Spatially Varied Flow

Open Channel Flow

Flow varies with longitudinal distance.

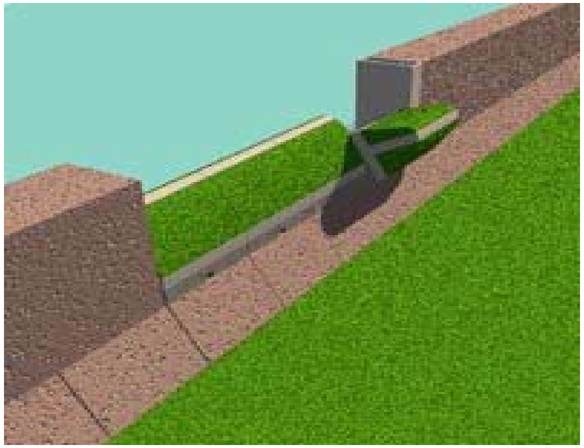
Examples: side-channel spillways, side weirs, channels with permeable boundaries, gutters for conveying storm water runoff, and drop structures in the bottom of channels.

Two types of flow:
- discharge increases with distance
- discharge decreases with distance

Different principles govern => different analysis approach

Side channel spillways



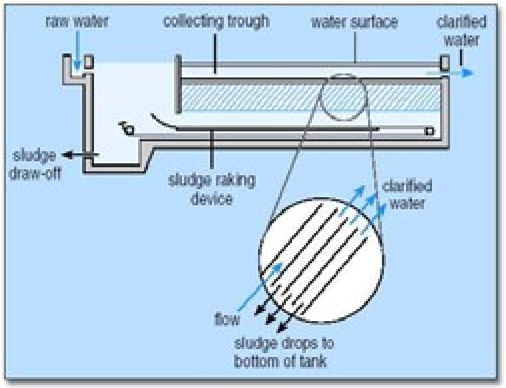


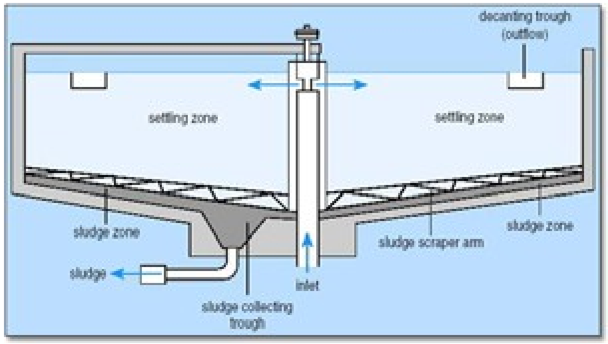

Side Weirs

- **Weirs from combined sewer systems**

Scale model of side weir (1:22)

Collecting Flumes – Settling Basins



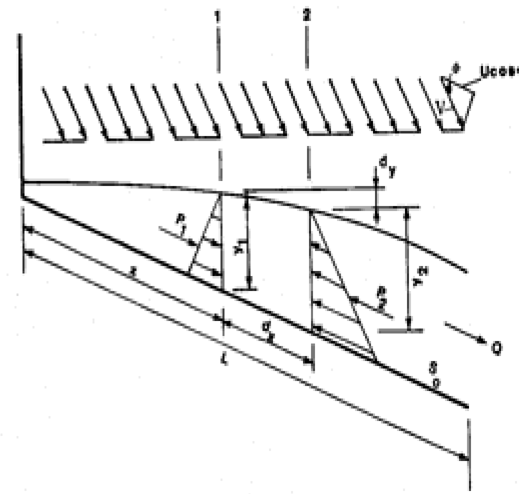


Increasing Discharge

- Example: side-channel spillway
- Significant energy loss from mixing between the
- water entering the channel and the water flowing
- in the channel.
- Hard to quantify the energy loss
- => Use momentum equation in the analysis

Analysis of Side-Channel Spillway

- Simplified analysis:
- hydrostatic pressure distribution
- small So
- inflow perpendicular to channel
- $q^* = dQ/dx = Q/x$ = constant
- Apply the momentum equation



$$P_1 - P_2 - \tau_o P dx - \rho g S_o A dx = M_2 - M_1 - M_\perp$$

$M_\perp$ = 0

- The momentum equation may be developed to yield:

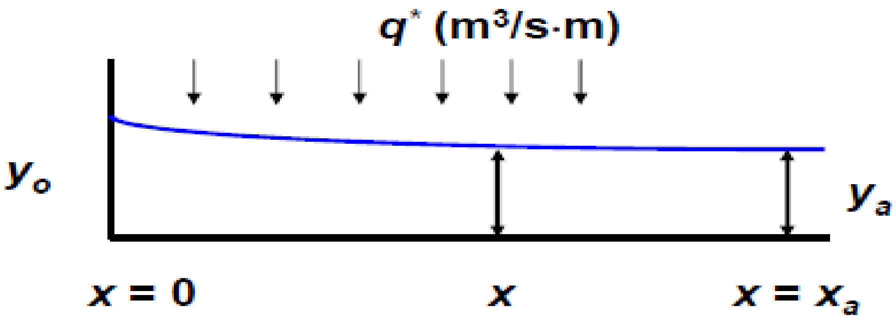
$$\frac{dy}{dx} = \frac{S_o - S_f - \dfrac{2Q(x)}{gA^2}\dfrac{dQ}{dx}}{1 - \dfrac{Q^2}{gA^2 A / T}}$$

Froude number

- *Sf* is computed from the Manning formula
- Solution proceeds from a control section.

Further simplifications:

- $S_o = 0$
- $S_f = 0$
- rectangular channel



Momentum equation (from $x = 0$ to $x = x_a$):

$$\frac{1}{2}\gamma y_o^2 b - \frac{1}{2}\gamma y_a^2 b = \rho Q(x_a)\overline{u}(x_a) - 0$$

$$Q(x_a) = q^* x_a$$

$$\overline{u}_a = \frac{Q(x_a)}{y_a b}$$

For arbitrary section (from *x* to x_a):

$$\frac{1}{2}\gamma y^2 b - \frac{1}{2}\gamma y_a^2 b = \rho Q(x_a)\overline{u}(x_a) - \rho Q(x)\overline{u}(x)$$

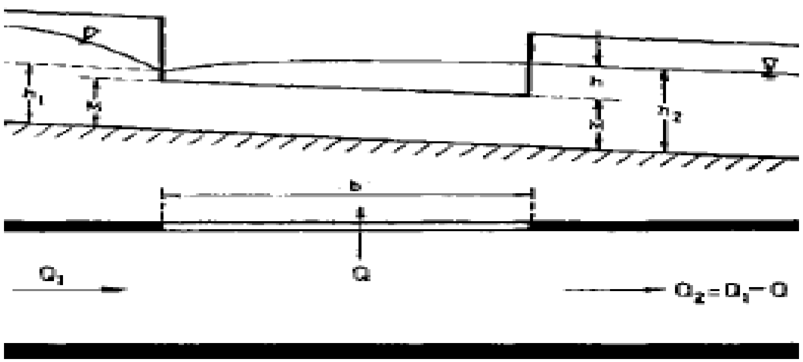
$$Q(x) = q^* x$$

$$\overline{u} = \frac{Q(x)}{yb}$$

Decreasing Discharge

- Example: side weirs
- No significant energy loss
- => Water surface profile can be estimated from the
- energy principle
- Total energy at channel section relative to a datum:

$$H = z + y + \frac{Q^2}{2gA^2}$$



Differentiating *H* with respect to *x*:

$$\frac{dH}{dx}=\frac{dz}{dx}+\frac{dy}{dx}+\frac{1}{2g}\left(\frac{2Q}{A^2}\frac{dQ}{dx}-\frac{2Q^2}{A^3}\frac{dA}{dx}\right)$$

$$S_o=-\frac{dz}{dx},\quad S_f=-\frac{dH}{dx},\quad \frac{dA}{dx}=\frac{dA}{dy}\frac{dy}{dx}=T\frac{dy}{dx}$$

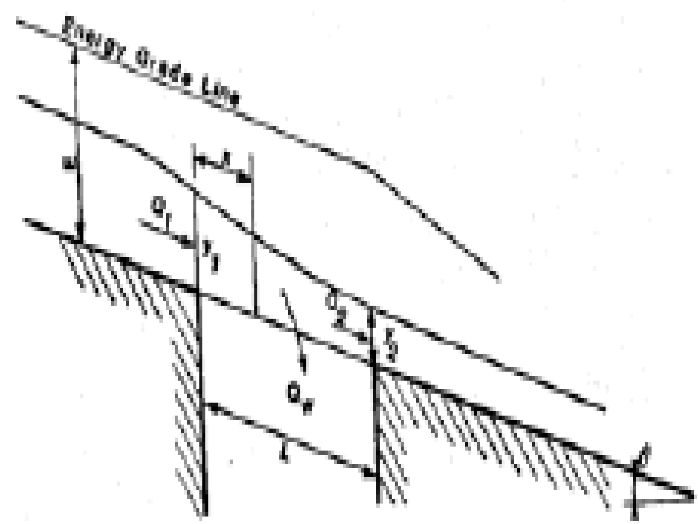
Substitute in relations:

New term

$$\frac{dy}{dx}=\frac{S_o-S_f-\left(Q/gA^2\right)\left(dQ/dx\right)}{1-\left(Q^2/gA^2D\right)}$$

Special case:

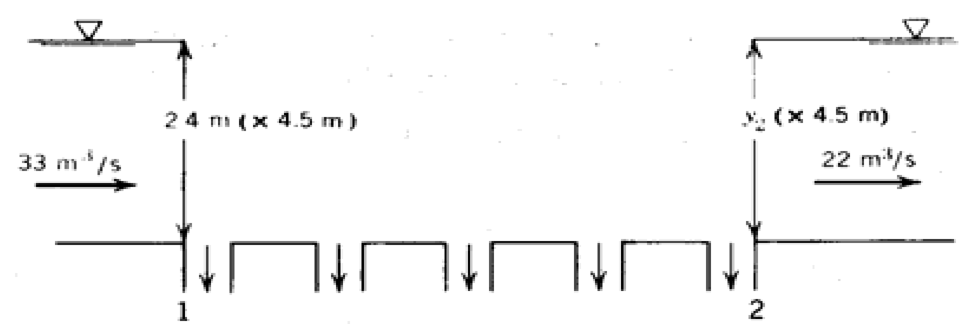
- $S_o = 0$
- $S_f = 0$
- rectangular channel



$$\frac{dy}{dx} = -\frac{Q(x)\,y\left(dQ/dx\right)}{gb^2y^3 - Q^2(x)}$$

Example: Outflow from Channel Bottom

Eleven cubic meters per second are diverted through ports in the bottom of the channel between sections 1 and 2. Neglecting head losses and assuming a horizontal channel, what depth of water is to be expected at section 2? What channel width at section 2 would be required to produce a depth of 2.5 m?



The specific energy is unchanged from 1 to 2 (no energy losses):

$$E = y_1 + \frac{u_1^2}{2g} = y_2 + \frac{u_2^2}{2g}$$

Flow per unit width:

$$q_1 = \frac{Q_1}{b_1} = \frac{33}{4.5} = 7.33 \text{ m}^3/\text{m}\cdot\text{s}$$

$$q_1 = \frac{Q_2}{b_2} = \frac{22}{4.5} = 4.89 \text{ m}^3/\text{m}\cdot\text{s}$$

Energy equation from 1 to 2:

$$y_1 + \left(\frac{q_1}{y_1}\right)^2 \frac{1}{2g} = y_2 + \left(\frac{q_2}{y_2}\right)^2 \frac{1}{2g}$$

$$2.4 + \left(\frac{7.33}{2.4}\right)^2 \frac{1}{2g} = y_2 + \left(\frac{4.89}{y_2}\right)^2 \frac{1}{2g}$$

Flow condition in section 1:

$$Fr_1 = \frac{u_1}{\sqrt{gy_1}} = \frac{q_1 / y_1}{\sqrt{gy_1}} = \frac{7.33/2.4}{\sqrt{g2.4}} = 0.63 < 1$$

Subcritical flow

Subcritical flow and q decreasing implies that the solution corresponding to subcritical flow in section 2 must be valid

→ y_2 = 2.72 m (found by trial-and-error)

Assume a depth of y_2 = 2.5 m. What channel width is required?

Employ the energy equation from 1 to 2:

$$2.4+\left(\frac{7.33}{2.4}\right)^2\frac{1}{2g}=2.5+\left(\frac{22}{2.5b_2}\right)^2\frac{1}{2g}$$

→ b_2 = 3.22 m