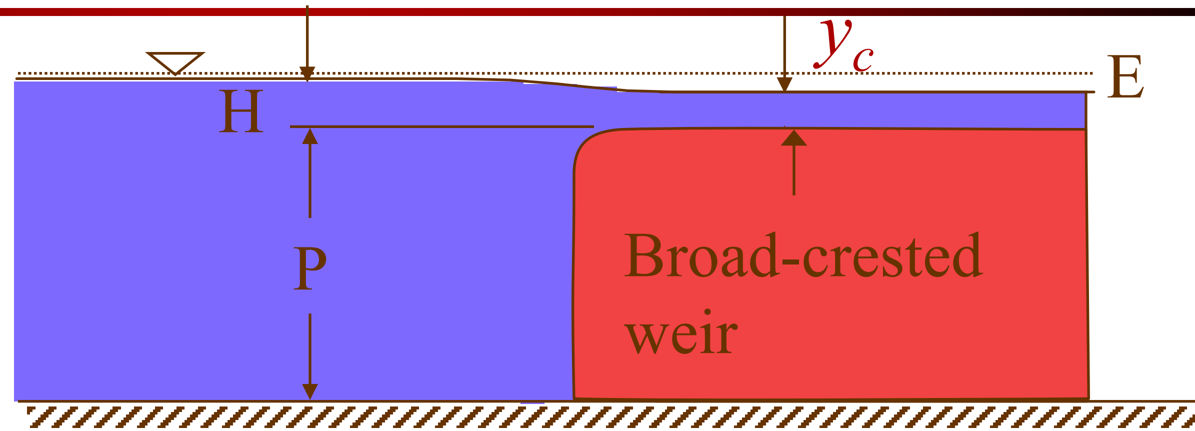
Broad-Crested Weir



$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$q = \sqrt{g y_c^3} \qquad Q = b\sqrt{g y_c^3}$$

$$y_c = \frac{2}{3}E$$

$$Q = b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} E^{3/2}$$

$$Q = C_d b\sqrt{g}\left(\frac{2}{3}H\right)^{3/2}$$

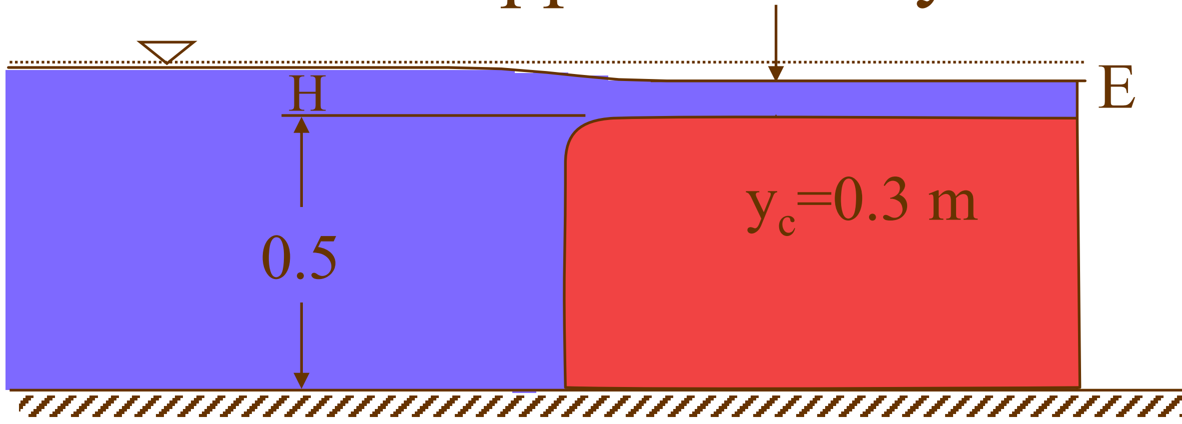
Hard to measure y_c

E measured from top of weir

C_d corrects for using H rather than E.

Broad-crested Weir: Example

- Calculate the flow and the depth upstream. The channel is 3 m wide. Is H approximately equal to E?



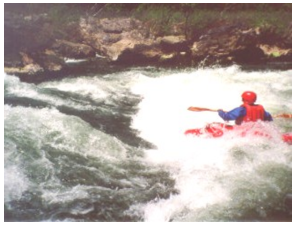
How do you find flow? Critical flow relation

How do you find H? Energy equation

Solution

Could a hydraulic jump be laminar?

Hydraulic Jump

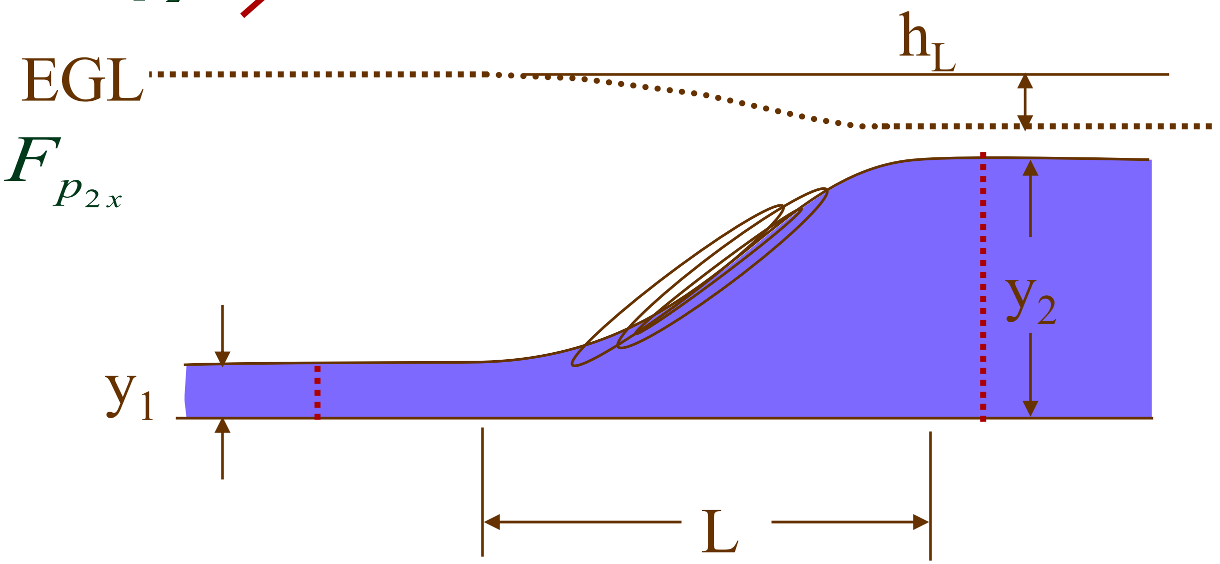


- Used for energy dissipation
- Occurs when flow transitions from supercritical to subcritical
  - base of spillway
  - Steep slope to mild slope
- We would like to know depth of water downstream from jump as well as the location of the jump
- Which equation, Energy or Momentum?

Hydraulic Jump

$$\mathbf{M}_1 + \mathbf{M}_2 = \mathbf{W} + \mathbf{F}_{p_1} + \mathbf{F}_{p_2} + \mathbf{F}_{ss}$$

Conservation of Momentum



$$M_{1x} + M_{2x} = F_{p_{1x}} + F_{p_{2x}}$$

$$M_{1x} = -\rho V_1^2 A_1$$

$$M_{2x} = \rho V_2^2 A_2$$

$$-\rho Q V_1 + \rho Q V_2 = \bar{p}_1 A_1 - \bar{p}_2 A_2$$

$$-\frac{Q^2}{A_1} + \frac{Q^2}{A_2} = \frac{g y_1 A_1}{2} - \frac{g y_2 A_2}{2}$$

$$\bar{p} = \frac{r\, g y}{2}$$

$$V = \frac{Q}{A}$$

Hydraulic Jump: Conjugate Depths

For a rectangular channel make the following substitutions

$$A = By \qquad Q = By_1V_1$$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}}$$ Froude number

Much algebra $\longrightarrow$ $$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right)$$

$$\frac{y_2}{y_1} = \frac{-1 + \sqrt{1 + 8Fr_1^2}}{2}$$

valid for slopes < 0.02

Hydraulic Jump: Energy Loss and Length

- Energy Loss $E_1 = E_2 + h_L$

$$E = y + \frac{q^2}{2gy^2} \xrightarrow{\text{algebra}} h_L = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

significant energy loss (to turbulence) in jump

- Length of jump

No general theoretical solution

Experiments show

$L = 6y_2$ for $4.5 < Fr_1 < 13$

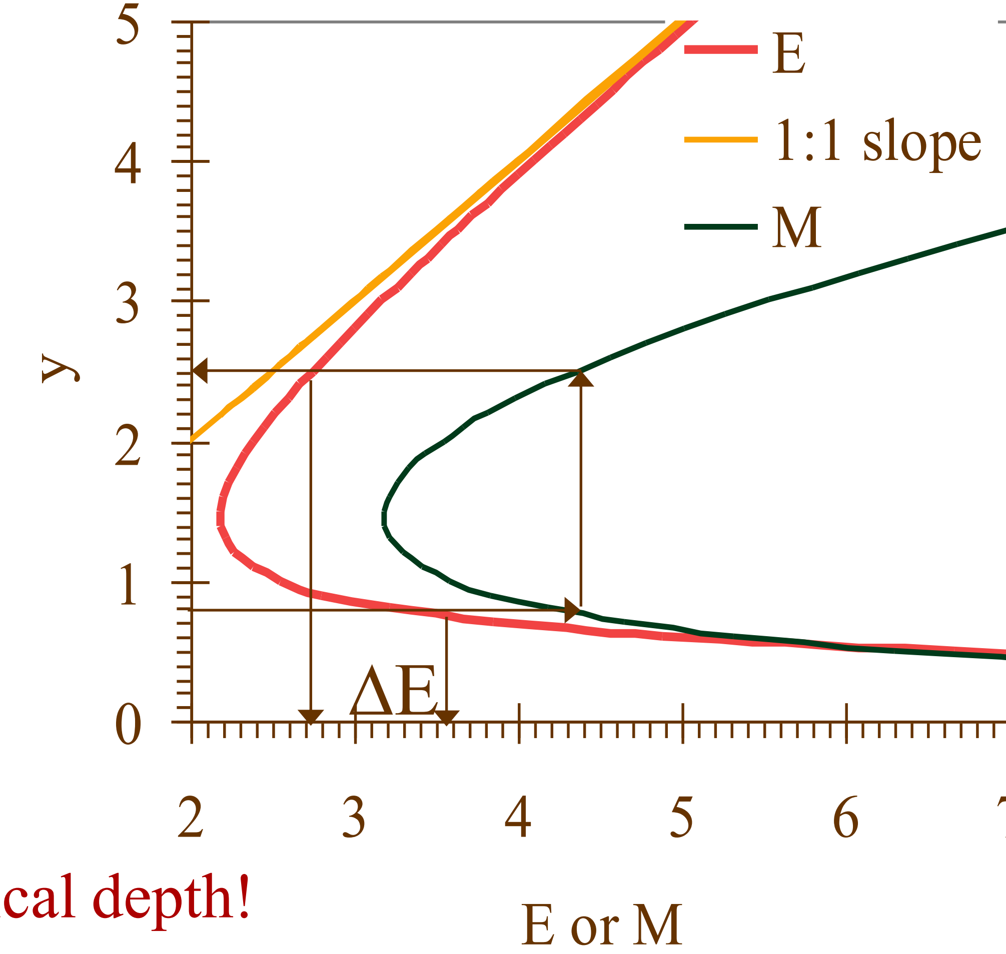
Specific Momentum

$$\frac{gy_1A_1}{2}+\frac{Q^2}{A_1}=\frac{gy_2A_2}{2}+\frac{Q^2}{A_2}$$

$$\frac{y_1A_1}{2}+\frac{Q^2}{A_1g}=\frac{y_2A_2}{2}+\frac{Q^2}{A_2g}$$

$$\frac{y_1^2}{2}+\frac{q^2}{y_1g}=\frac{y_2^2}{2}+\frac{q^2}{y_2g}$$

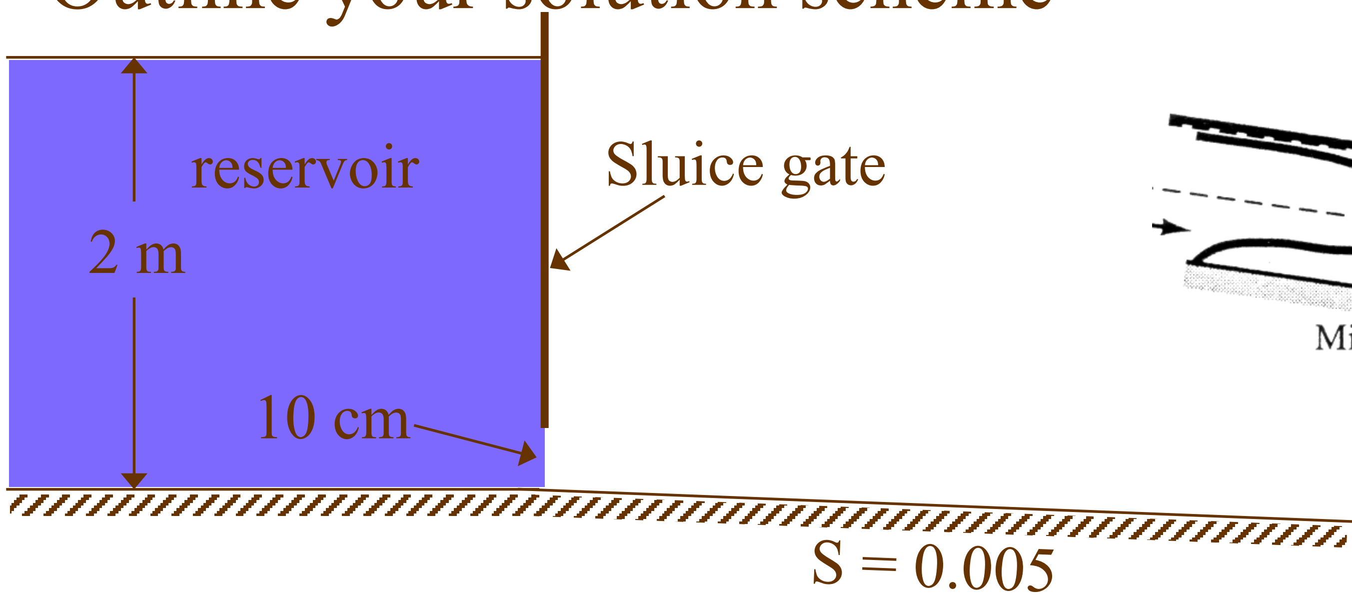
When is M minimum?

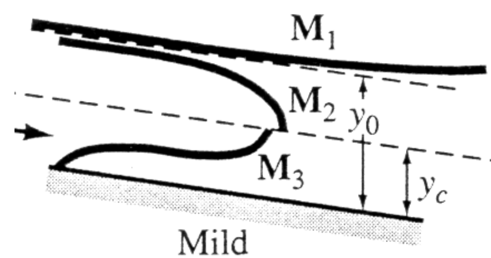


$$\frac{dM}{dy}=y+\frac{-q^2}{y^2g}$$ $$y=\left(\frac{q^2}{g}\right)^{1/3}$$ Critical depth!

Hydraulic Jump Location

- Suppose a sluice gate is located in a long channel with a mild slope. Where will the hydraulic jump be located?
- Outline your solution scheme





Energy Equation $y_1 + \frac{V_1^2}{2g} + S_o \Delta x = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$

- Specific Energy $E = y + \frac{V^2}{2g} = y + \frac{q^2}{2gy^2} = y + \frac{Q^2}{2gA^2}$
- Two depths with same energy!
  - How do we know which depth is the right one?
  - Is the path to the new depth possible?