Gradually Varied Flow: Find Change in Depth wrt x

$$y_1 + \frac{V_1^2}{2g} + S_o \Delta x = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

Energy equation for non-uniform, steady flow

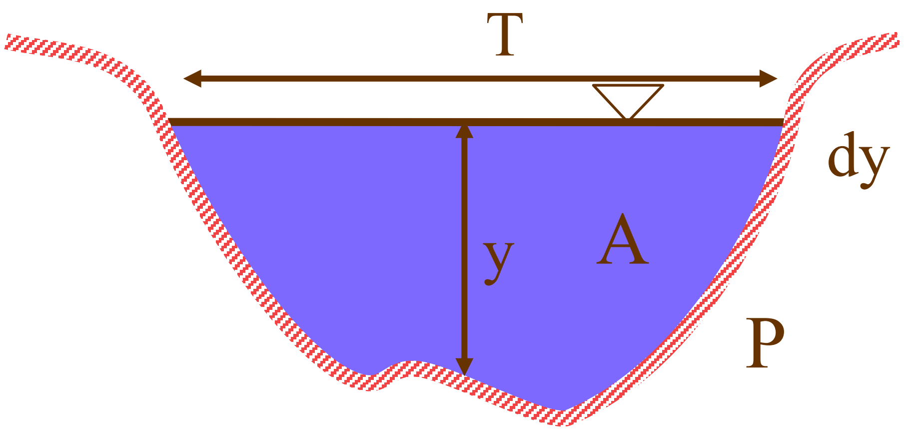
$$S_o dx = \left(y_2 - y_1\right) + \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) + S_f dx$$

$$dy = y_2 - y_1$$

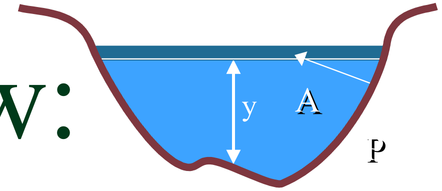
$$dy + d\left(\frac{V^2}{2g}\right) + S_f dx = S_o dx$$

$$\frac{dy}{dy} + \frac{d}{dy}\left(\frac{V^2}{2g}\right) + S_f \frac{dx}{dy} = S_o \frac{dx}{dy}$$

Shrink control volume



Gradually Varied Flow: Derivative of KE wrt Depth



$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right) = \left(\frac{-2Q^2}{2gA^3}\right)\cdot\frac{dA}{dy} = \left(\frac{-Q^2T}{gA^3}\right) = -Fr^2$$

Change in KE / Change in PE

$$dA = Tdy$$

$$\frac{dy}{dy} + \frac{d}{dy}\left(\frac{V^2}{2g}\right) + S_f\frac{dx}{dy} = S_o\frac{dx}{dy}$$

We are holding Q constant!

$$1 - Fr^2 + S_f\frac{dx}{dy} = S_o\frac{dx}{dy}$$

Does V=Q/A? Is V ⊥ A?

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

The water surface slope is a function of: bottom slope, friction slope, Froude number

Gradually Varied Flow: Governing equation

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

Governing equation for gradually varied flow

- Gives change of water depth with distance along channel
- Note
  - S_o and S_f are positive when sloping down in direction of flow
  - y is measured from channel bottom
  - dy/dx =0 means water depth is <u>constant</u>

y_n is when <u>$S_o = S_f$</u>

Surface Profiles

- **M**ild slope ($y_n > y_c$)
  - in a long channel subcritical flow will occur
- **S**teep slope ($y_n < y_c$)
  - in a long channel supercritical flow will occur
- **C**ritical slope ($y_n = y_c$)
  - in a long channel unstable flow will occur
- **H**orizontal slope ($S_o = 0$)
  - y_n undefined
- **A**dverse slope ($S_o < 0$)
  - y_n undefined

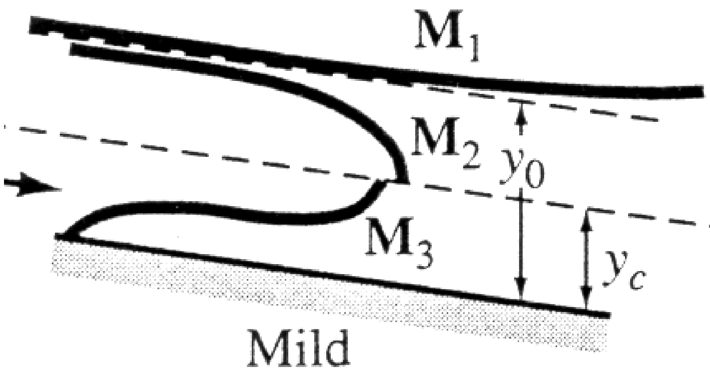
Note: These slopes are f(Q)!

Surface Profiles

Normal depth

Sluice gate

Steep slope

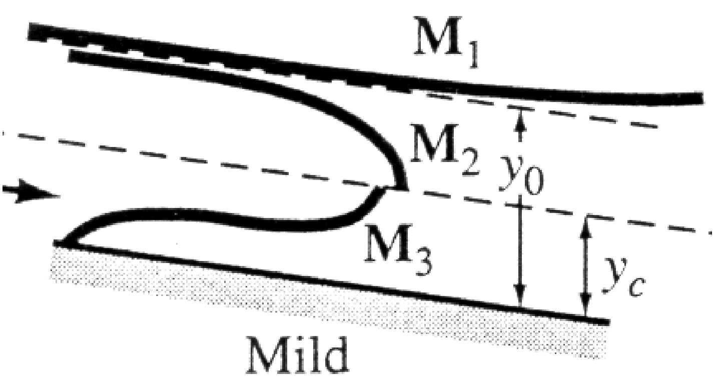


Obstruction

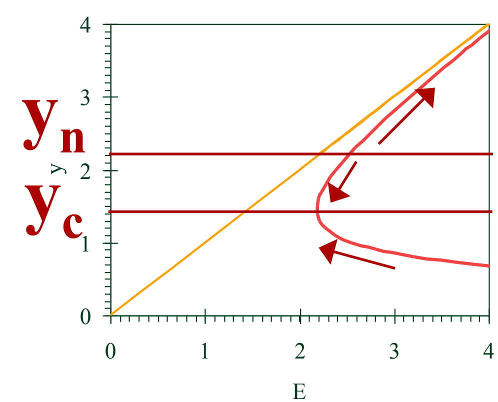
Steep slope (S_2)

Hydraulic Jump

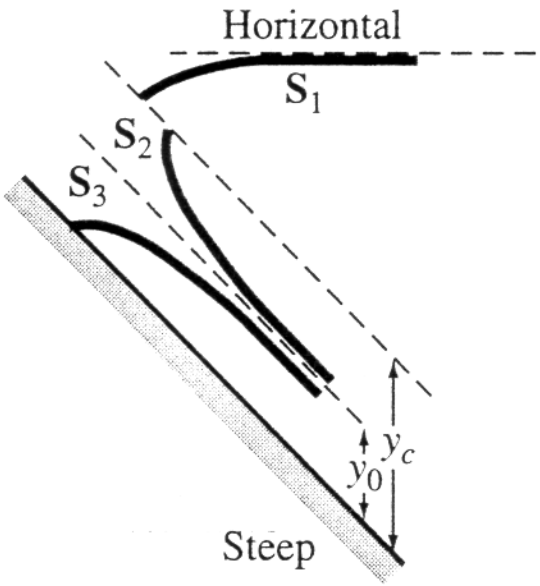
$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$



$S_0 - S_f$	$1 - Fr^2$	dy/dx
+	+	+
-	+	-
-	-	+

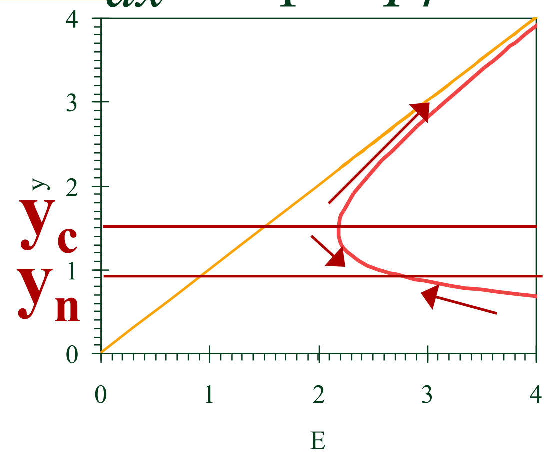


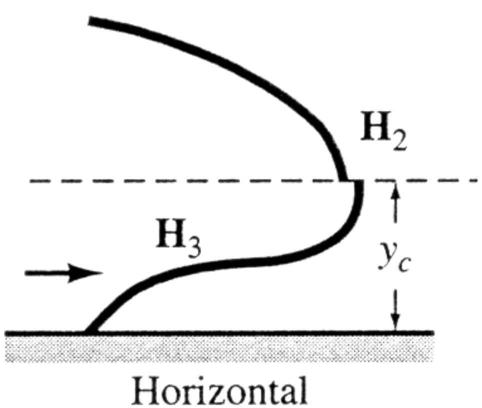
More Surface Profiles

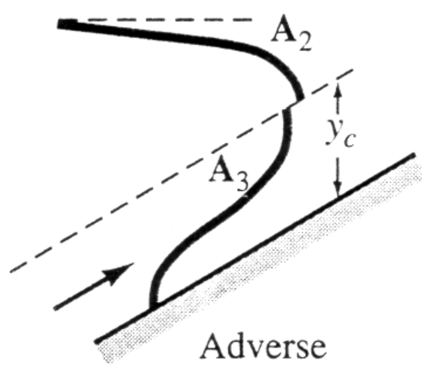


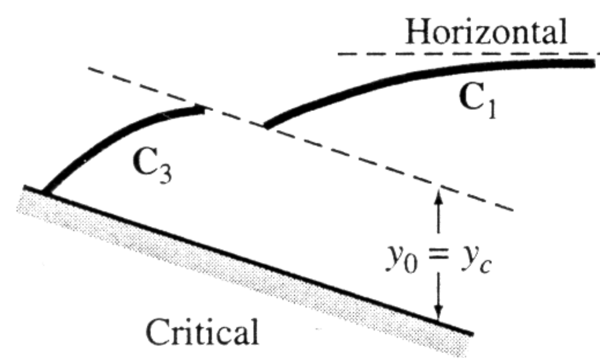
$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

	$S_0 - S_f$	$1 - Fr^2$	dy/dx
1	+	+	+
2	+	-	-
3	-	-	+









Direct Step Method

$$y_1 + \frac{V_1^2}{2g} + S_o \Delta x = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

energy equation

$$\Delta x = \frac{y_1 - y_2 + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}}{S_f - S_o}$$

solve for Δx

rectangular channel

$$V_1 = \frac{q}{y_1} \qquad V_2 = \frac{q}{y_2}$$

prismatic channel

$$V_2 = \frac{Q}{A_2} \qquad V_1 = \frac{Q}{A_1}$$

Direct Step Method
Friction Slope

Manning		Darcy-Weisbach
$S_f = \frac{n^2V^2}{R_h^{4/3}}$	SI units	$S_f = \mathrm{f}\frac{V^2}{8gR_h}$
$S_f = \frac{n^2V^2}{2.22R_h^{4/3}}$	English units	

Direct Step

- Limitation: channel must be prismatic (channel geometry is independent of x so that velocity is a function of depth only and not a function of x)
- Method
  - identify type of profile (determines whether Δy is + or -)
  - choose Δy and thus y_{i+1}
  - calculate hydraulic radius and velocity at y_i and y_{i+1}
  - calculate friction slope given y_i and y_{i+1}
  - calculate average friction slope
  - calculate Δx

Direct Step Method

=y*b+y^2*z

=2*y*(1+z^2)^0.5 +b

=A/P

=Q/A

=(n*V)^2/Rh^(4/3)

=y+(V^2)/(2*g)

=(G16-G15)/((F15+F16)/2-So)

$$\Delta x = \frac{y_1 - y_2 + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}}{S_f - S_o}$$

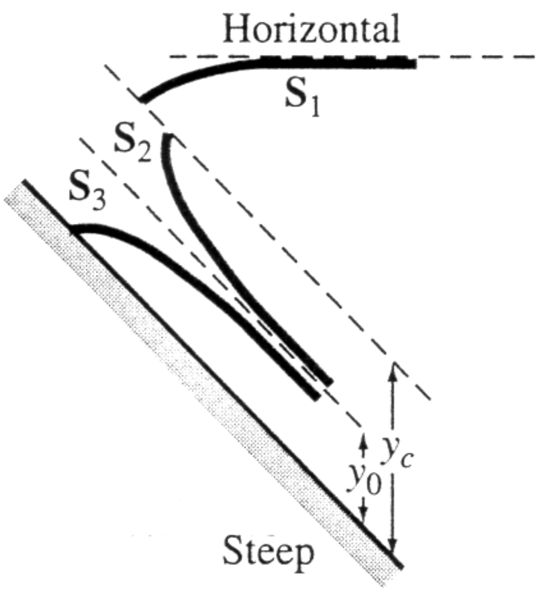
A	B	C	D	E	F	G	H	I	J	K	L	M
y	**A**	**P**	**Rh**	**V**	**Sf**	**E**	**Dx**	**x**	**T**	**Fr**	**bottom**	**surface**
0.900	1.799	4.223	0.426	0.139	0.00004	0.901		0	3.799	0.065	0.000	0.900
0.870	1.687	4.089	0.412	0.148	0.00005	0.871	0.498	0.5	3.679	0.070	0.030	0.900

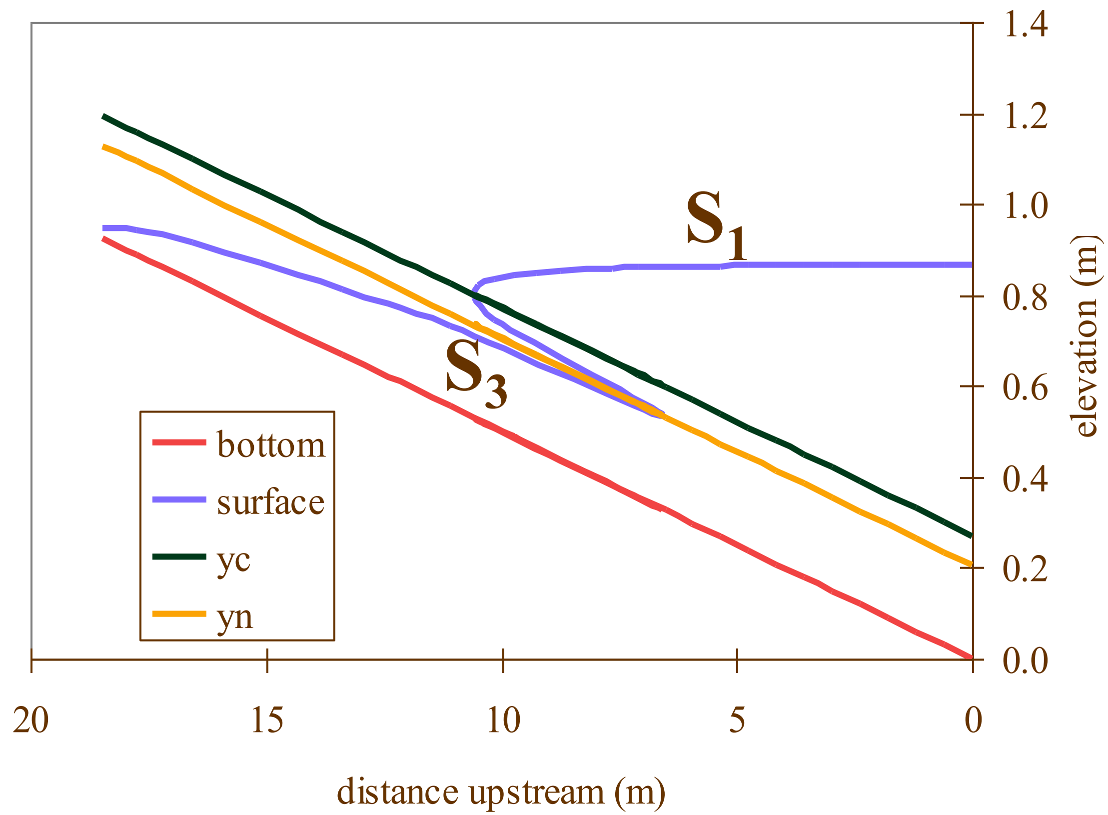
Standard Step

- Given a depth at one location, determine the depth at a second given location
- Step size (Δx) must be small enough so that changes in water depth aren't very large. Otherwise estimates of the friction slope and the velocity head are inaccurate
- Can solve in upstream or downstream direction
  - Usually solved upstream for subcritical
  - Usually solved downstream for supercritical
- Find a depth that satisfies the energy equation

$$y_1 + \frac{V_1^2}{2g} + S_o \Delta x = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

What curves are available? Steep Slope





Is there a curve between y_c and y_n that increases in depth in the downstream direction? _NO!_

Mild Slope

- If the slope is mild, the depth is less than the critical depth, and a hydraulic jump occurs, what happens next?

Rapidly varied flow!

When dy/dx is large then V isn't normal to cs

Hydraulic jump! Check conjugate depths