# Ground Water Flow and Well Mechanics

# Steady One-Dimensional Flow

- For ground water flow in the x-direction in a confined aquifer, the governing equation becomes:

  $$d^2h/dx^2 = 0$$

  and has the solution

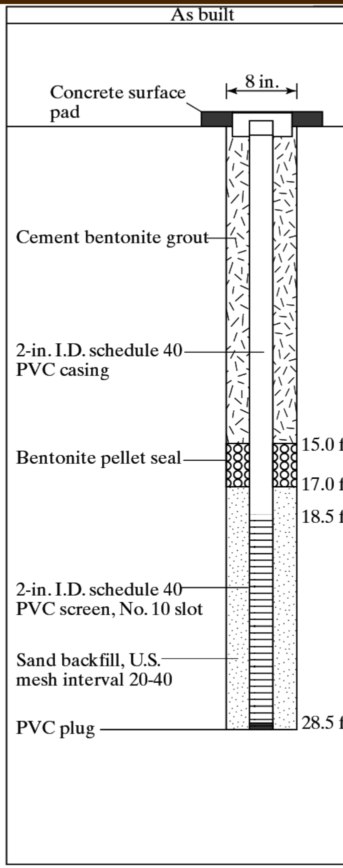
  $$h = -vx/K + h_0$$

  where $h = 0$ and $dh/dx = -v/K$, according to Darcy's law.

  This states that head varies linearly with flow in the x-direction.

# Water Supply Wells

Water supply wells, Floridan Aquifer near Tampa, Florida




Typical well designs for unconsolidated formations.

Piez Level

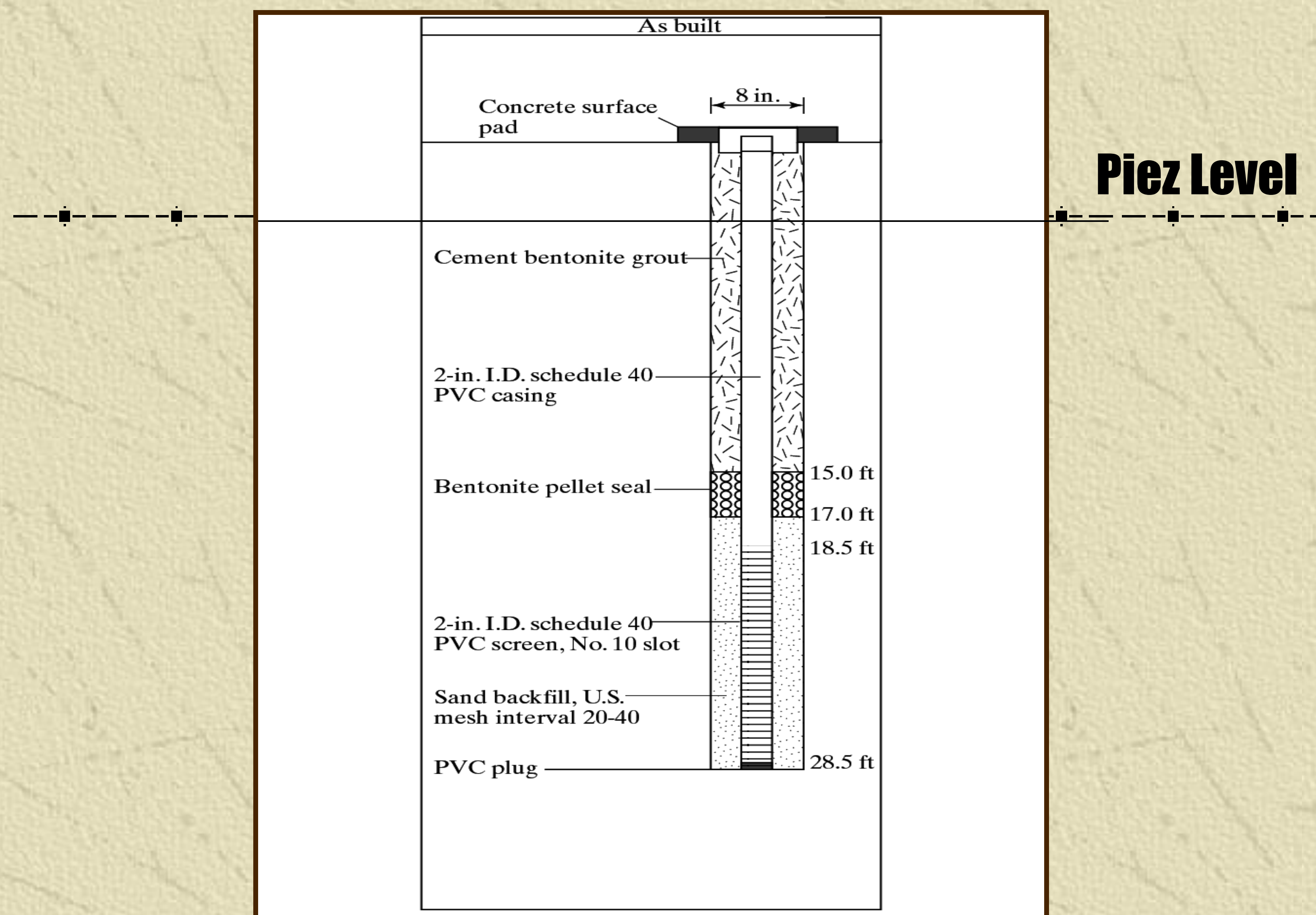



Figure 8.26

Typical well designs for unconsolidated formations.

# Steady Radial Flow to a Well- Confined

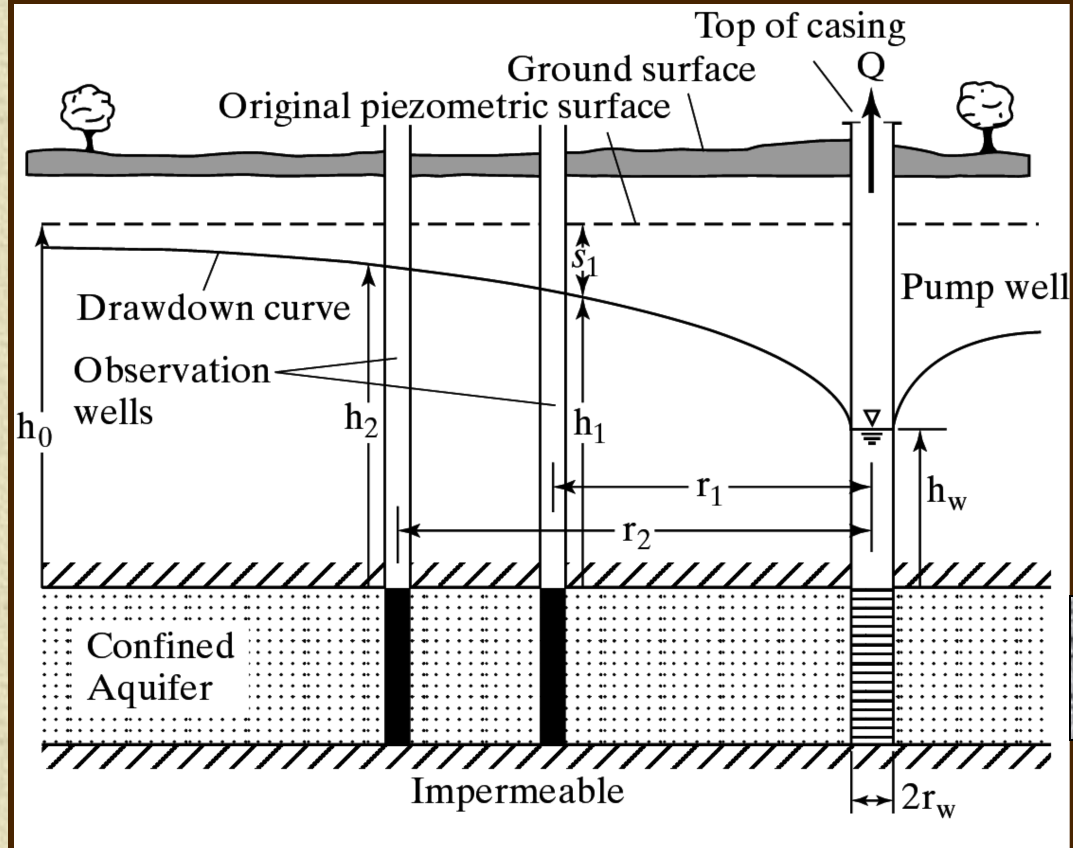
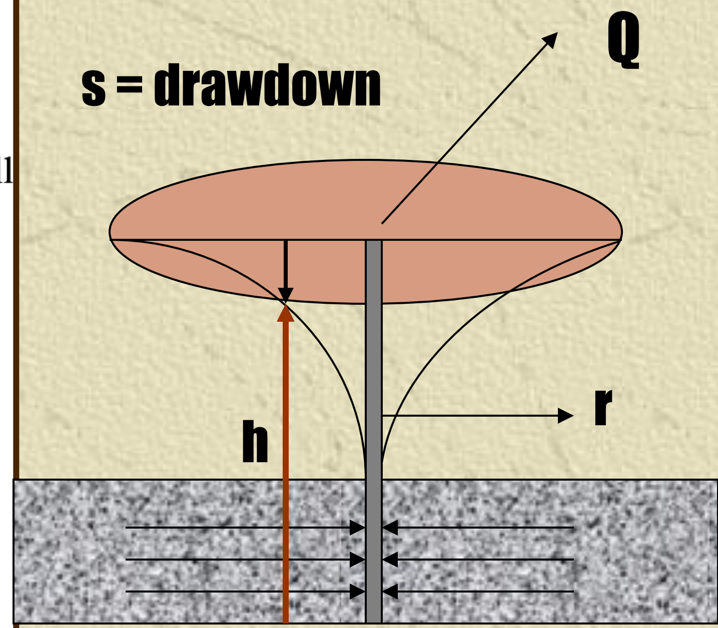



Figure 8.14

Radial flow to a well penetrating an extensive confined aquifer.

## Cone of Depression



# Steady Radial Flow to a Well-Confined

- In a confined aquifer, the drawdown curve or cone of depression varies with distance from a pumping well.
- For horizontal flow, Q at any radius r equals, from Darcy's law,

$$Q = -2\pi rbK\, dh/dr$$

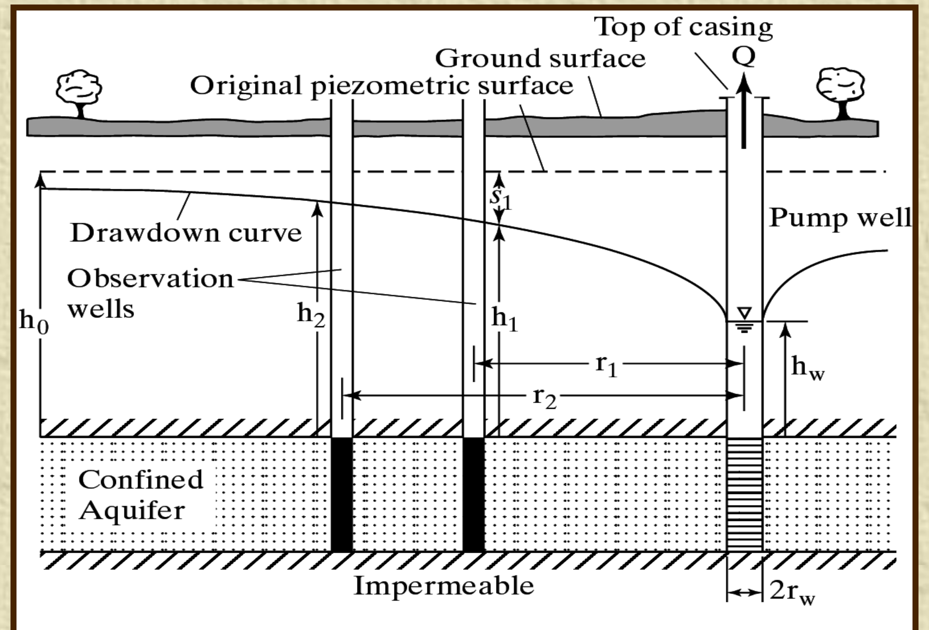
for steady radial flow to

a well where Q,b,K are const

# Steady Radial Flow to a Well-Confined

- Integrating after separation of variables, with $h = h_w$ at $r = r_w$ at the well, yields Thiem Eqn

$$Q = 2\pi Kb[(h-h_w)/(\ln(r/r_w))]$$




e 8.14

Radial flow to a well penetrating an extensive confined aquifer.

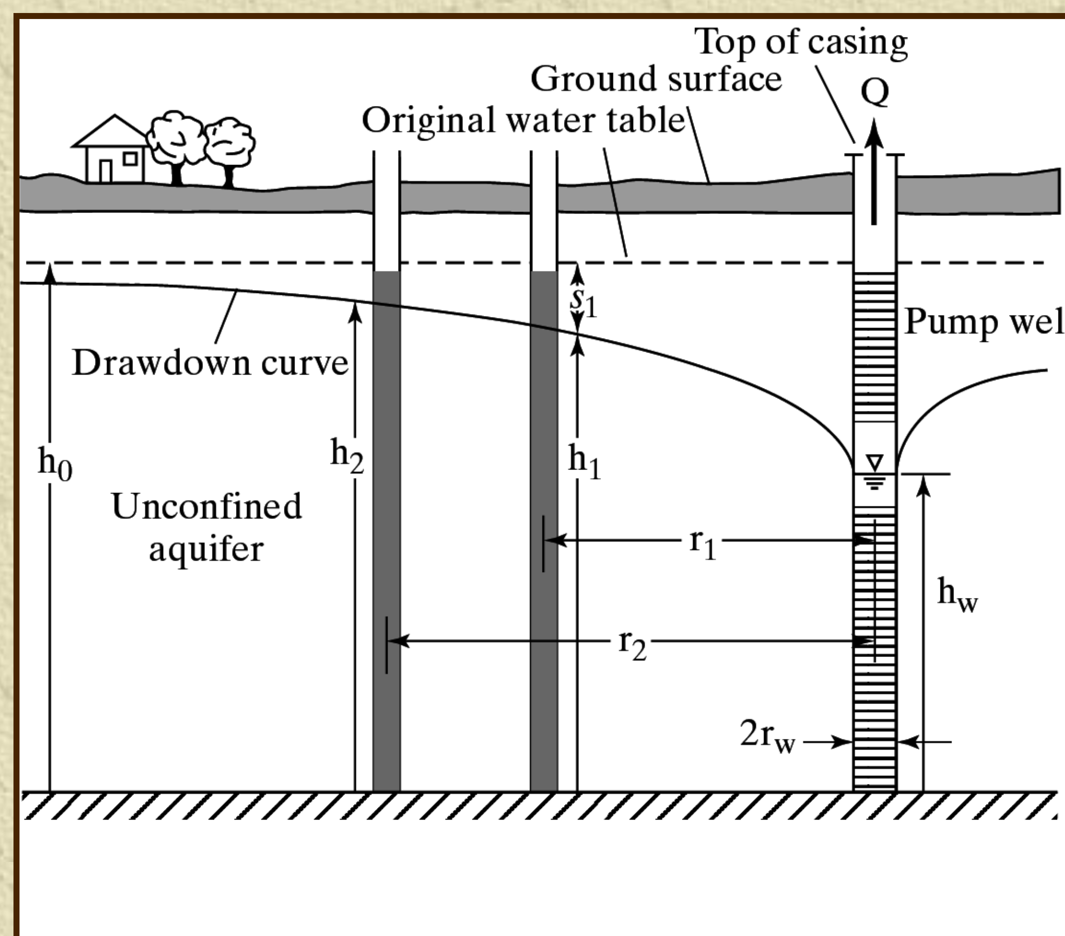
*Note*, *h* increases indefinitely with increasing *r*, yet the maximum head is $h_0$.

# Steady Radial Flow to a Well-Confined

- Near the well, transmissivity, *T*, may be estimated by observing heads $h_1$ and $h_2$ at two adjacent observation wells located at $r_1$ and $r_2$, respectively, from the pumping well

$$T = Kb = \frac{Q \ln(r_2 / r_1)}{2\pi(h_2 - h_1)}$$

# Steady Radial Flow to a Well- Unconfined




Radial flow to a well penetrating an unconfined aquifer.

# Steady Radial Flow to a Well- Unconfined

- Using Dupuit's assumptions and applying Darcy's law for radial flow in an unconfined, homogeneous, isotropic, and horizontal aquifer yields:

$$Q = -2\pi K h \, dh/dr$$

integrating,

$$Q = \pi K[(h_2^2 - h_1^2)/\ln(r_2/r_1)$$

solving for K,

$$K = [Q/\pi(h_2^2 - h_1^2)]\ln(r_2/r_1)$$

where heads $h_1$ and $h_2$ are observed at adjacent wells located distances $r_1$ and $r_2$ from the pumping well respectively.

# Multiple-Well Systems

- For multiple wells with drawdowns that overlap, the principle of superposition may be used for governing flows:

  drawdowns at any point in the area of influence of several pumping wells is equal to the sum of drawdowns from each well in a confined aquifer

# Multiple-Well Systems

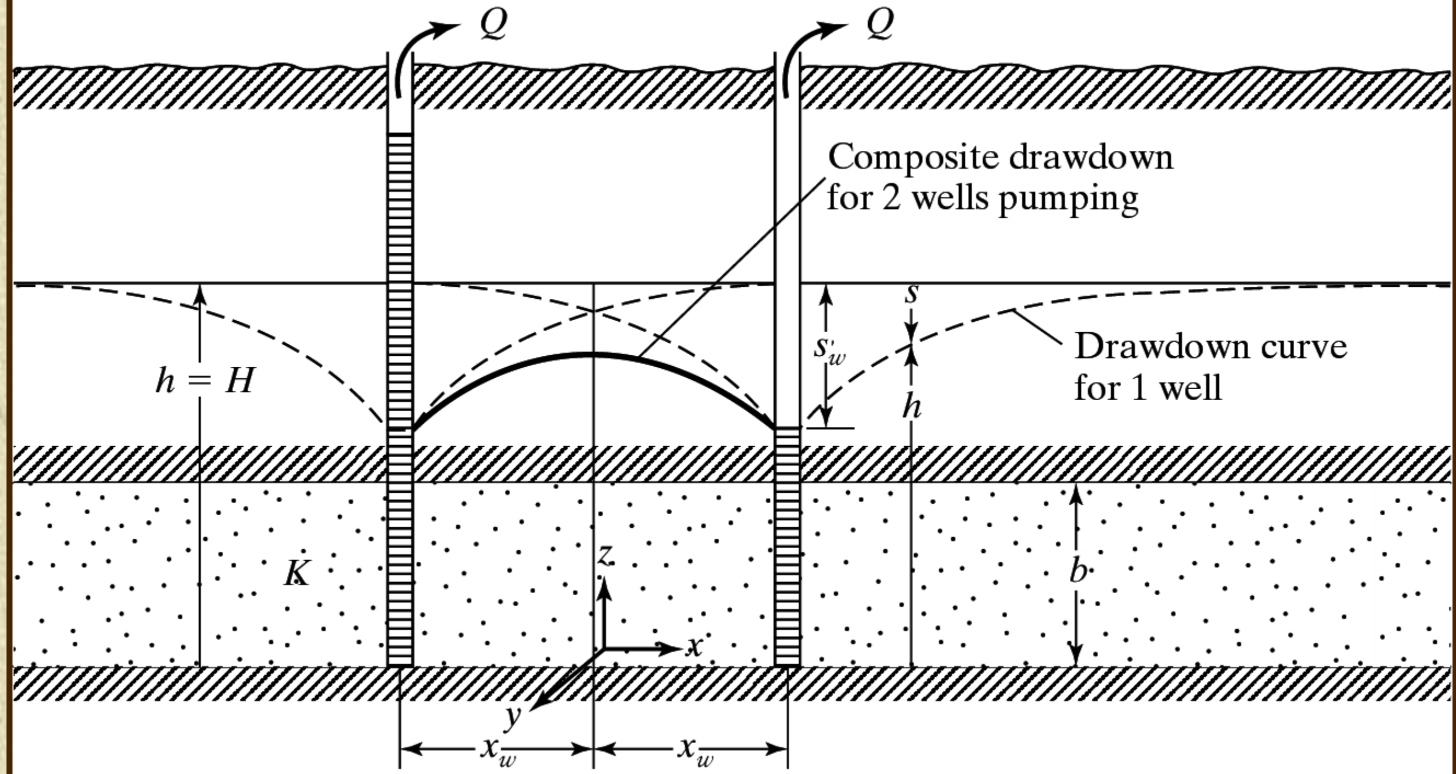



Figure 8.17

Individual and composite drawdown curves for two wells in a line.

# Injection-Pumping Pair of Wells

Pump

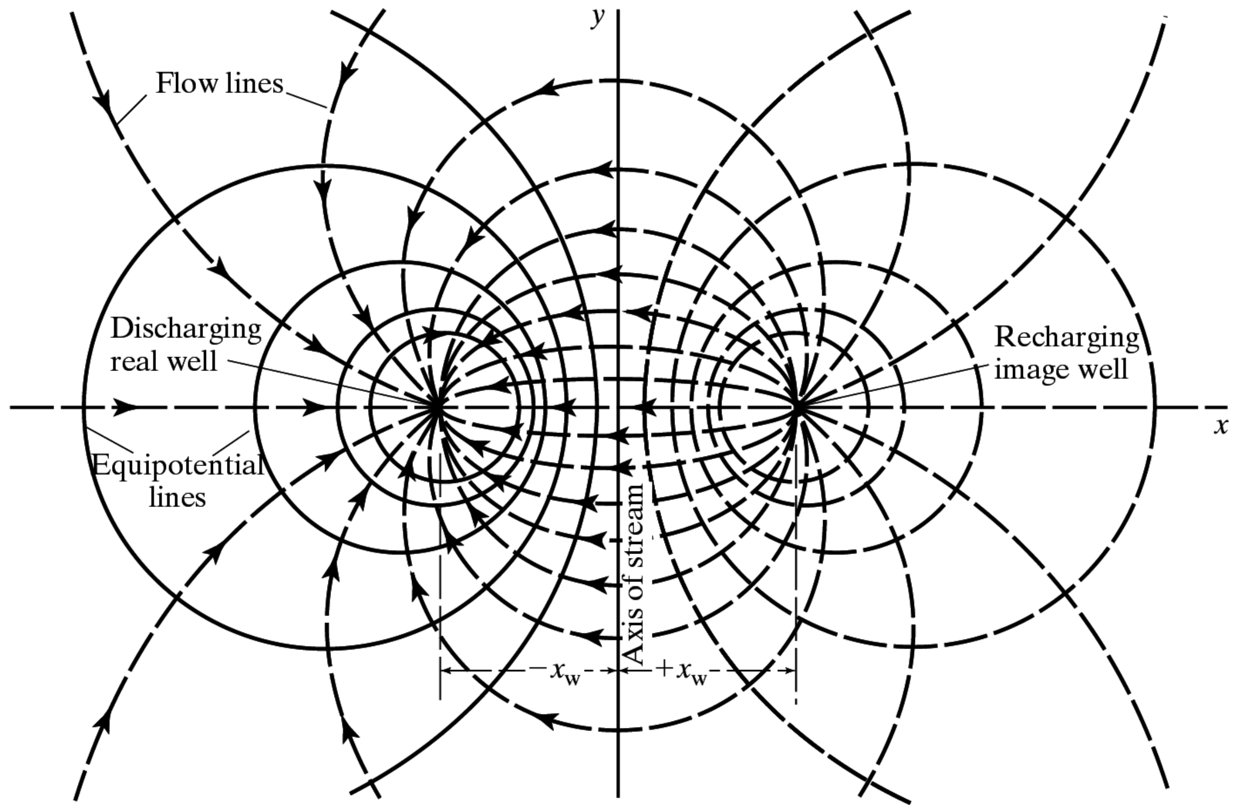



Figure 8.19

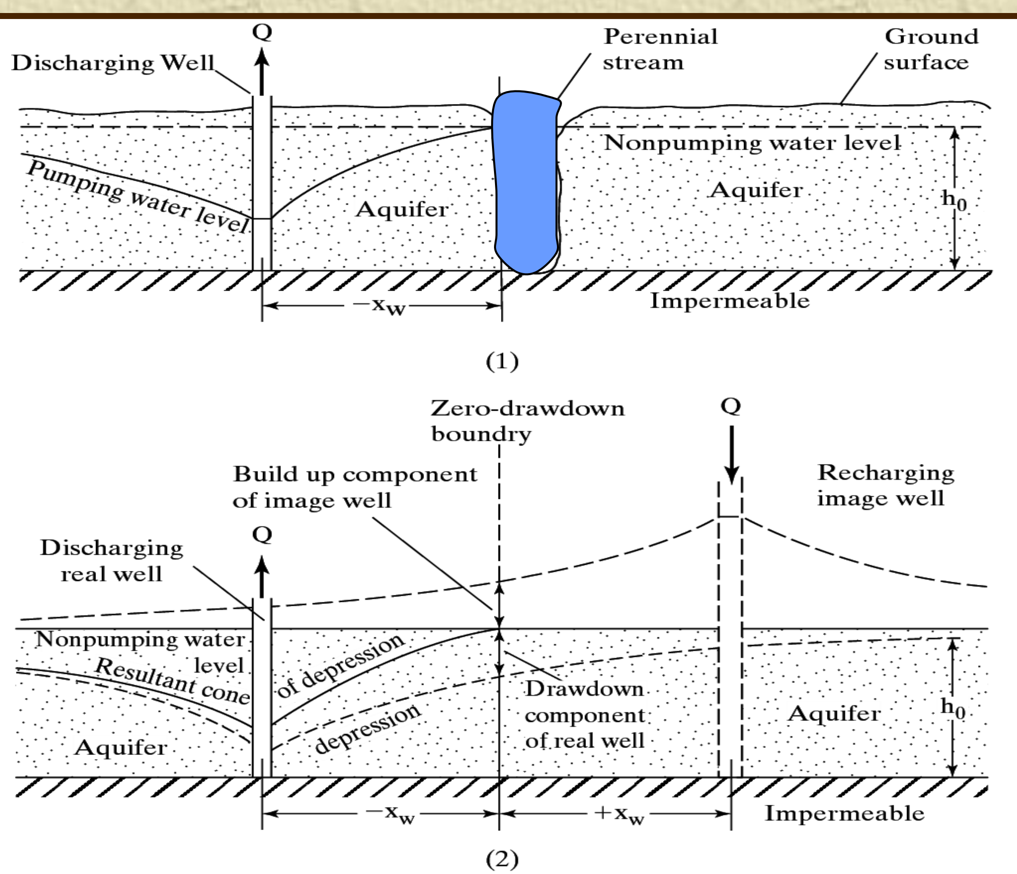
Flow net for a discharging real well and a recharging image well. (After Ferris et al., 1962.)

Inject

# Multiple-Well Systems

The same principle applies for well flow near a boundary

- Example:
  pumping near a fixed head stream

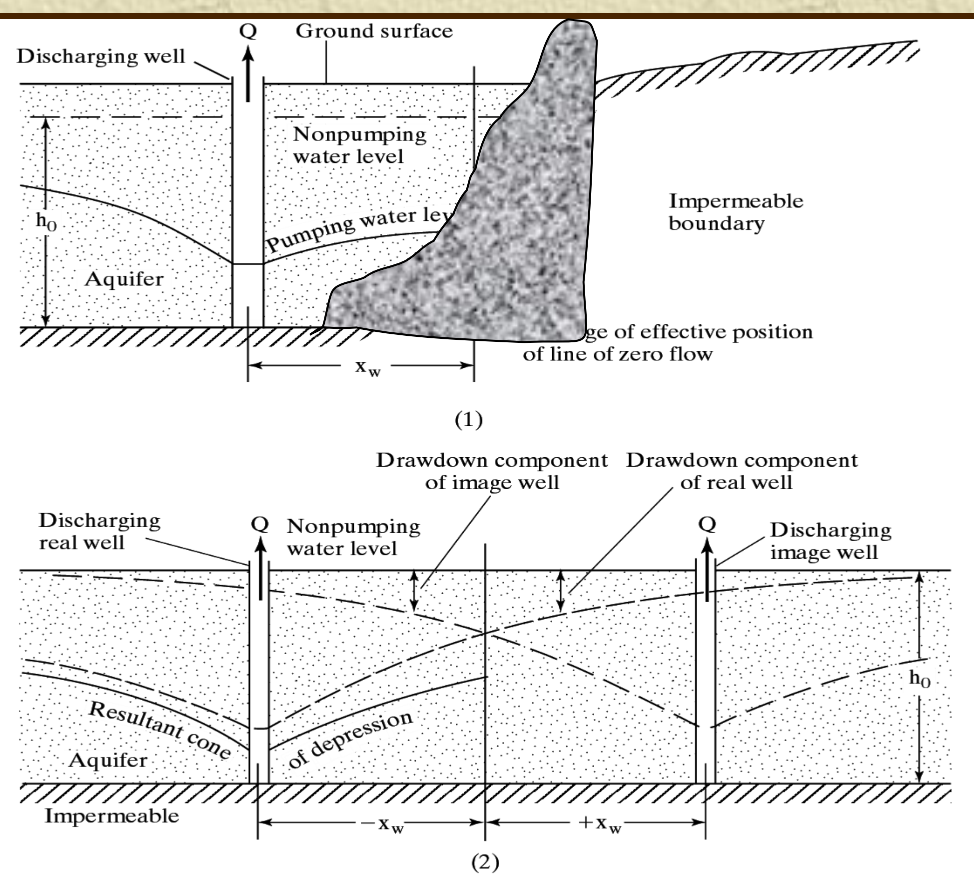



Figure

Sectional views. (1) Discharging well near a perennial stream.
(2) Equivalent hydraulic system in an aquifer of infinite areal extent.

# Multiple-Well Systems

- Another example: well pumping near an impermeable boundary



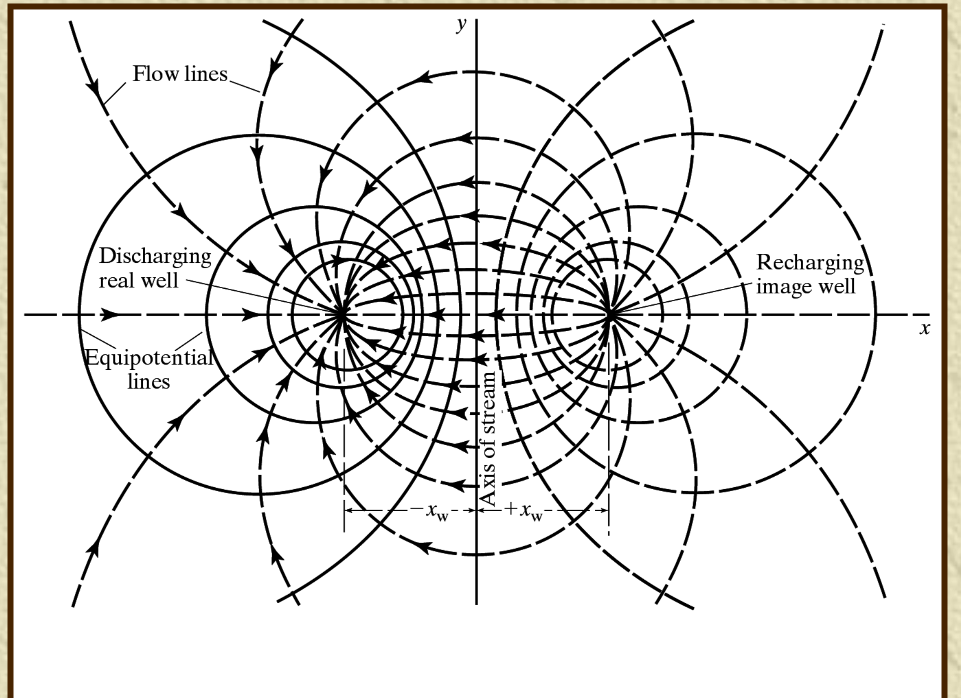

Figu

(1) Discharging well near an impermeable boundary. (2) Equivalent hydraulic system in an aquifer of infinite areal extent. (After Ferris et al., 1962.)

# Multiple-Well Systems

- The previously mentioned principles also apply for well flow near a boundary
- Image wells placed on the other side of the boundary at a distance $x_w$ can be used to represent the equivalent hydraulic condition
  - The use of image wells allows an aquifer of finite extent to be transformed into an infinite aquifer so that closed-form solution methods can be applied

# Multiple-Well Systems

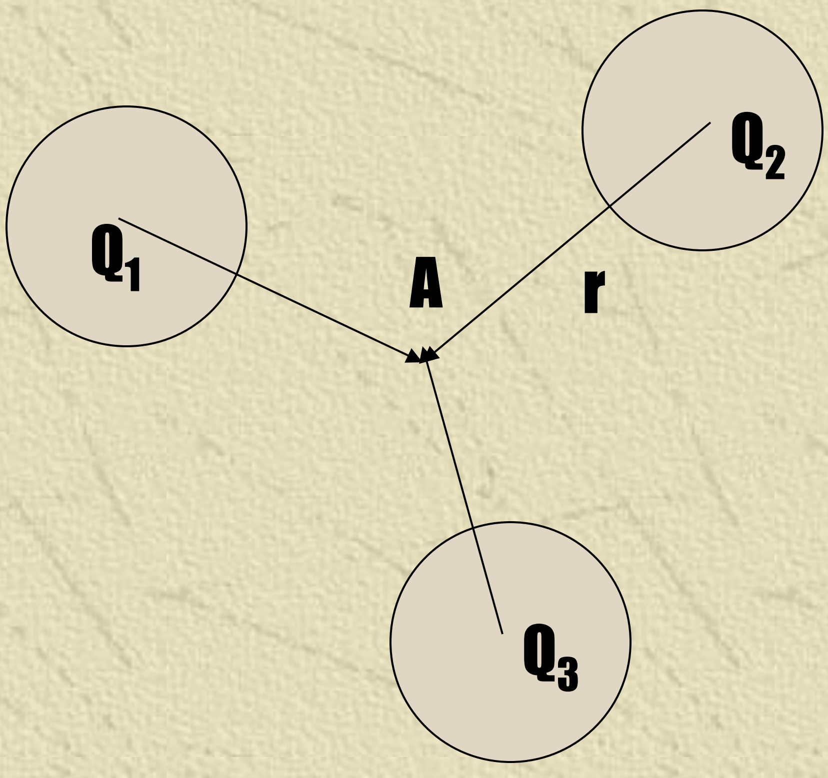



Flow net for a discharging real well and a recharging image well. (After Ferris et al., 1962.)

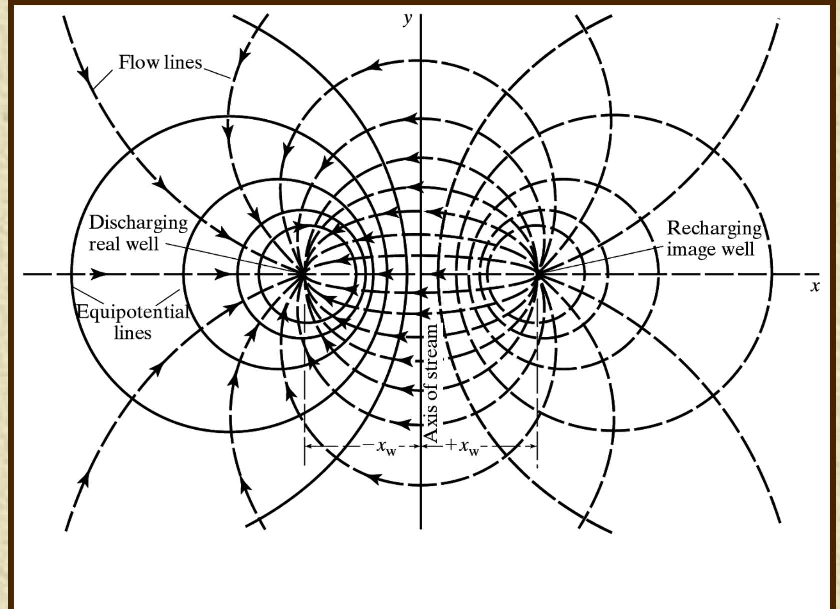
- **A flow net for a pumping well and a recharging image well**
  - **-indicates a line of constant head between the two wells**

# Three-Wells Pumping

Total Drawdown at A is sum of drawdowns from each well



# Multiple-Well Systems




Flow net for a discharging real well and ... ...age well. (After Ferris et al., 1962.)

**The steady-state drawdown s' at any point *(x,y)* is given by:**

$$s' = (Q/4\pi T)\ln \frac{(x + x_w)^2 + (y - y_w)^2}{(x - x_w)^2 + (y - y_w)^2}$$

where $(\pm x_w, y_w)$ are the locations of the recharge and discharge wells. For this case, $y_w = 0$.

# Multiple-Well Systems

The steady-state drawdown s' at any point *(x,y)* is given by

$$s' = (Q/4\pi T)[\ \ln \{(x + x_w)^2 + y^2\} - \ln \{(x - x_w)^2 + y^2\}\ ]$$

where the positive term is for the pumping well and the negative term is for the injection well. In terms of head,

$$h = (Q/4\pi T)[\ \ln \{(x - x_w)^2 + y^2\} - \ln \{(x + x_w)^2 + y^2\}] + H$$

Where H is the background head value before pumping.

Note how the signs reverse since $s' = H - h$

# Unsteady Well Hydraulics
# The Theis Equation

- The governing ground water flow equation for *h* in plane polar coordinates is:

$$\partial^2 h/\partial r^2 + (1/r)(\partial h/\partial r) = (S/T)(\partial h/\partial t)$$

where:

$r$ = radial distance from well

$S$ = storage coefficient, and

$T$ = transmissivity

*RHS* = transient term of storage

# The Theis Equation

- Theis obtained a solution to the governing equation by assuming that the well (pumping Q) is a sink of constant strength and by using boundary conditions:

$$h = h_0 \text{ for } t = 0 \text{ and,}$$

$$h \rightarrow h_0 \text{ as } r \rightarrow \infty \text{ for } t \geq 0$$

$$s' = (Q/4\pi T)\int_u^\infty e^{-u}/u\, du$$

$$s' = (Q/4\pi T)W(u) \qquad \textit{Theis Eqn.}$$

where $s'$ = drawdown

$Q$ = discharge at the well,

$u = r^2S/4Tt$

$W(u)$ = well function

# The Theis Equation

- The integral in the Theis equation is written as *W(u)* and is known as the exponential integral, or well function, which can be expanded as infinite series:

$$W(u) = -0.5772 - \ln(u) + u - u^2/2\cdot 2! + u^3/3\cdot 3! - u^4/4\cdot 4! + \ldots$$

The Theis equation can be used to obtain aquifer constants *S* and *T* by means of pumping tests at fully penetrating wells.

# The Theis Assumptions

1. Aquifer is homogeneous, isotropic, uniformly thick, and of infinite extent
2. Piezometric surface is horizontal initially
3. Fully penetrating well with $Q = C$
4. Flow horizontal within aquifer
5. Neglect storage within well
6. Water removed from storage responds instantaneously with declining head

# Theis Method - Graphical Soln

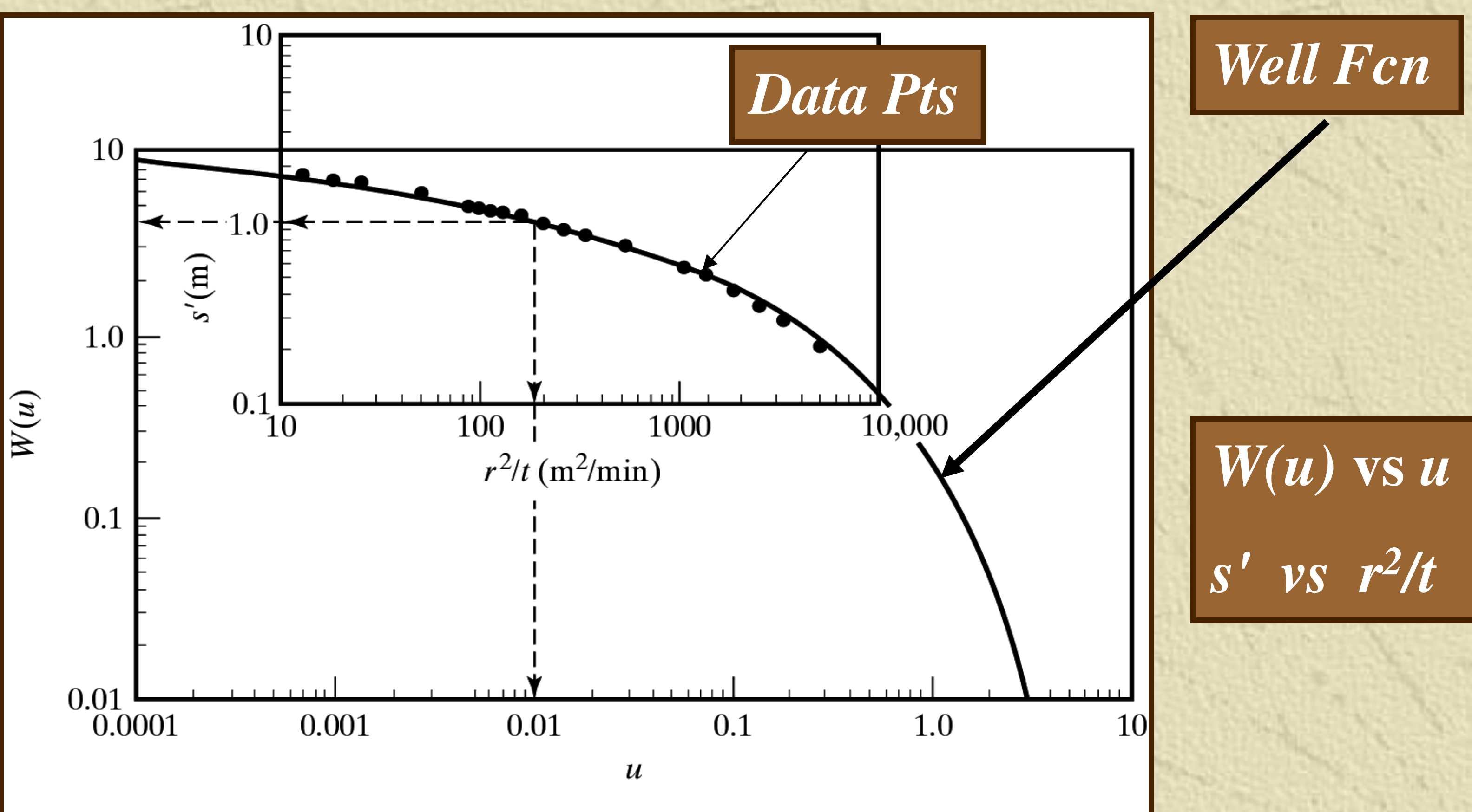



Figure 8.20

Theis method of superposition for solution of the nonequilibrium equation.

$W(u)$ vs $u$

$s'$ vs $r^2/t$

# The Theis Method

***W(u)* vs *u***

***s′* vs *r²/t***

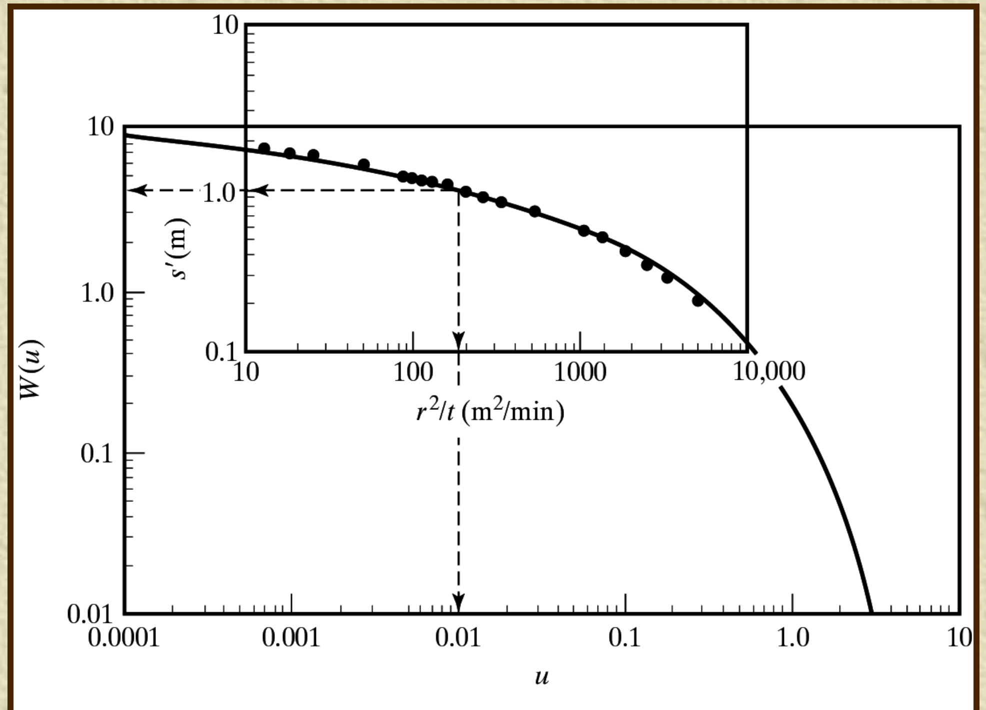



Figure 8.20

Theis method of superposition for solution of the nonequilibrium equation.

$$s' = (Q/4\pi T)W(u)$$

$$r^2/t = (4T/S)u$$

The relationship between *W(u)* and *u* must be the same as that between *s′* and $r^2/t$ because all other terms are constants.

- therefore, plotting:

*W(u)* vs. *u*, and

$s'$ vs. $r^2/t$

# The Theis Method

***s' = (Q/4πT)W(u)***

$r^2/t = (4T/S)u$

For a known S and T, one can use Theis to compute ***s'*** directly at a given ***r*** from the well as a function of time:

***First compute*** $u = r^2S / (4T\,t)$

***Then W(u) from Table 3.2***

***Finally s' = (Q/4πT)W(u)***

# Cooper-Jacob Method of Solution

Cooper and Jacob noted that for small values of ***r*** and large values of ***t***, the parameter $u = r^2S/4Tt$ becomes very small so that the infinite series can be approx. by: $W(u) = -0.5772 - \ln(u)$ ***(neglect higher terms)***

$$\text{Thus} \quad s' = (Q/4\pi T)\,[-0.5772 - \ln(r^2S/4Tt)]$$

Further rearrangement and conversion to decimal logs yields:

$$s' = (2.3Q/4\pi T)\,\log[(2.25Tt)/(r^2S)]$$

# Cooper-Jacob Method of Solution

A plot of drawdown *s′* vs. log of *t* forms a straight line as seen in adjacent figure.

A projection of the line back to *s′* = *0*, where *t* = $t_0$ yields the following relation:

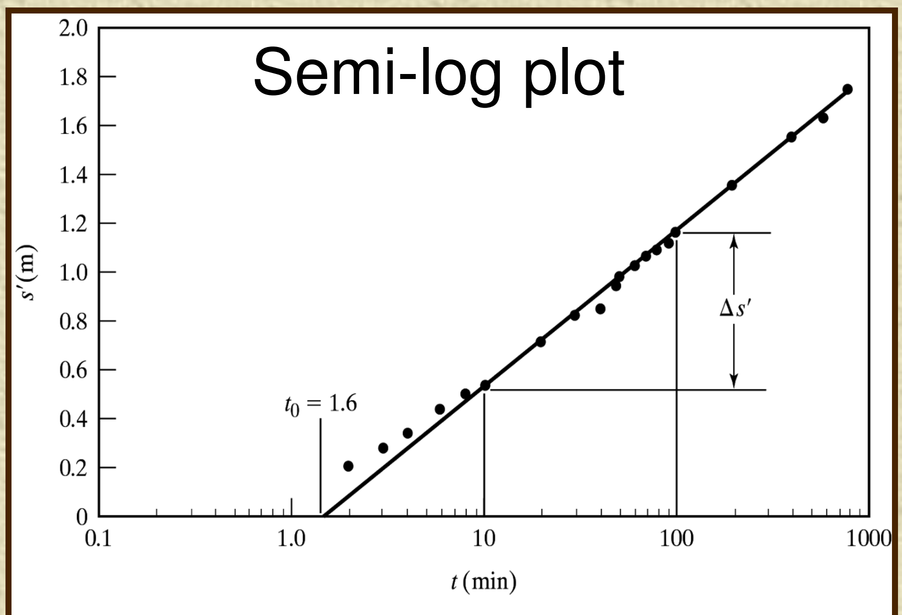



Figure 8.21

Cooper-Jacob method of analysis.

$$0 = (2.3Q/4\pi T) \log[(2.25Tt_0)/(r^2S)]$$

# Cooper-Jacob Method of Solution

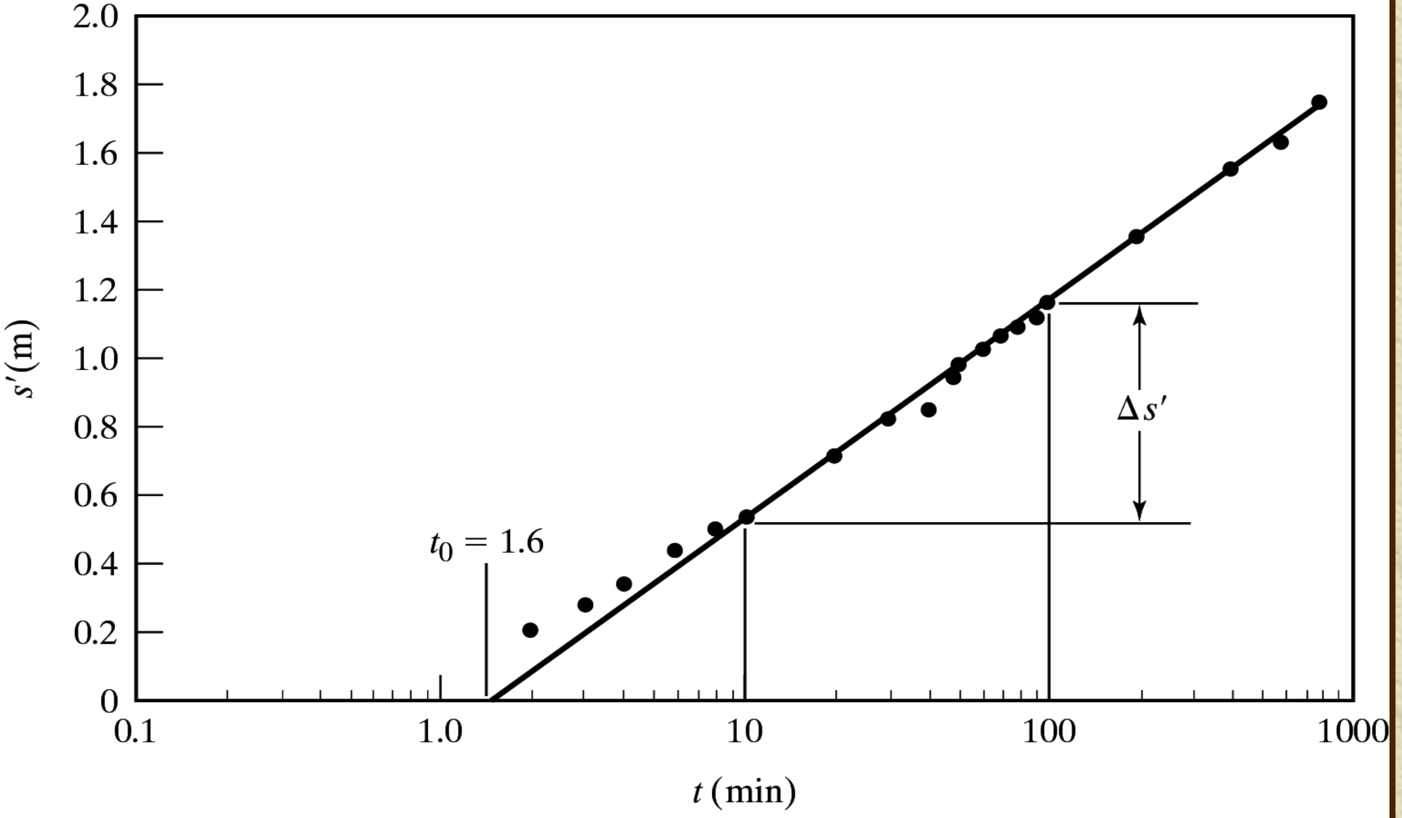



Figure 8.21

Cooper-Jacob method of analysis.

# Cooper-Jacob Method of Solution

So, since *log(1) = 0,* rearrangement yields

$$S = 2.25Tt_0/r^2$$

Replacing $s'$ by $\Delta s'$, where $\Delta s'$ is the drawdown difference per unit log cycle of $t$:

$$T = 2.3Q/4\pi\Delta s'$$

The Cooper-Jacob method first solves for $T$ and then for $S$ and is only applicable for small values of

$$u < 0.01$$

# Cooper-Jacob Example

For the data given in the Fig.

$t_0 = 1.6$ min and $\Delta s' = 0.65\ m$

$Q = 0.2\ m^3/sec$ and $r = 100\ m$

Thus:

$T = 2.3Q/4\pi\Delta s' = 5.63 \times 10^{-2}\ m^2/sec$

$T = 4864\ m^2/sec$

Finally, $S = 2.25Tt_0/r^2$

and $S = 1.22 \times 10^{-3}$

Indicating a confined aquifer

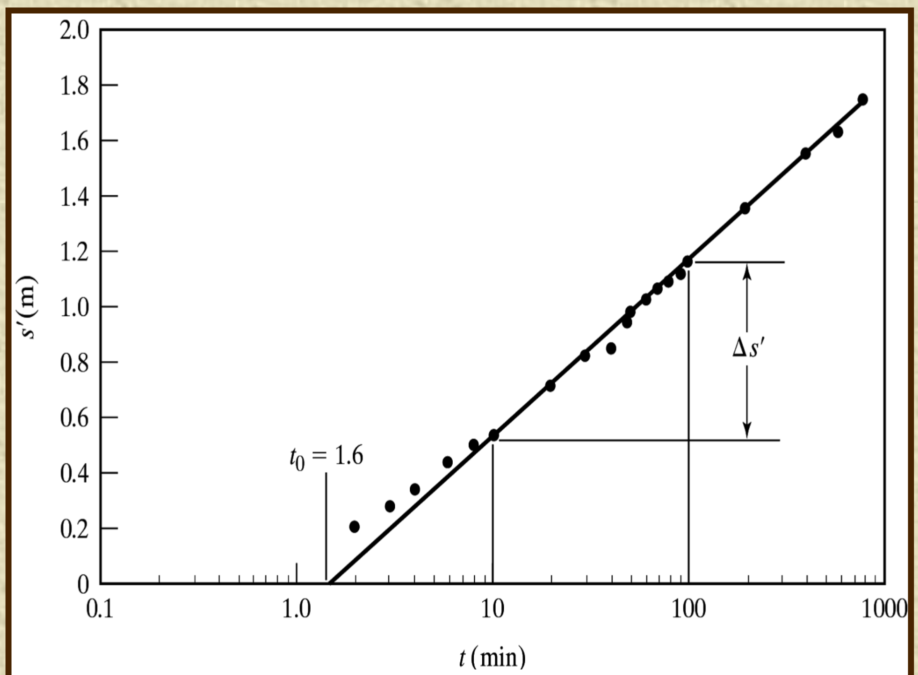



Figure 8.21

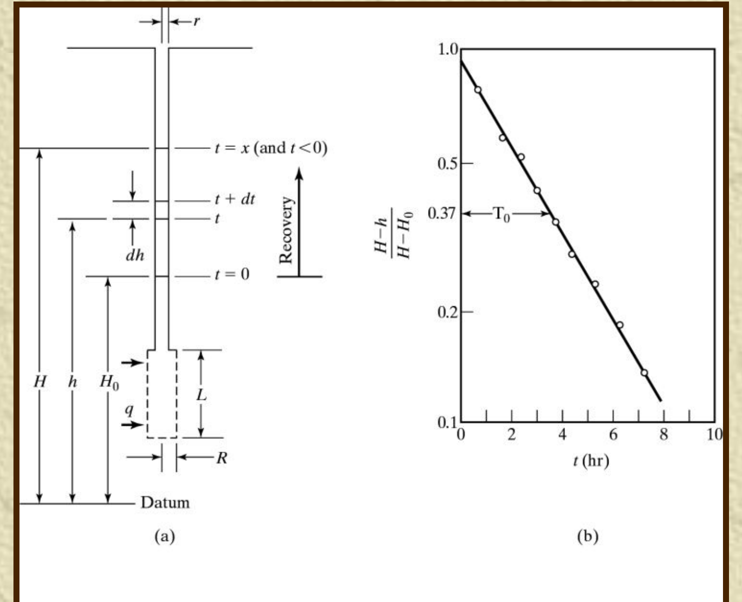
Cooper-Jacob method of analysis.

# Slug Tests

- Slug tests use a single well for the determination of aquifer formation constants
- Rather than pumping the well for a period of time, a volume of water is suddenly removed or added to the well casing and observations of recovery or drawdown are noted through time
- Slug tests are often preferred at hazardous waste sites, since no contaminated water has to be pumped out and then disposed.

# Hvorslev Piezometer Test

- Hvorslev used the recovery of water level over time to calculate hydraulic conductivity of the porous media
  - This method relates the flow *q(t)* at the piezometer at any time to the hydraulic conductivity and the unrecovered head distance, $H_0 - h$ by:



Hvorslev piezometer test. (a) Geometry. (b) Method of analysis.

$$q(t) = \pi r^2 \, dh/dt = FK(H_0 - h) \quad (1)$$

# Hvorslev Piezometer Test

- In the equation, $q(t) = \pi r^2\, dh/dt = FK(H_0 - h)$ **(1)**

  *F* is a factor that depends on the shape and dimensions of the piezometer intake

  - if $q = q_0$ at $t = 0$, then $q(t)$ will decrease toward zero as time increases

  - Hvorslev defined the basic time lag as:

  $$T_0 = \pi r^2 / FK$$

  and solved equation (1) with initial conditions $h = H_0$ at $t = 0$

  Thus,

  $$(H - h)/(H - H_0) = e^{-t/T_0}$$

# Hvorslev Piezometer Test

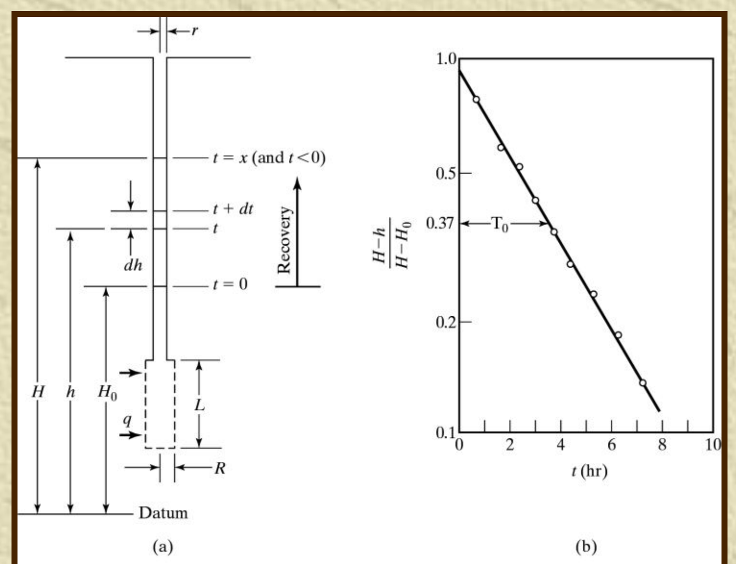



Figure 8.22
Hvorslev piezometer tes ) Method of analysis.

- By plotting recovery $(H-h)/(H-H_0)$ vs. time on semi-log graph paper, we find that $t = T_0$ where recovery equals 0.37

- For piezometer intake length divided by radius *(L/R)* greater than 8, Hvorslev has evaluated the shape factor *F* and obtained an equation for *K*.

$$K = r^2 \ln(L/R) / 2LT_0$$

# Other Slug Test Methods

- Other slug test methods have been developed by Cooper et al. (1967) and Papadopoulos et al. (1973) for confined aquifers that are similar to Theis's in that a curve-matching procedure is used to obtain S and T values for a given aquifer.
- However, the most common method for determining hydraulic conductivity is the Bouwer and Rice (1976) slug test.  This method may be used for unconfined aquifers and confined or stratified aquifers as long as the top of the screen is some distance below the upper confining layer.

# Bouwer and Rice Slug Test

- The Bouwer and Rice method is based on the following equation:

$$K = [r_c^2 \ln(R_e/r_w)] / (2L_e)(1/t)\ln(y_0/y_t)$$

**where:**

**$r_c$ = radius of casing**

**$y_0$ = vertical difference between water level inside well and water level outside at $t = 0$**

**$y_t$ = vertical difference between water level inside well and water table outside (drawdown) at time t**

**$R_e$ = effective radial distance over which y is dissipated, and varying with well geometry**

**$r_w$ = radial distance of undisturbed portion of aquifer from centerline (usually thickness of gravel pack)**

**$L_e$ = length of screened, perforated, or otherwise open section of well, and**

**$t$ = time**

# An Example

A screened, cased well penetrates a confined aquifer. The casing radius is 5 cm and the screen is 1 m long. A gravel pack 2.5 cm wide surrounds the well and a slug of water is injected that raises the water level by 0.28 m. The change in water level with time is as listed in the following table. Given that $R_e$ is 10 cm, calculate $K$ for the aquifer.

| t (sec) | $y_t$(m) |
|---|---|
| 1 | 0.24 |
| 2 | 0.19 |
| 3 | 0.16 |
| 4 | 0.13 |
| 6 | 0.07 |
| 9 | 0.03 |
| 13 | 0.013 |
| 19 | 0.005 |
| 20 | 0.002 |
| 40 | 0.001 |

# The Solution

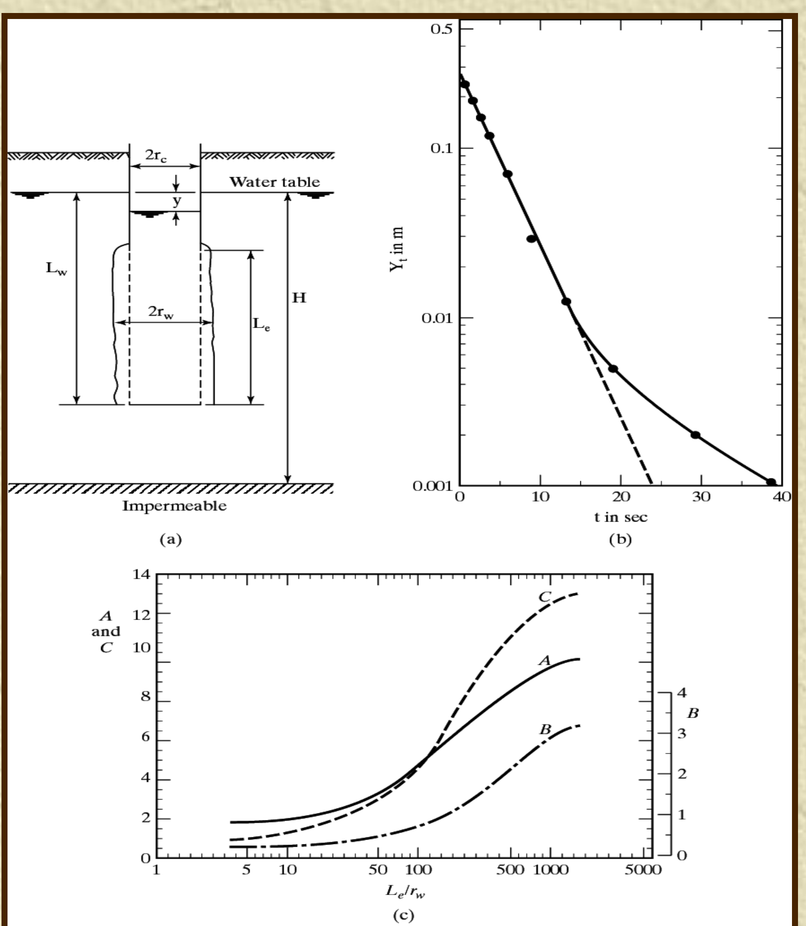



Illustration of the Bouwer and Rice slug test.

Data for *y* vs. *t* are plotted on semi-log paper as shown. The straight line from $y_0 = 0.28\ m$ to $y_t = 0.001$ m covers *2.4 log* cycles. The time increment between the two points is 24 seconds. To convert the log cycles to natural log, a factor of 2.3 is used. Thus, $1/t \ ln(y_0/y_t) = 2.3 \times 2.4/2.4 = 0.23$.

# The Solution

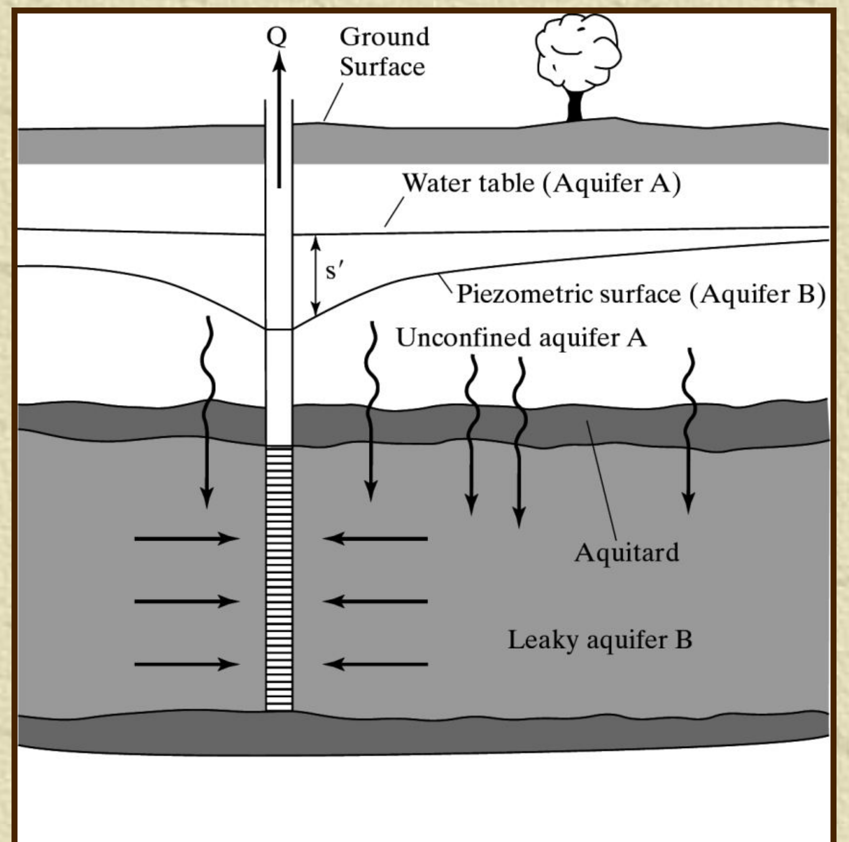
Using this value (0.23) in the Bouwer and Rice equation gives:

$K = [(5\ cm)^2 \ln(10\ cm/7.5\ cm)/(2 \times 100\ cm)](0.23\ sec^{-1})$

and,

$K = 8.27 \times 10^{-3}\ cm/s$

# Radial Flow in a Leaky Aquifer



Well pumping from a leaky aquifer.

- Leaky aquifers are complex because when they are pumped, water is withdrawn from both the lower aquifer and from the saturated portion of the overlying aquitard.
- By creating a lowered piezometric surface below the water table, ground water can migrate vertically downward and than move horizontally to the well

# Radial Flow in a Leaky Aquifer

- When pumping starts from a well in a leaky aquifer, drawdown of the piezometric surface can be given by:

$$s' = (Q/4\pi T)W(u,r/B)$$

where the quantity *r/B* is given by:

$$r/B = r/\sqrt{T/(K'/b')}$$

where:

*T* is transmissivity of the aquifer

*K'* is vertical hydraulic conductivity

*b'* is the thickness of the aquitard

# Radial Flow in a Leaky Aquifer

- Values of the function *W(u, r/B)* have been manipulated to create a family of type curves

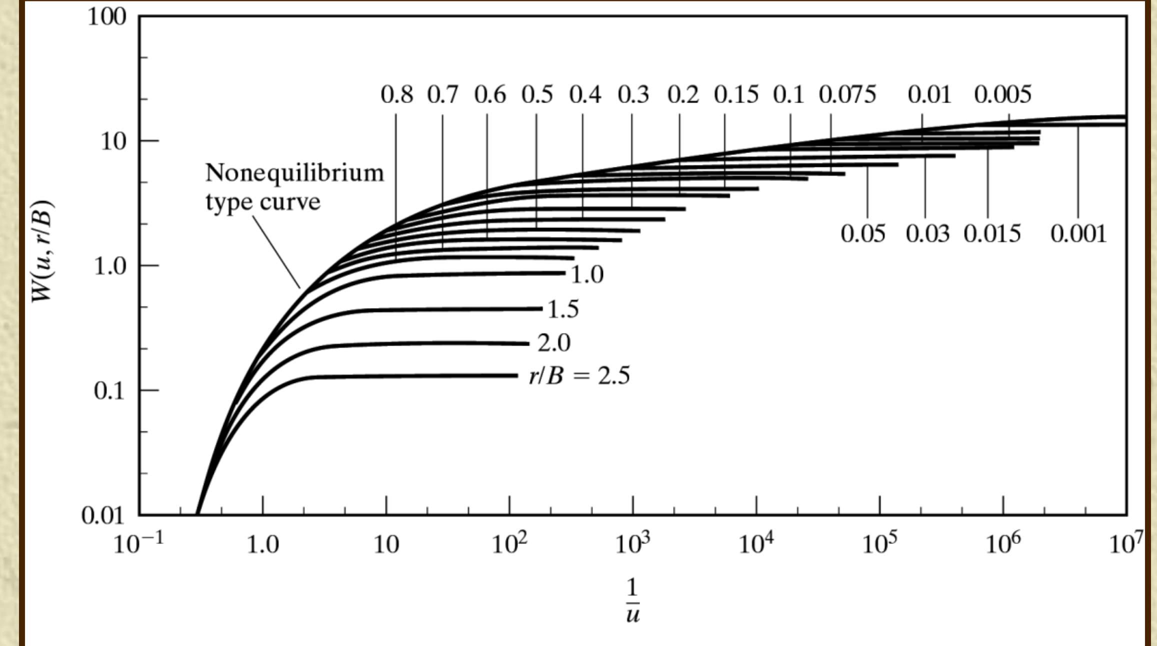


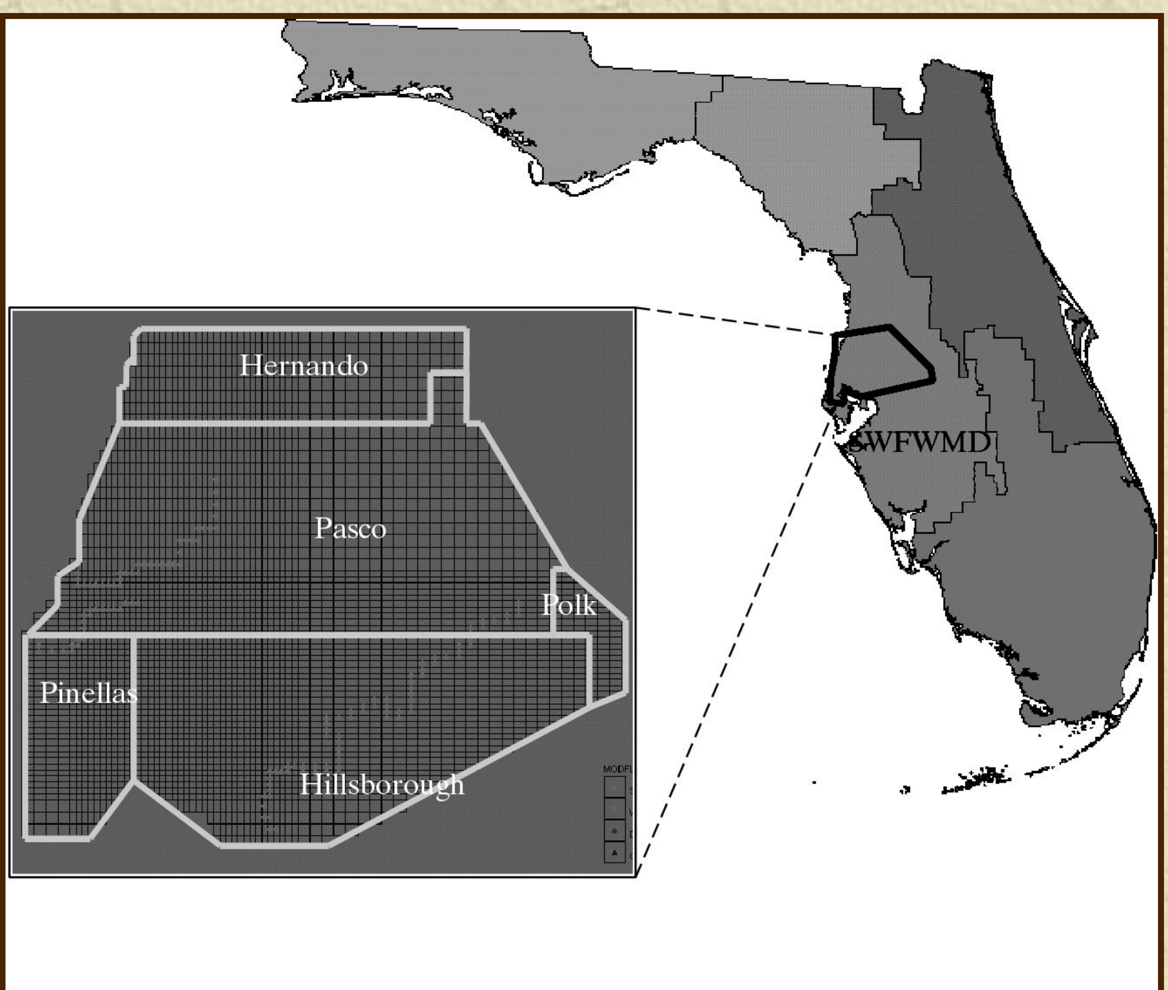

Figure 8.25

Type curves for analysis of pumping test data to evaluate storage coefficient and transmissivity of leaky aquifers. (After Walton, 1960, Illinois State Water Survey.)

# Radial Flow in a Leaky Aquifer

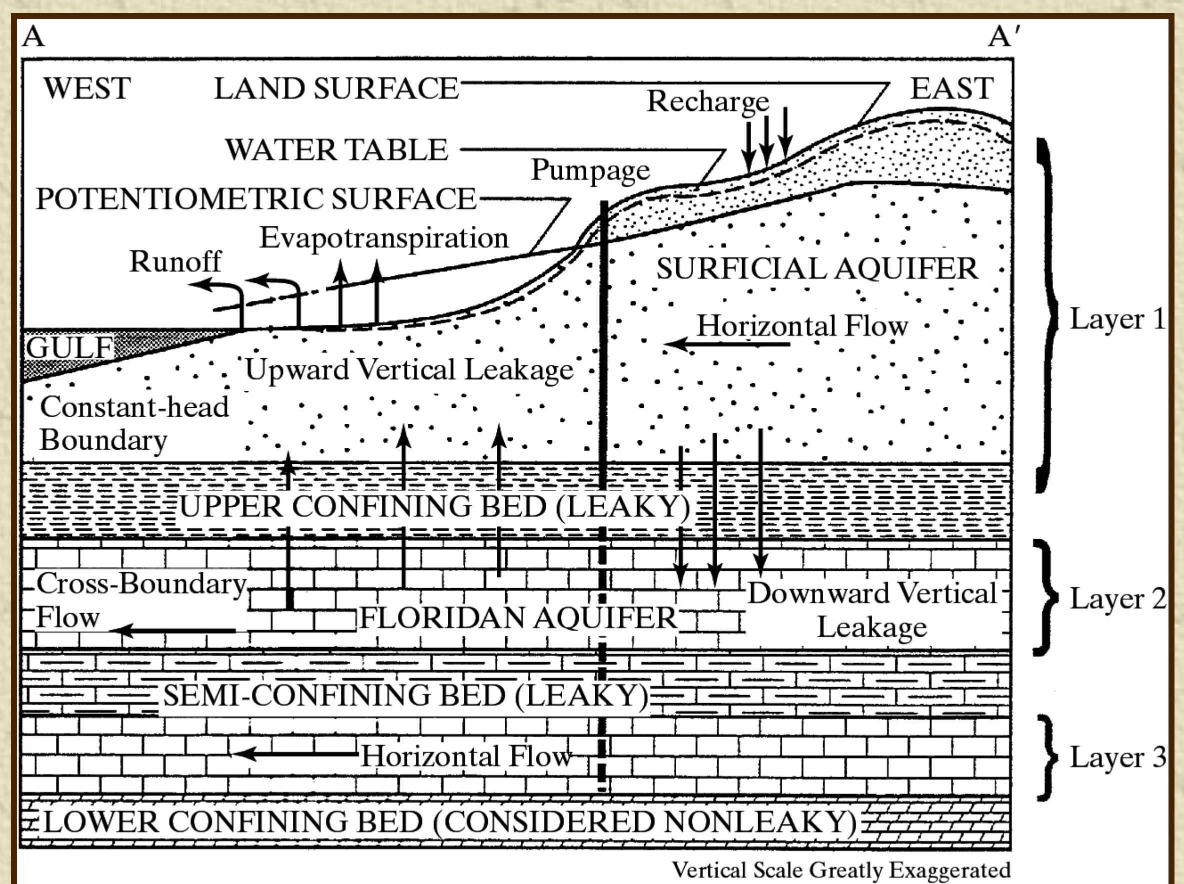
- This method of solution for the leaky aquifer is similar to the Theis method, except for $W(u,r/B)$
  - A curve of best fit is selected and values of $W$, $1/u$, $s'$, and $t$ are found, which allows $T$ and $S$ to be determined. This makes it possible to calculate $K'$ and $b'$.
  - Method is rarely used in practice since the assumptions are often violated in the field.
  - Better to use a numerical model (MODFLOW) that can handle variations more accurately.

# Florida: A Case Study




Counties encompassed in NTB wrap area model. (SWFWMD,1996.)

# Hydrogeology - Leaky Floridan Aquifer




Surface and ground water interactions. (SWFWMD, 1996.)