

DESIGN OF BEAM

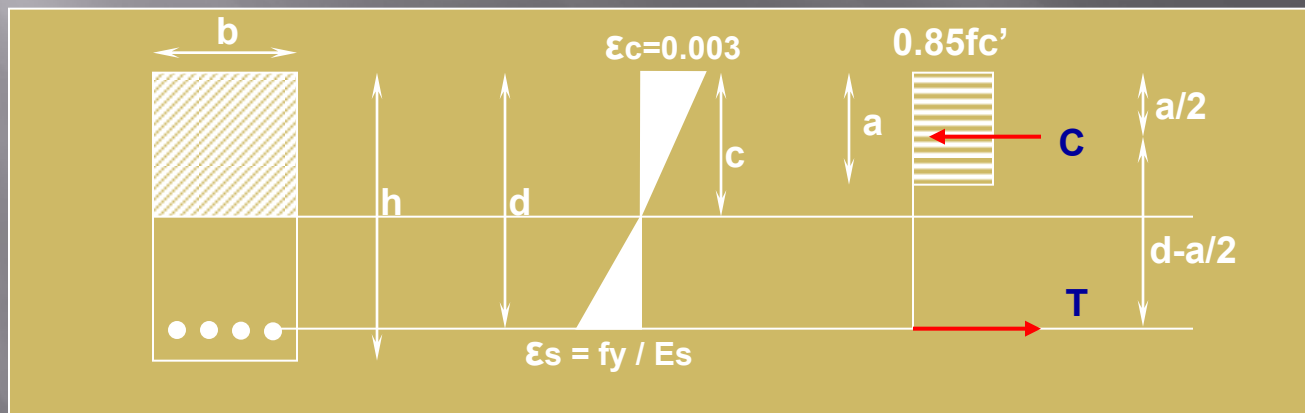
(AS PER ACI CODE)

CONTENT

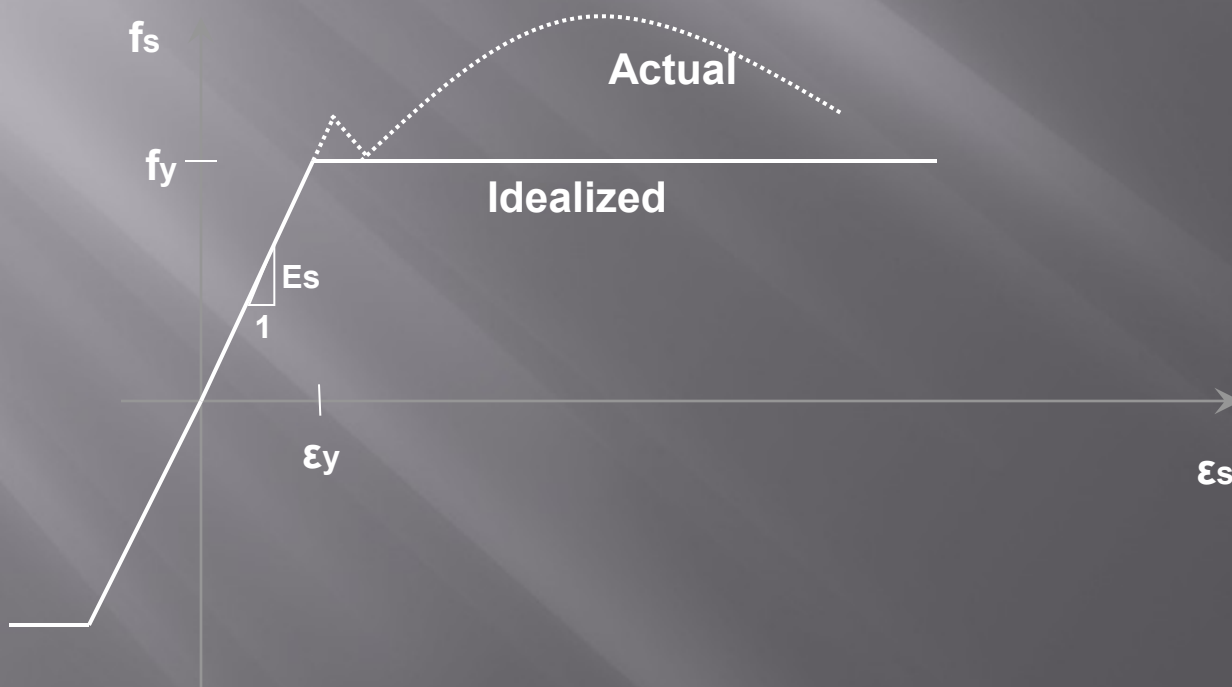
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ASSUMPTIONS

- Plane sections before bending remain plane and perpendicular to the N.A. after bending
- Strain distribution is linear both in concrete & steel and is directly proportional to the distance from N.A.
- Strain in the steel & surrounding concrete is the same prior to cracking of concrete or yielding of steel
- Concrete in the tension zone is neglected in the flexural analysis & design computation



- Concrete stress of $0.85f_c'$ is uniformly distributed over an equivalent compressive zone.
 f_c' = Specified compressive strength of concrete in psi.
- Maximum allowable strain of 0.003 is adopted as safe limiting value in concrete.
- The tensile strain for the balanced section is f_y/E_s
- Moment redistribution is limited to tensile strain of at least 0.0075



EVALUATION OF DESIGN PARAMETERS

- Total compressive force $C = 0.85f_c' ba$ (Refer stress diagram)
- Total Tensile force $T = A_s f_y$

$$C = T$$

$$0.85f_c' ba = A_s f_y$$

$$a = A_s f_y / (0.85f_c' b)$$

$$= \rho d f_y / (0.85 f_c') \quad \therefore \rho = A_s / bd$$

- Moment of Resistance, $M_n = 0.85f_c' ba (d - a/2)$ or

$$M_n = A_s f_y (d - a/2)$$

$$= \rho bd f_y [d - (\rho d f_y b / 1.7f_c')]$$

$$= \omega f_c' [1 - 0.59 \omega] bd^2$$

$$\therefore \omega = \rho f_y / f_c'$$

$$M_n = K_n bd^2 \quad \therefore K_n = \omega f_c' [1 - 0.59 \omega]$$

$$M_u = \phi M_n$$

$$= \phi K_n bd^2$$

TO SLIDE-7

ϕ = Strength Reduction Factor

■ **Balanced Reinforcement Ratio (ρ_b)**

From strain diagram, similar triangles

$$c_b / d = 0.003 / (0.003 + f_y / E_s) \quad ; \quad E_s = 29 \times 10^6 \text{ psi}$$

$$c_b / d = 87,000 / (87,000 + f_y)$$

Relationship between the depth 'a' of the equivalent rectangular stress block & depth 'c' of the N.A. is

$$a = \beta_1 c$$

$$\beta_1 = 0.85 \quad ; \quad f_c' \leq 4000 \text{ psi}$$

$$\beta_1 = 0.85 - 0.05(f_c' - 4000) / 1000 \quad ; \quad 4000 < f_c' \leq 8000$$

$$\beta_1 = 0.65 \quad ; \quad f_c' > 8000 \text{ psi}$$

$$\begin{aligned} \rho_b &= A_{sb} / bd \\ &= 0.85 f_c' a_b / (f_y \cdot d) \\ &= \beta_1 (0.85 f_c' / f_y) [87,000 / (87,000 + f_y)] \end{aligned}$$

- In case of statically determinate structure ductile failure is essential for proper moment redistribution. Hence, for beams the ACI code limits the max. amount of steel to 75% of that required for balanced section. For practical purposes, however the reinforcement ratio ($\rho = A_s / b d$) should not normally exceed 50% to avoid congestion of reinforcement & proper placing of concrete.

$$\rho \leq 0.75 \rho_b$$

- Min. reinforcement is greater of the following:

$$A_{s_{\min}} = 3\sqrt{f_c'} \times b_w d / f_y \quad \text{or} \quad 200 b_w d / f_y$$

$$\rho_{\min} = 3\sqrt{f_c'} / f_y \quad \text{or} \quad 200 / f_y$$

- For statically determinate member, when the flange is in tension, the b_w is replaced with $2b_w$ or b_f whichever is smaller
- The above min steel requirement need not be applied, if at every section, A_{st} provided is at least 1/3 greater than the analysis

DESIGN PROCEDURE FOR SINGLY REINFORCED BEAM

- Determine the service loads
- Assume `h` as per the support conditions according to [Table 9.5 \(a\) in the code](#)
- Calculate $d = h - \text{Effective cover}$
- Assume the value of `b` by the [rule of thumb.](#)
- Estimate self weight
- Perform preliminary elastic analysis and derive B.M (M), Shear force (V) values
- Compute ρ_{\min} and ρ_b
- Choose ρ between ρ_{\min} and ρ_b
- [Calculate \$\omega, Kn\$](#)
- [From \$Kn\$ & \$M\$ calculate `d' required \(Substitute b in terms of d\)](#)
- Check the required `d' with assumed `d'
- Revise & repeat the steps, if necessary

- With the final values of ρ , b , d determine the Total A_s required
- Design the steel reinforcement arrangement with appropriate cover and spacing stipulated in code. Bar size and corresponding no. of bars based on the bar size #n.
- Check crack widths as per codal provisions

DESIGN PROCEDURE FOR DOUBLY REINFORCED BEAM

■ Moment of resistance of the section

$$M_u = M_{u1} + M_{u2}$$

M_{u1} = M.R. of Singly reinforced section

$$= \phi A_{s1} f_y (d - a/2) \quad ; \quad A_{s1} = M_{u1} / [\phi f_y (d - a/2)]$$

$$M_{u2} = \phi A_{s2} f_y (d - d') \quad ; \quad A_{s2} = M_{u2} / [\phi f_y (d - d')]$$

$$M_u = \phi A_{s1} f_y (d - a/2) + \phi A_{s2} f_y (d - d')$$

If Compression steel yields,

$$\epsilon' \geq f_y / E_s$$

$$\text{i.e.,} \quad 0.003 [1 - (0.85 f_c' \beta_1 d') / ((\rho - \rho') f_y d)] \geq f_y / E_s$$

If compression steel does not yield,

$$f_s' = E_s \times 0.003 [1 - (0.85 f_c' \beta_1 d') / ((\rho - \rho') f_y d)]$$

Balanced section for doubly reinforced section is

$$\rho_b = \rho_{b1} + \rho' (f_s' / f_y)$$

ρ_{b1} = Balanced reinforcement ratio for S.R. section

END

DESIGN STRENGTH

$$M_u = \phi M_n$$

- The design strength of a member refers to the nominal strength calculated in accordance with the requirements stipulated in the code multiplied by a **Strength Reduction Factor ϕ** , which is always less than 1.

■ Why ϕ ?

- To allow for the probability of understrength members due to variation in material strengths and dimensions
- To allow for inaccuracies in the design equations
- To reflect the degree of ductility and required reliability of the member under the load effects being considered.
- To reflect the importance of the member in the structure

RECOMMENDED VALUE

Beams in Flexure.....	0.90
Beams in Shear & Torsion	0.85

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AS PER TABLE 9.5 (a)

Simply Supported	One End Continuous	Both End Continuous	Cantilever
$L / 16$	$L / 18.5$	$L / 21$	$L/8$

Values given shall be used directly for members with normal weight concrete ($W_c = 145 \text{ lb/ft}^3$) and Grade 60 reinforcement

- For structural light weight concrete having unit wt. In range 90-120 lb/ft³ the values shall be multiplied by $(1.65 - 0.005W_c)$ but not less than 1.09
- For f_y other than 60,000 psi the values shall be multiplied by $(0.4 + f_y/100,000)$
- `h` should be rounded to the nearest whole number

CLEAR COVER

- Not less than 1.5 in. when there is no exposure to weather or contact with the ground
- For exposure to aggressive weather 2 in.
- Clear distance between parallel bars in a layer must not be less than the bar diameter or 1 in.

RULE OF THUMB

- $d/b = 1.5$ to 2.0 for beam spans of 15 to 25 ft.
- $d/b = 3.0$ to 4.0 for beam spans > 25 ft.
- `b` is taken as an even number
- Larger the d/b , the more efficient is the section due to less deflection

BAR SIZE

- #n = n/8 in. diameter for n ≤ 8.

Ex. #1 = 1/8 in.

....

#8 = 8/8 i.e., 1 in.

Weight, Area and Perimeter of individual bars

Bar No	Wt.per Foot (lb)	Standard Nominal Dimensions			
		Diameter d_b		C/S Area, A_b (in ²)	Perimeter (in.)
		inch	mm		
3	0.376	0.375	9	0.11	1.178
4	0.668	0.500	13	0.20	1.571
5	1.043	0.625	16	0.31	1.963
6	1.502	0.750	19	0.44	2.356
7	2.044	0.875	22	0.60	2.749
8	2.670	1.000	25	0.79	3.142
9	3.400	1.128	28	1.00	3.544
10	4.303	1.270	31	1.27	3.990
11	5.313	1.410	33	1.56	4.430
14	7.650	1.693	43	2.25	5.319
18	13.600	2.257	56	4.00	7.091

CRACK WIDTH

$$w = 0.000091.f_s.\sqrt[3]{(d_c.A)}$$

Where,

w = Crack width

= 0.016 in. for an interior exposure condition

= 0.013 in. for an exterior exposure condition

f_s = 0.6 f_y , kips

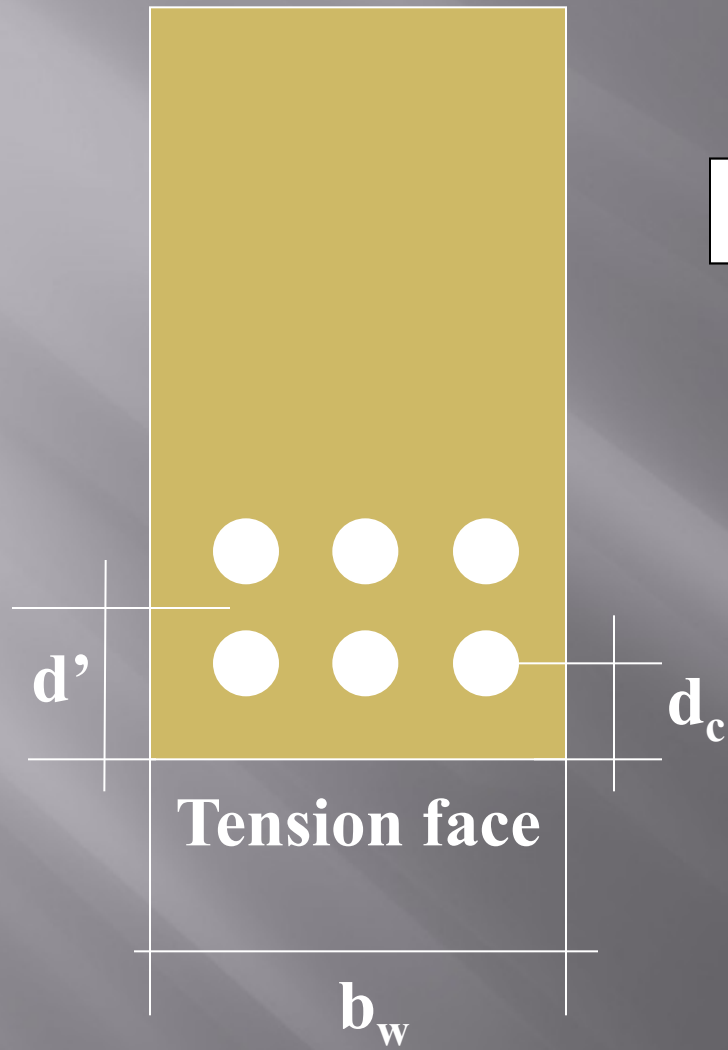
d_c = Distance from tension face to center of the row of bars closest to the outside surface

A = Effective tension area of concrete divided by the number of reinforcing bars

= A_{eff} / N

A_{eff} = Product of web width and a height of web equal to twice the distance from the centroid of the steel and tension surface

N = Total area of steel A_s / Area of larger bar



$$A_{\text{eff}} = b_w \times 2d'$$

FLANGED BEAMS

■ EFFECTIVE OVERHANG, r



T – BEAM

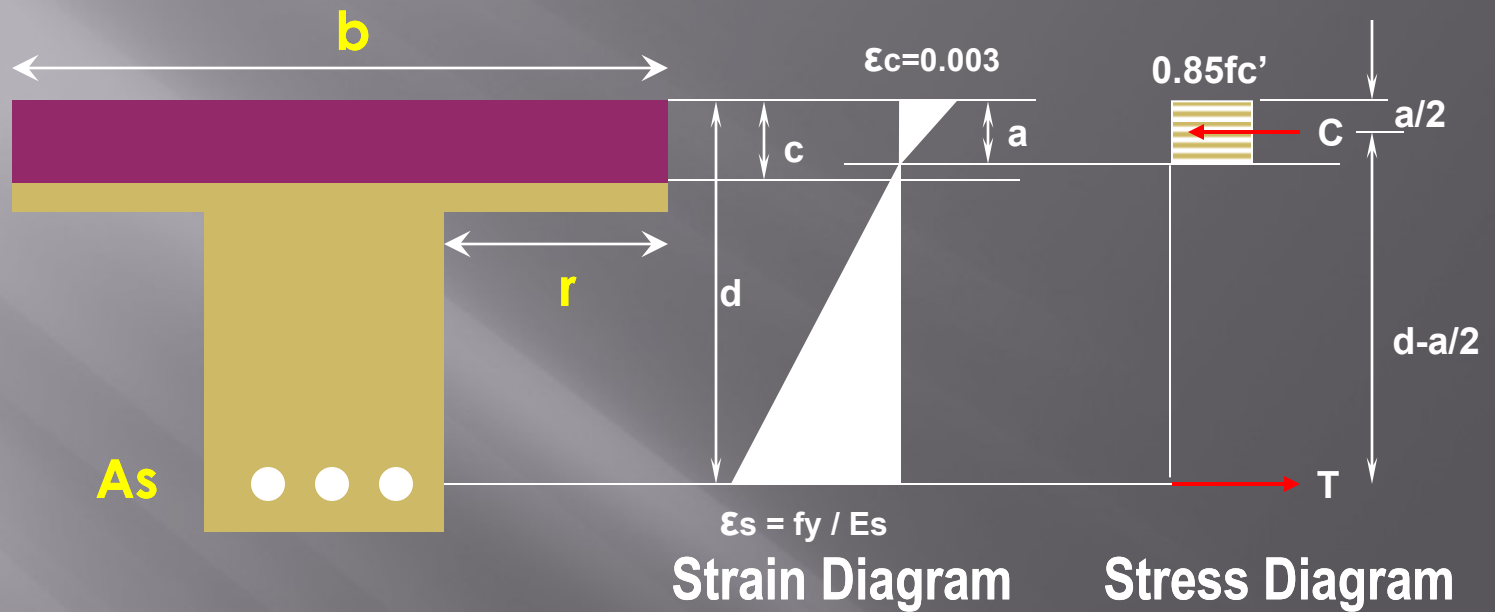
1. $r \leq 8 hf$
2. $r \leq \frac{1}{2} l_n$
3. $r \leq \frac{1}{4} L$



L – BEAM

1. $r \leq 6 hf$
2. $r \leq \frac{1}{2} l_n$
3. $r \leq \frac{1}{12} L$

Case-1: Depth of N.A 'c' < hf



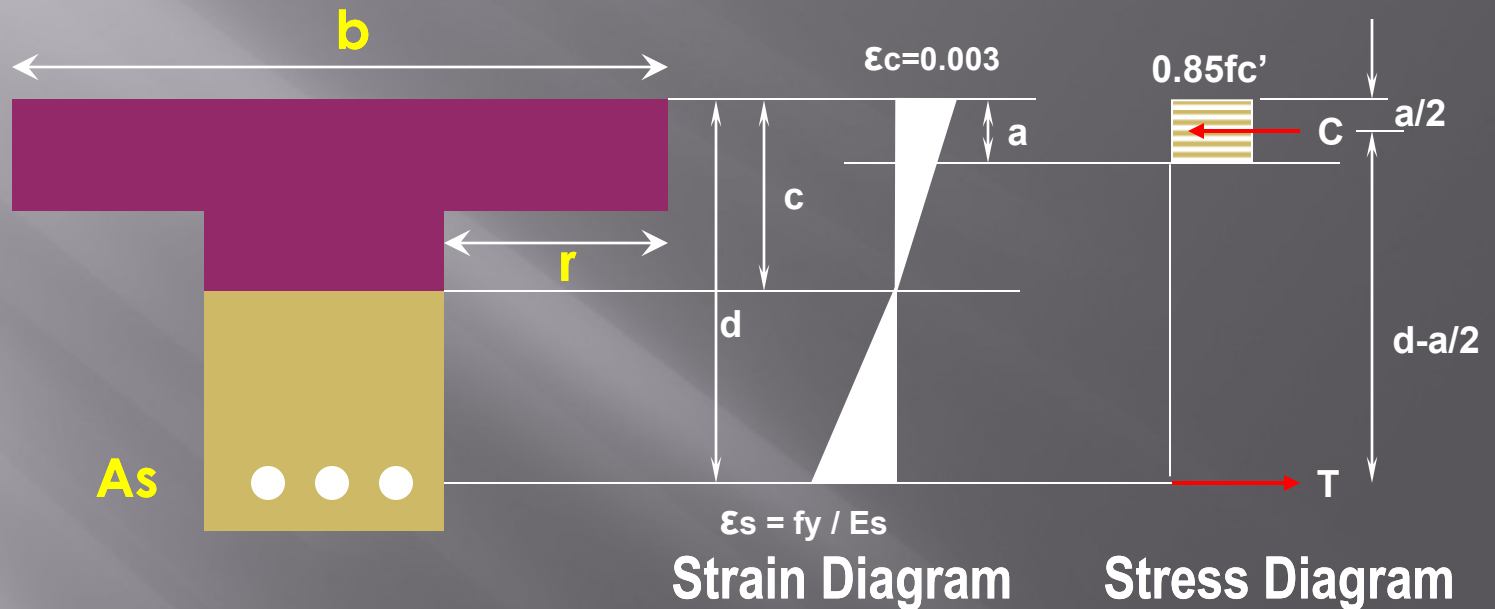
$$0.85f_c' b a = A_s f_y$$

$$a = A_s f_y / [0.85f_c' b]$$

$$M_n = A_s f_y (d - a/2)$$

Case-2: Depth of N.A 'c' > hf

i) $a < hf$



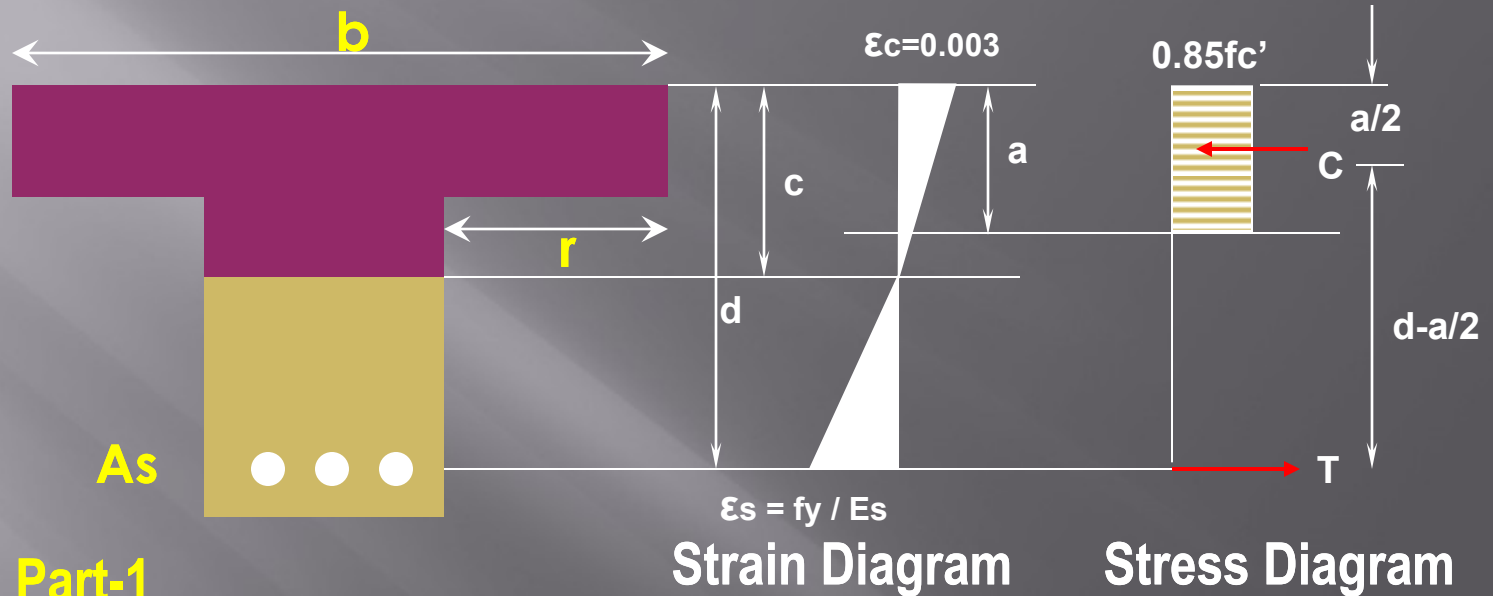
$$0.85fc' b a = A_s f_y$$

$$a = A_s f_y / [0.85fc' b]$$

$$M_n = A_s f_y (d - a/2)$$

Case-2: Depth of N.A 'c' > hf

ii) $a > hf$



$$0.85f_c' b_w a = A_{s_1} f_y$$

Part-2

$$0.85f_c' (b - b_w) hf = A_{s_2} f_y$$

$$0.85f_c' b_w a +$$

$$0.85f_c' (b - b_w) hf = A_s f_y$$

$$a = [A_s f_y - 0.85f_c' (b - b_w) hf] / [0.85f_c' b_w]$$

- **Moment of resistance of the section**

$$M_n = M_{n1} + M_{n2}$$

$$M_{n1} = A_{s1} f_y (d - a / 2)$$

$$M_{n2} = A_{s2} f_y (d - h_f / 2)$$

- **Moment Redistribution**

For continuous beam members,

Code permits Max of 20%

when $e_t \geq 0.0075$ at that section

■ Balanced Reinforcement Ratio (ρ_b)

$$\rho_b = (b_w / b) [\rho_{\underline{b}} + \rho_f]$$

$$\begin{aligned}\rho_{\underline{b}} &= A_{sb} / b_w d \\ &= 0.85 f_c' a_b / (f_y \cdot d) \\ &= \beta_1 (0.85 f_c' / f_y) [87,000 / (87,000 + f_y)]\end{aligned}$$

$$\rho_{\underline{f}} = 0.85 f_c' (b - b_w) h_f / (f_y b_w d)$$

$$\rho \leq 0.75 \rho_b$$

- Min. reinforcement is greater of the following:

$$\rho_w = 3\sqrt{f_c'} / f_y \quad \text{or} \quad 200 / f_y \quad ; \quad \text{for +ve Reinf.}$$

$$\rho_{\min} = 6\sqrt{f_c'} / f_y \quad \text{or} \quad 200 / f_y \quad ; \quad \text{for -ve Reinf.}$$

THANK YOU