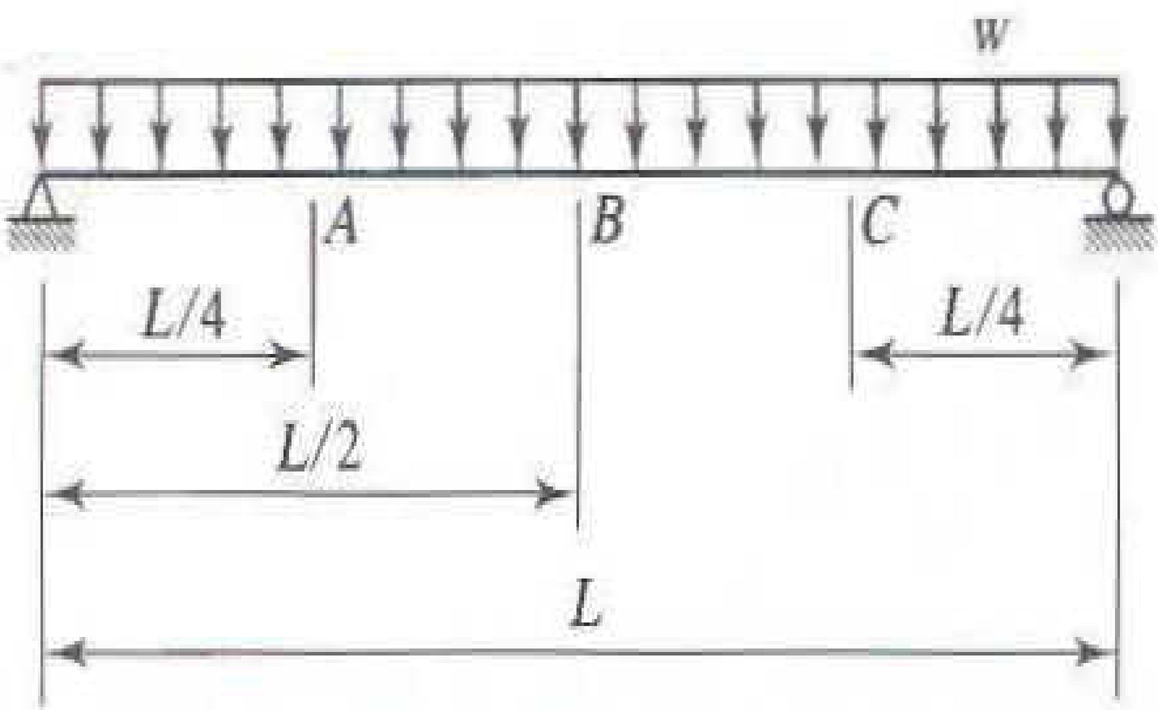
The moment capacity M_n for the case of non-uniform bending moment = $M_n = C_b \times \{M_n$ for the case of uniform B.M$\} \le M_p$

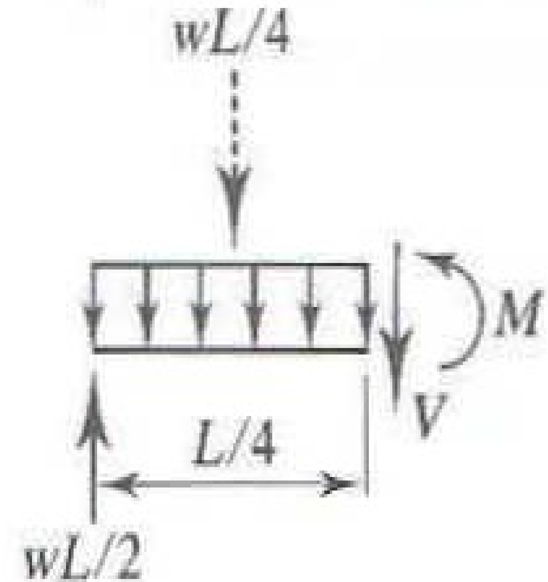
Example 5.4

Determine C_b for a uniformly loaded, simply supported beam with lateral support at its ends only.

Because of symmetry, the maximum moment is at midspan, so

$$M_{\max} = M_B = \frac{1}{8}wL^2$$





Also because of symmetry, the moment at the quarter point equals the moment at the three-quarter point. From Figure 5.14,

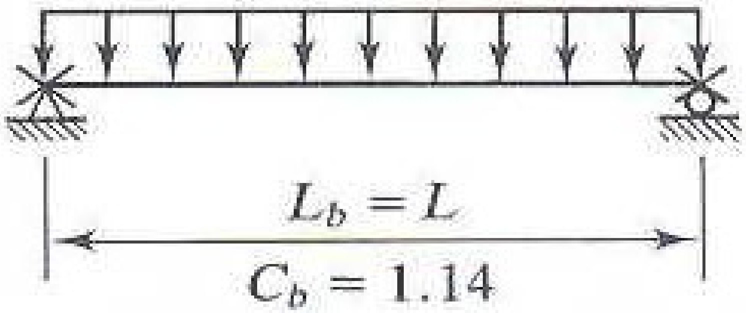
$$M_A = M_C = \frac{wL}{2}\left(\frac{L}{4}\right) - \frac{wL}{4}\left(\frac{L}{8}\right) = \frac{wL^2}{8} - \frac{wL^2}{32} = \frac{3}{32}wL^2$$

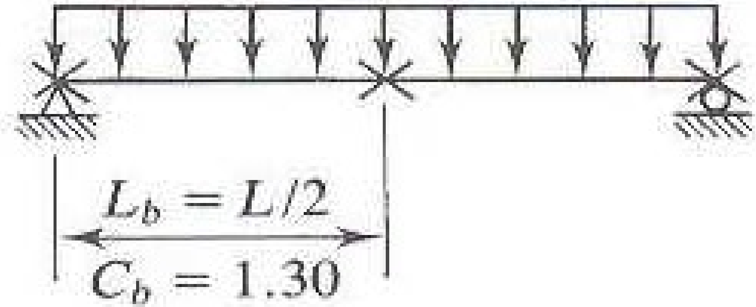
$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

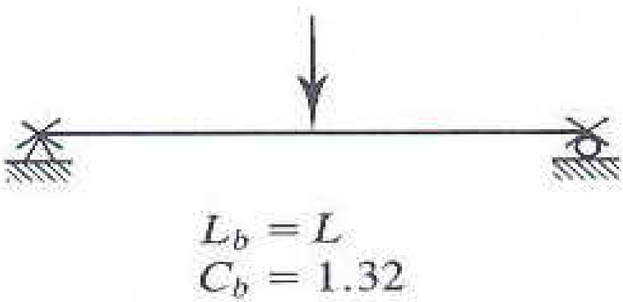
$$= \frac{12.5\left(\frac{1}{8}\right)}{2.5\left(\frac{1}{8}\right) + 3\left(\frac{3}{32}\right) + 4\left(\frac{1}{8}\right) + 3\left(\frac{3}{32}\right)} = 1.14$$

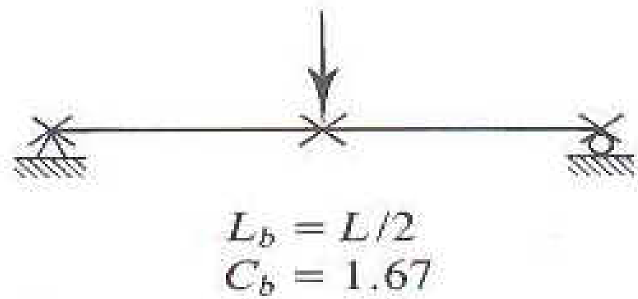
Answer $C_b = 1.14.$

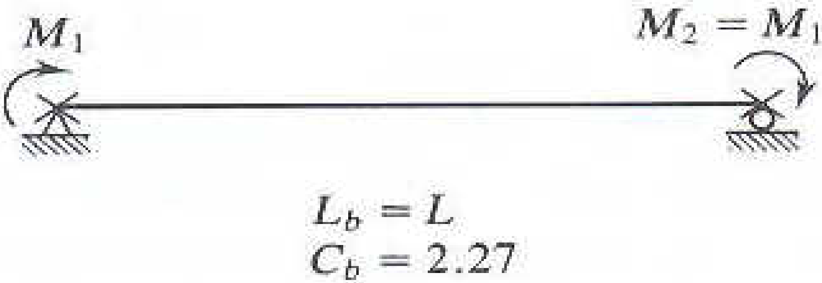
The following Figures shows typical values of C_b.

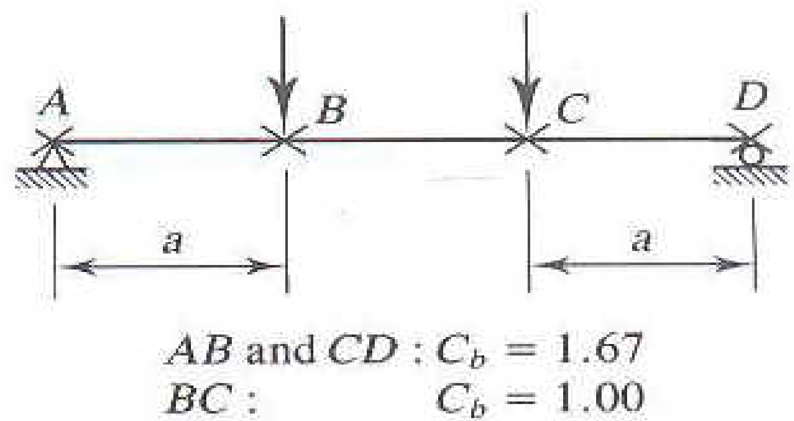

$L_b = L$
$C_b = 1.14$


$L_b = L/2$
$C_b = 1.30$


$L_b = L$
$C_b = 1.32$


$L_b = L/2$
$C_b = 1.67$


M_1
$M_2 = M_1$
$L_b = L$
$C_b = 2.27$


A
B
C
D
a
a
AB and CD : $C_b = 1.67$
BC : $C_b = 1.00$

$L_b = L$

$C_b = 1.67$

The complete specification of nominal moment strength for compact shapes can now be summarized.

For $L_b \le L_p$,

$$M_n = M_p \le 1.5M_y \qquad \text{(AISC Equation F1-1)}$$

For $L_p < L_b \le L_r$,

$$M_n = C_b\left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

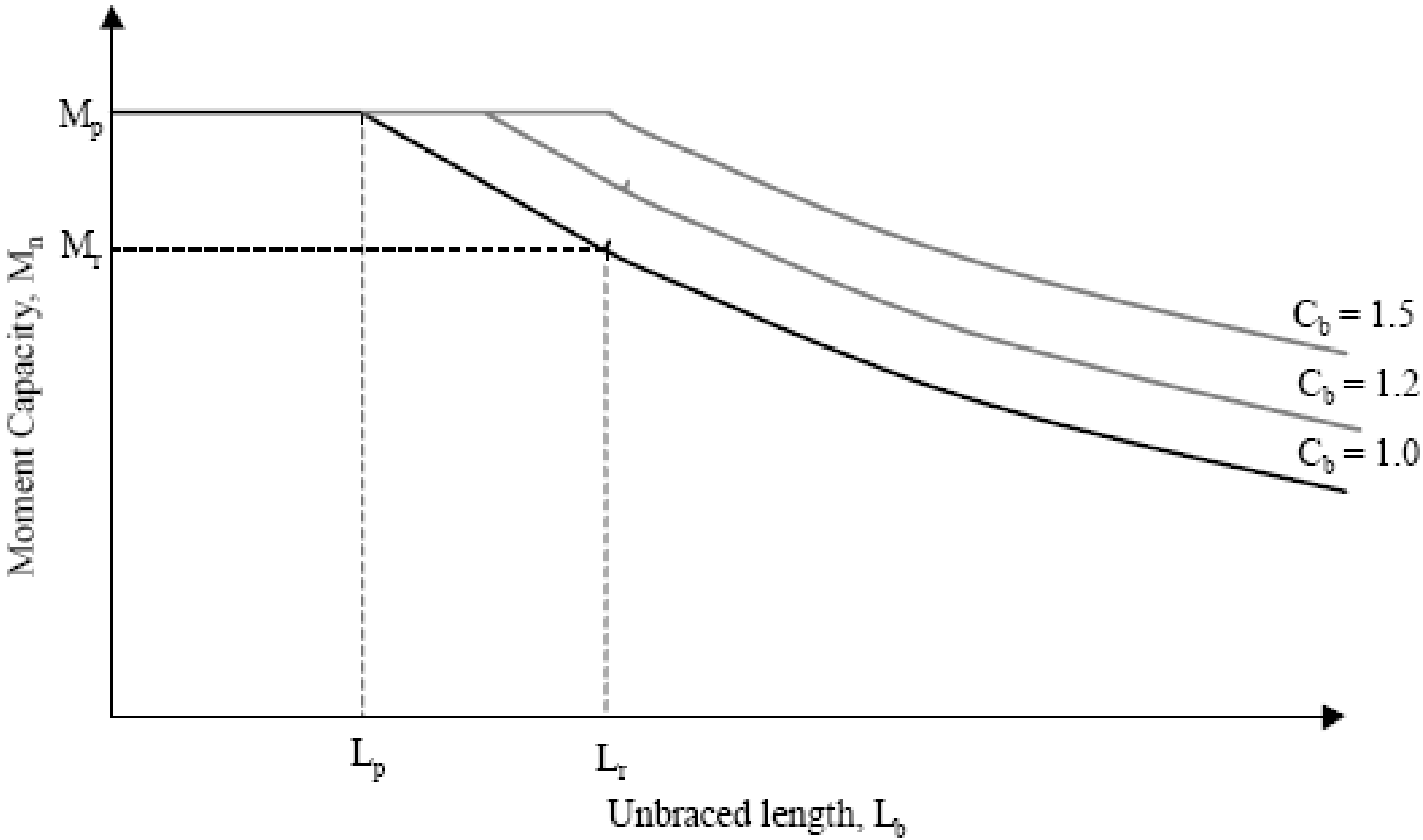
For $L_b > L_r$,

$$M_n = M_{cr} \le M_p$$

where

$$M_{cr} = C_b \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$$

$$= \frac{C_bS_xX_1\sqrt{2}}{L_b/r_y}\sqrt{1 + \frac{X_1^2X_2}{2(L_b/r_y)^2}}$$



Moment capacity versus L_b for non-uniform moment case.

Example 5.5:

Determine the design strength $\phi_b M_n$ for a W14 × 68 of A242 steel subject to

a. continuous lateral support.
b. unbraced length = 20 ft; C_b = 1.0.
c. unbraced length = 20 ft; C_b = 1.75.

Answer:

a. From Part 2 of the *Manual*, a W14 × 68 is in shape group 2 and is therefore available with a yield stress, F_y, of 50 ksi. Determine whether this shape is compact, noncompact, or slender:

$$\frac{b_f}{2t_f} = 6.97 < 0.38\sqrt{\frac{29{,}000}{50}} = 9.15$$

This shape is compact and thus

$$M_n = M_p = F_y Z_x = 50(115) = 5750 \text{ in.-kips} = 479.2 \text{ ft-kips}$$

$$\phi_b M_n = 0.90(479.2) = 431 \text{ ft-kips}$$

b. $L_b = 20$ ft and $C_b = 1.0$. Compute L_p and L_r:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76(2.46)\sqrt{\frac{29{,}000}{50}} = 104.3 \text{ in.} = 8.689 \text{ ft}$$

From the torsion properties tables in Part 1 of the *Manual*,

$$J = 3.01 \text{ in.}^4 \quad \text{and} \quad C_w = 5370 \text{ in.}^6$$

Although X_1 and X_2 are tabulated in the dimensions and properties tables in Part 1 of the *Manual*, we compute them here for illustration:

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}} = \frac{\pi}{103}\sqrt{\frac{29{,}000(11{,}200)(3.01)(20.0)}{2}} = 3016 \text{ ksi}$$

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2 = 4\left(\frac{5370}{121}\right)\left(\frac{103}{11{,}200 \times 3.01}\right)^2 = 0.001657 \text{ (ksi)}^{-2}$$

$$L_r = \frac{r_y X_1}{(F_y - F_r)} \sqrt{1 + \sqrt{1 + X_2 (F_y - F_r)^2}}$$

$$= \frac{2.46(3016)}{(50 - 10)} \sqrt{1 + \sqrt{1 + 0.001657(50 - 10)^2}} = 316.5 \text{ in.} = 26.37 \text{ ft}$$

Since $L_p < L_b < L_r$, the strength is based on inelastic LTB and

$$M_r = (F_y - F_r)S_x = \frac{(50 - 10)(103)}{12} = 343.3 \text{ ft-kips}$$

$$M_n = C_b \left[M_p - (M_p - M_r) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right]$$

$$= 1.0 \left[479.2 - (479.2 - 343.3) \left(\frac{20 - 8.689}{26.37 - 8.689} \right) \right]$$

$$= 392.3 \text{ ft-kips} < M_p$$

Answer $\phi_b M_n = 0.90(392.3) = 353$ ft-kips

c. $L_b = 20$ ft and $C_b = 1.75$. The design strength for $C_b = 1.75$ is 1.75 times the design strength for $C_b = 1.0$. Therefore,

$$M_n = 1.75(392.3) = 686.5 \text{ ft-kips} > M_p = 479.2 \text{ ft-kips}$$

The nominal strength cannot exceed M_p; hence use a nominal strength of M_n = 479.2 ft-kips:

Answer $\phi_b M_n = 0.90(479.2) = 431$ ft-kips.

5.6 BENDING STRENGTH OF NONCOMPACT SHAPES

Beam may fail by:

Lateral-torsional buckling. (LTB)

Flange local buckling (FLB).

Web local buckling (WLB).

Any of these failures can be in either the elastic range or the inelastic range.

The strength corresponding to each of these three limit states must be computed.

The smallest value will control.

For flange local buckling

If $\lambda_p \le \lambda \le \lambda_r$, the flange is noncompact, buckling will be inelastic, and

$$M_n = M_p - (M_p - M_r)\left[\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right]$$

where

$$\lambda = \frac{b_f}{2t_f}, \quad \lambda_p = 0.38\sqrt{\frac{E}{F_y}}, \quad \lambda_r = 0.83\sqrt{\frac{E}{F_y - 10}}$$

$$M_r = (F_y - 10)S_x$$

For flange local buckling

If $\lambda_p \leq \lambda \leq \lambda_r$, the flange is noncompact, buckling will be inelastic, and

$$M_n = M_p - (M_p - M_r)\left[\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right]$$

where

$$\lambda = \frac{h}{t_w}, \quad \lambda_p = 3.76\sqrt{\frac{E}{F_y}}, \quad \lambda_r = 5.70\sqrt{\frac{E}{F_y}}$$

$$M_r = F_y S_x$$

Note that M_r definition is different for the flange local buckling

Example 5.6

a simply supported beam with a span length of 40 feet is laterally supported at its ends and is subjected to 400 Ib/ft D.L and 1000 Ib/ft L.L. if F_y =50 ksi, is W 14 x 90 adequate?

Solution:

Factored load = 1.2*0.4 + 1.6*1.0 = 2.080 kips/ft

$M_u=(2.08 * (40)^2)/8 = 416.0$ ft.kips

determine whether the shape is compact, or noncompact, or slender

$$\lambda = \frac{b_f}{2t_f} = 10.2$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29000}{50}} = 9.15$$

$$\lambda_r = 0.83\sqrt{\frac{E}{F_y - 10}} = 0.83\sqrt{\frac{29000}{50-10}} = 22.3$$

since $\lambda_p \le \lambda \le \lambda_r$, this shape is noncompact. Check the capacity based on the limit state of flange local buckling

$$M_p = F_y Z_x = \frac{50 * 157}{12} = 654.2\ ft.kips$$

$$M_r = (F_y - 10)S_x = \frac{(50-10)143}{12} = 476.7\ ft.kips$$

$$M_n = M_p - (M_p - M_r)\left[\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right]$$

$$M_n = 654.2 - (654.2 - 476.7)\left[\frac{10.2 - 9.15}{22.3 - 915}\right]$$

Check the capacity based on the limit state of LTB. From the Z_x table,

L_p= 15.1 ft and L_r = 38.4 ft

L_b = 40.0 ft > L_r so failure is by elastic LTB.

From Manual:

I_y = 362 in^4 , J = 4.06 in^4 and C_w = 16.000 in^6

for a uniformly loaded, simply supported beam with lateral support at the ends, C_b = 1.14

$$M_n = C_b \frac{\pi}{L_b} \sqrt{EI_y GJ + \left[\frac{\pi E}{L_b}\right]^2 I_y C_w} \leq M_p$$

$$M_n = 1.14\left[\frac{\pi}{40*12} \sqrt{29000*362*11200*4.06 + \left[\frac{\pi 29000}{40*12}\right]^2 362*16000}\right]$$

$$M_n = 1.14*5421 = 6180.0 \ in.kips = 515.0 \ ft.kips$$

$$M_p = 654.2 > 515.0 \quad (OK)$$

Because 5150 < 640.0, LTB controls, and

$\Phi M_n = 0.90 * 515.0 = 464.0$ ft.kips > $M_u = 416.0$ ft.kips

Since $M_u < \Phi M_n$, the beam has adequate moment strength

5.7 SUMMARY OF MOMENT STRENGTH

Please read it.