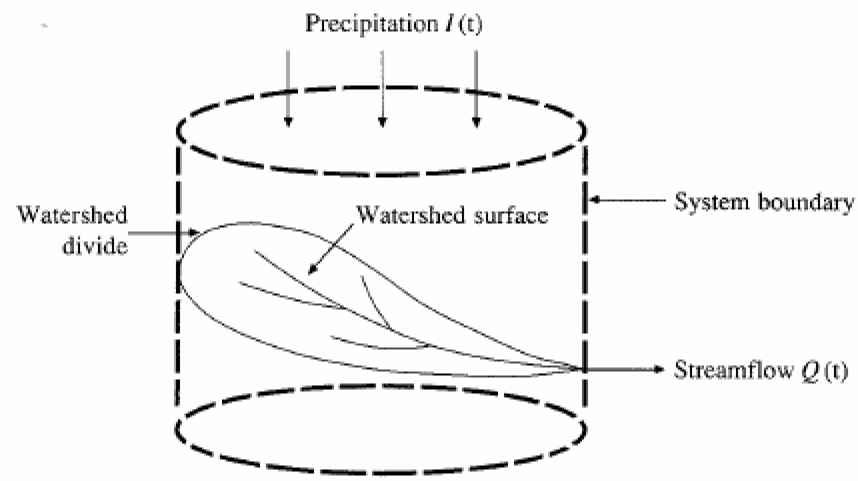
Runoff Hydrograph and Flow Routing

Quote for Today: "Can we afford clean water? Can we afford rivers and lakes and streams and oceans which continue to make possible life on this planet? Can we afford life itself? Those questions were never asked as we destroyed the waters of our nation, and they deserve no answers as we finally move to restore and renew them. These questions answer themselves."

Hydrologic Analysis



The watershed as a hydrologic system.

$$Q(t) = \Omega I(t)$$

$$\Omega = \frac{Q(t)}{I(t)}$$

Change in storage w.r.t. time = inflow - outflow $\dfrac{dS}{dt} = I(t) - Q(t)$

In the case of a linear reservoir, $S = kQ$

$$k\frac{dQ}{dt} + Q(t) = I(t)$$

$$\Omega = \frac{Q(t)}{I(t)} = \frac{1}{1 + kD}$$

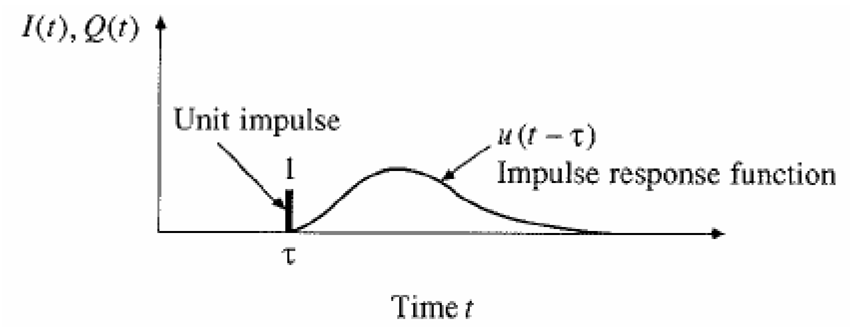
Transfer function for a linear system ($S = kQ$).

Proportionality and superposition

- Linear system (k is constant in $S = kQ$)
  - Proportionality
    - *If* $I_1 \rightarrow Q_1$ then $C^*I_2 \rightarrow C^*Q_2$
  - Superposition
    - *If* $I_1 \rightarrow Q_1$ and $I_2 \rightarrow Q_2$, t*hen* $I_1 + I_2 \rightarrow Q_1 + Q_2$

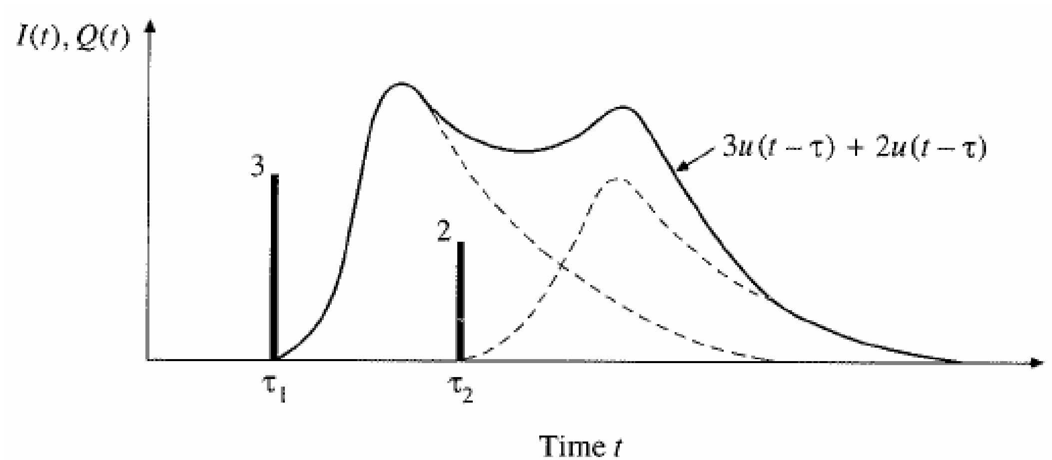
Impulse response function

Impulse input: an input applied instantaneously (spike) at time τ and zero everywhere else



An unit impulse at τ produces as unit impulse response function u(t-τ)

Principle of proportionality and superposition



Convolution integral

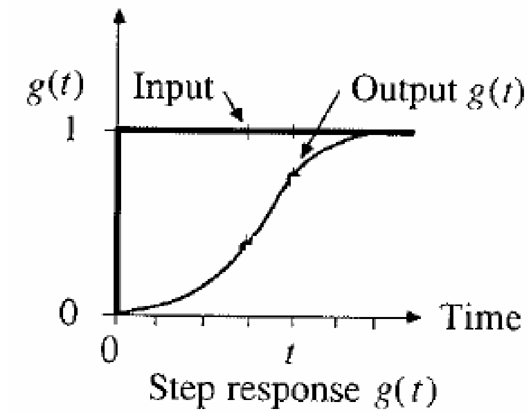
- For an unit impulse, the response of the system is given by the unit impulse response function $u(t-\tau)$
- An impulse of 3 units produces the $3u(t-\tau)$
- If $I(\tau)$ is the precipitation intensity occurring for a time period of $d\tau$, the response of the system (direct runoff) is $I(\tau)u(t-\tau)d\tau$
- The complete response due to the input function $I(\tau)$ is [illegible] ution integral

$$Q(t) = \int_0^t I(\tau)u(t-\tau)d\tau$$

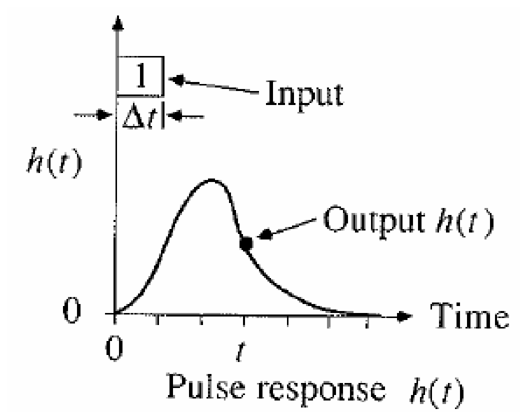
- Response of a linear system is the sum (convolution) of the responses to inputs that have happened in the past.

Step and pulse inputs

- A unit step input is an input that goes from 0 to 1 at time 0 and continues indefinitely thereafter



- A unit pulse is an input of unit amount occurring in duration Δt and 0 elsewhere.



Precipitation is a series of pulse inputs!

Unit Hydrograph Theory

- Direct runoff hydrograph resulting from a unit depth of excess rainfall occurring uniformly on a watershed at a constant rate for a specified duration.
- Unit pulse response function of a linear hydrologic system
- Can be used to derive runoff from any excess rainfall on the watershed.

Unit hydrograph assumptions

- Assumptions
  - Excess rainfall has constant intensity during duration
  - Excess rainfall is uniformly distributed on watershed
  - Base time of runoff is constant
  - Ordinates of unit hydrograph are proportional to total runoff (linearity)
  - Unit hydrograph represents all characteristics of watershed (lumped parameter) and is time invariant (stationarity)

Discrete Convolution

Continuous $$Q(t) = \int_0^t I(\tau)u(t-\tau)d\tau$$

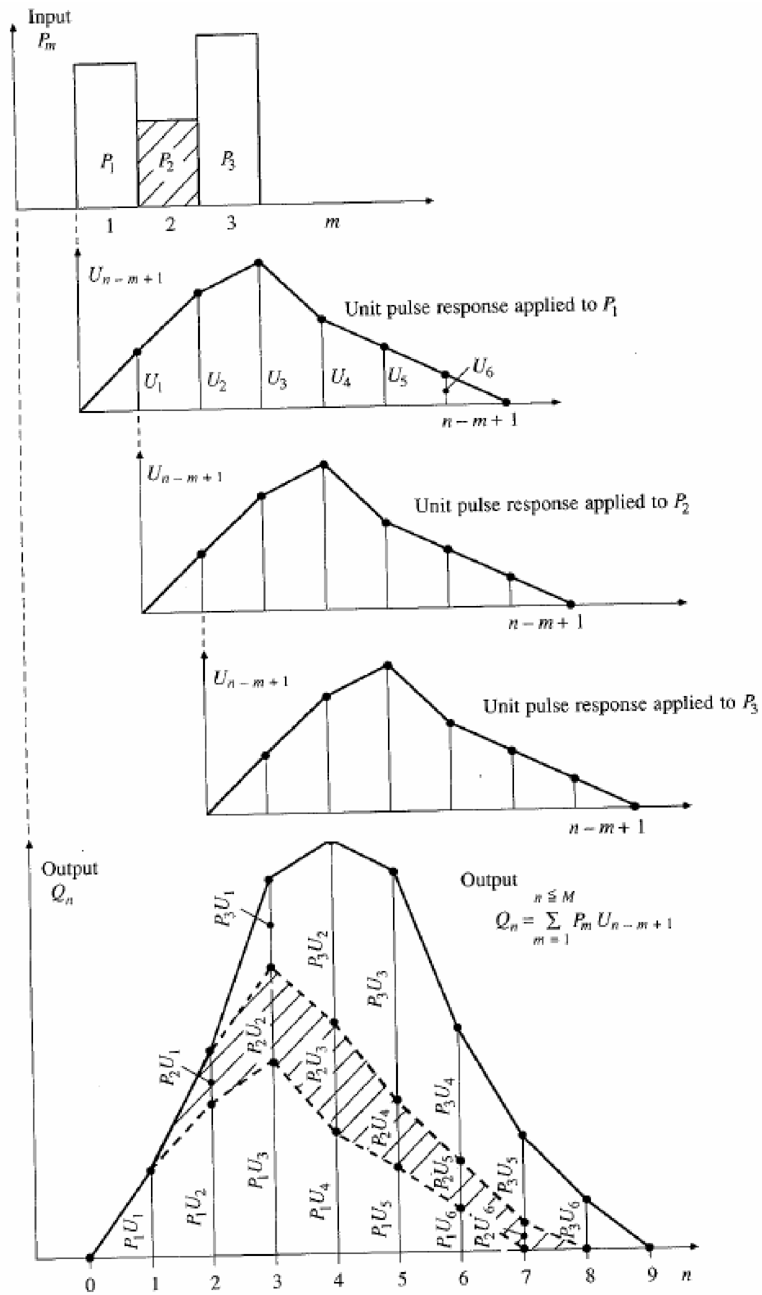
Discrete $$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1}$$

Q is flow, P is precipitation and U is unit hydrograph

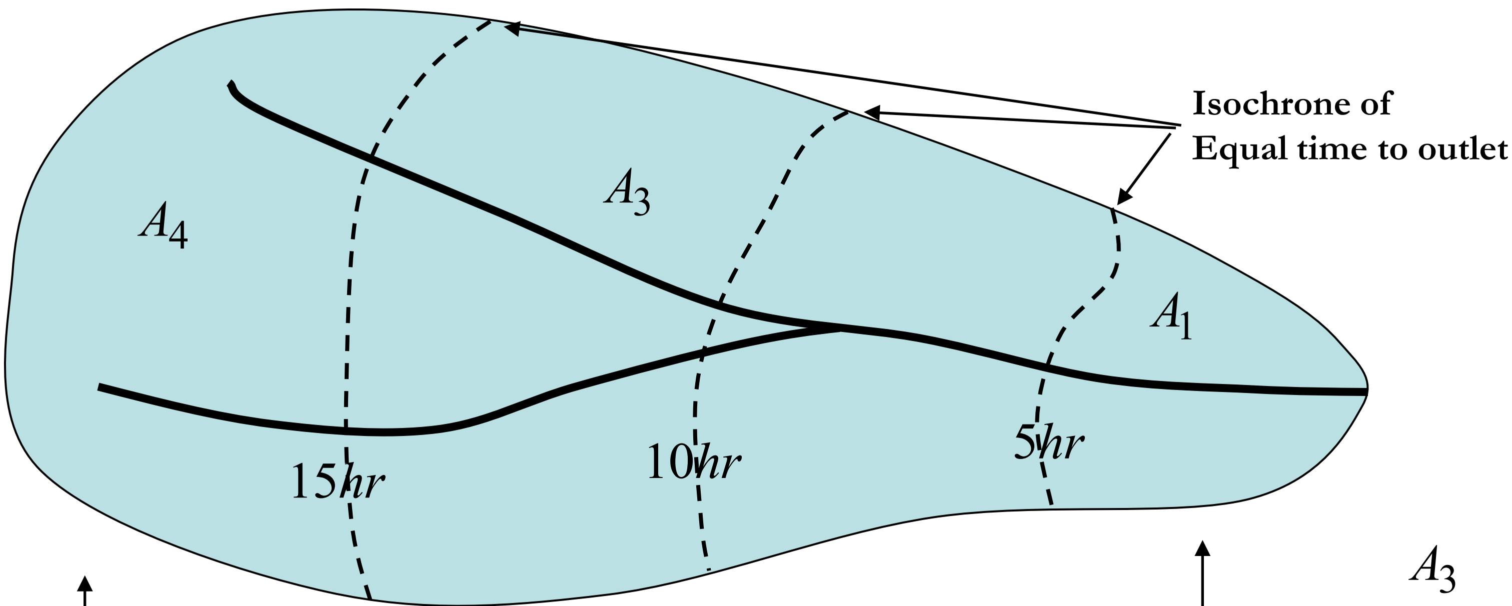
M is the number of precipitation pulses, n is the number of flow rate intervals

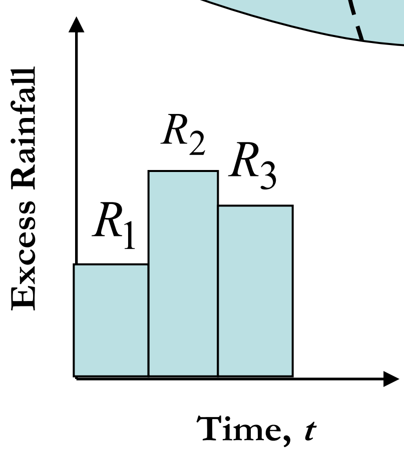
The unit hydrograph has N-M+1 pulses

Application of convolution to the output from a linear system

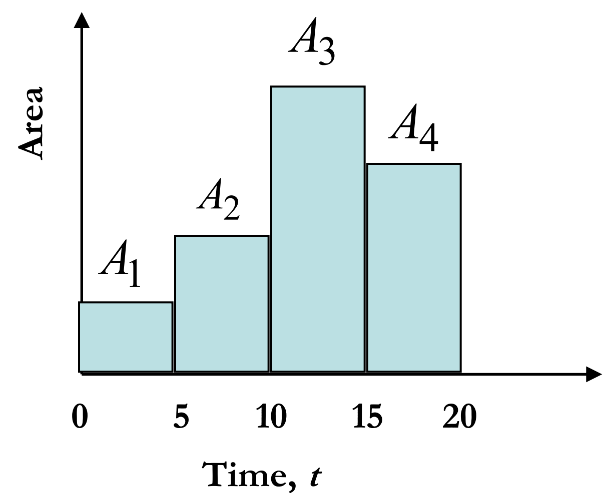


Time – Area Relationship





$$Q_n = R_i A_1 + R_{i-1} A_2 + \ldots + R_1 A_j$$



Application of UH

- Once a UH is derived, it can be used/applied to find direct runoff and stream flow hydrograph from other storm events.

Given:

Ex. 7.5.1

P_1 = 2 in, P_2 = 3 in and P_3 = 1 in, baseflow = 500 cfs and watershed area is 7.03 mi^2. Given the Unit Hydrograph below, determine the streamflow hydrograph

Unit hydrograph

n	1	2	3	4	5	6	7	8	9
U_n (cfs/in)	404	1079	2343	2506	1460	453	381	274	173

7.5.1 solution (cont'd)

Time ($\frac{1}{2}$-h)	Excess Precipitation (in)	Unit hydrograph ordinates (cfs/in)									Direct runoff (cfs)	Streamflow (cfs)
		1	2	3	4	5	6	7	8	9		
		404	1079	2343	2506	1460	453	381	274	173		
$n =$ 1	2.00	808									808	1308
2	3.00	1212	2158								3370	3870
3	1.00	404	3237	4686							8327	8827
4			1079	7029	5012						13,120	13,620
5				2343	7518	2920					12,781	13,281
6					2506	4380	906				7792	8292
7						1460	1359	762			3581	4081
8							453	1143	548		2144	2644
9								381	822	346	1549	2049
10									274	519	793	1293
11										173	173	673
										Total	54,438	

*Baseflow = 500 cfs.

See another example at: http://www.egr.msu.edu/~northco2/BE481/UHD.htm

Gauged and ungauged watersheds

- Gauged watersheds
  – Watersheds where data on precipitation, streamflow, and other variables are available
- Ungauged watersheds
  – Watersheds with no data on precipitation, streamflow and other variables.

Need for synthetic UH

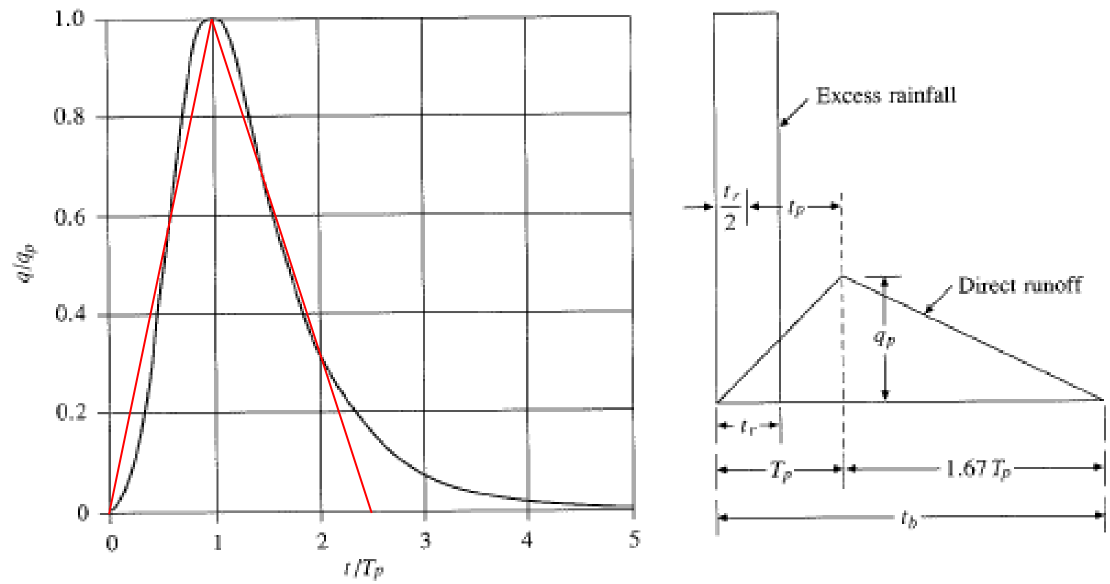
- UH is applicable only for gauged watershed and for the point on the stream where data are measured
- For other locations on the stream in the same watershed or for nearby (ungauged) watersheds, synthetic procedures are used.

Synthetic UH

- Synthetic hydrographs are derived by
  - Relating hydrograph characteristics such as peak flow, base time etc. with watershed characteristics such as area and time of concentration.
  - Using dimensionless unit hydrograph
  - Based on watershed storage

SCS dimensionless hydrograph

- Synthetic UH in which the discharge is expressed by the ratio of q to q_p and time by the ratio of t to T_p
- If peak discharge and lag time are known, UH can be estimated.



T_c: time of concentration

C = 2.08 (483.4 in English system)

A: drainage area in km² (mi²)

$$t_p \cong 0.6T_c \qquad t_b \cong 2.67T_p$$

$$T_p = \frac{t_r}{2} + t_p \qquad q_p = \frac{CA}{T_p}$$

Ex. 7.7.3

- Construct a 10-min SCS UH. A = 3.0 km^2 and T_c = 1.25 h

$$t_r = 10 \;\; \text{min} = 0.166 \;\; h$$

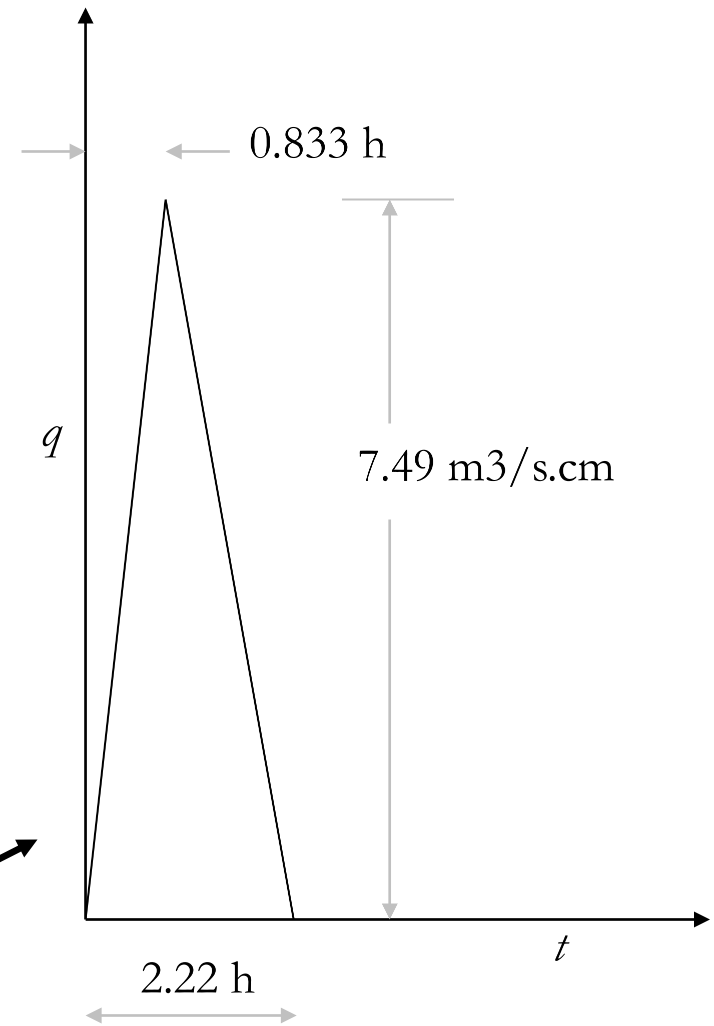
$$t_p = 0.6T_c = 0.6 \times 1.25 = 0.75 \;\; h$$

$$T_p = \frac{t_r}{2} + t_p$$

$$T_p = \frac{0.166}{2} + 0.75 = 0.833 \;\; h$$

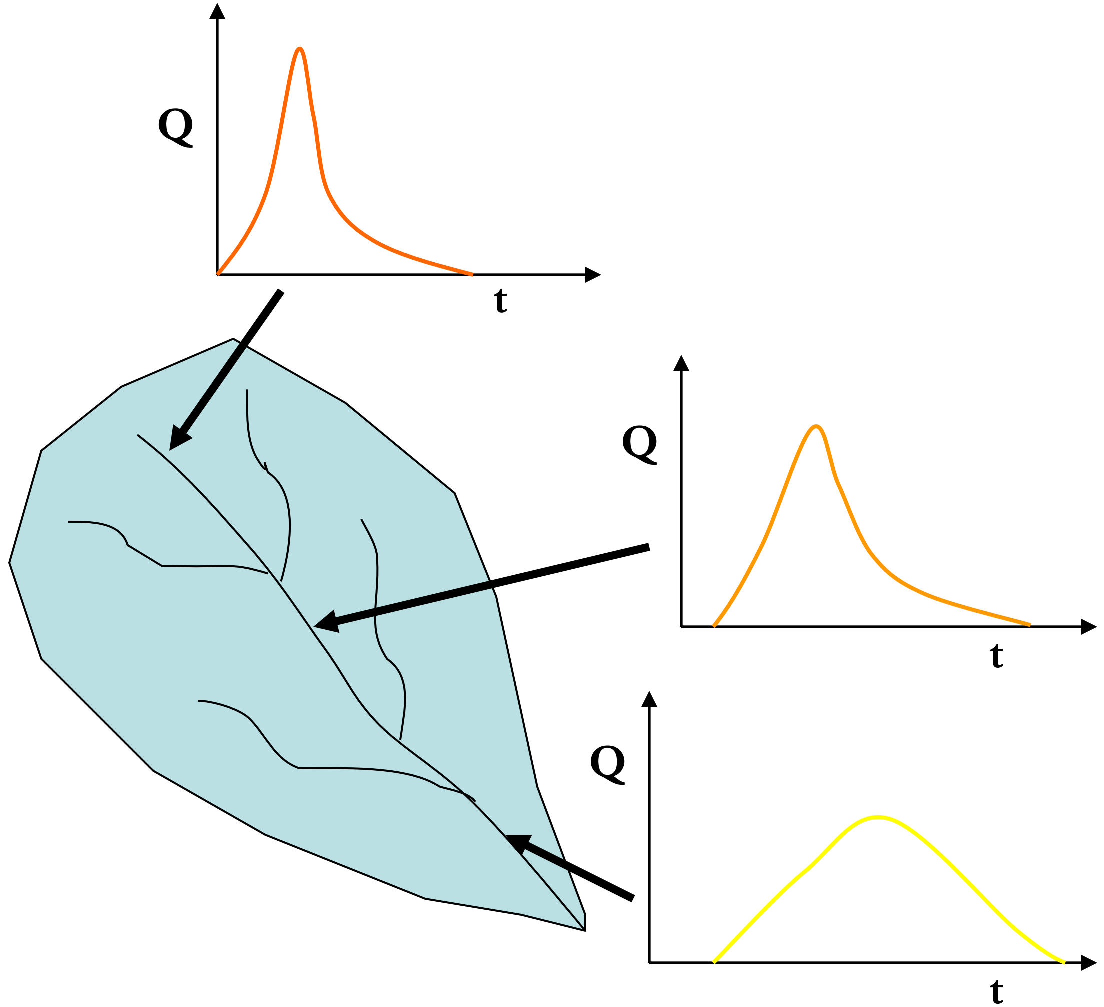
$$q_p = \frac{CA}{T_p} = \frac{2.08 \times 3}{0.833} = 7.49 \;\; m^3 / s.cm$$

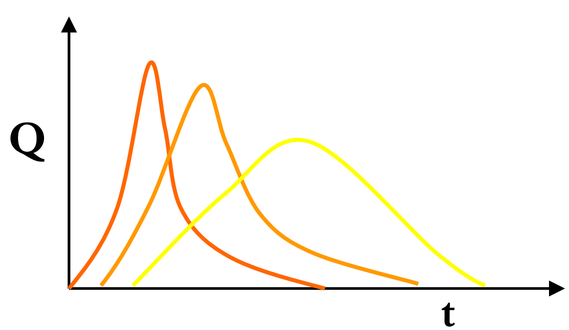
Multiply y-axis of SCS hydrograph by q_p and x-axis by T_p to get the required UH, or construct a triangular UH



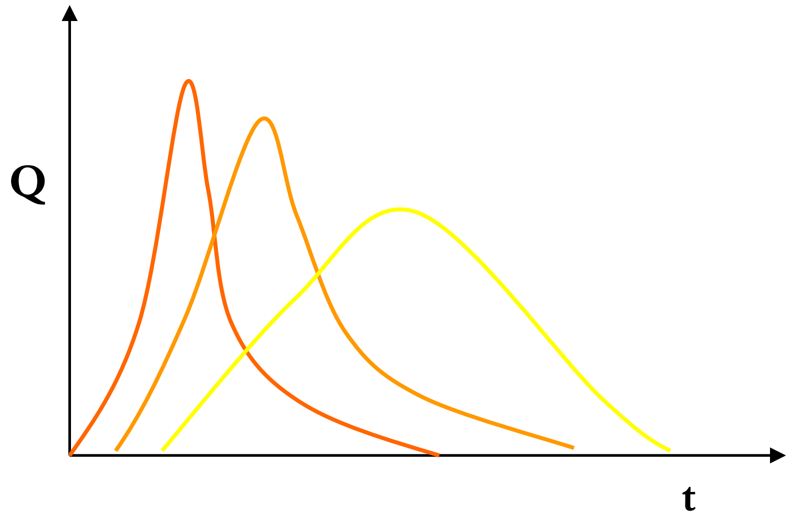
Flow Routing

- Procedure to determine the flow hydrograph at a point on a watershed from a known hydrograph upstream
- As the hydrograph travels, it
  - attenuates
  - gets delayed





Why route flows?

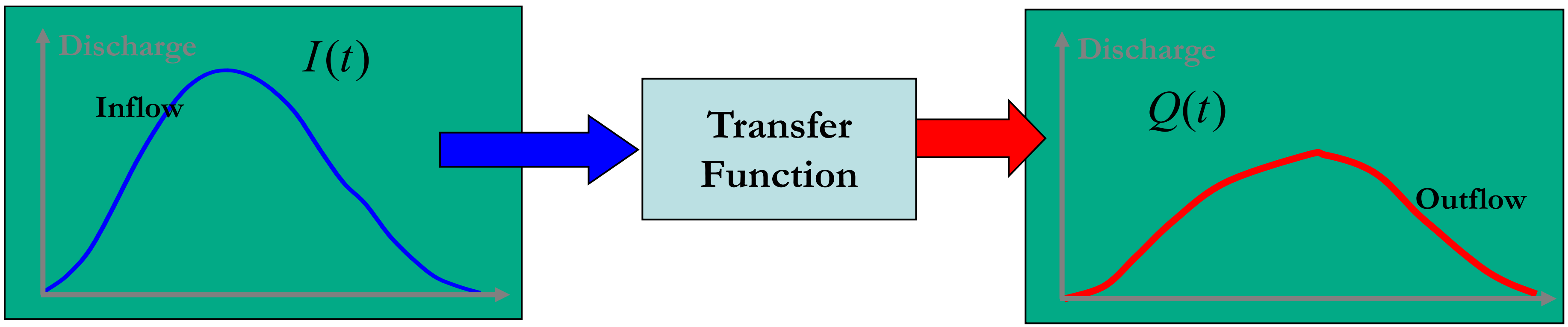


- Account for changes in flow hydrograph as a flood wave passes downstream
- This helps in
  - Accounting for storages
  - Studying the attenuation of flood peaks

Types of flow routing

- Lumped/hydrologic
  - Flow is calculated as a function of time alone at a particular location
  - Governed by continuity equation and flow/storage relationship
- Distributed/hydraulic
  - Flow is calculated as a function of space and time throughout the system
  - Governed by continuity and momentum equations

Hydrologic Routing



$I(t) = \text{Inflow}$

Upstream hydrograph

$Q(t) = \text{Outflow}$

Downstream hydrograph

Input, output, and storage are related by continuity equation:

$$\frac{dS}{dt} = I(t) - Q(t)$$ Q and S are unknown

Storage can be expressed as a function of I(t) or Q(t) or both

$$S = f\left(I, \frac{dI}{dt}, \cdots, Q, \frac{dQ}{dt}, \cdots\right)$$

For a linear reservoir, S=*k*Q

$$k\frac{dQ}{dt} + Q(t) = I(t)$$

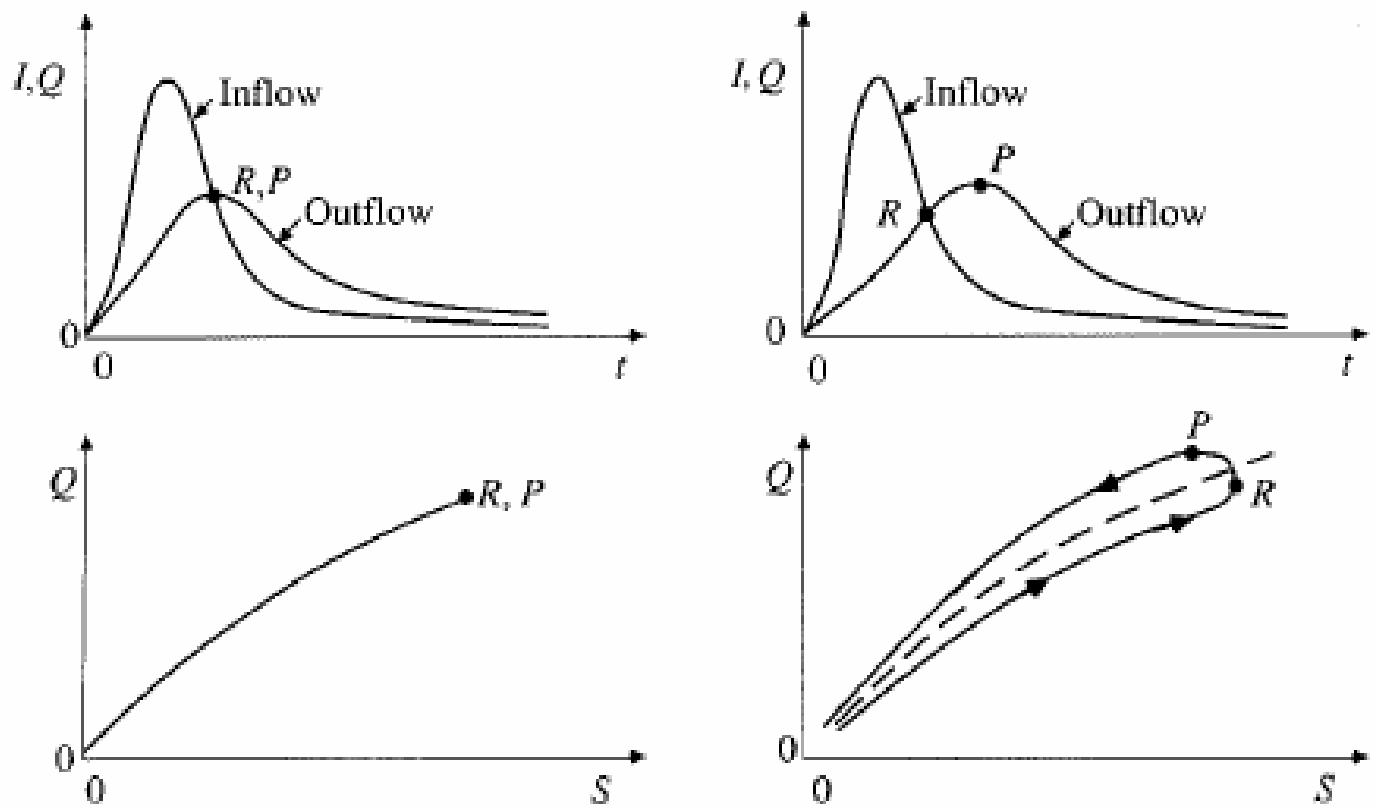
$$\Omega = \frac{Q(t)}{I(t)} = \frac{1}{1 + kD}$$

Lumped flow routing

- Three types

1. Level pool method (Modified Puls)
   - Storage is nonlinear function of Q
2. Muskingum method
   - Storage is linear function of I and Q
3. Series of reservoir models
   - Storage is linear function of Q and its time derivatives

S and Q relationships



(a) Invariable relationship

(b) Variable relationship

Level pool routing

- Procedure for calculating outflow hydrograph Q(t) from a reservoir with horizontal water surface, given its inflow hydrograph I(t) and storage-outflow relationship

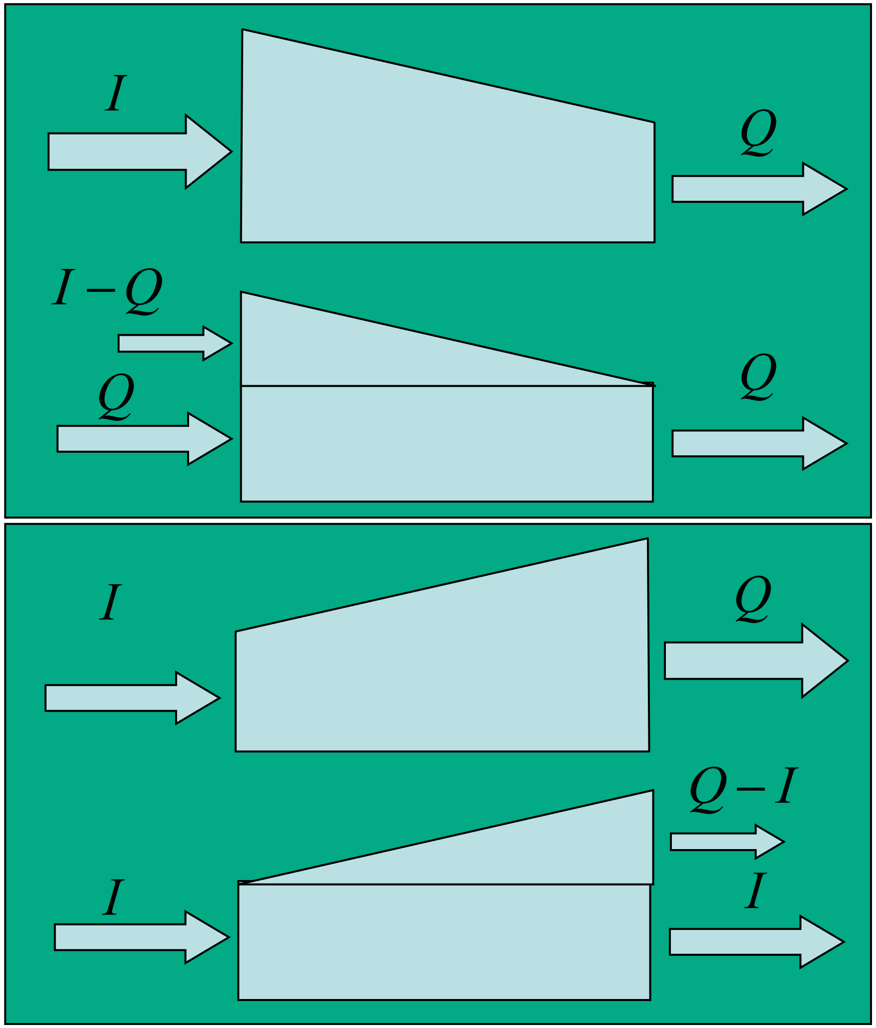
Hydrologic river routing (Muskingum Method)

Wedge storage in reach

$$S_{Prism} = KQ$$

$$S_{Wedge} = KX(I-Q)$$

Advancing Flood Wave
I* > *Q



K = travel time of peak through the reach
X = weight on inflow versus outflow (0 ≤ X ≤ 0.5)
X = 0 ➔ Reservoir, storage depends on outflow, no wedge
X = 0.0 - 0.3 ➔ Natural stream

Receding Flood Wave
Q* > *I

$$S = KQ + KX(I-Q)$$

$$\boxed{S = K[XI + (1-X)Q]}$$

Muskingum Method (Cont.)

$$S = K[XI + (1-X)Q]$$

$$S_{j+1} - S_j = K\{[XI_{j+1} + (1-X)Q_{j+1}] - [XI_j + (1-X)Q_j]\}$$

Recall:

$$S_{j+1} - S_j = \frac{I_{j+1} + I_j}{2}\Delta t - \frac{Q_{j+1} + Q_j}{2}\Delta t$$

Combine:

$$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j$$

$$C_1 = \frac{\Delta t - 2KX}{2K(1-X) + \Delta t}$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1-X) + \Delta t}$$

$$C_3 = \frac{2K(1-X) - \Delta t}{2K(1-X) + \Delta t}$$

If *I(t)*, *K* and *X* are known, *Q(t)* can be calculated using above equations

Muskingum - Example

- Given:
  - Inflow hydrograph
  - K = 2.3 hr, X = 0.15, Δt = 1 hour, Initial Q = 85 cfs
- Find:
  - Outflow hydrograph using Muskingum routing method

Period (hr)	Inflow (cfs)
1	93
2	137
3	208
4	320
5	442
6	546
7	630
8	678
9	691
10	675
11	634
12	571
13	477
14	390
15	329
16	247
17	184
18	134
19	108
20	90

$$C_1 = \frac{\Delta t - 2KX}{2K(1-X)+\Delta t} = \frac{1-2*2.3*0.15}{2*2.3(1-0.15)+1} = 0.0631$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1-X)+\Delta t} = \frac{1+2*2.3*0.15}{2*2.3(1-0.15)+1} = 0.3442$$

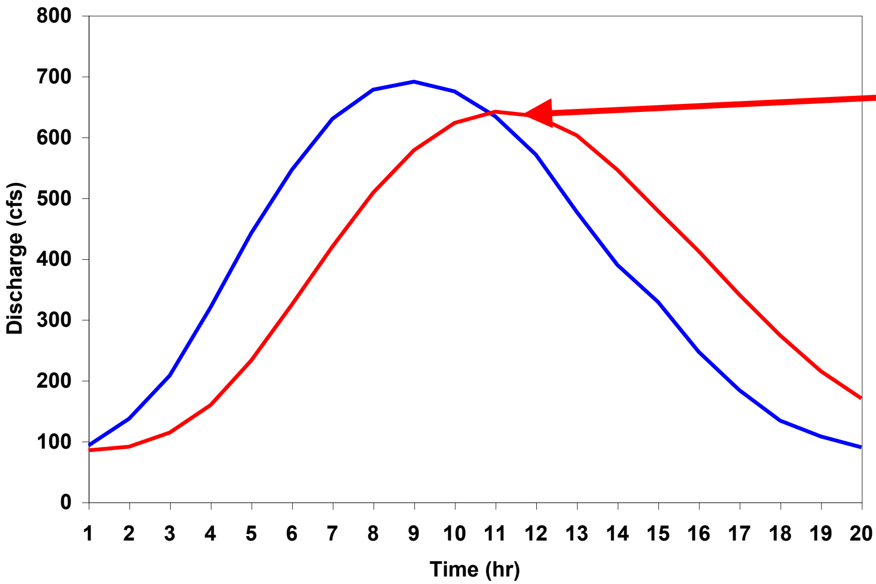
$$C_3 = \frac{2K(1-X)-\Delta t}{2K(1-X)+\Delta t} = \frac{2*2.3*(1-0.15)-1}{2*2.3(1-0.15)+1} = 0.5927$$

Muskingum – Example (Cont.)

$$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j$$

$C_1 = 0.0631$, $C_2 = 0.3442$, $C_3 = 0.5927$

Period (hr)	Inflow (cfs)	$C_1 I_{j+1}$	$C_2 I_j$	$C_3 Q_j$	Outflow (cfs)
1	93	0	0	0	85
2	137	9	32	50	91
3	208	13	47	54	114
4	320	20	72	68	159
5	442	28	110	95	233
6	546	34	152	138	324
7	630	40	188	192	420
8	678	43	217	249	509
9	691	44	233	301	578
10	675	43	238	343	623
11	634	40	232	369	642
12	571	36	218	380	635
13	477	30	197	376	603
14	390	25	164	357	546
15	329	21	134	324	479
16	247	16	113	284	413
17	184	12	85	245	341
18	134	8	63	202	274
19	108	7	46	162	215
20	90	6	37	128	170



Distributed Flow routing in channels

- Distributed Routing
- St. Venant equations
  - Continuity equation

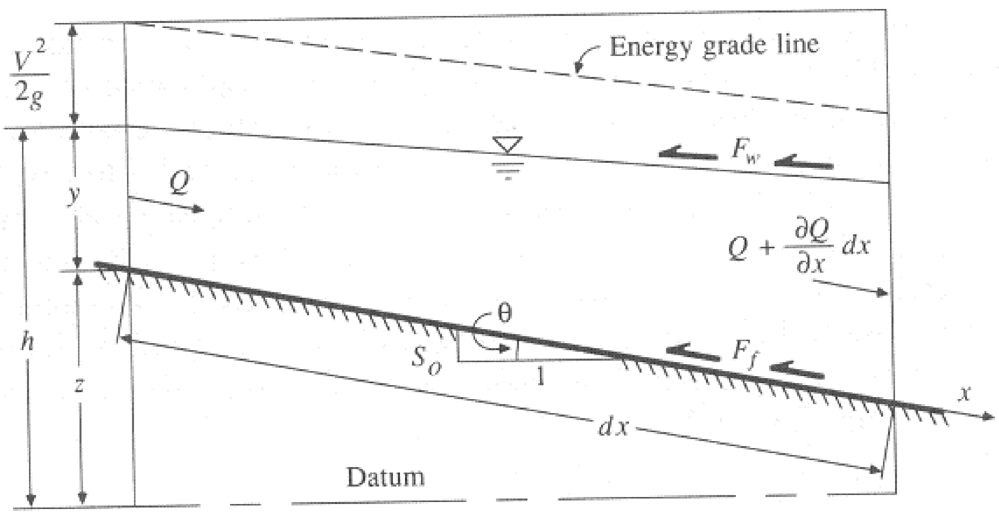
$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

  - Momentum Equation

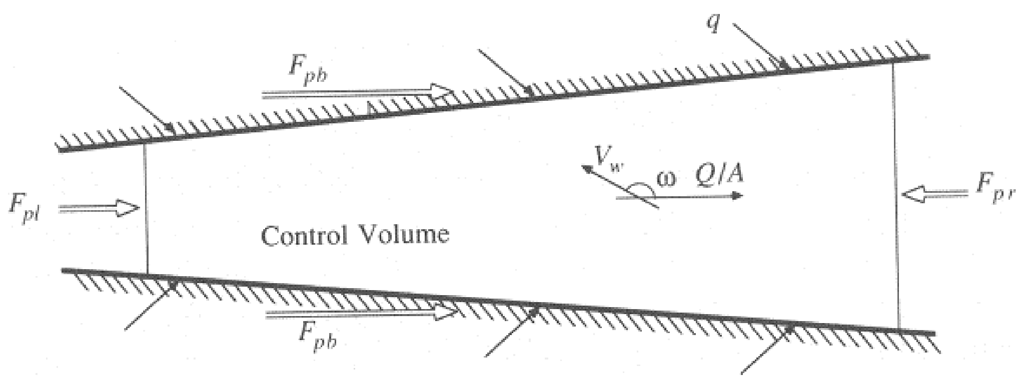
$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0$$

What are all these terms, and where are they coming from?

Continuity Equation



Elevation View



Plan View

Q = inflow to the control volume

q = lateral inflow

$\frac{\partial Q}{\partial x}$ Rate of change of flow with distance

$Q + \frac{\partial Q}{\partial x} dx$ Outflow from the C.V.

$\frac{\partial(\rho A dx)}{\partial t}$ Change in mass

Reynolds transport theorem

$$0 = \frac{d}{dt} \iiint_{c.v.} \rho d\forall + \iint_{c.s.} \rho V.dA$$

Continuity Equation (2)

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

Conservation form

$$\frac{\partial (Vy)}{\partial x} + \frac{\partial y}{\partial t} = 0$$

$$V\frac{\partial y}{\partial x} + y\frac{\partial V}{\partial x} + \frac{\partial y}{\partial t} = 0$$

Non-conservation form (velocity is dependent variable)

Momentum Equation

- From Newton's 2[nd] Law:
- Net force = time rate of change of momentum

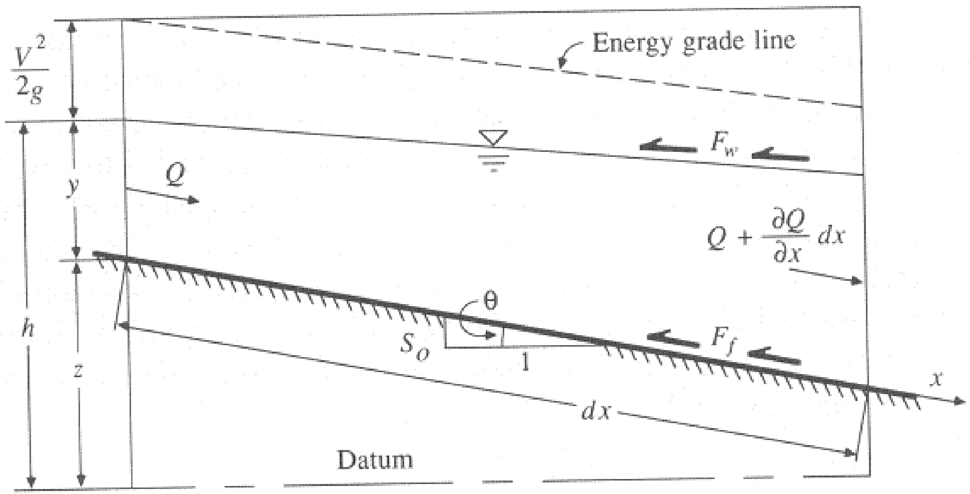
$$\sum F = \frac{d}{dt}\iiint_{c.v.} V\rho d\forall + \iint_{c.s.} V\rho V.dA$$

Sum of forces on the C.V.

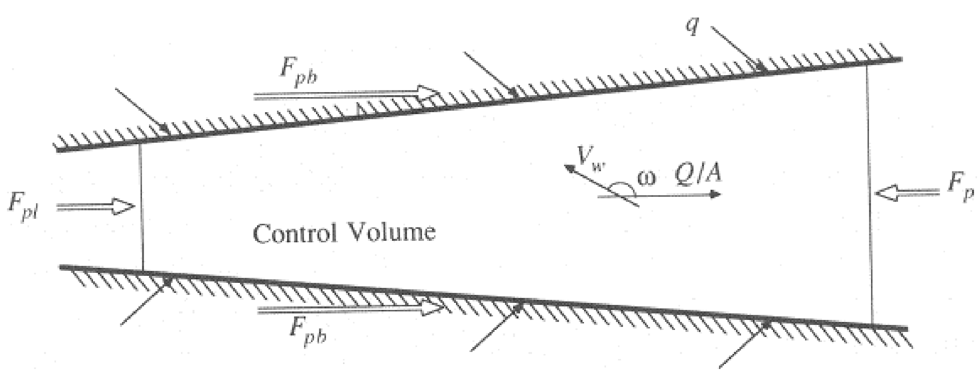
Momentum stored within the C.V

Momentum flow across the C. S.

Forces acting on the C.V.



Elevation View



Plan View

- F_g = Gravity force due to weight of water in the C.V.
- F_f = friction force due to shear stress along the bottom and sides of the C.V.
- F_e = contraction/expansion force due to abrupt changes in the channel cross-section
- F_w = wind shear force due to frictional resistance of wind at the water surface
- F_p = unbalanced pressure forces due to hydrostatic forces on the left and right hand side of the C.V. and pressure force exerted by banks

Momentum Equation

$$\sum F = \frac{d}{dt}\iiint_{c.v.} V\rho d\forall + \iint_{c.s.} V\rho V.dA$$

Sum of forces on the C.V.

Momentum stored within the C.V

Momentum flow across the C. S.

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0$$

Momentum Equation(2)

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0$$

Local acceleration term	Convective acceleration term	Pressure force term	Gravity force term	Friction force term

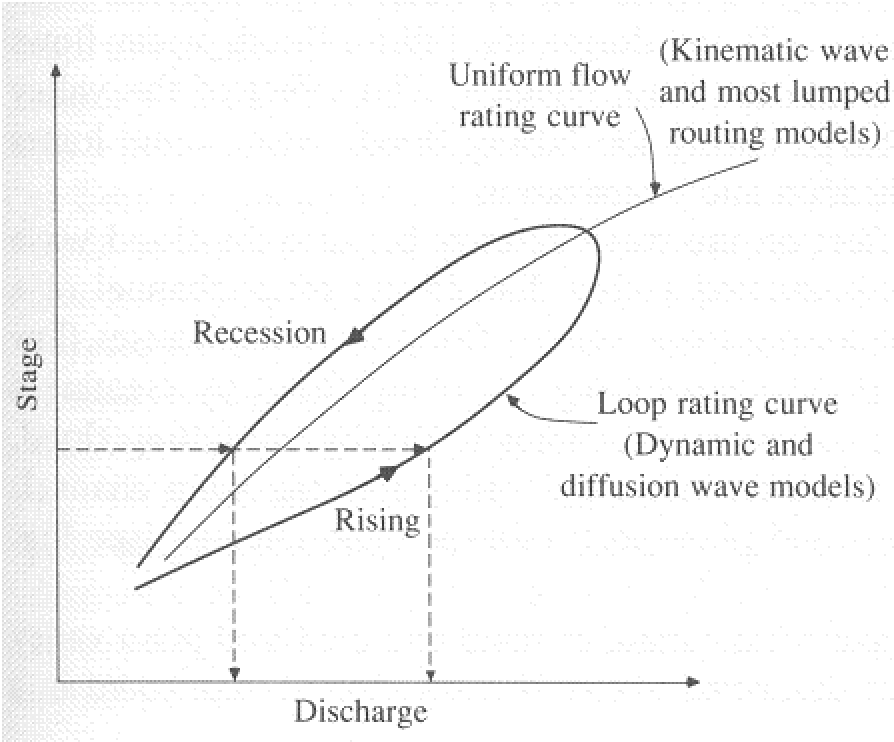
$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0$$

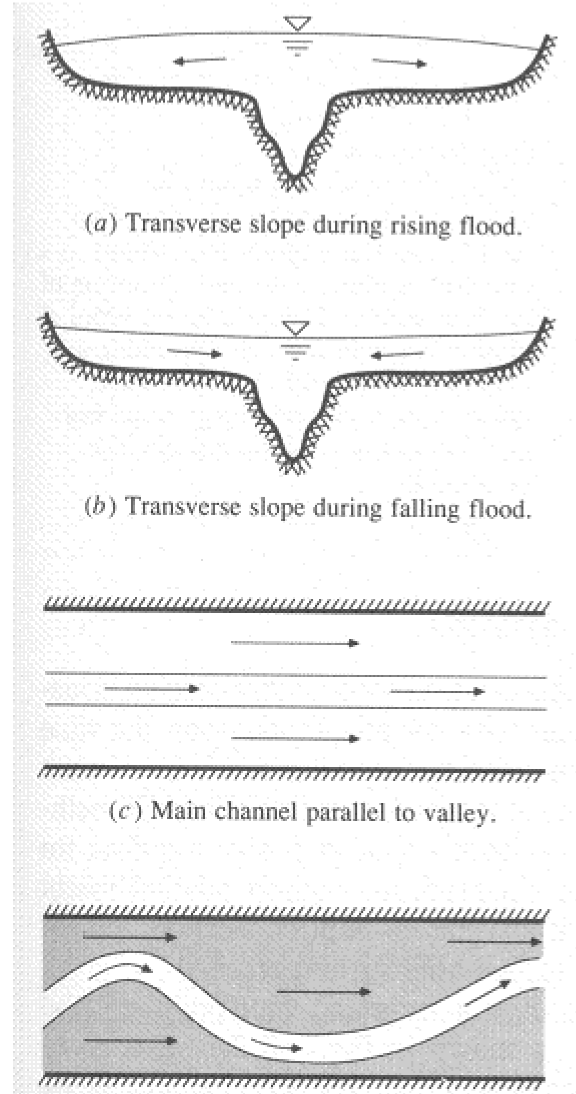
Kinematic Wave

Diffusion Wave

Dynamic Wave

Dynamic Wave Routing



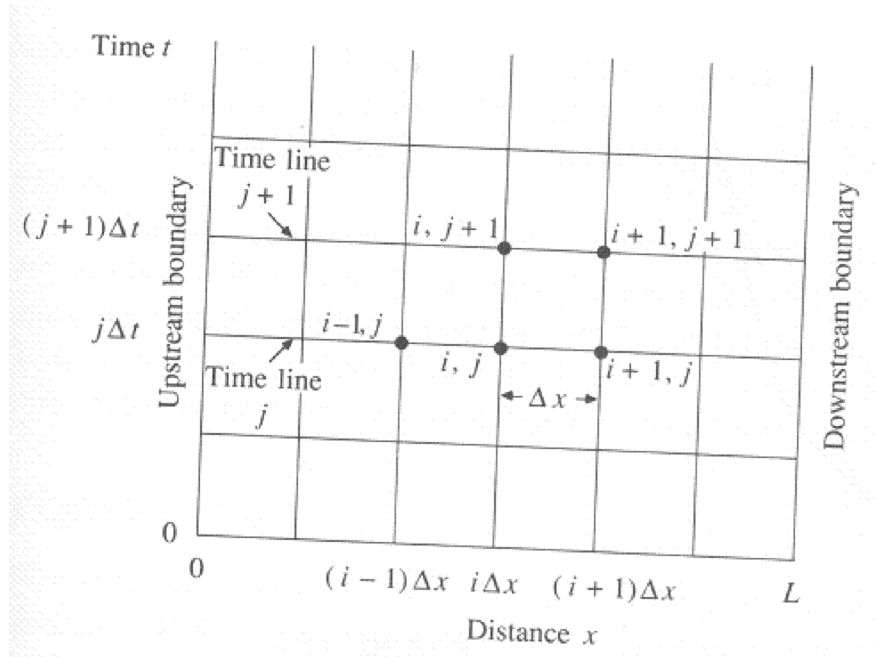

(a) Transverse slope during rising flood.
(b) Transverse slope during falling flood.
(c) Main channel parallel to valley.
(d) Meandering main channel.

Flow in natural channels is unsteady, non-uniform with junctions, tributaries, variable cross-sections, variable resistances, variable depths, etc etc.

Solving St. Venant equations

- Analytical
  - Solved by integrating partial differential equations
  - Applicable to only a few special simple cases of kinematic waves

- Numerical
  - Finite difference approximation
  - Calculations are performed on a grid placed over the (x,t) plane
  - Flow and water surface elevation are obtained for incremental time and distances along the channel



x-t plane for finite differences calculations

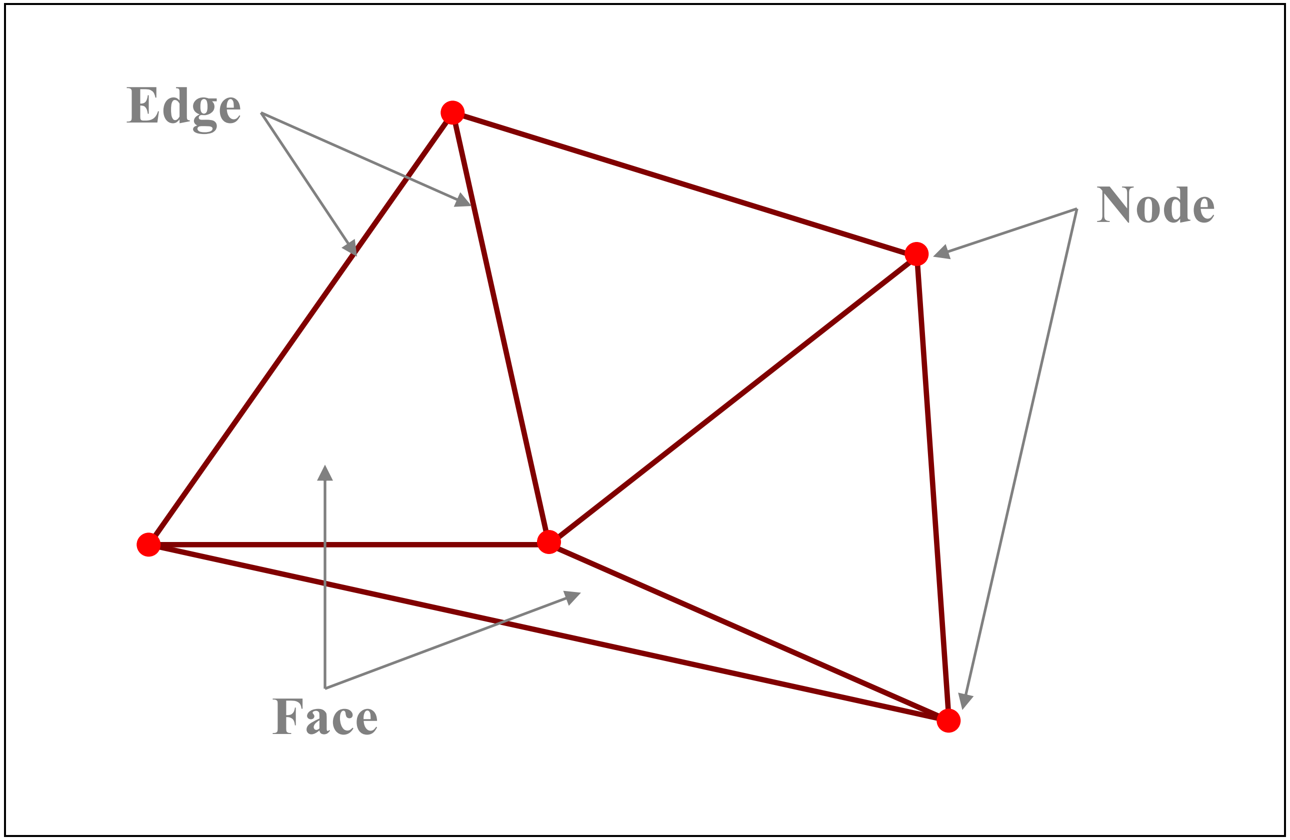
Obtaining river cross-sections

Traditional methods

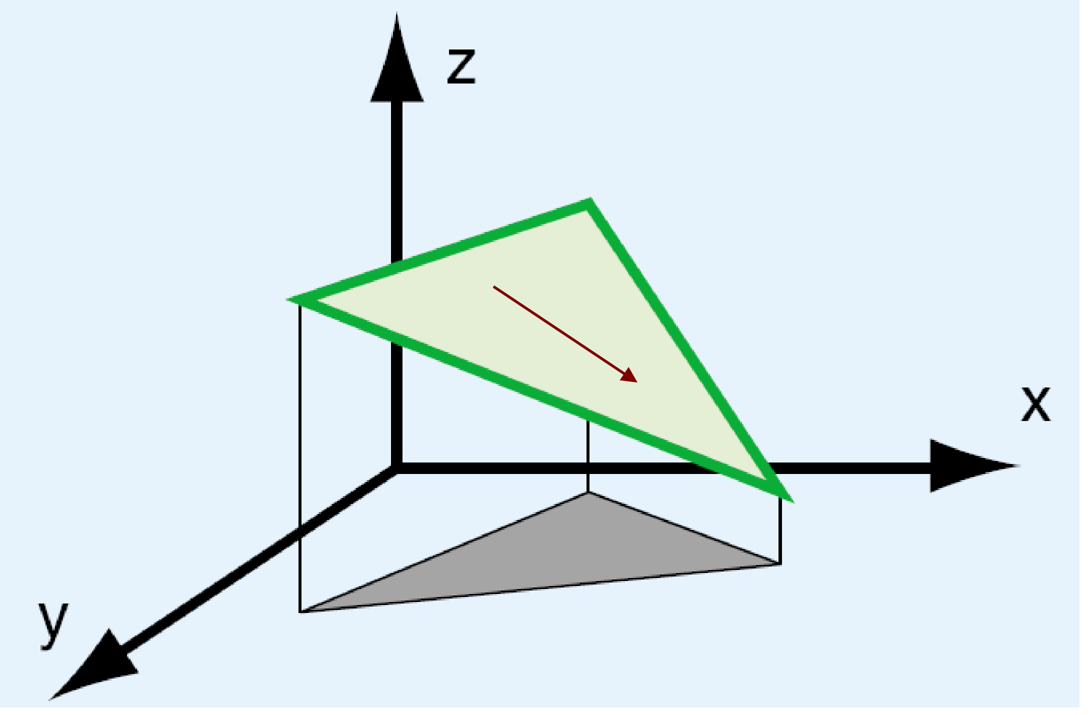
Depth sounder and GPS

Cross-sections are also extracted from a contour map, DEM, and TIN

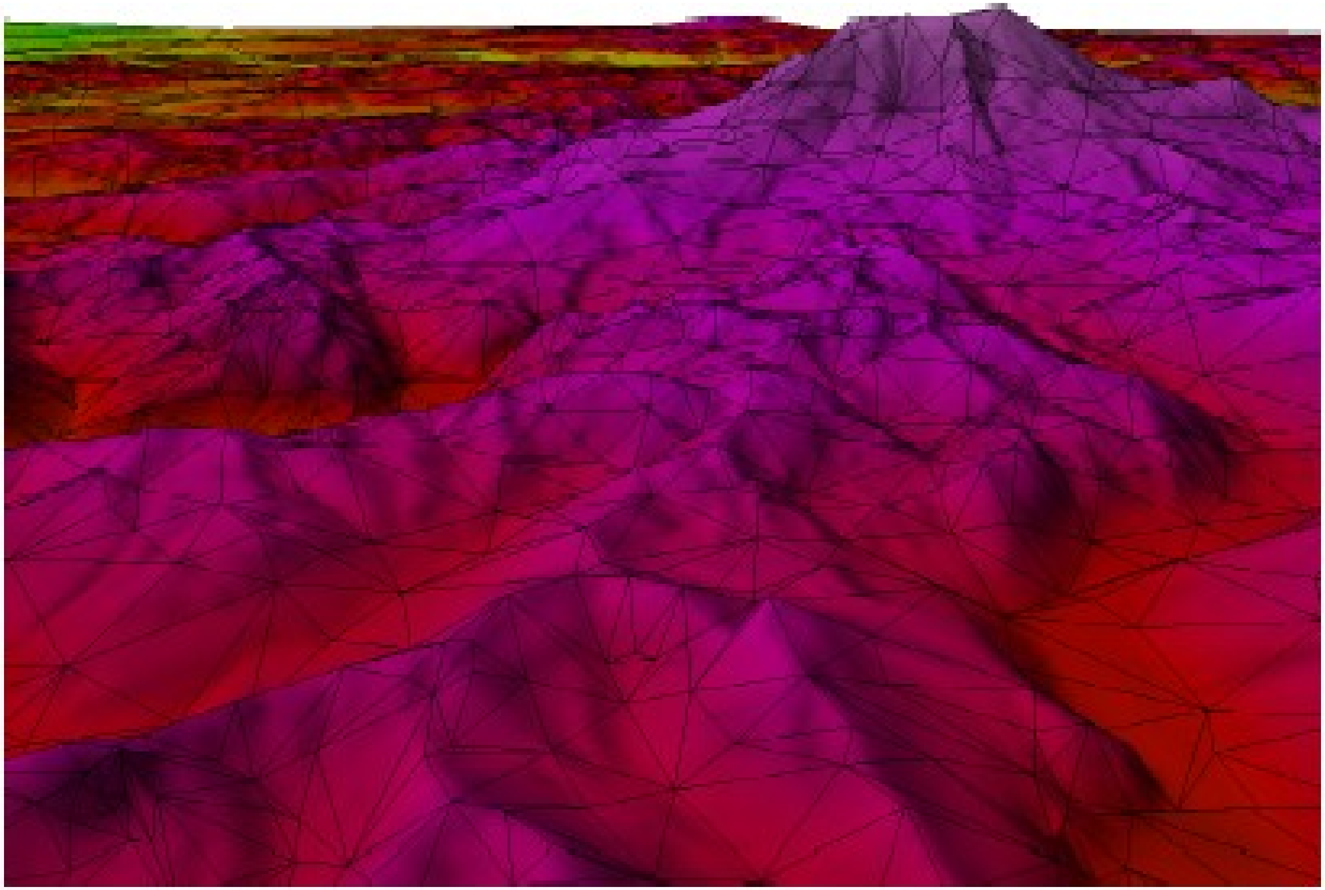
Triangulated Irregular Network



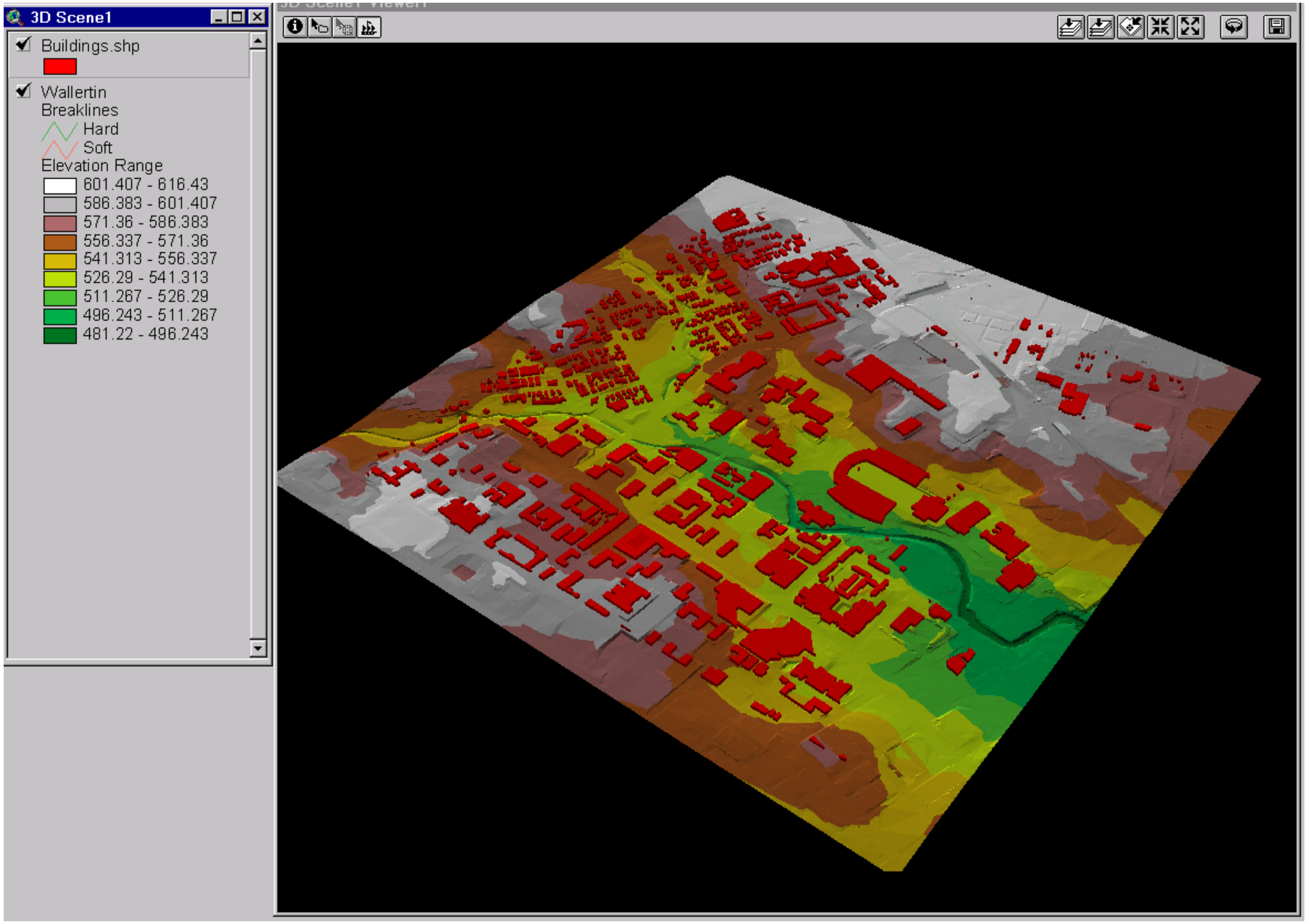
3D Structure of a TIN



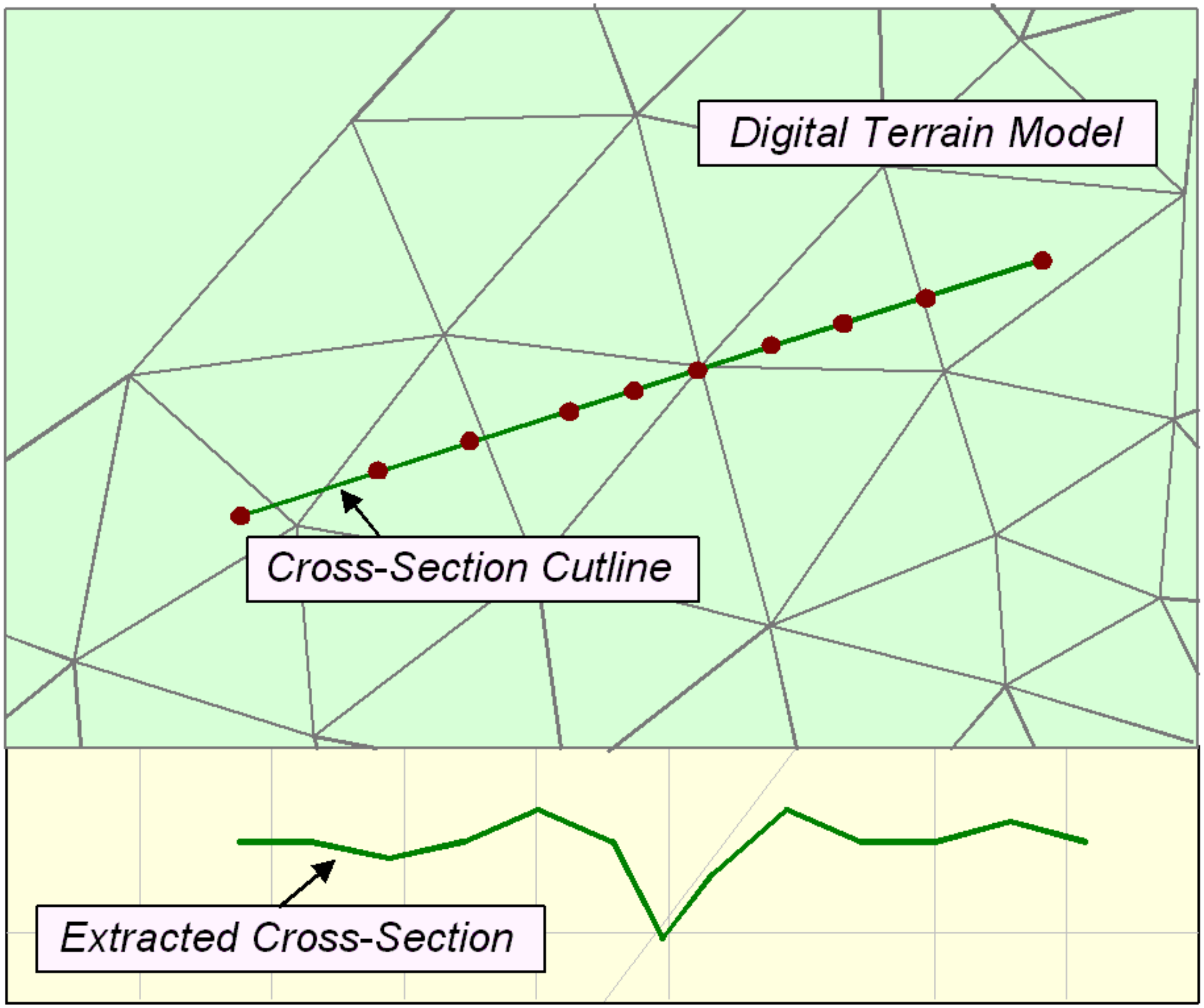
Real TIN in 3D!



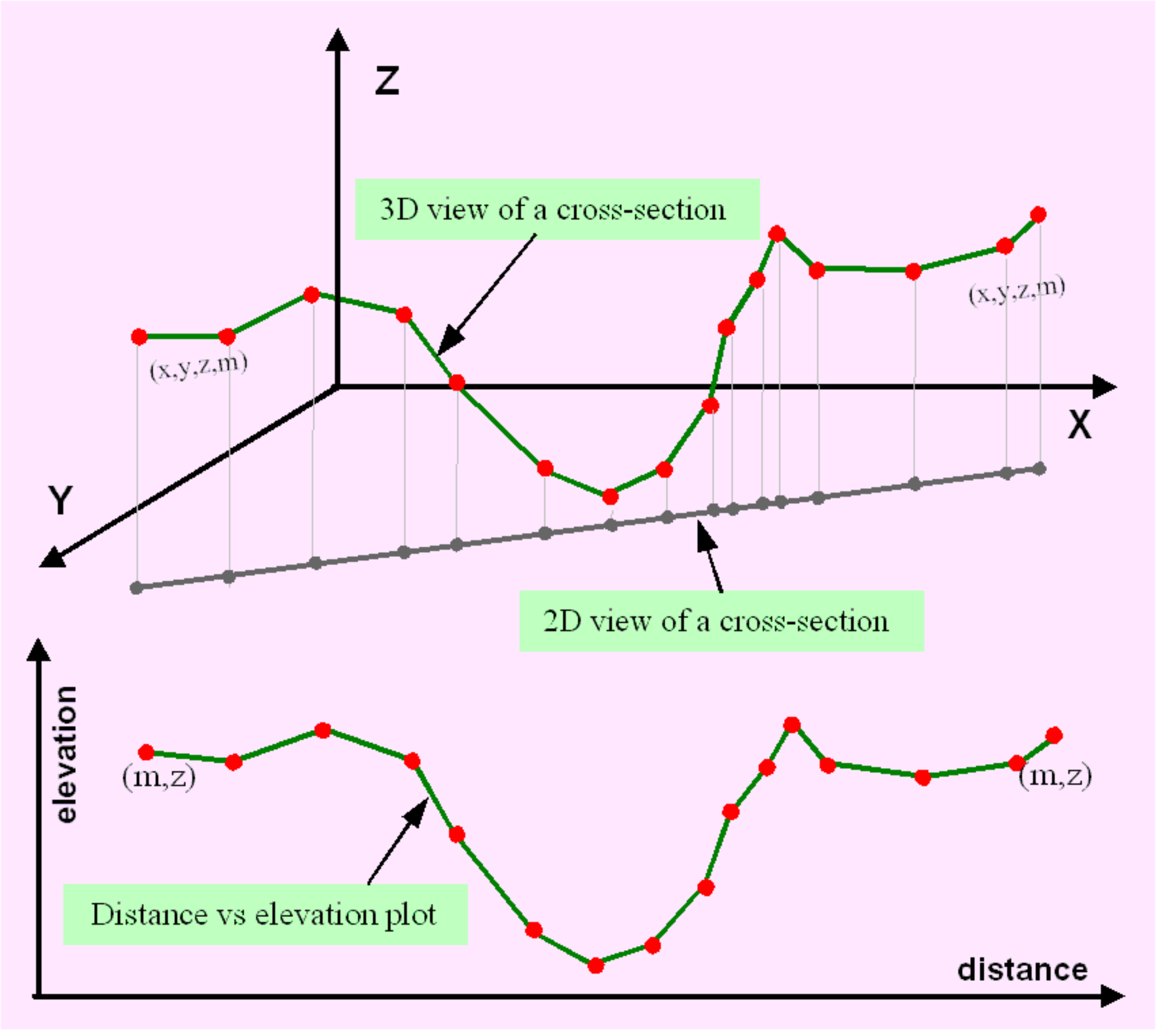
TIN for UT campus



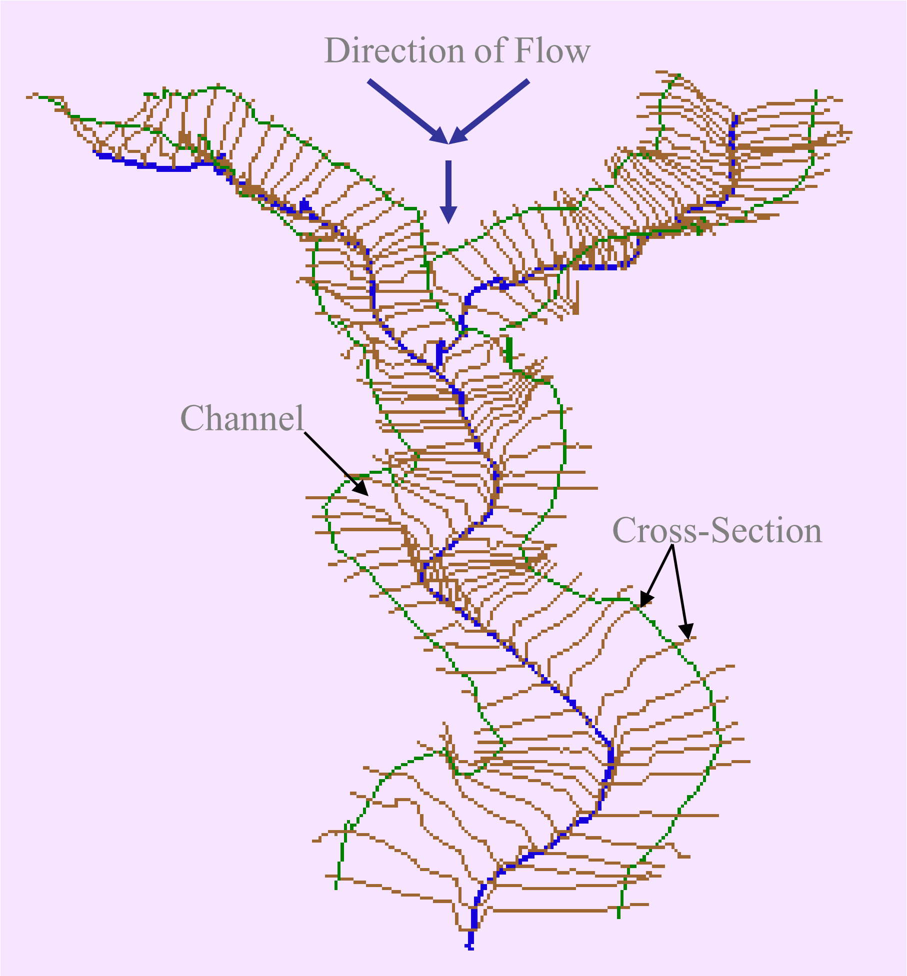
TIN as a source of cross-sections



CrossSections



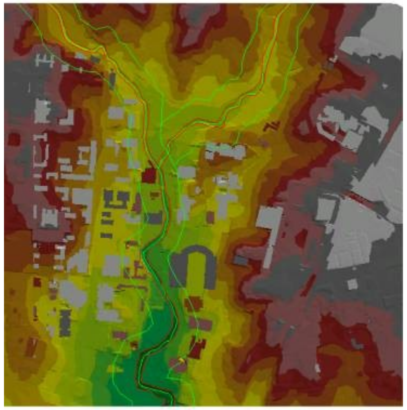
Channel and Cross-Section



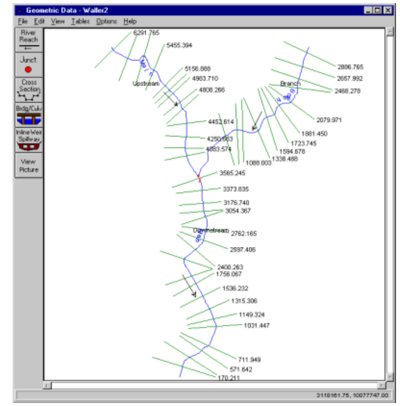
HEC GeoRAS

- A set of ArcGIS tools for processing of geospatial data for
  - Export of geometry HEC-RAS
  - Import of HEC-RAS output for display in GIS
- Available from HEC at http://www.hec.usace.army.mil/software/hec-ras/hec-georas.html

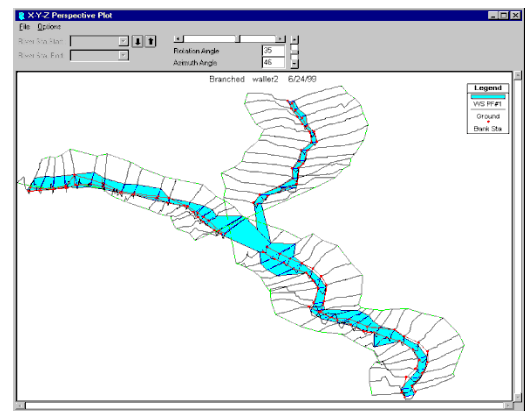
Hydraulic Modeling with Geo-RAS



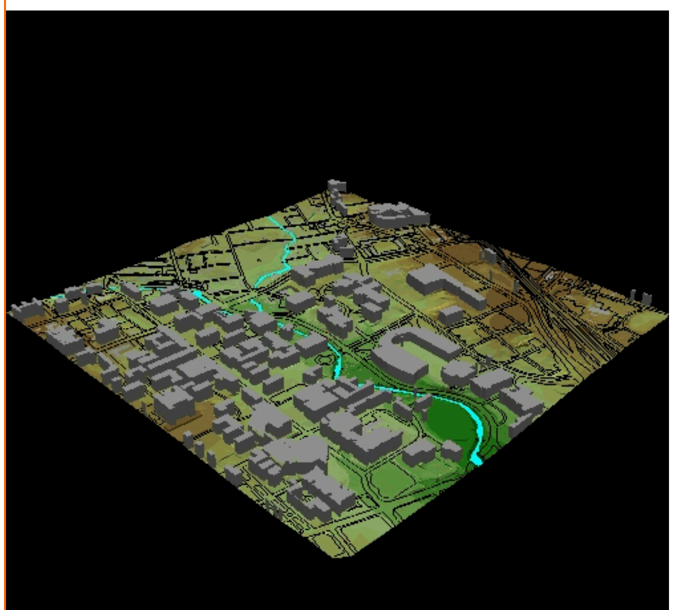
GIS data



HEC-RAS Geometry



HEC-RAS Flood Profiles



Flood display in GIS