Open Channel Flow

- Liquid (water) flow with a _free surface_ (interface between water and air)

- relevant for
  - natural channels: rivers, streams
  - engineered channels: canals, sewer lines or culverts (partially full), storm drains
- of interest to hydraulic engineers
  - location of free surface
  - velocity distribution
  - discharge - stage (_depth_) relationships
  - optimal channel design

Topics in Open Channel Flow

- Uniform Flow   normal depth
  - Discharge-Depth relationships
- Channel transitions
  - Control structures (sluice gates, weirs…)
  - Rapid changes in bottom elevation or cross section
- Critical, Subcritical and Supercritical Flow
- Hydraulic Jump
- Gradually Varied Flow
  - Classification of flows
  - Surface profiles

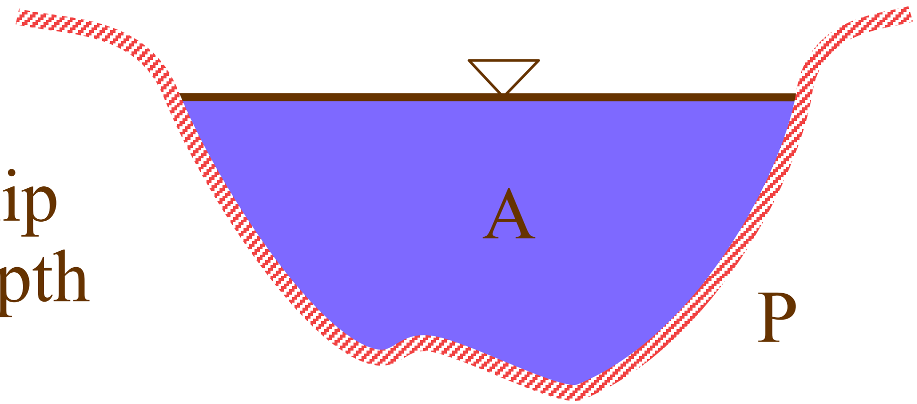
Classification of Flows

- Steady and Unsteady (Temporal)
  - Steady: velocity at a given point does not change with time
- Uniform, Gradually Varied, and Rapidly Varied (Spatial)
  - Uniform: velocity at a given time does not change within a given length of a channel
  - Gradually varied: gradual changes in velocity with distance
- Laminar and Turbulent
  - Laminar: flow appears to be as a movement of thin layers on top of each other
  - Turbulent: packets of liquid move in irregular paths

Momentum and Energy Equations

- Conservation of Energy
  - "losses" due to conversion of turbulence to heat
  - useful when energy losses are known or small
    - Contractions
  - Must account for losses if applied over long distances
    - We need an equation for losses
- Conservation of Momentum
  - "losses" due to shear at the boundaries
  - useful when energy losses are unknown
    - Expansion

Open Channel Flow: Discharge/Depth Relationship



- Given a long channel of constant slope and cross section find the relationship between discharge and depth
- Assume
  - Steady Uniform Flow - no acceleration
  - prismatic channel (no change in geometry with distance)
- Use Energy, Momentum, Empirical or Dimensional Analysis?
- What controls depth given a discharge?
- Why doesn't the flow accelerate? Force balance

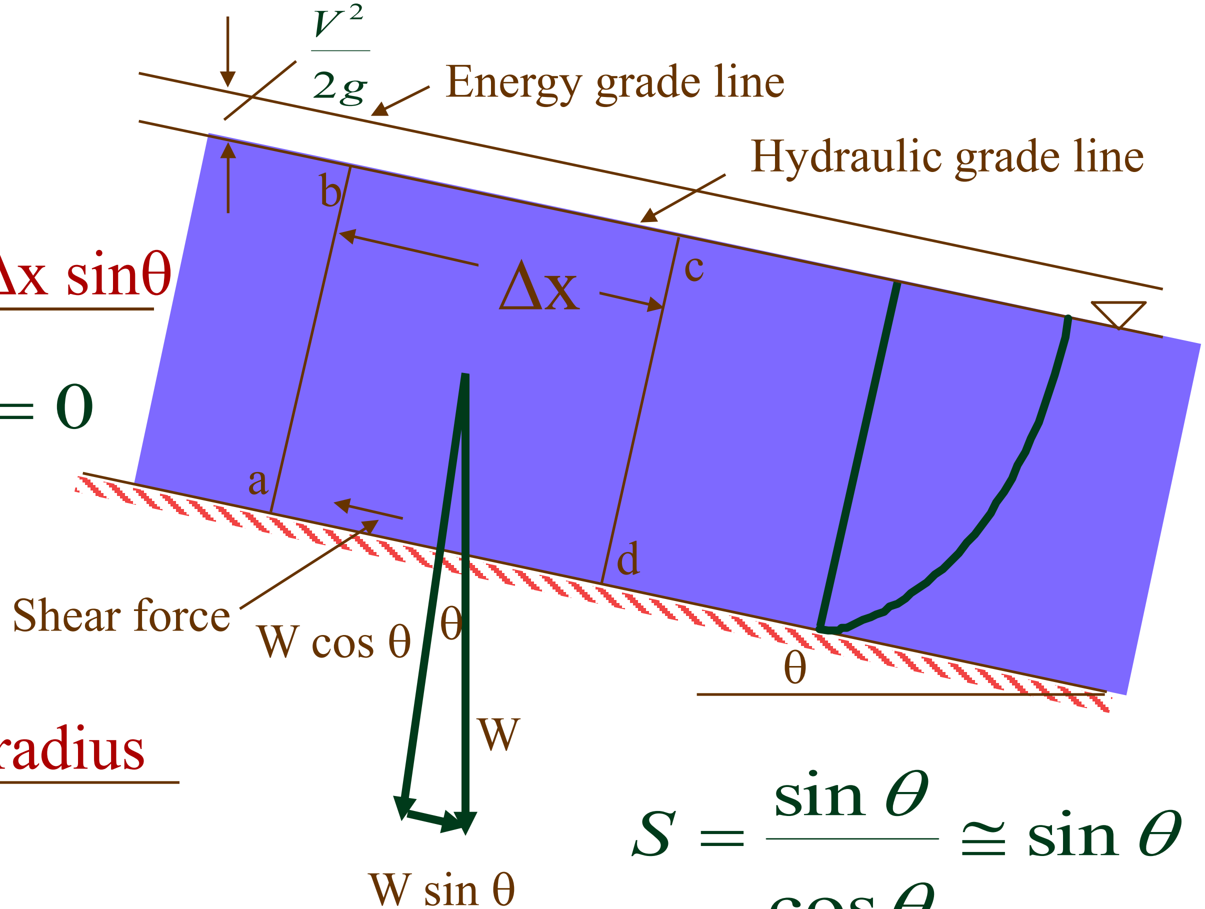
$$\tau_0 = -\frac{\gamma h_l d}{4l}$$

Steady-Uniform Flow: Force Balance

Shear force = $\tau_o P \Delta x$

Wetted perimeter = P

Gravitational force = $\gamma A \Delta x \sin\theta$



$$\gamma A \Delta x \sin\theta - \tau_o P \Delta x = 0$$

$$\tau_o = \gamma \frac{A}{P} \sin\theta$$

$$\frac{A}{P} = R_h$$ Hydraulic radius

$$t_o = g R_h S$$

$$S = \frac{\sin\theta}{\cos\theta} \cong \sin\theta$$

Relationship between shear and velocity? Turbulence

Pressure Coefficient for Open Channel Flow?

$$C_p = \frac{-2\Delta p}{\rho V^2}$$

Pressure Coefficient

(Energy Loss Coefficient)

$$-\Delta p = \gamma h_l$$

$$C_{h_l} = \frac{2 g h_l}{V^2}$$

Head loss coefficient

$$h_l = S_f l$$

Friction slope

Slope of EGL

$$C_{S_f} = \frac{2 g S_f l}{V^2}$$

Friction slope coefficient

Chezy Equation (1768)

- Introduced by the French engineer Antoine Chezy in 1768 while designing a canal for the water-supply system of Paris

$$V = C\sqrt{R_h S_f} \qquad \text{compare} \qquad V = \sqrt{\frac{2g}{f}}\sqrt{S_f R_h}$$

where C = Chezy coefficient

$$60\,\frac{\sqrt{m}}{s} < C < 150\,\frac{\sqrt{m}}{s}$$

$$0.0054 > f > 0.00087$$

$$0.022 > f > 0.0035$$

For a pipe

$$d = 4R_h$$

where 60 is for rough and 150 is for smooth

also a function of **R** (like f in Darcy-Weisbach)

Manning Equation (1891)

- Most popular in U.S. for open channels

$$V = \frac{1}{n} R_h^{2/3} S_o^{1/2}$$

(MKS units!)

Dimensions of *n*? $T/L^{1/3}$

Is *n* only a function of roughness? NO!

$$V = \frac{1.49}{n} R_h^{2/3} S_o^{1/2}$$

(English system)

Bottom slope

$$Q = VA$$

$$Q = \frac{1}{n} A R_h^{2/3} S_o^{1/2}$$

very sensitive to *n*

Values of Manning *n*

Lined Canals	n
Cement plaster	0.011
Untreated gunite	0.016
Wood, planed	0.012
Wood, unplaned	0.013
Concrete, trowled	0.012
Concrete, wood forms, unfinished	0.015
Rubble in cement	0.020
Asphalt, smooth	0.013
Asphalt, rough	0.016
Natural Channels	
Gravel beds, straight	0.025
Gravel beds plus large boulders	0.040
Earth, straight, with some grass	0.026
Earth, winding, no vegetation	0.030
Earth , winding with vegetation	0.050

n = f(surface roughness, channel irregularity, stage...)

$$n = 0.031 d^{1/6}$$ d in ft

$$n = 0.038 d^{1/6}$$ d in m

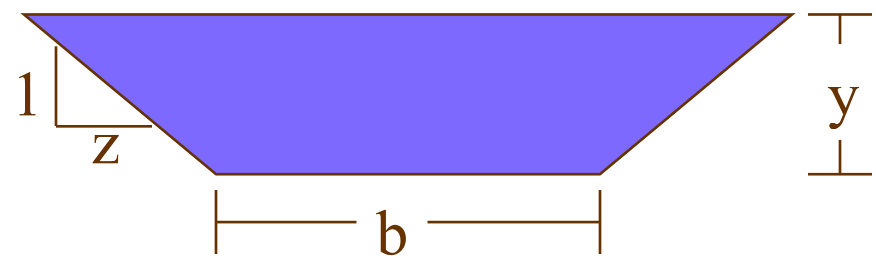
d = median size of bed material

Trapezoidal Channel

$$Q = \frac{1}{n} A R_h^{2/3} S_o^{1/2}$$

- Derive P = f(y) and A = f(y) for a trapezoidal channel
- How would you obtain y = f(Q)?

$$A = yb + y^2 z$$



Flow in Round Conduits

$$\theta = \arccos\left(\frac{r-y}{r}\right)$$

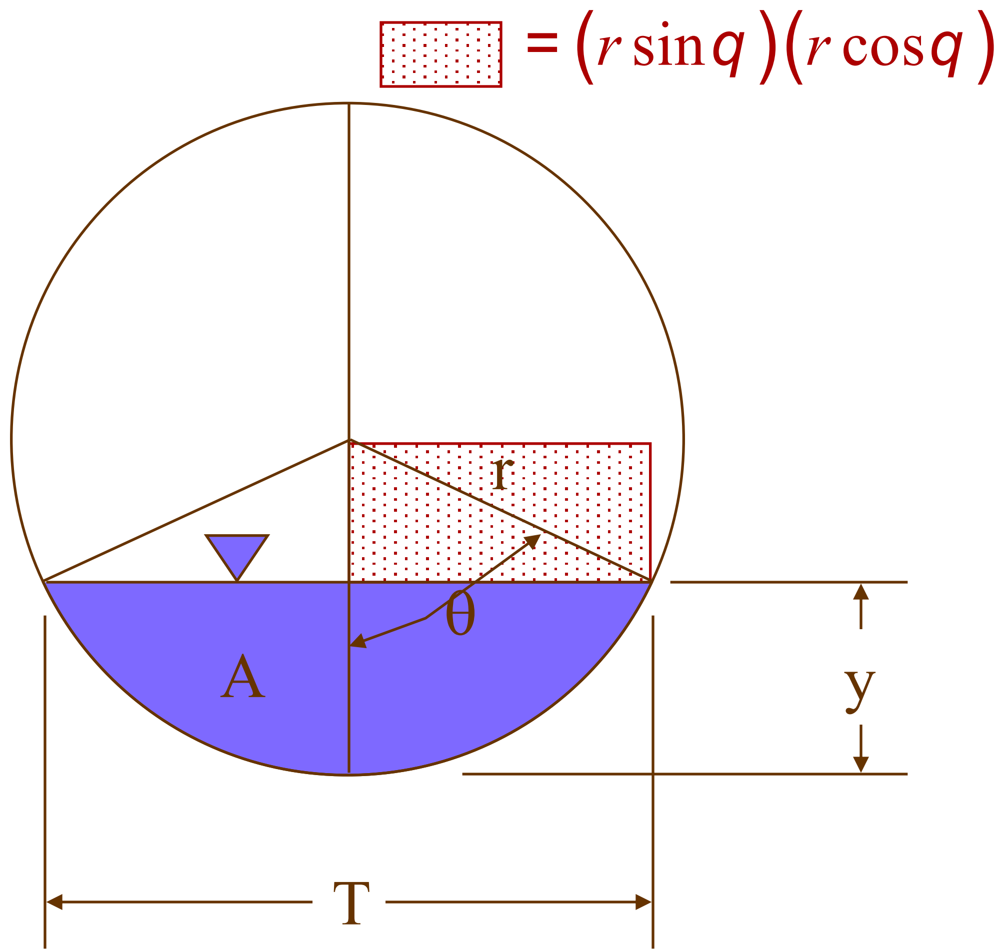
radians

$$A = r^2\left(\theta - \sin\theta\cos\theta\right)$$

$$T = 2r\sin\theta$$

$$P = 2r\theta$$

Maximum discharge when y = 0.938d

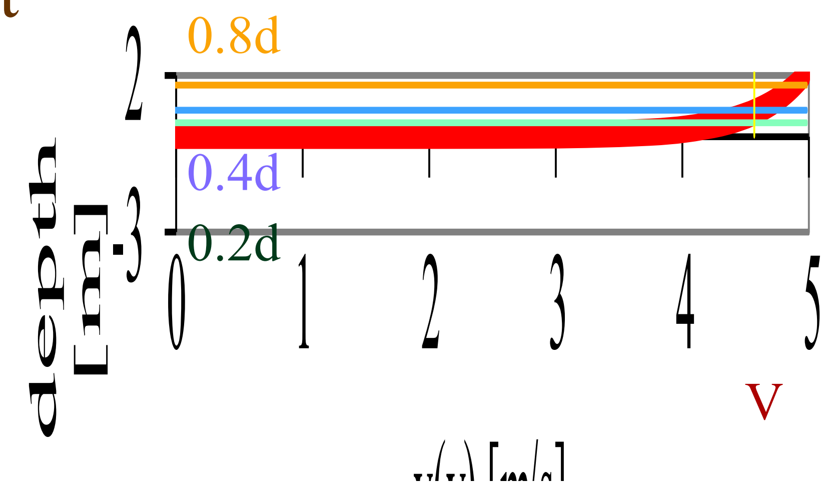


Velocity Distribution

$$v(y) = V + \frac{1}{\kappa}\sqrt{gdS_0}\left(1 + \ln\frac{y}{d}\right)$$

For channels wider than 10d

k » 0.4 Von Kármán constant



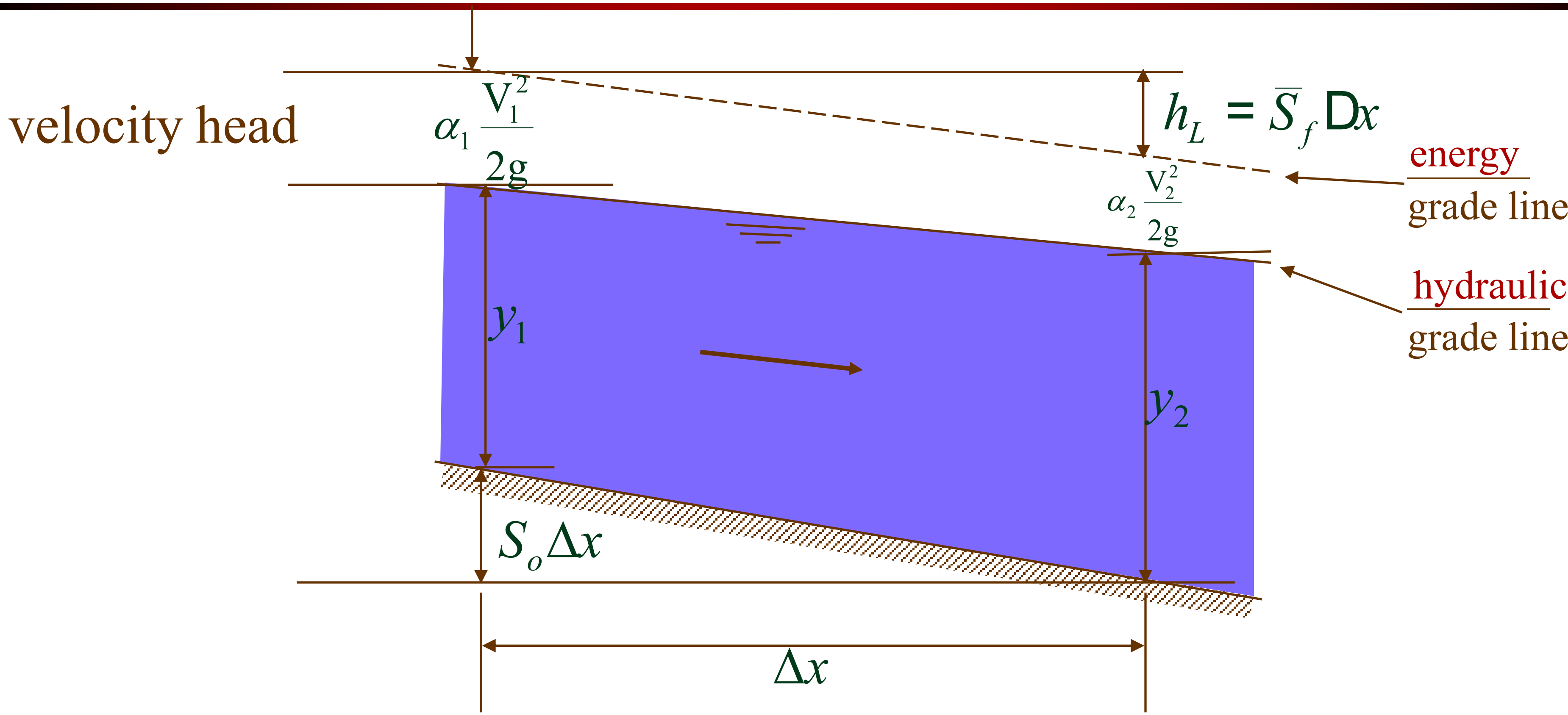
V = average velocity

d = channel depth

At what elevation does the velocity equal the average velocity?

$$-1 = \ln\frac{y}{d} \qquad y = \frac{1}{e}d \qquad 0.368d$$

Open Channel Flow: Energy Relations



Bottom slope (S_o) not necessarily equal to EGL slope (S_f)

Energy Relationships

$$\frac{p_1}{g} + z_1 + \alpha_1 \frac{V_1^2}{2g} = \frac{p_2}{g} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_L$$

Pipe flow

z - measured from horizontal datum

From diagram on previous slide...

$$y_1 + S_o \Delta x + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

Turbulent flow ($\alpha \cong 1$)

y - depth of flow

Energy Equation for Open Channel Flow

$$y_1 + \frac{V_1^2}{2g} + S_o \Delta x = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

Specific Energy

- The sum of the depth of flow and the velocity head is the specific energy:

$$E = y + \frac{V^2}{2g}$$

y - potential energy

$\frac{V^2}{2g}$ - kinetic energy

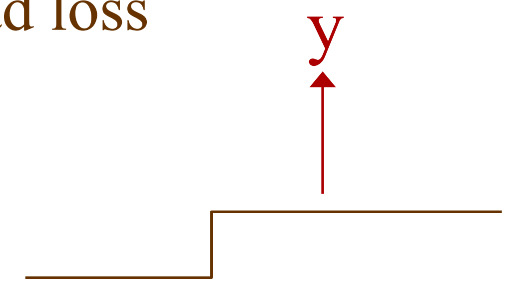
$$E_1 + S_o \Delta x = E_2 + S_f \Delta x$$

If channel bottom is horizontal and no head loss

$$E_1 = E_2$$

For a change in bottom elevation

$$E_1 - Dy = E_2$$



Specific Energy

In a channel with constant discharge, Q

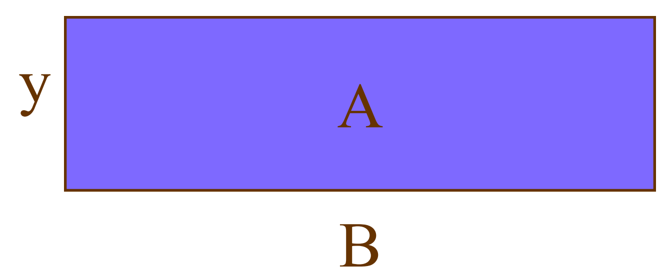
$$Q = A_1 V_1 = A_2 V_2$$

$$E = y + \frac{V^2}{2g} \longrightarrow E = y + \frac{Q^2}{2gA^2}$$ where A=f(y)

Consider rectangular channel (A = By) and Q = qB

q is the discharge per unit width of channel

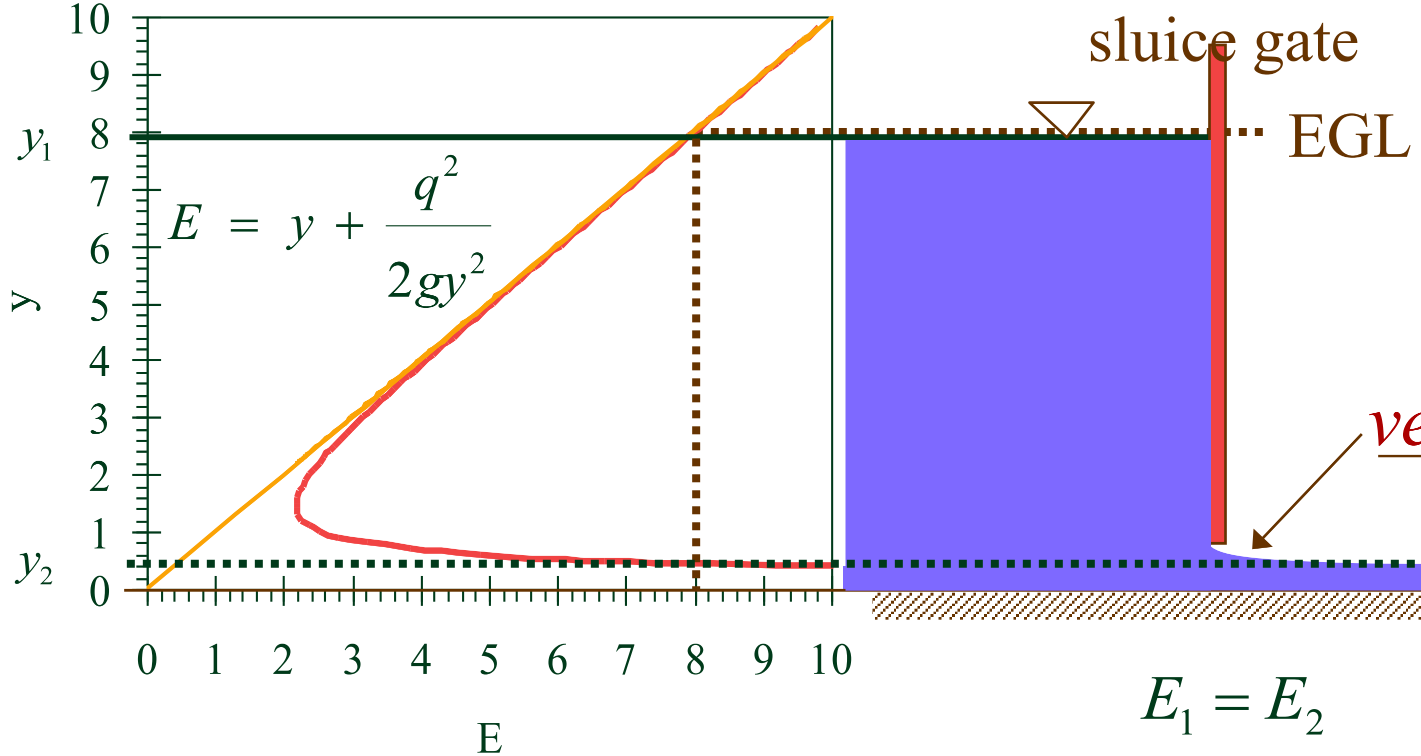
$$E = y + \frac{q^2}{2gy^2}$$



3 roots (one is negative)

How many possible depths given a specific energy? ___2___

Specific Energy: Sluice Gate



q = 5.5 m²/s

y_2 = 0.45 m

V_2 = 12.2 m/s

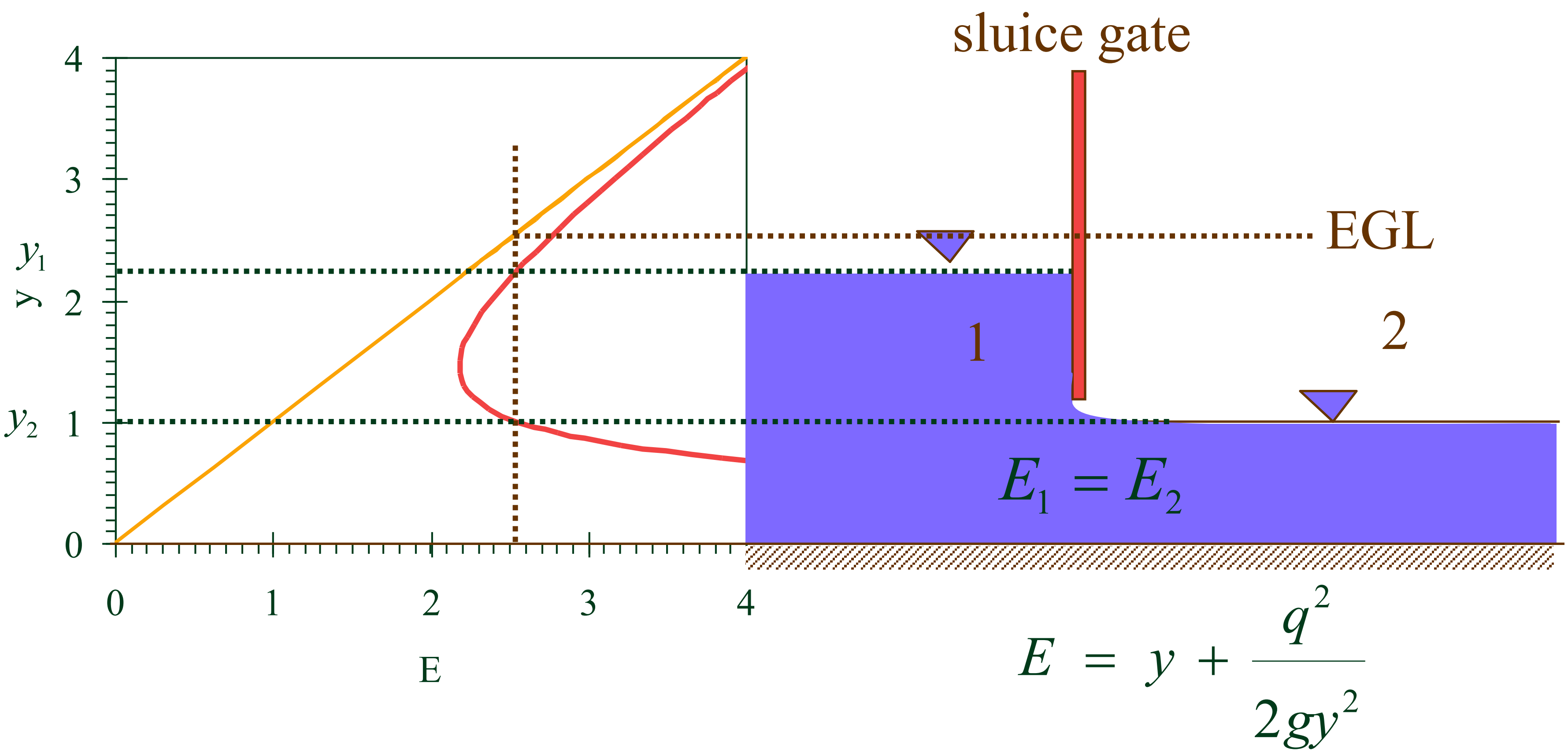
E_2 = 8 m

Given downstream depth and discharge, find upstream depth.

y_1 and y_2 are <u>alternate</u> depths (same specific energy)

Why not use momentum conservation to find y_1?

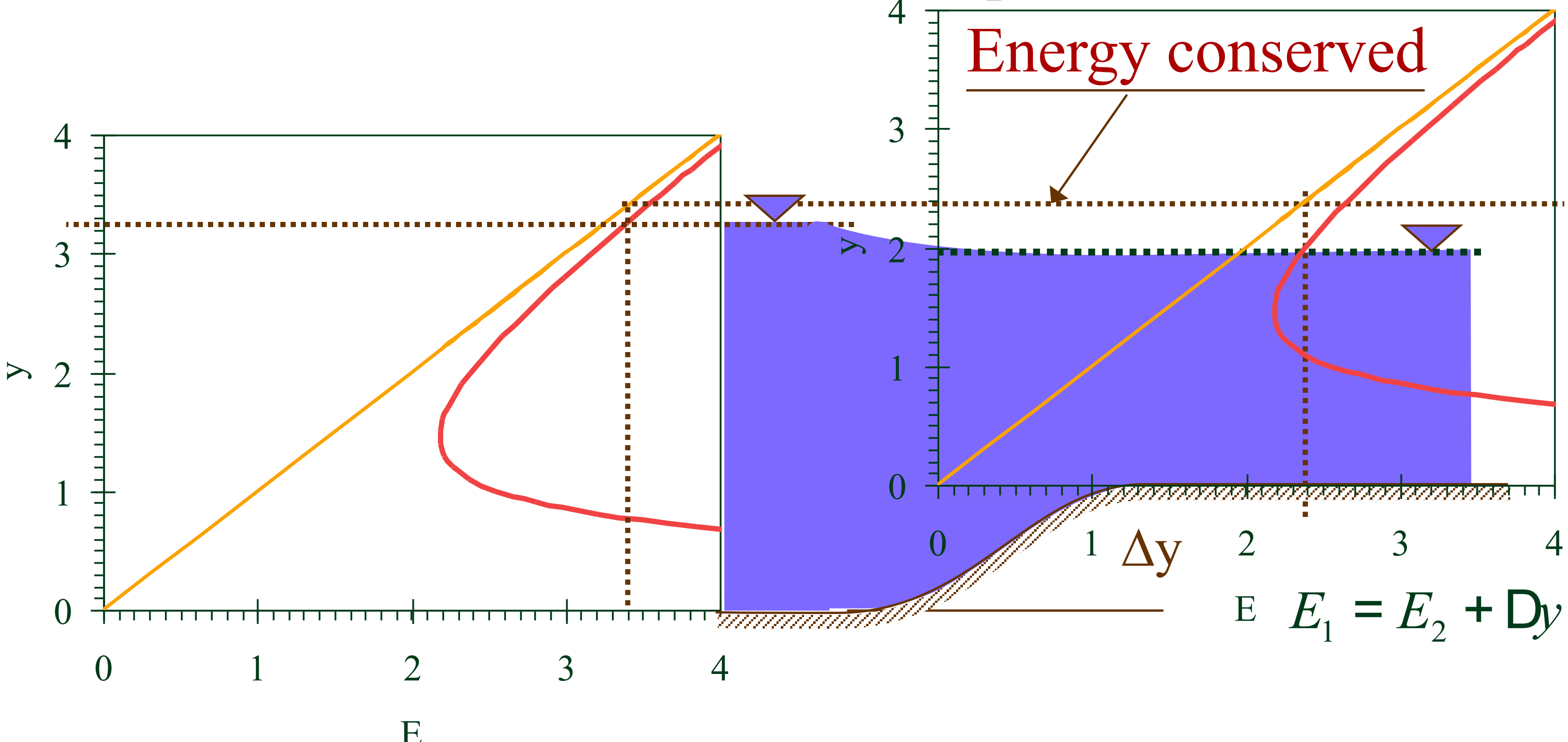
Specific Energy: Raise the Sluice Gate



as sluice gate is raised y_1 approaches y_2 and E is minimized: Maximum discharge for given energy.

Step Up with Subcritical Flow

Short, smooth step with rise Δy in channel
Given upstream depth and discharge find y_2

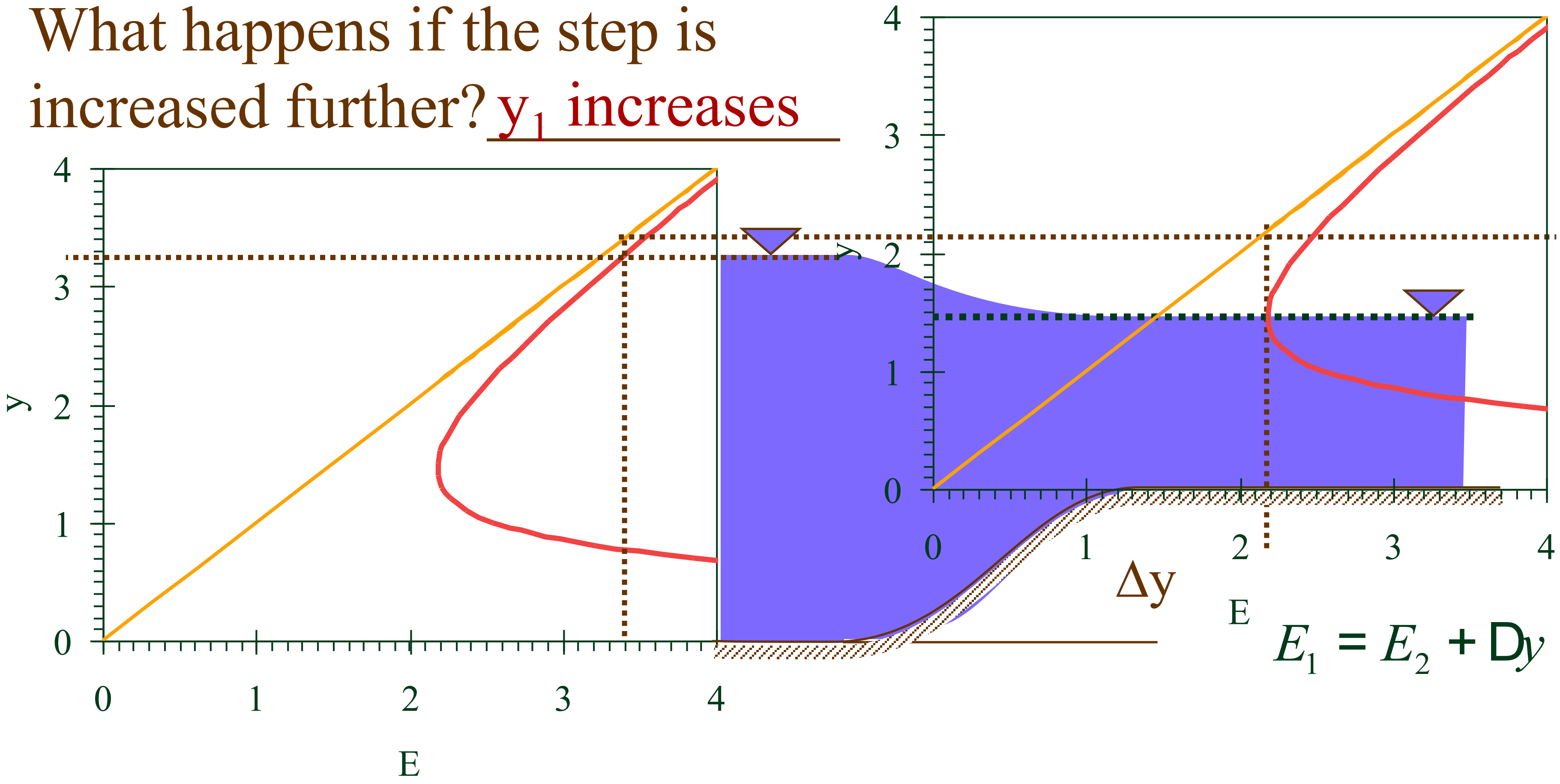


Is alternate depth possible? NO! Calculate depth along step.

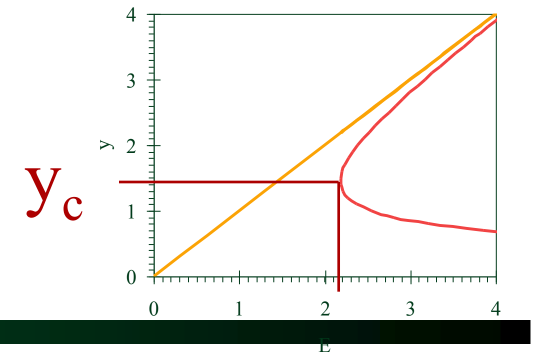
Max Step Up

Short, smooth step with maximum rise Δy in channel

What happens if the step is increased further? y_1 increases



Critical Flow



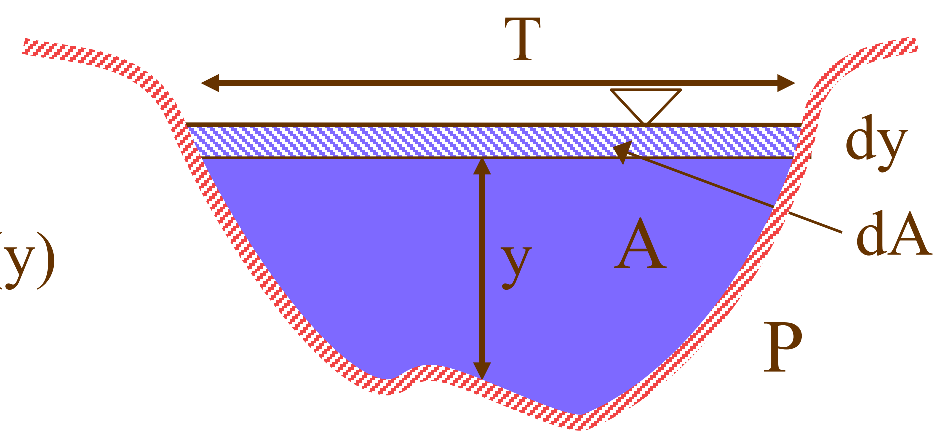
Find critical depth, y_c

$$\frac{dE}{dy} = 0$$

$$E = y + \frac{Q^2}{2gA^2}$$

A=f(y)

Arbitrary cross-section



$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dy} = 0$$

$dA = Tdy$

T=surface width

More general definition of Fr

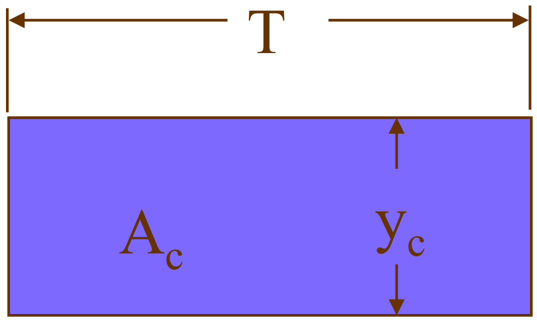
$$1 = \frac{Q^2 T_c}{gA_c^3} \qquad \frac{Q^2 T}{gA^3} = Fr^2 \qquad \frac{V^2 T}{gA} = Fr^2 \qquad \frac{A}{T} = D$$

Hydraulic Depth

Critical Flow: Rectangular channel

$$1 = \frac{Q^2 T_c}{g A_c^3} \qquad T = T_c$$

$$Q = qT \qquad A_c = y_c T$$



$$1 = \frac{q^2 T^3}{g y_c^3 T^3} = \frac{q^2}{g y_c^3}$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

Only for rectangular channels!

$$q = \sqrt{g y_c^3}$$

Given the depth we can find the flow!

Critical Flow Relationships: Rectangular Channels

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_c^3 = \left(\frac{V_c^2 y_c^2}{g}\right)$$ because $q = V_c y_c$

$$\frac{V_c}{\sqrt{y_c g}} = 1$$

Froude number

$$\frac{\text{inertial force}}{\text{gravity force}}$$

$$\sqrt{\frac{\textit{Kinetic energy}}{\textit{Potential energy}}}$$

$$y_c = \frac{V_c^2}{g} \longrightarrow \frac{y_c}{2} = \frac{V_c^2}{2g}$$

velocity head = 0.5 (depth)

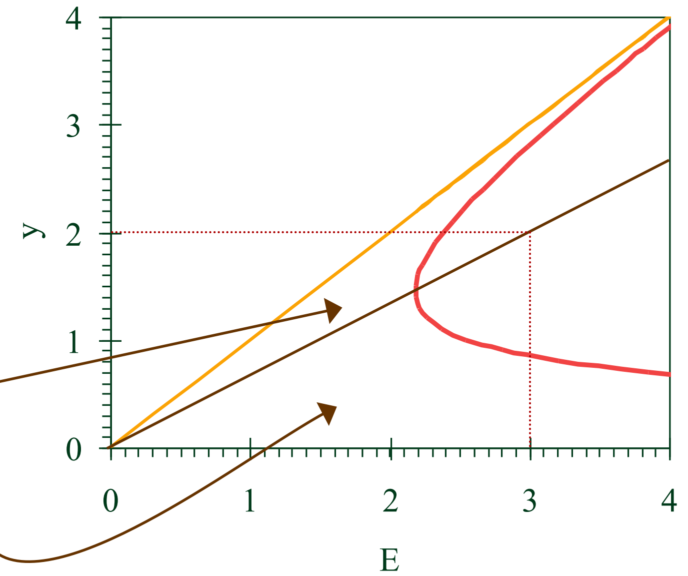
$$E = y + \frac{V^2}{2g} \longrightarrow E = y_c + \frac{y_c}{2} \longrightarrow y_c = \frac{2}{3}E$$

Critical Depth

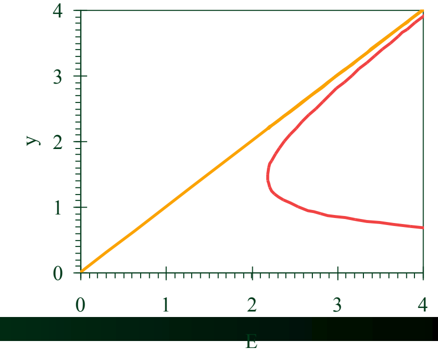
- Minimum energy for a given q
  - Occurs when $\frac{dE}{dy} = \underline{\;0\;}$
  - When kinetic = potential! $\frac{V_c^2}{2g} = \frac{y_c}{2}$
  - Fr=1

$$Fr = \frac{V_c}{\sqrt{y_c g}} = \frac{q}{\sqrt{g y_c^3}} = Q\sqrt{\frac{T}{gA^3}}$$

  - Fr>1 = Super critical
  - Fr<1 = Sub critical



Critical Flow



- Characteristics

$$\frac{dE}{dy} = 0$$

  - Unstable surface
  - Series of standing waves

Difficult to measure depth

- Occurrence
  - Broad crested weir (and other weirs)
  - Channel Controls (rapid changes in cross-section)
  - Over falls
  - Changes in channel slope from mild to steep
- Used for flow measurements
  - Unique relationship between depth and discharge