Hydrograph

: Hydrographs

- Previous Chapter estimation of long-term runoff was examined

- the present chapter examines in detail the short-term runoff phenomenon by the *storm hydrograph* or *flood hydrograph or simply Hydrograph*

- The runoff measured at the stream-gauging station will give a typical hydrograph as shown in Fig. 6.1

- The flood hydrograph is formed as a result of uniform rainfall of duration, T_r,. over a catchment.

- The Hydrograph (Figure 6.1) has three characteristic regions:

  (i) the rising limb *AB,* joining point A, the starting point of the rising curve and point *B,* the point of inflection,

  (ii) the crest segment *BC* between the two points of inflection with a peak *P* in between,

  (iii) the falling limb or *depletion curve CD* starting from the second point of inflection C.

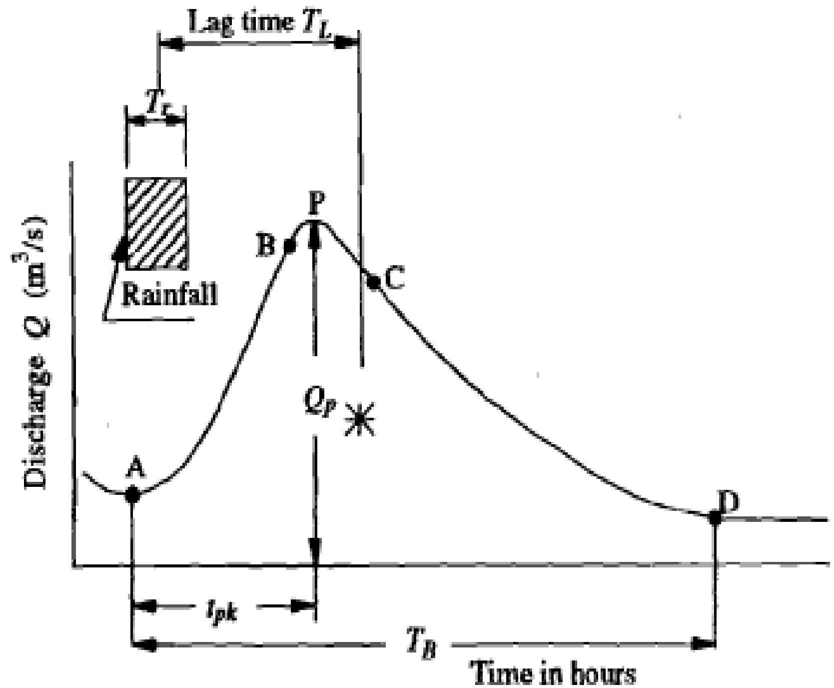


Fig. 6.1 Elements of a Flood Hydrograph

- Timing of the Hydrograph

1. t_{pk} : the time to peak (Q_p) from the starting point A,
2. *lag time* T_L : the time interval from the centre of mass of rainfall to the centre of mass of hydrograph,
3. T_B : the time base of the hydrograph

- Factor Influencing the Hydrograph

1. Watershed Characteristics such as
   size, shape, slope, storage
2. Infiltration Characteristics
   soil and land use and cover
3. Climactic Factors
   - rainfall intensity and pattern
   - aerial distribution
   - duration
   - type (rainfall vs snowmelt)

- Generally, the climatic factors control the rising limb
- catchment characteristics determine the recession limb

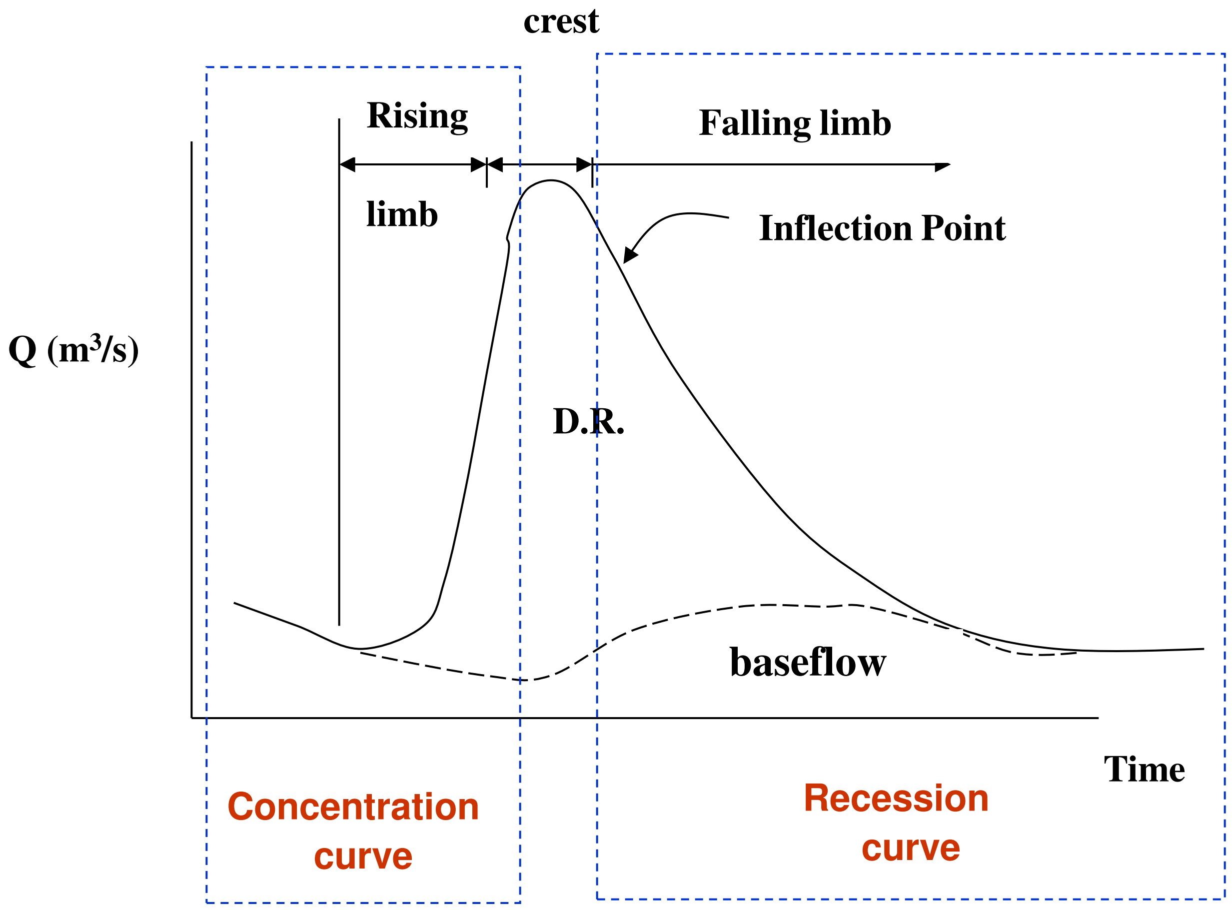
6.3 COMPONENTS OF A HYDROGRAPH

the essential components of a hydrograph are:

(i) the rising limb,

(ii) the crest segment, and

(iii) the recession limb.



Rising Limb

- The rising limb of a hydrograph (concentration curve) represents the increase in discharge due to the gradual building up of storage in channels and over the catchment surface.
- As the storm continues more and more flow from distant parts reach the basin outlet. At the same time the infiltration losses also decrease with time.

Crest

- The peak flow occurs when the runoff from various parts of the catchment at the same time contribute the maximum amount of flow at the basin outlet.
- Generally for large catchments, the peak flow occurs after the end of rainfall,
- the time interval from the centre of mass of rainfall to the peak being essentially controlled by basin and storm characteristics.

Recession Limb

- It extends from the point of inflection at the end of the crest segment to the start of the natural groundwater flow

- It represents the withdrawal of water from the storage built up in the basin during the earlier phases of the hydrograph.

- The starting point of the recession limb (the point of inflection) represents the condition of maximum storage.

- Since the depletion of storage takes place after the end of rainfall, the shape of this part of the hydrograph is independent of storm characteristics and depends entirely on the basin characteristics.

- The storage of water in the basin exists as

  - *surface storage, which includes both surface detention and channel storage,*
  - *interflow storage, and*
  - *groundwater storage, i.e. base-flow storage.*

- Barnes (1940) showed that the recession of a storage can be expressed as

$$Q_t = Q_0 K_r^t$$

which Q_0: the initial discharge and

Q_t :are discharges at a time interval of t days;

K: is a recession constant of value less than unity.

- Previous Equation can also be expressed in an alternative form of the exponential decay as

$$Q_t = Q_0 e^{-at}$$

where $a = -\ln K$,

- The recession constant K; can be considered to be made up of three components to take care of the three types of storages as:

$K = K_{rs} \cdot K_{ri} \cdot K_{rb}$

where K_{rs} = recession constant for surface storage (0.05 to 0.20),

K_{ri} = recession constant for interflow (0.50 to 0.85) and

K_{rb} = recession constant for base flow (0.85 to 0.99)

Example 6.1

6.4 Base Flow Separation

- The surface hydrograph is obtained from the total storm hydrograph by separating the quick-response flow from the slow response runoff.

- The base flow is to be deducted from the total storm hydrograph to obtain the surface flow hydrograph in three methods

Method I: Straight line method

- Draw a horizontal line from start of runoff to intersection with recession limb (**Point A**).
- Extend from time of peak to intersect with recession limb using a lag time, N.

$$N = 0.83\ A^{0.2}$$

Where: A = the drainage area in Km^2 and N = days where **Point B** can be located and determine the end of the direct runoff .

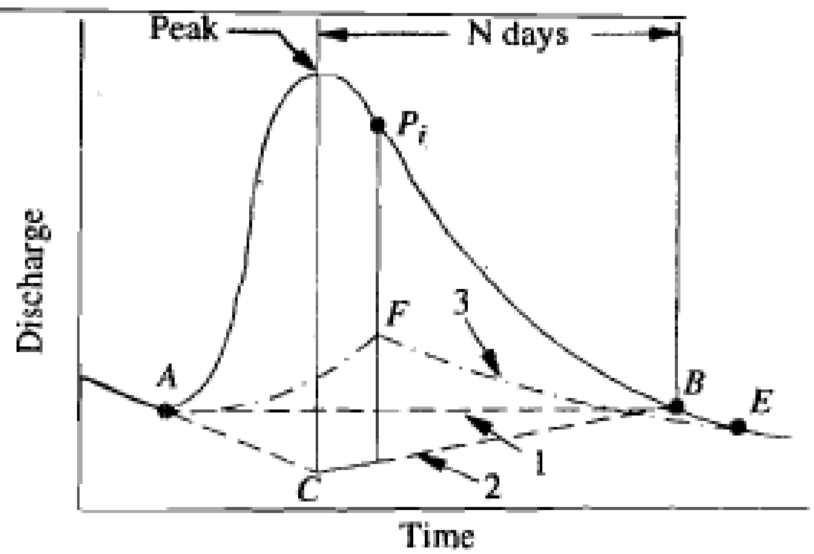


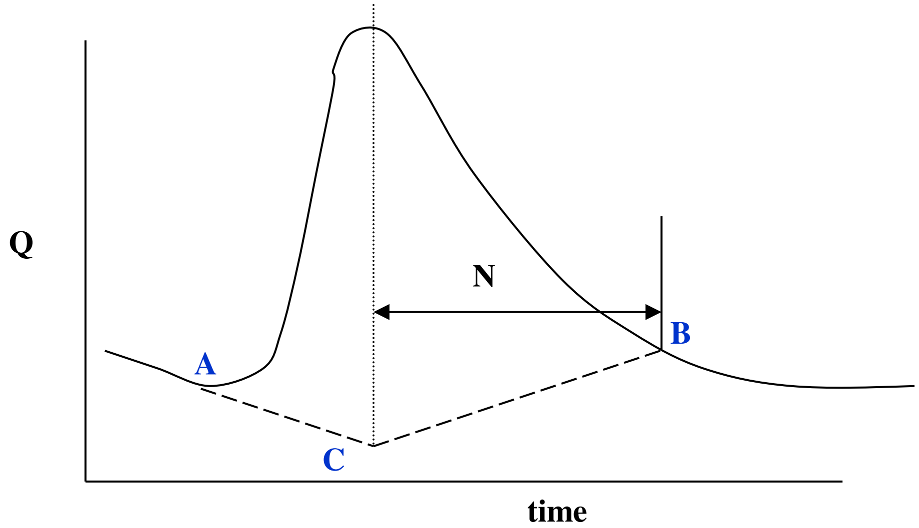
Fig. 6.5 Base flow seperation methods

Method II:

- In this method the base flow curve existing prior to the beginning of the surface runoff is extended till it intersects the ordinate drawn at the peak (point C in Fig, 6.5). This point is joined to point *B* by a straight line.

- Segment *AC* and *CB* separate the base flow and surface runoff.

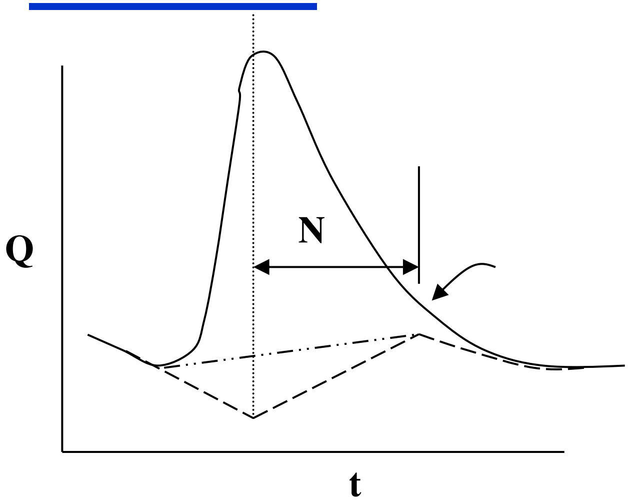
- This is probably the most widely used base-flow separation procedure.



Method III

- In this method the base flow recession curve after the depletion of the flood water is extended backwards till it intersects the ordinate at the point of inflection (line *EF* in Fig. 6.5), Points *A* and *F* are joined by an arbitrary smooth curve.
- This method of base-flow separation is realistic in situations where the groundwater contributions are significant and reach the stream quickly.
- The selection of anyone of the three methods depends upon the local practice and successful predictions achieved in the past.
- The surface runoff hydrograph obtained after the base-flow separation is also known as *direct runoff hydrograph* (DRH).

Method V



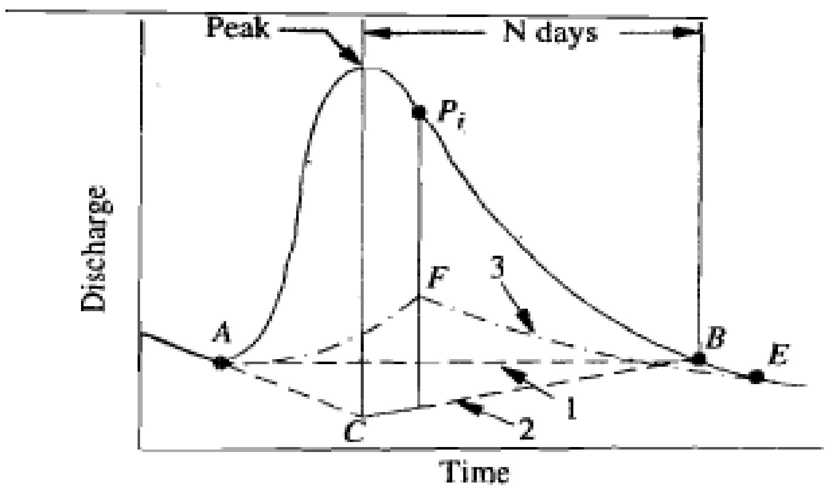


Fig. 6.5 Base flow seperation methods

6.5 EFFECTIVE RAINFALL

- Figure 6.6. show, the hyetograph of a storm. The initial loss and infiltration losses are subtracted from it. The resulting hyetograph is known' as *effective rainfall hyetograph* (ERH). It is also known as *hyetograph of rainfall excess* or *supra rainfall*.
- *Both DRH and ERH represent the same Total quantity but in different units*
- *ERH is usually in cm/h against time*
- *The area multiplied by the catchment Area gives the total volume of the direct runoff (total area of DRH)*

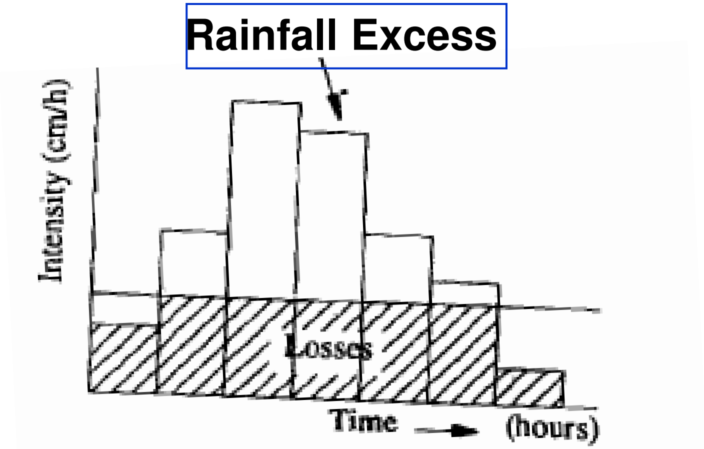


Fig. 6.6 Effective rainfall hyetograph (ERH)

EXAMPLE 6.2. *Rainfall of magnitude 3.8 cm and 2.8 cm occurring on two consecutive 4-h durations on a catchment of area 27 km² produced the following hydrograph of flow at the outlet of the catchment. Estimate the rainfall excess and φ index.*

Time from start of rainfall (h)	–6	0	6	12	18	24	30	36	42	48	54	60	66
Observed flow (m^3/s)	6	5	13	26	21	16	12	9	7	5	5	4.5	4.5

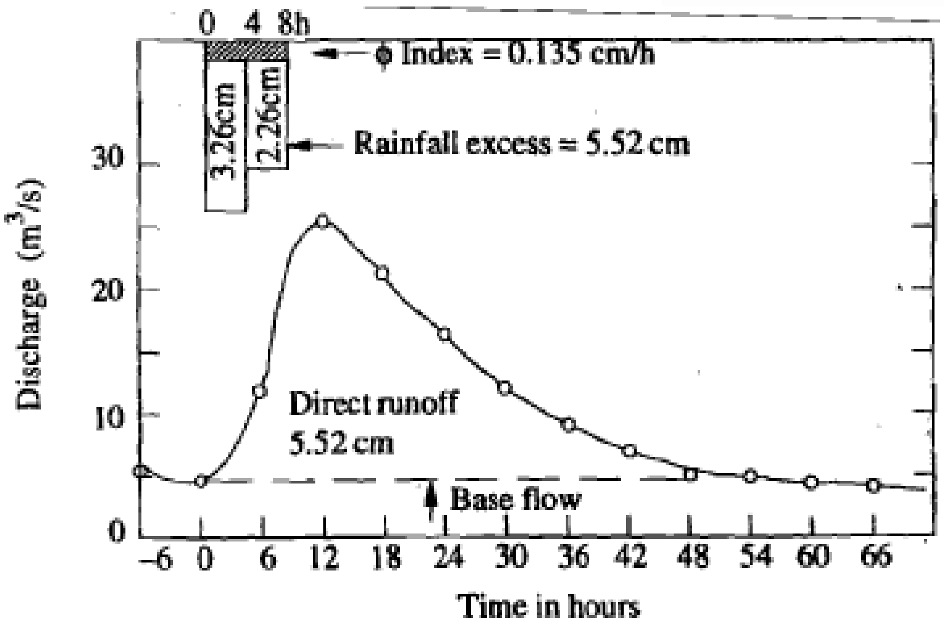


Fig. 6.7 Base flow seperation—Example 6.2

6.6 **UNIT HYDROGRAPH**

- A large number of methods are proposed to solve this problem and of them probably the most popular and widely used method is the *unit-hydrograph method.*

- A unit hydrograph is defined as the hydrograph of direct runoff resulting from one unit depth (1 cm) of rainfall excess occurring uniformly over the basin and at a uniform rate for a specified duration (D hours).

- The term unit here refers to a unit depth of rainfall excess which is usually taken as 1 cm.
- The duration, being a very important characteristic, is used as indication to a specific unit hydro graph. Thus one has a 6-h unit hydrograph, 12-h unit hydrograph, etc. and in general a *D-h* unit hydrograph applicable to a given catchment.

<u>The definition of a unit hydrograph implies the following:</u>

- It relates only the direct runoff to the rainfall excess. Hence the volume of water contained in the unit hydrograph must be equal to the rainfall excess.

As 1 cm depth of rainfall excess is considered the area of the unit hydrograph is equal to a volume given by 1cm over the catchment.

- The rainfall is considered to have an average intensity of *excess rainfall* (ER) of *l/D* cm/h for the duration *D-h* of the storm.

- The distribution of the storm is considered to be uniform all over the catchment.

- Fig 6.9 shows a typical 6-h unit hydrograph. Here the duration of the rainfall excess is 6 h

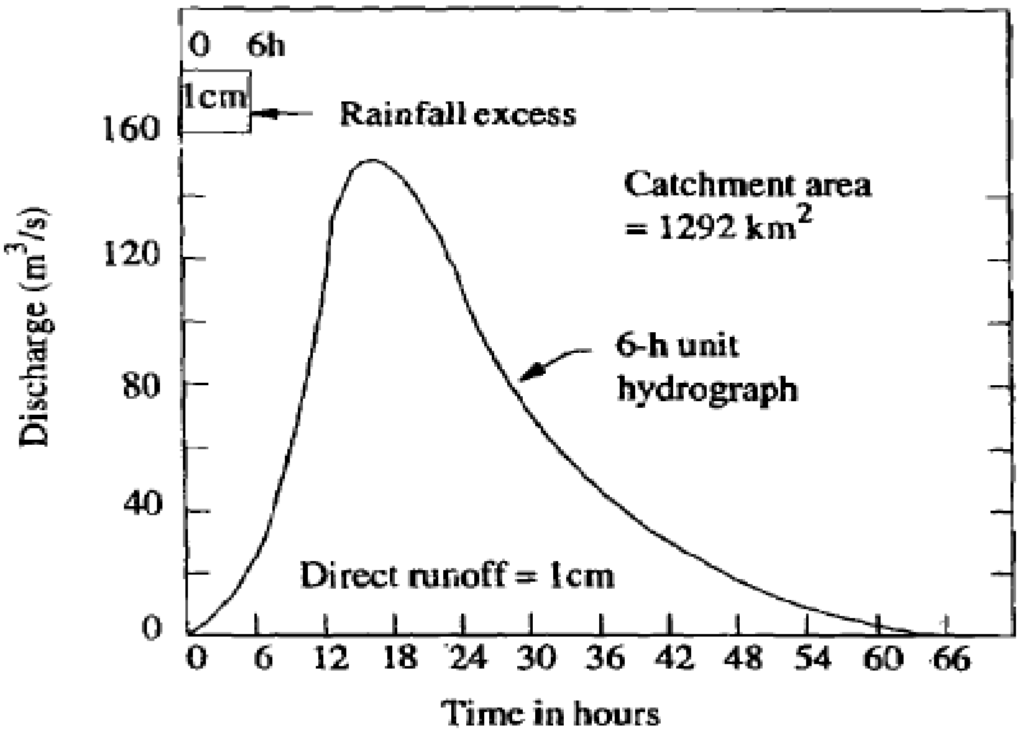


Fig. 6.9 Typical 6–h unit hydrograph

Area under the unit hydrograph $= 12.92 \times 10^6\ m^3$

Two basic assumptions constitute the foundations for the unit-hydrograph theory:

(i) the time invariance and

(ii) the linear response.

Time Invariance

This first basic assumption is that the direct-runoff response to a given effective rainfall in a catchment is time-invariant. This implies that the DRH for a given ER in a catchment is always the same irrespective of when it occurs.

Linear Response

- The direct-runoff response to the rainfall excess is assumed to be linear. This is the most important assumption of the unit-hydrograph theory.

- Linear response means that if an input x_l *(t)* causes an output y_l *(t)* and an input $.x_2$ *(t)* causes an output y_2 *(t)*, then an input x_l *(t)* $+x_2$ *(t)* gives an output y_1 *(t)* $+y_2$*(t)*.

- Consequently, if x_2 *(t)* = *r* X_l *(t)*, then y_z *(t)* = *r* y_l *(t)*.

- Thus if the rainfall excess in a duration D is *r* times the unit depth, the resulting DRH will have ordinates bearing ratio *r* to those of the corresponding *D-h* unit hydrograph.

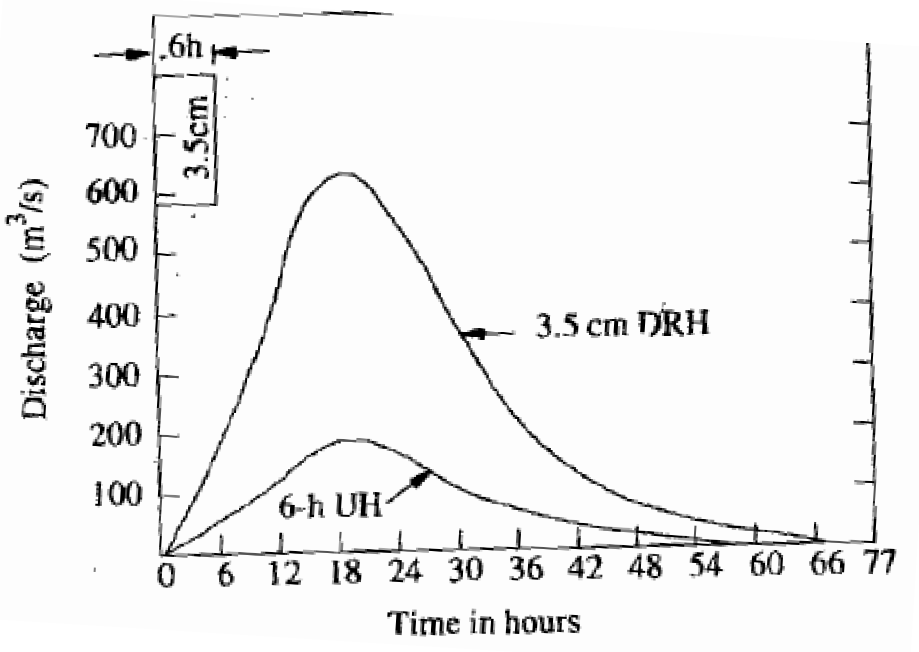
- Since the area of the resulting DRH should increase by the ratio *r,* the base of the DRH will be the same as that of the unit hydrograph.

- If two rainfall excess of *D-h* duration each occur consecutively, their combined effect is obtained by superposing the respective DRHs with due care being taken to account for the proper sequence of events. **(The method of superposition)**

EXAMPLE 6.4 *Given below are the ordinates of a 6-h unit hydrograph for a catchment. Calculate the ordinates of the DRH due to a rainfall excess of 3.5 cm occurring in 6 hr.*

Time (h)	0	3	6	9	12	15	18	24	30	36	42	48	54	60	69
UH ordinate (m^3/s)	0	25	50	85	125	160	185	160	110	60	36	25	16	8	0

- The desired ordinates of the DRH are obtained by multiplying the ordinates of the unit hydrograph by a factor of 3.5 as in Table 6.3.
- Note that the time base of DRH is not changed and remains the same as that of the unit hydrograph.



EXAMPLE 6.5 *Two storms each of 6-h duration and having rainfall excess values of 3.0 and 2.0 cm respectively occur successively. The 2-cm ER rain follows the 3-cm rain. The 6-h unit hydrograph for the catchment is the same as given in Example 6.4. Calculate the resulting DRH.*

TABLE 6.4 CALCULATION OF DRH BY METHOD OF SUPERPOSITION — *EXAMPLE 6.5*

Time (h)	Ordinate of 6-h UH (m^3/s)	Ordinate of 3-cm DRH (col.2) × 3	Ordinate of 2-cm DRH (col.2 lagged by 6 h) × 2	Ordinate of 5-cm DRH (col. 3 + col.4) (m^3/s)	Remarks
1	2	3	4	5	6
0	0	0	0	0	
3	25	75	0	75	
6	50	150	0	150	
9	85	255	50	305	
12	125	375	100	475	
15	160	480	170	650	
18	185	555	250	805	
(21)	(172.5)	(517.5)	(320)	(837.5)	Interpolated value
24	160	480	370	850	
30	110	330	320	650	
36	60	180	220	400	
42	36	108	120	228	
48	25	75	72	147	
54	16	48	50	98	
60	8	24	32	56	
(66)	(2.7)	(8.1)	(16)	(24.1)	Interpolated value
69	0	0	(10.6)	(10.6)	Interpolated value
75	0	0	0	0	

Note : 1. The entries in col.4 are shifted by 6 h in time relative to col.2.

2. Due to unequal time interval of ordinates a few entries have to be interpolated to complete the table. These interpolated values are shown in parentheses.

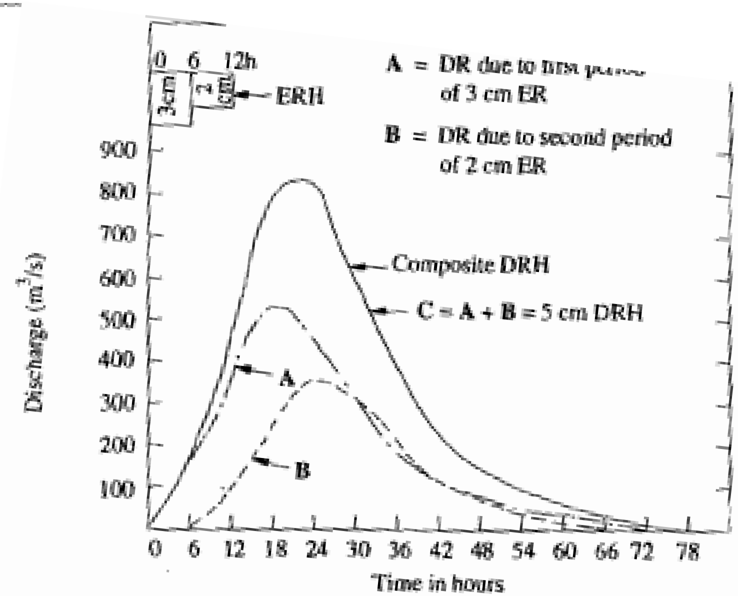


Fig. 6.10 (b) Principle of superposition—Example 6.5

Application of U-Hydrograph

- D-h U-hydrograph and storm hyetograph are available
- ERH is obtained by deducting the losses
- ERH is divided by M blocks of D-h duration
- Rainfall excesses is operated upon unit hydrograph successively to get different DHR curves

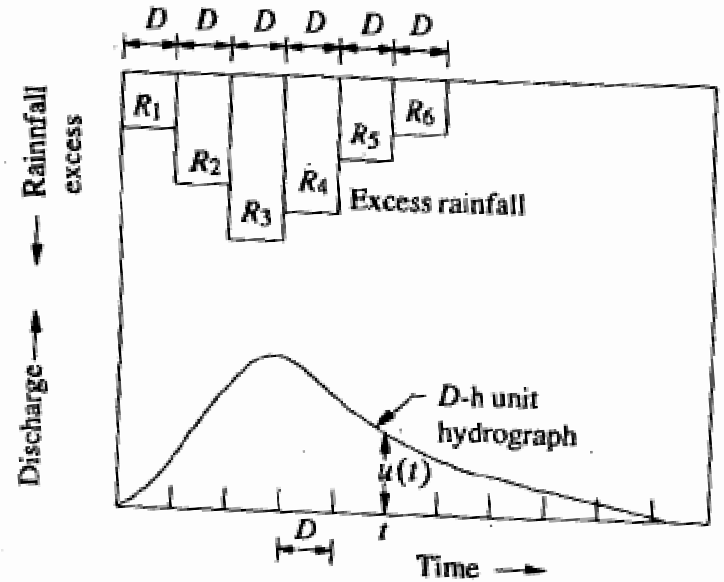


Fig. 6.11 DRH due to an ERH

The direct runoff due to R_1 at time t is

$$Q_1 = R_1 \cdot u[t]$$

The direct runoff due to R_2 at time $(t - D)$ is

$$Q_2 = R_2 \cdot u[t - D]$$

Similarly, $$Q_i = R_i \cdot u[t - (i-1)D]$$

and $$Q_m = R_m \cdot u[t - (M-1)D]$$

Thus at any time t, the total direct runoff is

$$Q_t = \sum_{i=1}^{M} Q_i = \sum_{i=1}^{M} R_i \cdot u[t - (i-1)D]$$

EXAMPLE 6.6 *The ordinates of a 6- hour unit hydrograph of a catchment is given below.*

Time (h)	0	3	6	9	12	15	18	24	30	36	42	48
Ordinate of 6-h UH	0	25	50	85	125	160	185	160	110	60	36	25

Time (h)	54	60	69
Ordinate of 6-h UH	16	8	0

Derive the flood hydrograph in the catchment due to the storm given below:

Time from start of storm (h)	0	6	12	18
Accumulated rainfall (cm)	0	3.5	11.0	16.5

The storm loss rate (φ – index) for the catchment is estimated as 0.25 cm/h. The base flow can be assumed to be 15 m^3/s *at the beginning and increasing by 2.0* m^3/s *for every 12 hours till the end of the direct–runoff hydrograph.*

Solution of Ex.6.6

Interval	1st 6 hours	2nd 6 hours	3rd 6 hours
Rainfall depth (cm)	3.5	(11.0-3.5) = 7.5	(16.5-11.0) = 5.5
Loss @ 0.25 cm/h for 6 h	1.5	1.5	1.5
Effective rainfall (cm)	2.0	6.0	4.0

Time	Ordinaters of U.H.	DRH due to 2 cm ER Col. 2 × 2.0	DRH due to 2 cm ER × 6.0 (Advanced by 6 h)	DRH due to 4 cm ER Col.2 × 4.0 (Advanced by 12 h)	Ordinates of final DRH (Col 3+ 4+5)	Base flow (m^3/s)	Ordinates of flood hydrograph (m^3/s) (Col. 6 + 7)
1	2	3	4	5	6	7	8
0	0	0	0	0	0	15	15
3	25	50	0	0	50	15	65
6	50	100	0	0	100	15	115
9	85	170	150	0	320	15	335
12	125	250	300	0	550	17	567
15	160	320	510	100	930	17	947
18	185	370	750	200	1320	17	1337
(21)	(172.5)	(345)	960	340	1645	(17)	1662
24	160	320	1110	500	1930	19	1949
(27)	(135)	(270)	(1035)	640	1945	19	1964
30	110	220	960	740	1920	19	1939
36	60	120	660	640	1420	21	1441
42	36	72	360	440	872	21	893
48	25	50	216	240	506	23	529
54	16	32	150	144	326	23	349
60	8	16	96	100	212	25	237
66	(2.7)	(5.4)	48	64	117	25	142
69	0	0	—	—	—	—	—
72		0	16	32	48	27	75
75		0	0	—	—	—	—
78		0	0	(10.8)	(11)	27	49
81				0	0	27	27
84						27	27

6.7Derivation of Unit Hydrographs

- The area under each DRH is evaluated and the volume of the direct runoff obtained is divided by the catchment area to obtain the depth of ER.
- The ordinates of the various DHRs are divided by the respective ER values to obtain the ordinates of the unit hydrograph.

EXAMPLE 6.7 *Following are the ordinates of a storm hydrograph of a river draining a catchment area of 423 km² due to a 6-h isolated storm. Derive the ordinates of a 6-h unit hydrograph for the catchment.*

Time from start of storm (h)	−6	0	6	12	18	24	30	36	42	48
Discharge (m^3/s)	10	10	30	87.5	115.5	102.5	85.0	71.0	59.0	47.5

Time from start of storm (h)	54	60	66	72	78	84	90	96	102
Discharge (m^3/s)	39.0	31.5	26.0	21.5	17.5	15.0	12.5	12.0	12.0

Solution of Example 6.7

SOLUTION: The storm hydrograph is plotted to scale (Fig. 6.13). Denoting the tim beginning of storm as t, by inspection of Fig. 6.12,

A = beginning of DRH $t = 0$

B = end of DRH $t = 90$ h

P_m = peak $t = 20$ h

Hence

$$N = (90 - 20) = 70 \text{ h} = 2.91 \text{ days}$$

By Eq. (6.4),

$$N = 0.83\,(423)^{0.2} = 2.78 \text{ days},$$

- However, N =2.91 days is adopted for convenience.
- A straight line joining A and B is taken as the divide line for base-flow separation.
- The ordinates of DRH are obtained by Subtracting the base flow from the ordinates of the storm hydrograph.

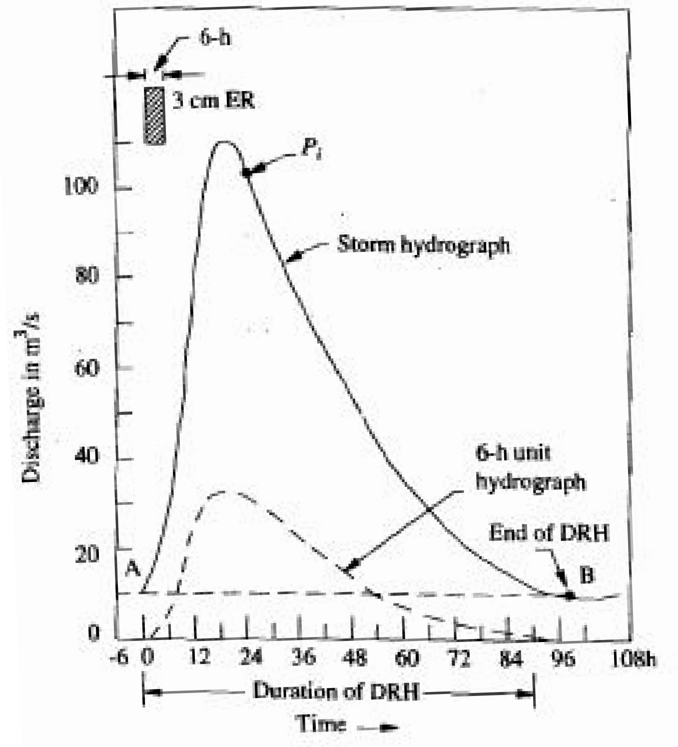


Fig. 6.13 Derivation of unit hydrograph from a storm hydrograph

$$\text{Volume of DRH} = 60 \times 60 \times 6 \times (\text{sum of DRH ordinates})$$

$$= 60 \times 60 \times 6 \times 587 = 12.68 \text{ Mm}^3$$

$$\text{Drainage area} = 423 \text{ km}^2 = 423 \text{ Mm}^2$$

$$\text{Runoff depth} = \text{ER depth} = \frac{12.68}{423} = 0.03 \text{ m} = 3 \text{ cm.}$$

The ordinates of DRH (col. 4) are divided by 3 to obtain the ordinates of the 6-h unit hydrograph,, see Table 6.6

TABLE 6.6 CALCULATION OF THE ORDINATES OF A 6–h UNIT HYDROGRAPH—*EXAMPLE 6.7*

Time from beginning of storm (h)	Ordinate of storm hydrograph (m^3/s)	Base flow (m^3/s)	Ordinate of DRH (m^3/s)	Ordinate of 6-h unit hydrograph (Col. 4 ÷ 3)
1	2	3	4	5
–6	10.0	10.0	0	0
0	10.0	10.0	0	0
6	30.0	10.0	20.0	6.7
12	87.5	10.5	77.0	25.7
18	111.5	10.5	101.0	33.7
24	102.5	10.5	101.0	33.7
30	85.0	11.0	74.0	24.7
36	71.0	11.0	60.0	20.0
42	59.0	11.0	48.0	16.0
48	47.5	11.5	36.0	12.0

6.8 Unit Hydrographs of Different Duration

- Unit hydrograph are derived from simple isolated storms and if the duration of the various storms do not differ two much (20% D) >>>>> make the average duration of *D* h.

- In practice the unit hydrographs of different duration are needed (*n*D).

- Two methods are available
  1. Method of Superposition
  2. the S-Curve

6.8 Unit Hydrographs of Different Duration

- **Unit hydrograph are derived from simple isolated storms and if the duration of the various storms do not differ two much (20% D) >>>>> make the average duration of *D* h.**

- **In practice the unit hydrographs of different duration are needed (*n*D).**

- **Two methods are available**

  1. **Method of Superposition**
  2. **the S-Curve**

1. Method of Superposition

- D-H unit duration is available and it is needed to make UH of nDH, where n is and integer
- Superposing n UH with each graph separated from the previous one by D h.

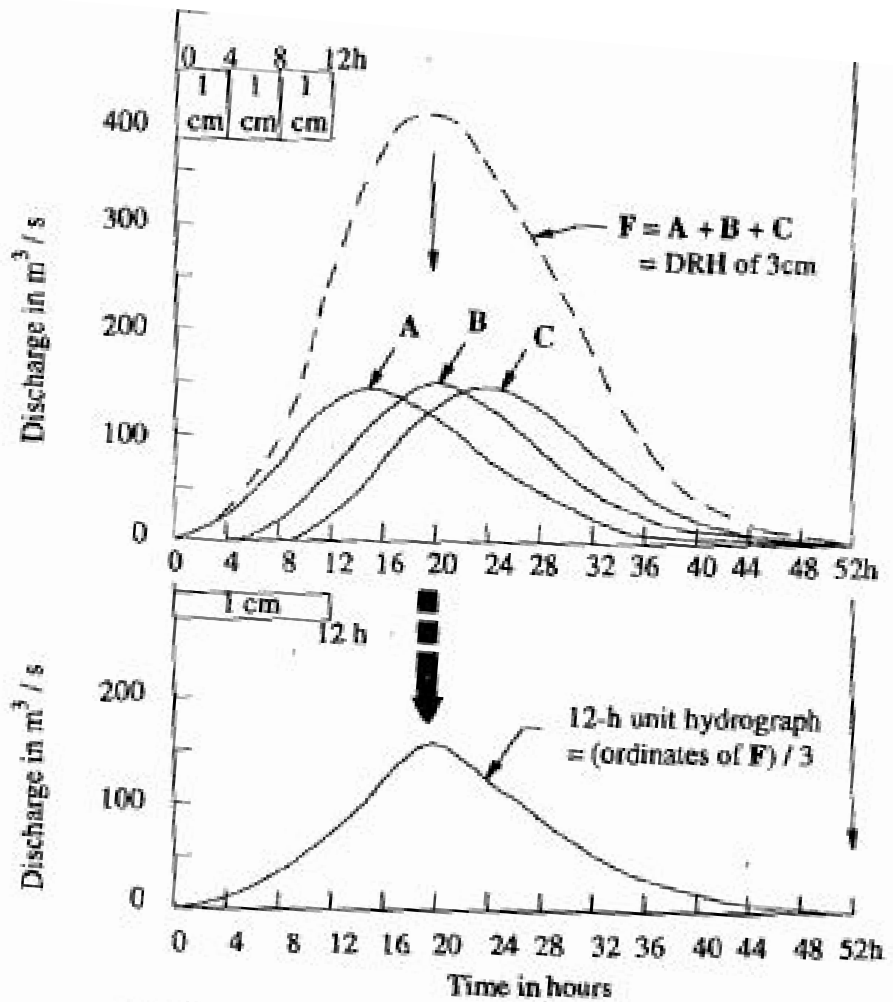


Fig. 6.15 Construction of a 12–h unit hydrograph from a 4–h unit hydrograph

EXAMPLE 6.9 *Given the ordinates of a 4-h unit hydrograph as below derive the ordinates of a 12-h unit hydrograph for the same catchment.*

Time (h)	0	4	8	12	16	20	24	28	32	36	40	44
Ordinate of 4–h UH	0	20	80	130	150	130	90	52	27	15	5	0

SOLUTION: The calculations are performed in a tabular form in Table 6.7. In this

Column 3 = ordinates of 4-h UH lagged by 4 h

Column 4 = ordinates of 4-h UH lagged by 8 h

Column 5 = ordinates of DRH representing 3 cm ER in 12 h

Column 6 = ordinates of 12-h UH = (Column 5)/3

The 12-h unit hydrograph is shown in Fig. 6.15.

TABLE 6.7 CALCULATION OF A 12-h UNIT HYDROGRAPH FROM A 4-h UNIT HYDROGRAPH—*EXAMPLE 6.9*

Time (h)	Ordinates of 4-h UH (m^3/s) A	B Lagged by 4 h	C Lagged by 8 h	DRH of 3 cm in 12-h (m^3/s) (Col.2+3+4)	Ordinate of 12-h UH (m^3/s) (Col. 5)/3
1	2	3	4	5	6
0	0	—	—	0	0
4	20	0	—	20	6.7
8	80	20	0	100	33.3
12	130	80	20	230	76.7
16	150	130	80	360	120.0
20	130	150	130	410	136.7
24	90	130	150	370	123.3
28	52	90	130	272	90.7
32	27	52	90	169	56.3
36	15	27	52	94	31.3
40	5	15	27	47	15.7
44	0	5	15	20	6.7
48		0	5	5	1.7
52			0	0	0

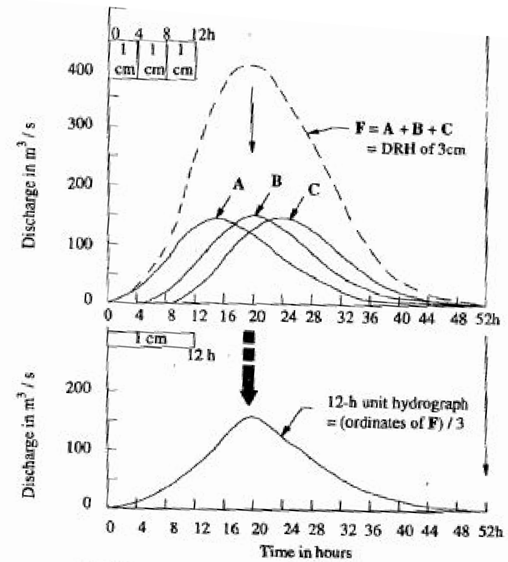


Fig. 6.15 Construction of a 12–h unit hydrograph from a 4–h unit hydrograph

2- The S-Curve

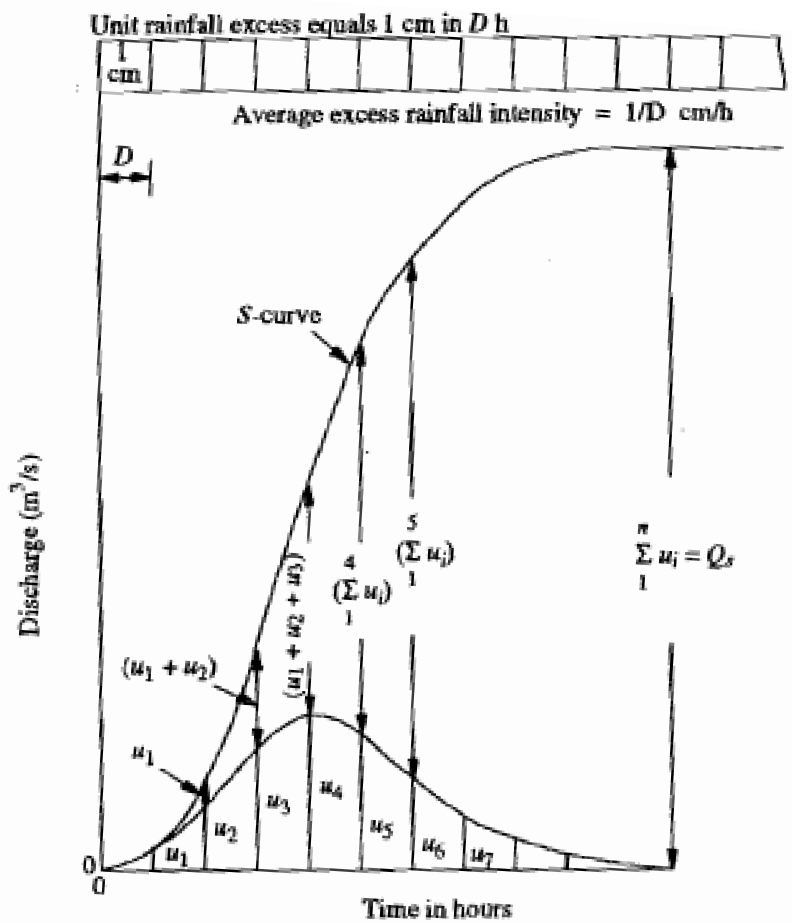


Fig. 6.16(a) S–curve

Solution of Example 6.9 by S-curve method

Time (h)	Ordinate of 4–h UH (m^3/s)	S-curve addition (m^3/s)	S-curve ordinate (m^3/s) (Col.2 + Col. 3)	S-curve lagged by 12 h (m^3/s)	(Col. 4– Col. 5)	Col. 6 / (12/4) = 12–h UH ordinates (m^3/s)
1	2	3	4	5	6	7
0	0	—	0	—	0	0
4	20	0	20	—	20	6.7
8	80	20	100	—	100	33.3
12	130	100	230	0	230	76.7
16	150	230	380	20	360	120.0
20	130	380	510	100	410	136.7
24	90	510	600	230	370	123.3
28	52	600	652	380	272	90.7
32	27	652	679	510	169	56.3
36	15	679	694	600	94	31.3
40	5	694	699	652	47	15.7
44	0	699	699	679	20	6.7
48		699	699	694	5	1.7
52			699	699	0	0