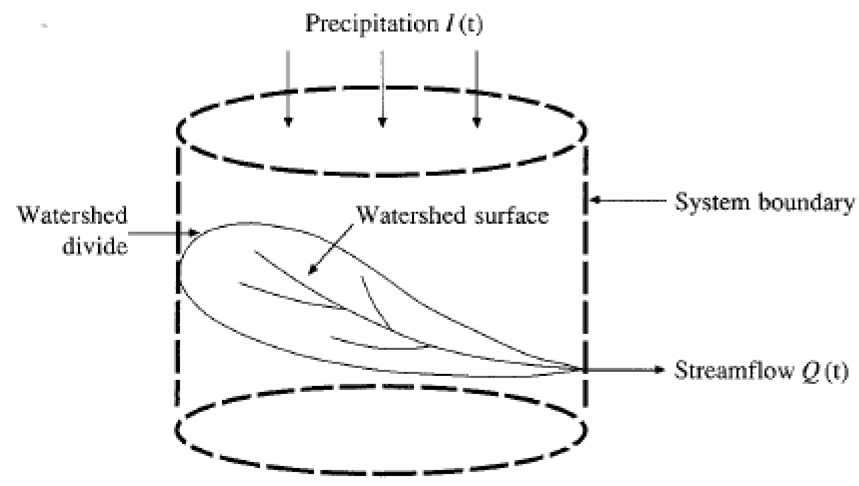
Unit Hydrograph

Hydrologic Analysis



The watershed as a hydrologic system.

$$Q(t) = \Omega I(t)$$

$$\Omega = \frac{Q(t)}{I(t)}$$

Change in storage w.r.t. time = inflow - outflow

$$\frac{dS}{dt} = I(t) - Q(t)$$

In the case of a linear reservoir, $S = kQ$

$$k\frac{dQ}{dt} + Q(t) = I(t)$$

$$\Omega = \frac{Q(t)}{I(t)} = \frac{1}{1 + kD}$$

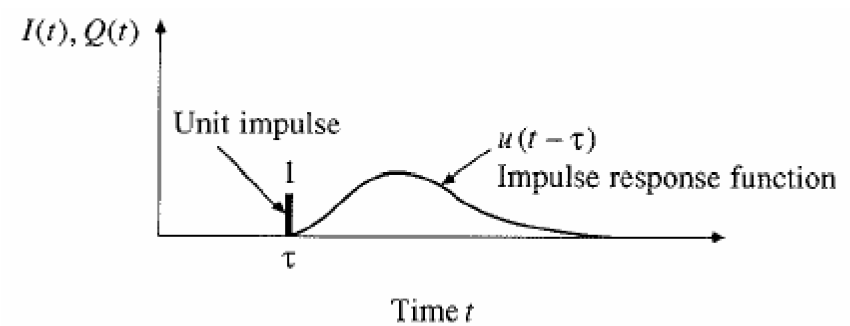
Transfer function for a linear system ($S = kQ$).

Proportionality and superposition

- Linear system (k is constant in $S = kQ$)
  - Proportionality
    - *If* $I_1 \rightarrow Q_1$ then $C^*I_2 \rightarrow C^*Q_2$
  - Superposition
    - *If* $I_1 \rightarrow Q_1$ and $I_2 \rightarrow Q_2$, t*hen* $I_1 + I_2 \rightarrow Q_1 + Q_2$

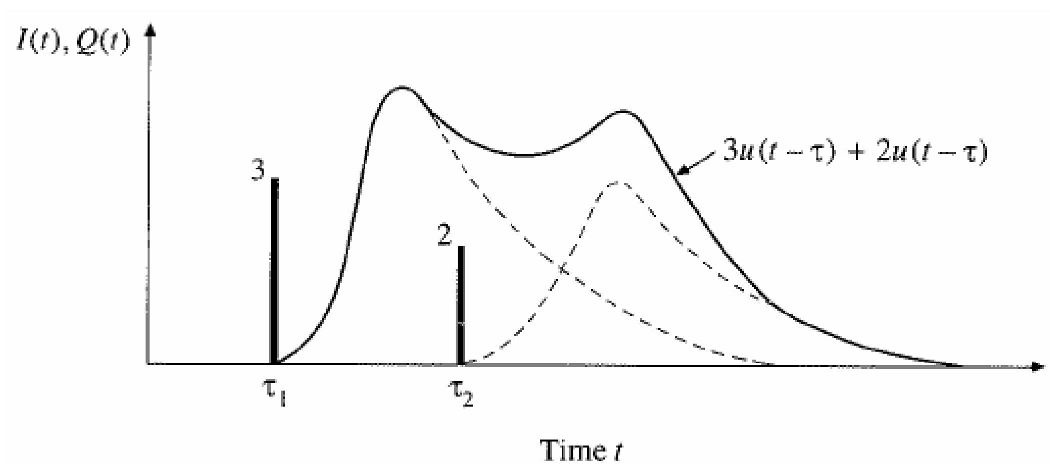
Impulse response function

Impulse input: an input applied instantaneously (spike) at time τ and zero everywhere else



An unit impulse at τ produces as unit impulse response function u(t-τ)

Principle of proportionality and superposition



Convolution integral

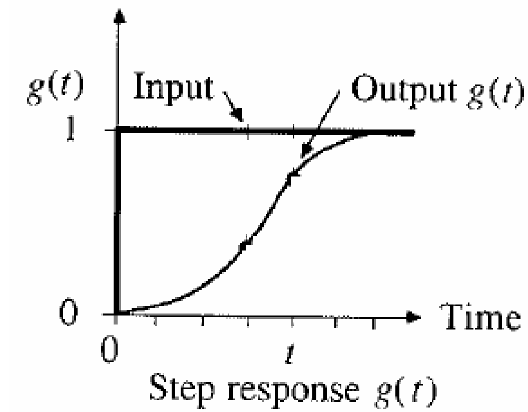
- For an unit impulse, the response of the system is given by the unit impulse response function u(t-τ)
- An impulse of 3 units produces the 3u(t-τ)
- If I(τ) is the precipitation intensity occurring for a time period of dτ, the response of the system (direct runoff) is I(τ)u(t-τ)dτ
- The complete response due to the input function I(τ) is …ution integral

$$Q(t) = \int_0^t I(\tau) u(t-\tau) d\tau$$

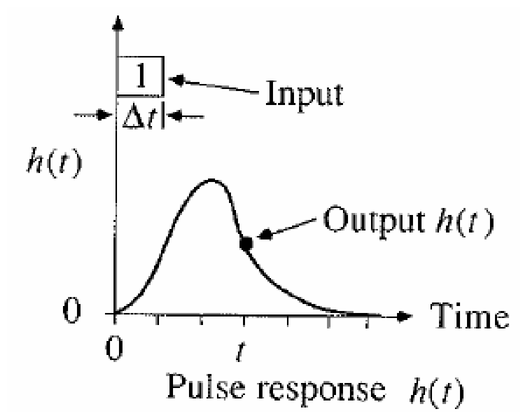
- Response of a linear system is the sum (convolution) of the responses to inputs that have happened in the past.

Step and pulse inputs

- A unit step input is an input that goes from 0 to 1 at time 0 and continues indefinitely thereafter



- A unit pulse is an input of unit amount occurring in duration Δt and 0 elsewhere.



Precipitation is a series of pulse inputs!

Unit Hydrograph Theory

- Direct runoff hydrograph resulting from a unit depth of excess rainfall occurring uniformly on a watershed at a constant rate for a specified duration.
- Unit pulse response function of a linear hydrologic system
- Can be used to derive runoff from any excess rainfall on the watershed.

Unit hydrograph assumptions

- Assumptions
  - Excess rainfall has constant intensity during duration
  - Excess rainfall is uniformly distributed on watershed
  - Base time of runoff is constant
  - Ordinates of unit hydrograph are proportional to total runoff (linearity)
  - Unit hydrograph represents all characteristics of watershed (lumped parameter) and is time invariant (stationarity)

Discrete Convolution

Continuous $$Q(t) = \int_0^t I(\tau)u(t-\tau)d\tau$$

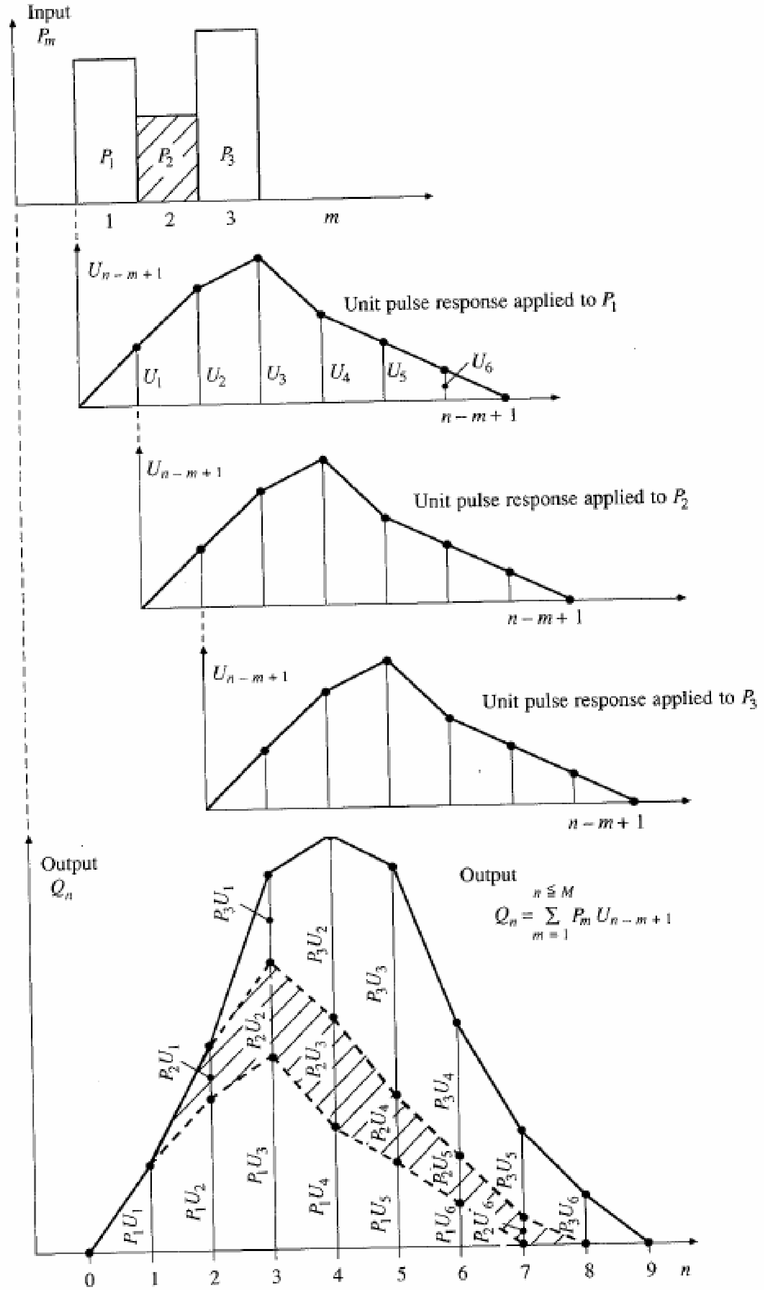
Discrete $$Q_n = \sum_{m=1}^{n\leq M} P_m U_{n-m+1}$$

Q is flow, P is precipitation and U is unit hydrograph

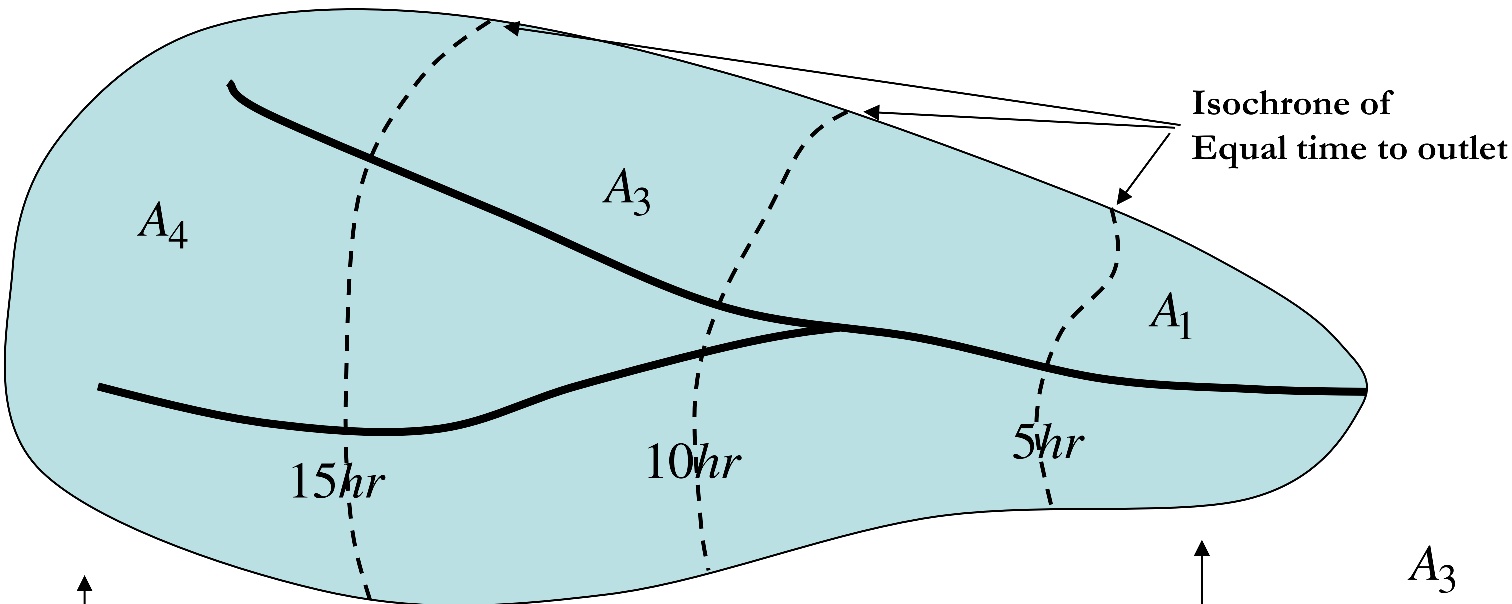
M is the number of precipitation pulses, n is the number of flow rate intervals

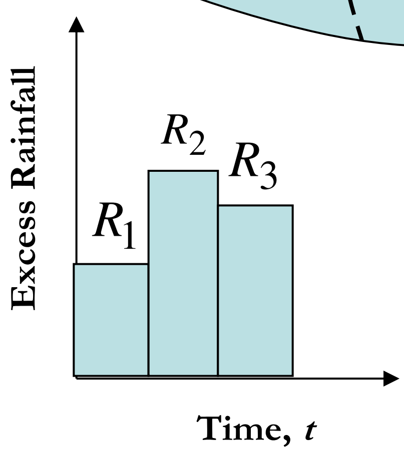
The unit hydrograph has N-M+1 pulses

Application of convolution to the output from a linear system

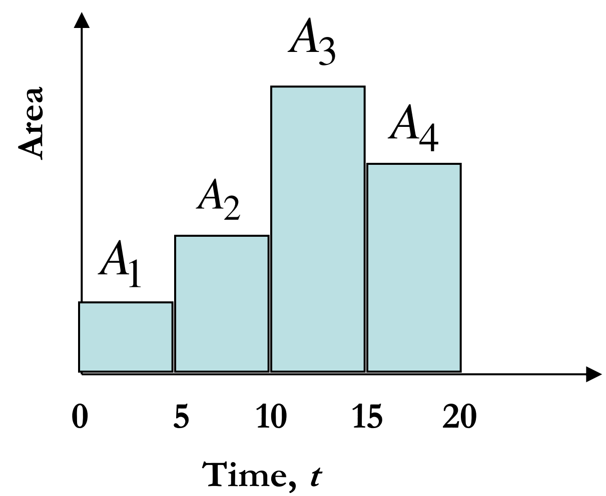


Time – Area Relationship





$$Q_n = R_i A_1 + R_{i-1} A_2 + \ldots + R_1 A_j$$



Application of UH

- Once a UH is derived, it can be used/applied to find direct runoff and stream flow hydrograph from other storm events.

Given: Ex. 7.5.1

P_1 = 2 in, P_2 = 3 in and P_3 = 1 in, baseflow = 500 cfs and watershed area is 7.03 mi^2. Given the Unit Hydrograph below, determine the streamflow hydrograph

Unit hydrograph

n	1	2	3	4	5	6	7	8	9
U_n (cfs/in)	404	1079	2343	2506	1460	453	381	274	173

7.5.1 solution (cont'd)

Time ($\frac{1}{2}$-h)	Excess Precipitation (in)	Unit hydrograph ordinates (cfs/in)									Direct runoff (cfs)	Streamflow (cfs)
		1	2	3	4	5	6	7	8	9		
		404	1079	2343	2506	1460	453	381	274	173		
$n =$ 1	2.00	808									808	1308
2	3.00	1212	2158								3370	3870
3	1.00	404	3237	4686							8327	8827
4			1079	7029	5012						13,120	13,620
5				2343	7518	2920					12,781	13,281
6					2506	4380	906				7792	8292
7						1460	1359	762			3581	4081
8							453	1143	548		2144	2644
9								381	822	346	1549	2049
10									274	519	793	1293
11										173	173	673
										Total	54,438	

*Baseflow = 500 cfs.

See another example at: http://www.egr.msu.edu/~northco2/BE481/UHD.htm

Gauged and ungauged watersheds

- Gauged watersheds
  - Watersheds where data on precipitation, streamflow, and other variables are available
- Ungauged watersheds
  - Watersheds with no data on precipitation, streamflow and other variables.

Need for synthetic UH

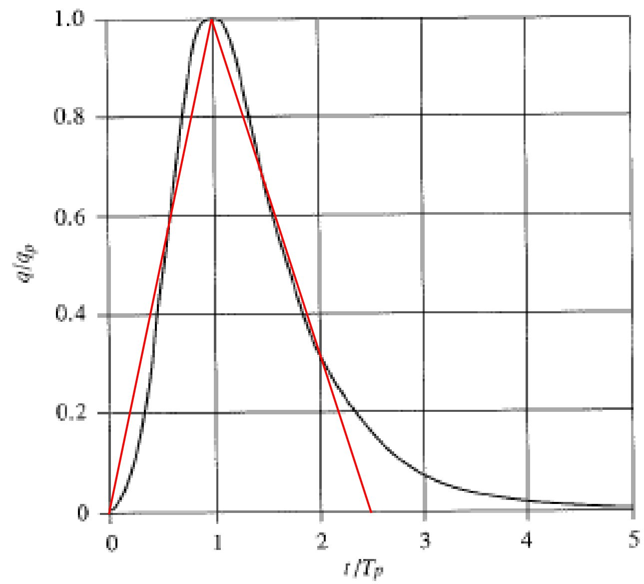
- UH is applicable only for gauged watershed and for the point on the stream where data are measured
- For other locations on the stream in the same watershed or for nearby (ungauged) watersheds, synthetic procedures are used.

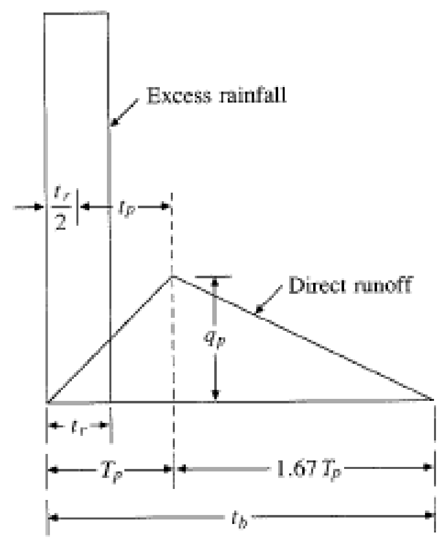
Synthetic UH

- Synthetic hydrographs are derived by
  - Relating hydrograph characteristics such as peak flow, base time etc. with watershed characteristics such as area and time of concentration.
  - Using dimensionless unit hydrograph
  - Based on watershed storage

SCS dimensionless hydrograph

- Synthetic UH in which the discharge is expressed by the ratio of q to q_p and time by the ratio of t to T_p
- If peak discharge and lag time are known, UH can be estimated.





T_c: time of concentration

C = 2.08 (483.4 in English system)

A: drainage area in km^2 (mi^2)

$$t_p \cong 0.6T_c \qquad t_b \cong 2.67T_p$$

$$T_p = \frac{t_r}{2} + t_p \qquad q_p = \frac{CA}{T_p}$$

Ex. 7.7.3

- Construct a 10-min SCS UH. A = 3.0 km^2 and T_c = 1.25 h

$$t_r = 10 \ \text{min} = 0.166 \ h$$

$$t_p = 0.6T_c = 0.6 \times 1.25 = 0.75 \ h$$

$$T_p = \frac{t_r}{2} + t_p$$

$$T_p = \frac{0.166}{2} + 0.75 = 0.833 \ h$$

$$q_p = \frac{CA}{T_p} = \frac{2.08 \times 3}{0.833} = 7.49 \ m^3 / s.cm$$

Multiply y-axis of SCS hydrograph by q_p and x-axis by T_p to get the required UH, or construct a triangular UH