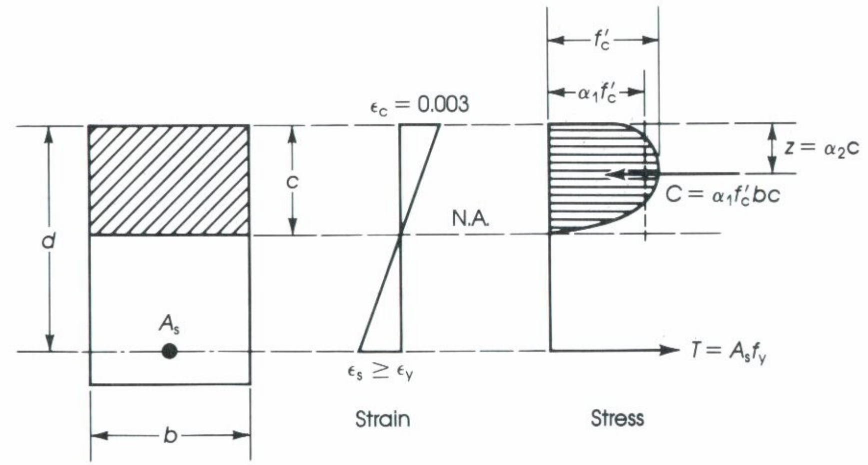
Lecture Goals

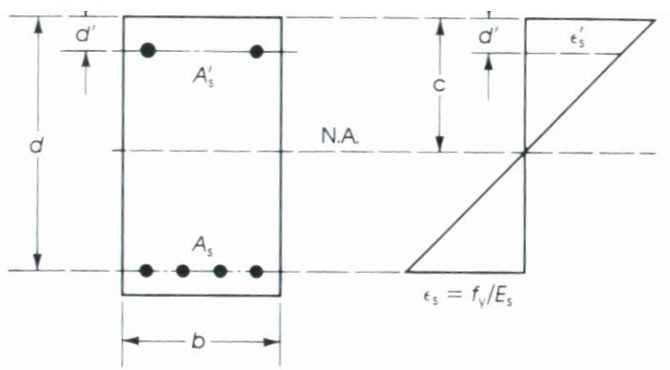
- Doubly Reinforced beams
- T Beams and L Beams

Analysis of Doubly Reinforced Sections

Effect of Compression Reinforcement on the Strength and Behavior

Less concrete is needed to resist the T and thereby moving the neutral axis (NA) up.





$$T = A_s f_y$$

$$C = T$$

Analysis of Doubly Reinforced Sections

Effect of Compression Reinforcement on the Strength and Behavior

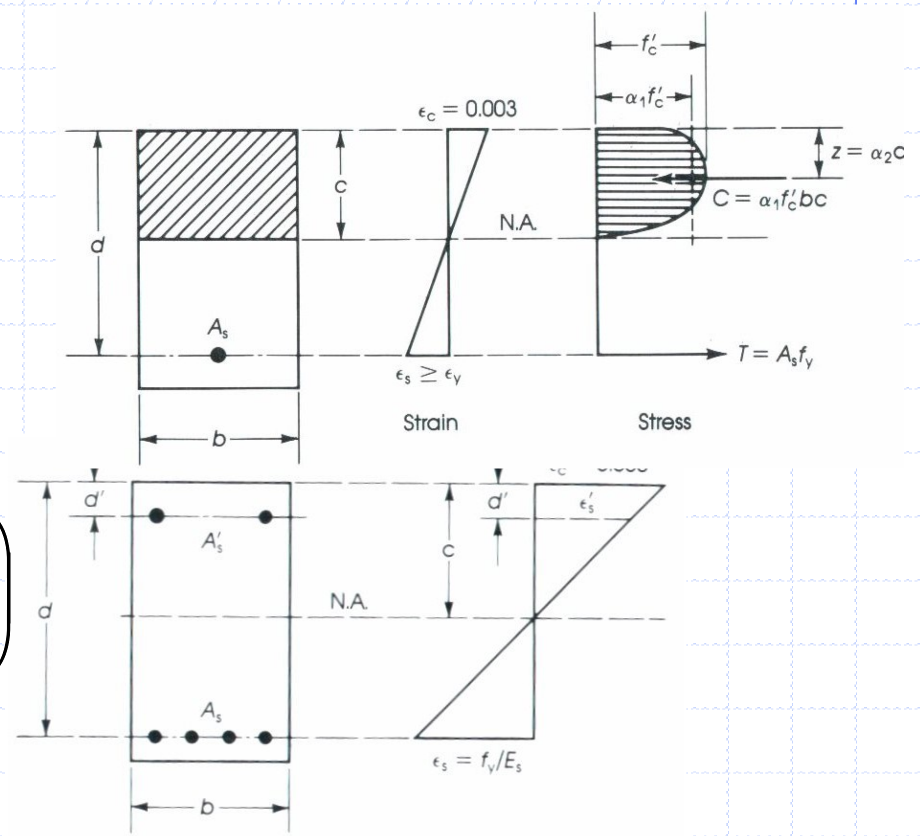
Singly Reinforced $\Rightarrow$

$$C = C_c \;;\; M_n = A_s f_y \left(d - \frac{a_1}{2} \right)$$

Doubly Reinforced $\Rightarrow$

$$C = C_c + C'_s \;;\; M_n = A_s f_y \left(d - \frac{a_2}{2} \right)$$

and $(a_2 < a_1)$

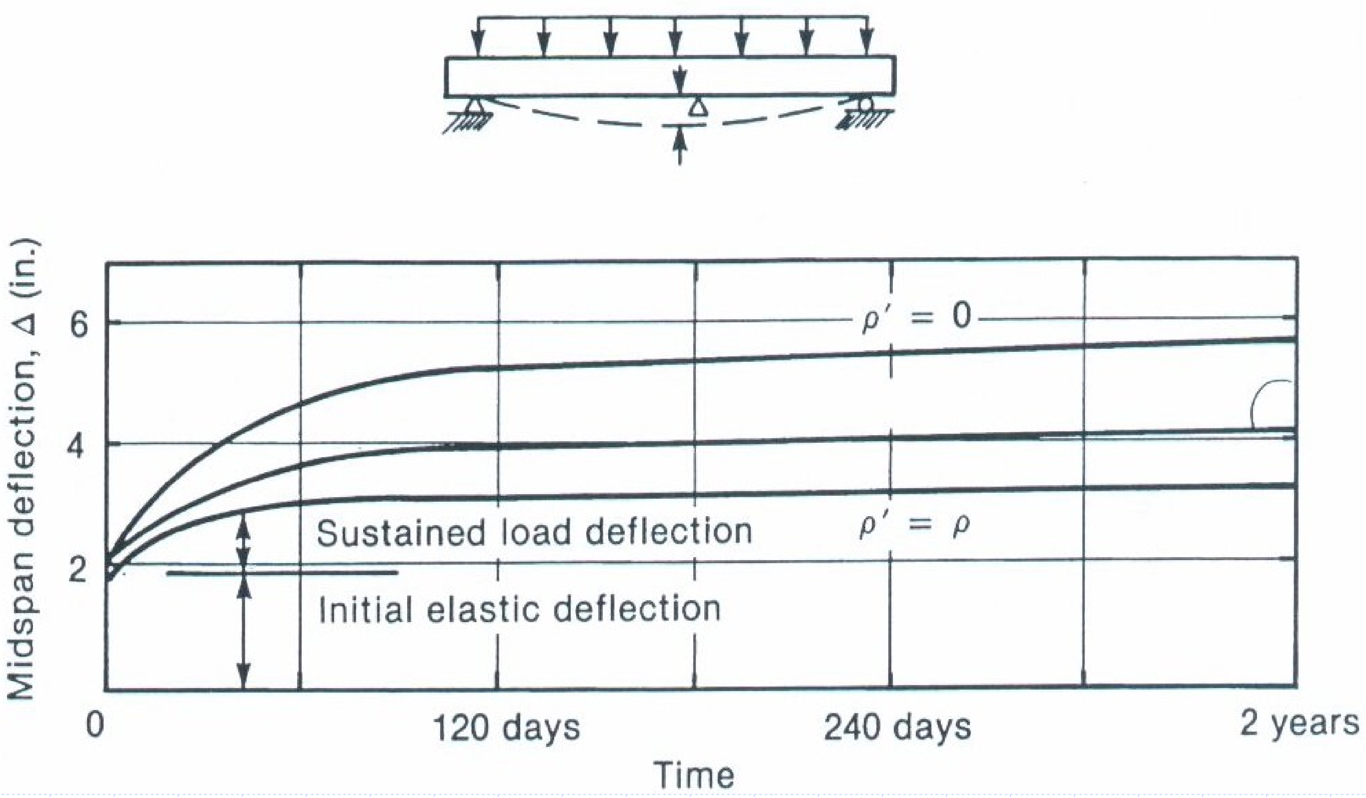


Reasons for Providing Compression Reinforcement

- Reduced sustained load deflections.
  - Creep of concrete in compression zone
  - transfer load to compression steel
  - reduced stress in concrete
  - less creep
  - less sustained load deflection

Reasons for Providing Compression Reinforcement

Effective of compression reinforcement on sustained load deflections.



Reasons for Providing Compression Reinforcement

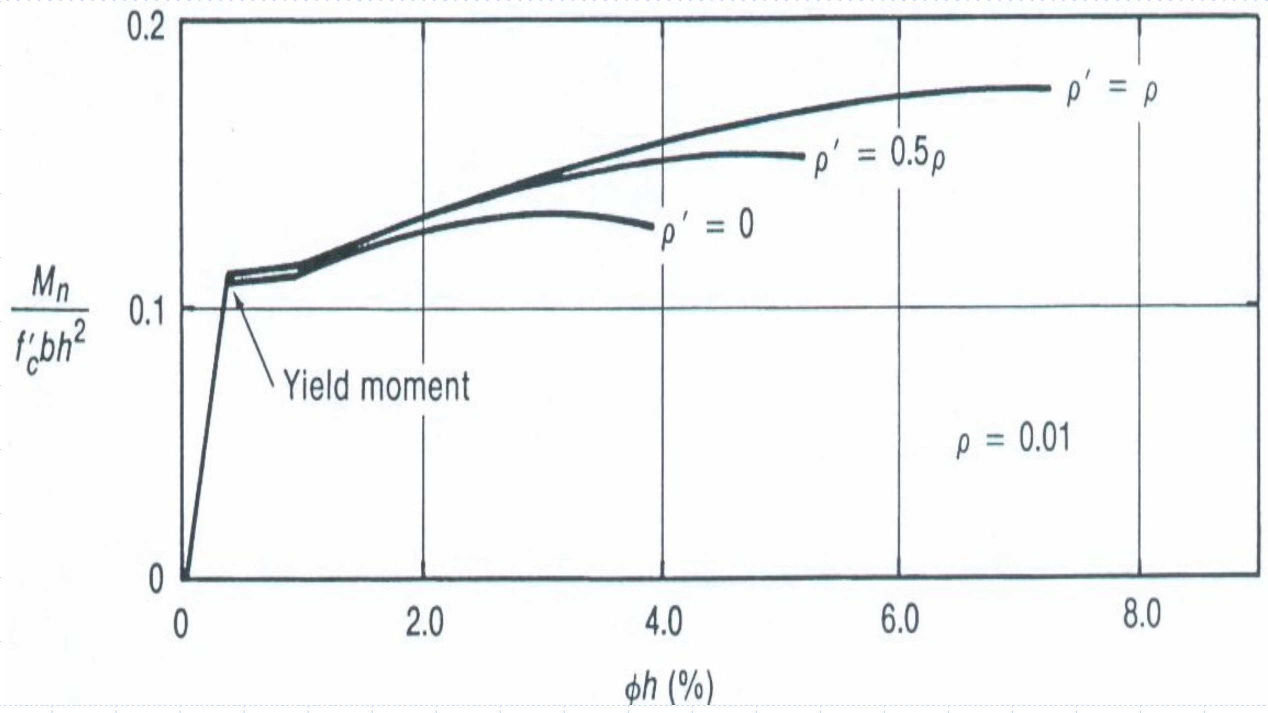
- Increased Ductility

reduced stress block depth → increase in steel strain larger curvature are obtained.

Reasons for Providing Compression Reinforcement

Effect of compression reinforcement on strength and ductility of under reinforced beams.

$\rho < \rho_b$



Reasons for Providing Compression Reinforcement

- Change failure mode from compression to tension. When $\rho > \rho_{bal}$ addition of A_s strengthens.

Compression zone → allows tension steel to yield before crushing of concrete.

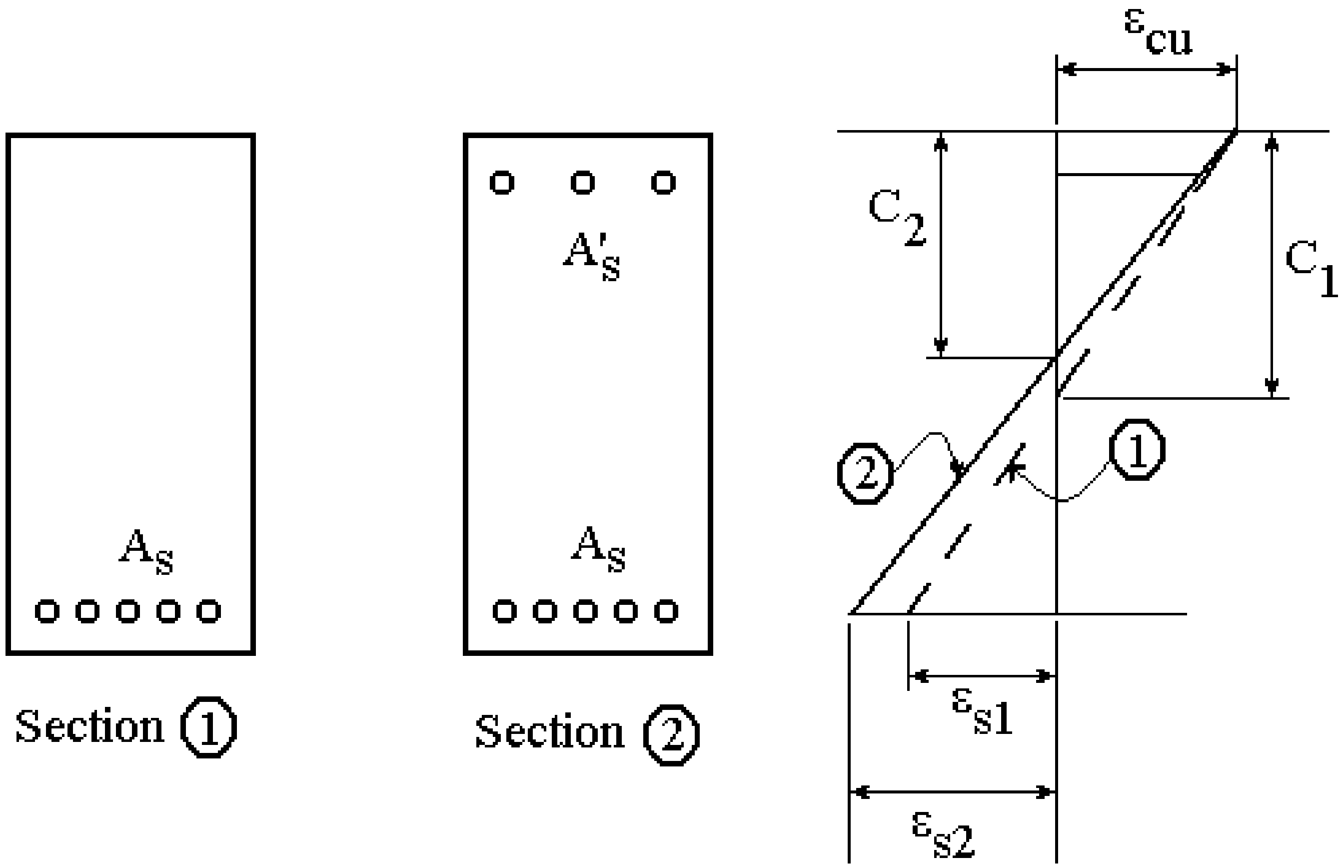
Effective reinforcement ratio $= (\rho - \rho')$

Reasons for Providing Compression Reinforcement

- Eases in Fabrication
  - Use corner bars to hold & anchor stirrups.

Effect of Compression Reinforcement

- Compare the strain distribution in two beams with the same A_s



Effect of Compression Reinforcement

Section 1:

$$T = A_s f_s$$

$$T = C_{c1} = 0.85 f_c' ba = 0.85 f_c' b \beta_1 c_1$$

$$c_1 = \frac{A_s f_s}{0.85 f_c' b \beta_1}$$

Section 2:

$$T = A_s f_s$$

$$T = C_s' + C_{c1}$$

$$= A_s' f_s' + 0.85 f_c' b a_2$$

$$= A_s' f_s' + 0.85 f_c' b \beta_1 c_2$$

$$c_2 = \frac{A_s f_s - A_s' f_s'}{0.85 f_c' b \beta_1}$$

Addition of A'_s strengthens compression zone so that less concrete is needed to resist a given value of T. ⟶ NA goes up ($c_2 < c_1$) and ε_s increases ($\varepsilon_{s2} > \varepsilon_{s1}$).

Doubly Reinforced Beams

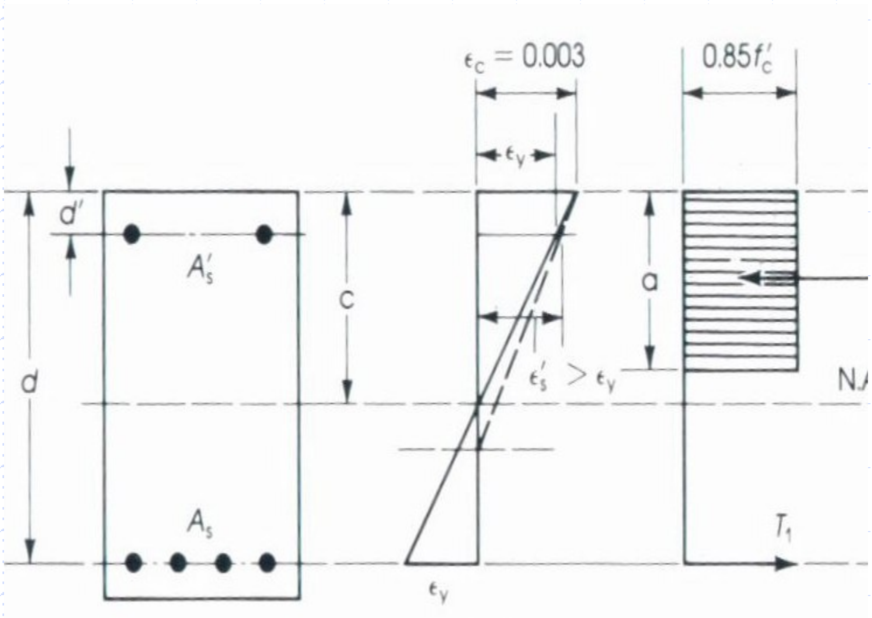
Four Possible Modes of Failure

- Under reinforced Failure
  - (Case 1) Compression and tension steel yields
  - (Case 2) Only tension steel yields
- Over reinforced Failure
  - (Case 3) Only compression steel yields
  - (Case 4) No yielding Concrete crushes

Analysis of Doubly Reinforced Rectangular Sections

Strain Compatibility Check

Assume ε_s’ using similar triangles



$$\frac{\varepsilon_s'}{(c-d')} = \frac{0.003}{c} \Rightarrow$$

$$\varepsilon_s' = \frac{(c-d')}{c} * 0.003$$

Analysis of Doubly Reinforced Rectangular Sections

Strain Compatibility

Using equilibrium and find a

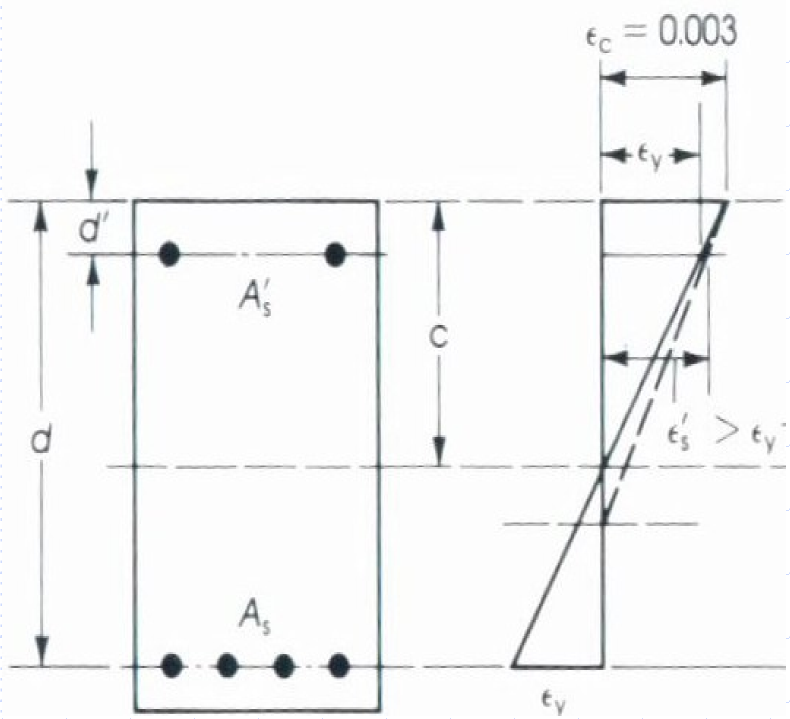
$$T = C_c' + C_s' \Rightarrow a = \frac{\left(A_s - A_s'\right) f_y}{0.85 f_c' b}$$

$$c = \frac{a}{\beta_1} = \frac{\left(A_s - A_s'\right) f_y}{\beta_1 \left(0.85 f_c' b\right)} = \frac{\left(\rho - \rho'\right) d\, f_y}{\beta_1 \left(0.85 f_c'\right)}$$

Analysis of Doubly Reinforced Rectangular Sections

Strain Compatibility

The strain in the compression steel is



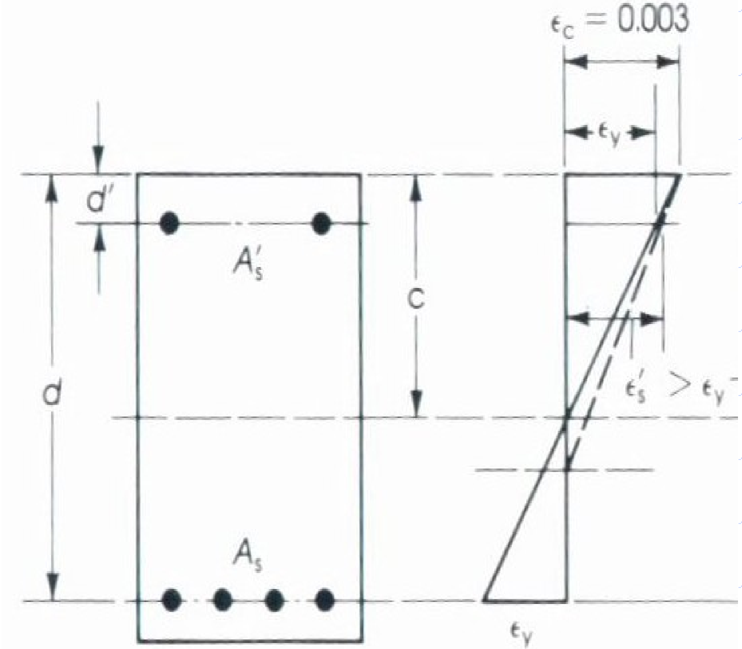
$$\varepsilon'_{\mathrm{s}} = \left(1 - \frac{d'}{c}\right)\varepsilon_{\mathrm{cu}}$$

$$= \left(1 - \frac{\beta_1 \left(0.85 f'_{\mathrm{c}}\right) d'}{\left(\rho - \rho'\right) d\, f_{\mathrm{y}}}\right) 0.003$$

Analysis of Doubly Reinforced Rectangular Sections

Strain Compatibility

Confirm



$$\varepsilon'_s \geq \varepsilon_y = \frac{f_y}{E_s}; \qquad \varepsilon_s \geq \varepsilon_y$$

$$\varepsilon'_s = \left(1 - \frac{\beta_1 (0.85 f'_c) d'}{(\rho - \rho') d\, f_y}\right) 0.003 \geq \frac{f_y}{E_s} = \frac{f_y}{29 \text{ x } 10^3 \text{ ksi}}$$

Analysis of Doubly Reinforced Rectangular Sections

Strain Compatibility

Confirm

$$-\frac{\beta_1\left(0.85 f_c'\right)d'}{\left(\rho-\rho'\right)d\, f_y} \geq \frac{f_y-87}{87}$$

$$\left(\rho-\rho'\right) \geq \left(\frac{\beta_1\left(0.85 f_c'\right)d'}{d\, f_y}\right)\left(\frac{87}{87-f_y}\right)$$

Analysis of Doubly Reinforced Rectangular Sections

Find c

$$A'_{\mathrm{s}} f_{\mathrm{y}} + 0.85 f'_{\mathrm{c}}\, ba = A_{\mathrm{s}} f_{\mathrm{y}}$$

$$c = \frac{\left(A_{\mathrm{ss}} - A'_{\mathrm{s}}\right) f_{\mathrm{y}}}{0.85 f'_{\mathrm{c}}\, b \beta_1} \qquad \Rightarrow a = \beta_1 c$$

confirm that the tension steel has yielded

$$\varepsilon_{\mathrm{s}} = \left(\frac{d-c}{c}\right)\varepsilon_{\mathrm{cu}} \geq \varepsilon_{\mathrm{y}} = \frac{f_{\mathrm{y}}}{\mathrm{E_s}}$$

Analysis of Doubly Reinforced Rectangular Sections

If the statement is true than

$$M_n = \left(A_s - A_s'\right) f_y \left(d - \frac{a}{2}\right) + A_s' f_y \left(d - d'\right)$$

else the strain in the compression steel

$$f_s = E\varepsilon_s'$$

Analysis of Doubly Reinforced Rectangular Sections

Return to the original equilibrium equation

$$A_{\mathrm{s}} f_{\mathrm{y}} = A'_{\mathrm{s}} f_{\mathrm{s}} + 0.85 f_{\mathrm{c}} ba$$

$$= A'_{\mathrm{s}} E_{\mathrm{s}} \varepsilon_{\mathrm{s}}' + 0.85 f_{\mathrm{c}} b \beta_1 c$$

$$= A'_{\mathrm{s}} E_{\mathrm{s}} \left(1 - \frac{d'}{c}\right) \varepsilon_{\mathrm{cu}} + 0.85 f_{\mathrm{c}} b \beta_1 c$$

Analysis of Doubly Reinforced Rectangular Sections

Rearrange the equation and find a quadratic equation

$$A_{\mathrm{s}} f_{\mathrm{y}} = A'_{\mathrm{s}} E_{\mathrm{s}} \left(1 - \frac{d'}{c}\right) \varepsilon_{\mathrm{cu}} + 0.85 f_{\mathrm{c}} b \beta_1 c$$

$$\Rightarrow 0.85 f_{\mathrm{c}} b \beta_1 c^2 + \left(A'_{\mathrm{s}} E_{\mathrm{s}} \varepsilon_{\mathrm{cu}} - A_{\mathrm{s}} f_{\mathrm{y}}\right) c - A'_{\mathrm{s}} E_{\mathrm{s}} \varepsilon_{\mathrm{cu}} d' = 0$$

Solve the quadratic and find c.

Analysis of Doubly Reinforced Rectangular Sections

Find the f_s'

$$f_s' = \left(1 - \frac{d'}{c}\right) E_s \varepsilon_{cu} = \left(1 - \frac{d'}{c}\right) 87 \text{ ksi}$$

Check the tension steel.

$$\varepsilon_s = \left(\frac{d - c}{c}\right) \varepsilon_{cu} \geq \varepsilon_y = \frac{f_y}{E_s}$$

Analysis of Doubly Reinforced Rectangular Sections

Another option is to compute the stress in the compression steel using an iterative method.

$$f_s' = 29 \text{ x } 10^3 \left(1 - \frac{\beta_1 \left(0.85 f_c' \right) d'}{\left(\rho - \rho' \right) d \, f_y} \right) 0.003$$

Analysis of Doubly Reinforced Rectangular Sections

Go back and calculate the equilibrium with f_s'

$$T = C_c' + C_s' \Rightarrow a = \frac{\left(A_s f_y - A_s' f_s\right)}{0.85 f_c' b}$$

$$c = \frac{a}{\beta_1}$$

$$f_s' = \left(1 - \frac{d'}{c}\right) 87 \text{ ksi}$$

Iterate until the c value is adjusted for the f_s' until the stress converges.

Analysis of Doubly Reinforced Rectangular Sections

Compute the moment capacity of the beam

$$M_{\mathrm{n}} = \left(A_{\mathrm{s}} f_{\mathrm{y}} - A'_{\mathrm{s}} f'_{\mathrm{s}}\right)\left(d - \frac{a}{2}\right) + A'_{\mathrm{s}} f'_{\mathrm{s}}\left(d - d'\right)$$

Limitations on Reinforcement Ratio for Doubly Reinforced beams

Lower limit on ρ

$$\rho_{\min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y} \qquad \text{(ACI 10.5)}$$

same as for single reinforce beams.

Example: Doubly Reinforced Section

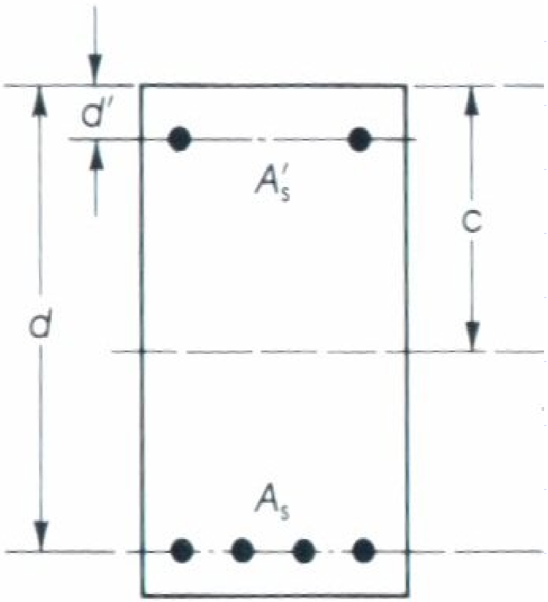
Given:

$f'_c = 4000$ psi $f_y = 60$ ksi

$A'_s = 2$ #5 $A_s = 4$ #7

$d' = 2.5$ in. $d = 15.5$ in

$h = 18$ in. $b = 12$ in.



Calculate M_n for the section for the given compression steel.

Example: Doubly Reinforced Section

Compute the reinforcement coefficients, the area of the bars #7 (0.6 in^2) and #5 (0.31 in^2)

$$A_s = 4\left(0.6\ \text{in}^2\right) = 2.4\ \text{in}^2$$

$$A_s' = 2\left(0.31\ \text{in}^2\right) = 0.62\ \text{in}^2$$

$$\rho = \frac{A_s}{bd} = \frac{2.4\ \text{in}^2}{(12\ \text{in.})(15.5\ \text{in.})} = 0.0129$$

$$\rho' = \frac{A_s'}{bd} = \frac{0.62\ \text{in}^2}{(12\ \text{in.})(15.5\ \text{in.})} = 0.0033$$

Example: Doubly Reinforced Section

Compute the effective reinforcement ratio and minimum ρ

$$\rho_{eff} = \rho - \rho' = 0.0129 - 0.0033 = 0.00957$$

$$\rho = \frac{200}{f_y} = \frac{200}{60000} = 0.00333$$

$$\text{or } \frac{3\sqrt{f_c}}{f_y} = \frac{3\sqrt{4000}}{60000} = 0.00316$$

$$\rho \geq \rho_{min} \Rightarrow 0.0129 \geq 0.00333 \text{ OK!}$$

Example: Doubly Reinforced Section

- Compute the effective reinforcement ratio and minimum ρ

$$(\rho - \rho') \geq \left(\frac{\beta_1 \left(0.85 f_c'\right) d'}{d\, f_y} \right) \left(\frac{87}{87 - f_y} \right)$$

$$\geq \left(\frac{0.85\big(0.85(4\text{ ksi})\big)(2.5\text{ in.})}{60\text{ ksi}(15.5\text{ in.})} \right) \left(\frac{87}{87-60} \right) = 0.0398$$

$0.00957 \not\geq 0.0398$ Compression steel has not yielded.

Example: Doubly Reinforced Section

- Instead of iterating the equation use the quadratic method

$$0.85 f_c b \beta_1 c^2 + \left(A_s' E_s \varepsilon_{cu} - A_s f_y \right) c - A_s' E_s \varepsilon_{cu} d' = 0$$

$$0.85 \left(4 \text{ ksi}\right)\left(12 \text{ in.}\right)\left(0.85\right) c^2 +$$

$$+ \left[\left(\left(0.62 \text{ in}^2\right)\left(29000 \text{ ksi}\right)\left(0.003\right) - \left(2.4 \text{ in}^2\right)\left(60 \text{ ksi}\right) \right) \right] c$$

$$- \left(0.62 \text{ in}^2\right)\left(29000 \text{ ksi}\right)\left(0.003\right)\left(2.5 \text{ in.}\right) = 0$$

$$34.68 c^2 - 90.06 c - 134.85 = 0$$

$$c^2 - 2.5969 c - 3.8884 = 0$$

Example: Doubly Reinforced Section

Solve using the quadratic formula

$$c^2 - 2.5969c - 3.8884 = 0$$

$$c = \frac{2.5969 \pm \sqrt{\left(-2.5969\right)^2 - 4\left(-3.8884\right)}}{2}$$

$$c = 3.6595 \text{ in.}$$

Example: Doubly Reinforced Section

Find the f_s'

$$f_s' = \left(1 - \frac{d'}{c}\right) E_s \varepsilon_{cu} = \left(1 - \frac{2.5 \text{ in.}}{3.659 \text{ in.}}\right) 87 \text{ ksi}$$

$$= 27.565 \text{ ksi}$$

Check the tension steel.

$$\varepsilon_s = \left(\frac{15.5 \text{ in.} - 3.659 \text{ in.}}{3.659 \text{ in.}}\right) 0.003 = 0.00971 \geq 0.00207$$

Example: Doubly Reinforced Section

Check to see if c works

$$c = \frac{A_s f_y - A_s' f_s'}{0.85 f_c \beta_1 b} = \frac{\left(2.4 \text{ in}^2\right)\left(60 \text{ ksi}\right) - \left(0.62 \text{ in}^2\right)\left(27.565 \text{ ksi}\right)}{0.85\left(4 \text{ ksi}\right)\left(0.85\right)\left(12 \text{ in.}\right)}$$

$$c = 3.659 \text{ in.}$$

The problem worked

Example: Doubly Reinforced Section

Compute the moment capacity of the beam

$$M_n = \left(A_s f_y - A_s' f_s'\right)\left(d - \frac{a}{2}\right) + A_s' f_s' \left(d - d'\right)$$

$$= \begin{pmatrix} \left(2.4\ \text{in}^2\right)\left(60\ \text{ksi}\right) \\ -\left(0.62\ \text{in}^2\right)\left(27.565\ \text{ksi}\right) \end{pmatrix}\left(15.5\ \text{in.} - \frac{0.85\left(3.659\ \text{in.}\right)}{2}\right)$$

$$+\left(0.62\ \text{in}^2\right)\left(27.565\ \text{ksi}\right)\left(15.5\ \text{in.} - 2.5\ \text{in.}\right)$$

$$= 1991.9\ \text{k - in.} \Rightarrow 166\ \text{k - ft}$$

Example: Doubly Reinforced Section

If you want to find the M_u for the problem

$$\frac{c}{d} = \frac{3.66 \text{ in.}}{15.5 \text{ in.}} = 0.236$$

From ACI (figure R9.3.2)or figure (pg 100 in your text)

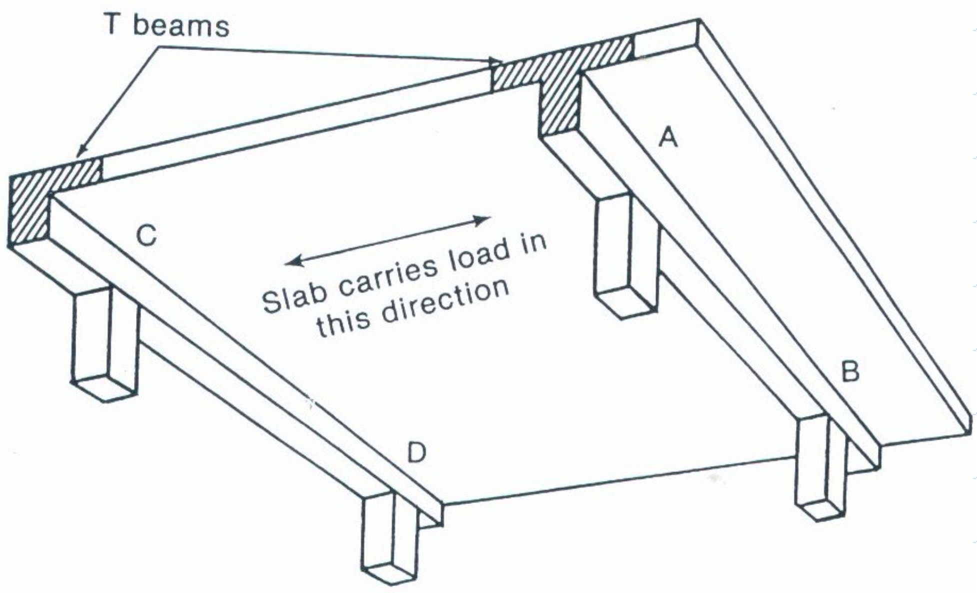
$$0.375 > \frac{c}{d} \quad \Rightarrow \quad \phi = 0.9$$

The resulting ultimate moment is

$$M_{\mathrm{u}} = \phi M_{\mathrm{u}} = 0.9\left(166 \text{ k-ft}\right)$$
$$= 149.4 \text{ k-ft}$$

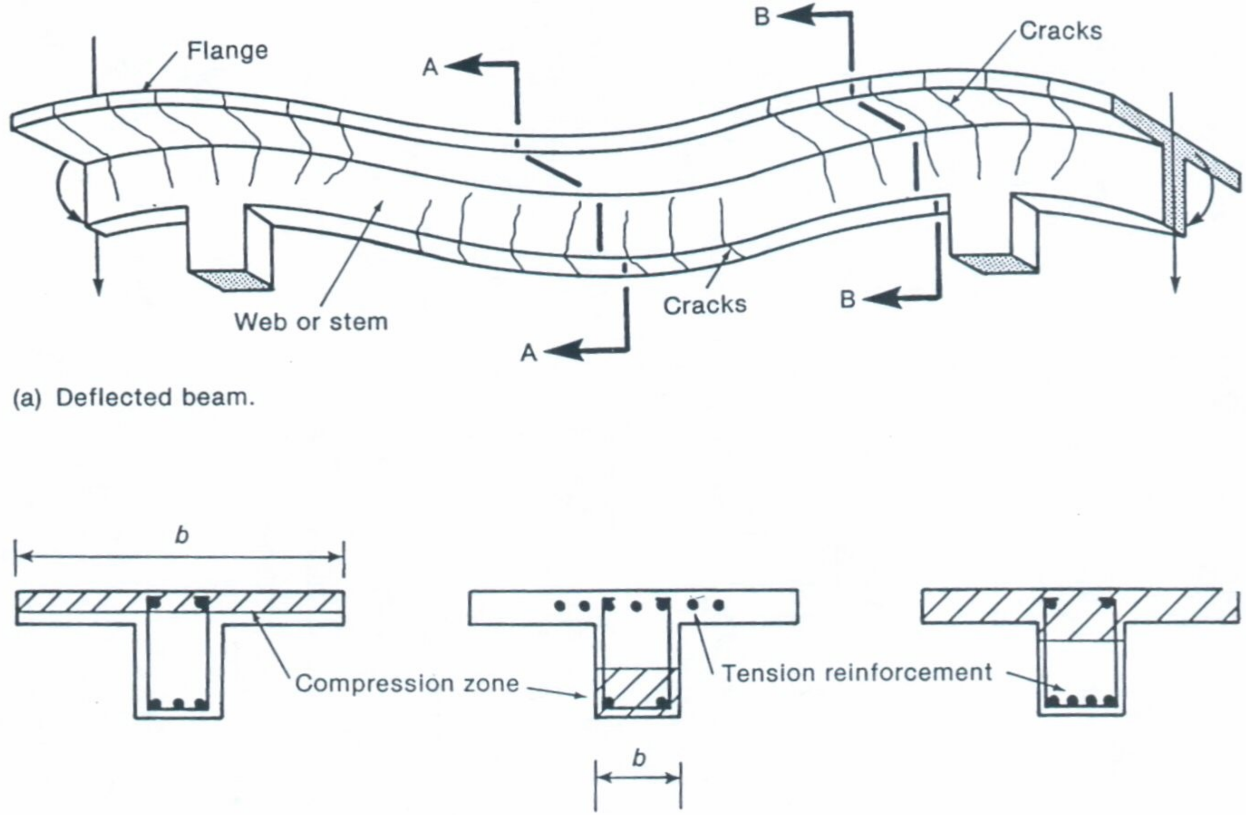
Analysis of Flanged Section

- Floor systems with slabs and beams are placed in monolithic pour.
- Slab acts as a top flange to the beam; ***T-beams***, and ***Inverted L(Spandrel) Beams***.



Analysis of Flanged Sections

Positive and Negative Moment Regions in a T-beam



(a) Deflected beam.

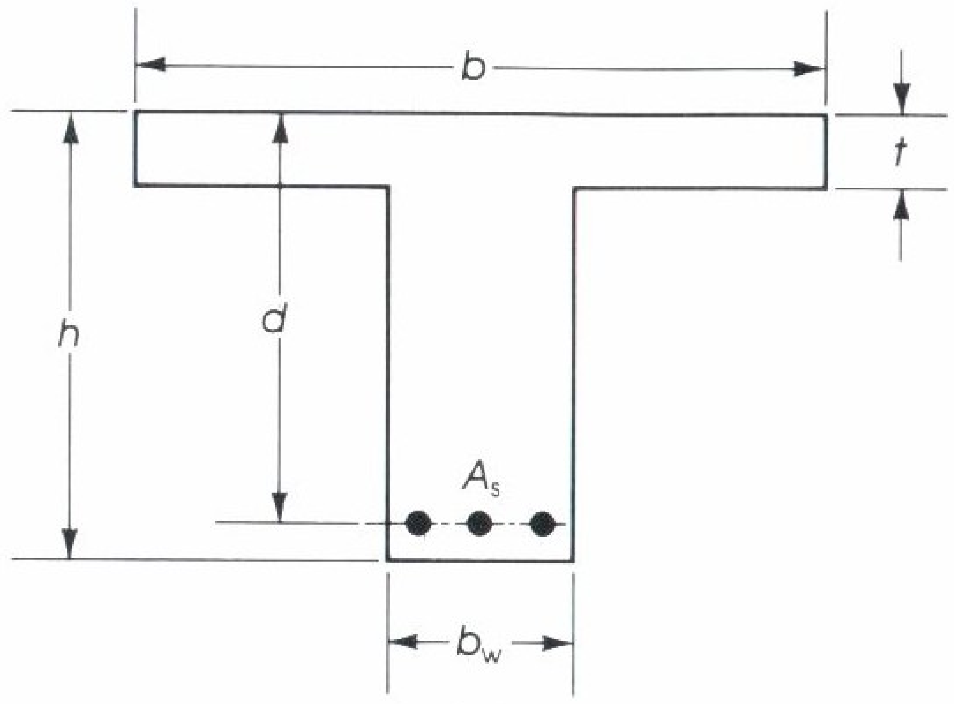
(b) Section A–A (rectangular compression zone).

(c) Section B–B (negative moment).

(d) Section A–A (T-shaped compression zone).

Analysis of Flanged Sections

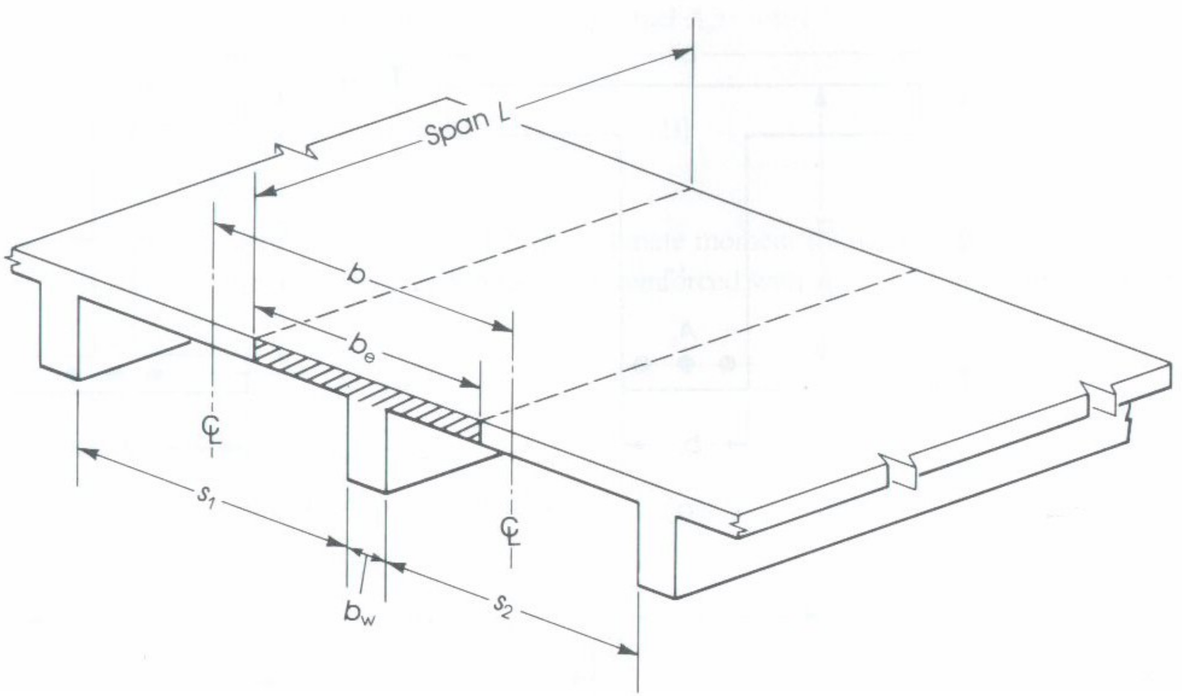
If the neutral axis falls within the slab depth analyze the beam as a rectangular beam, otherwise as a T-beam.



Analysis of Flanged Sections

Effective Flange Width

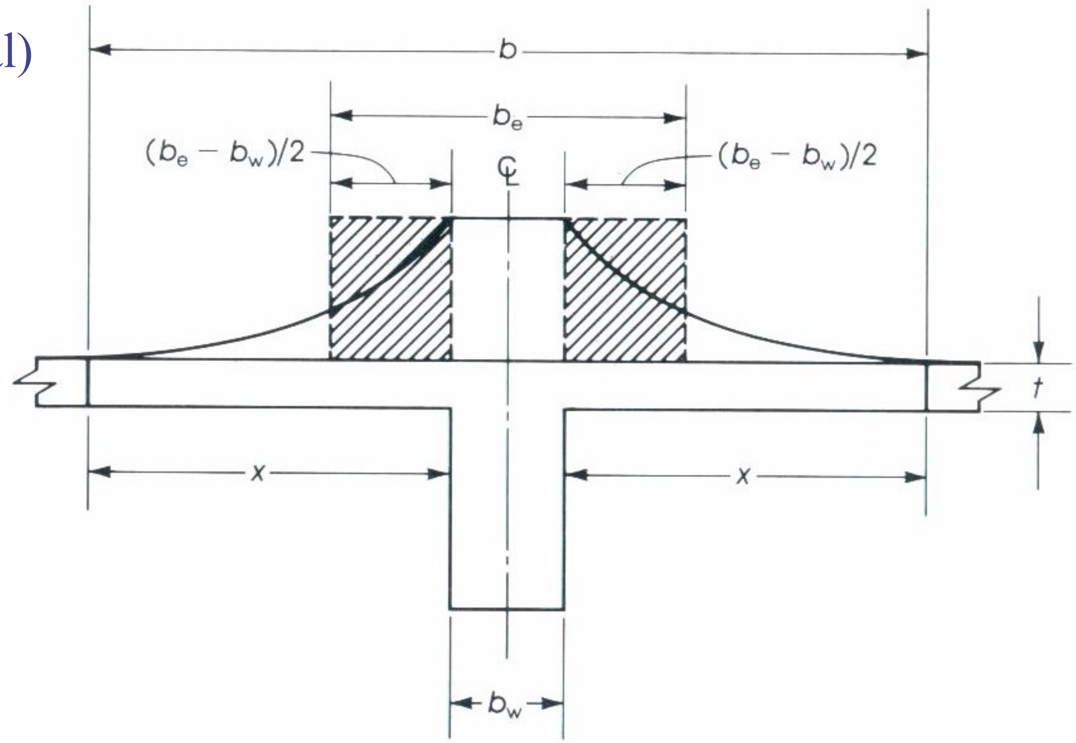
Portions near the webs are more highly stressed than areas away from the web.



Analysis of Flanged Sections

Effective width (b_{eff})

b_{eff} is width that is stressed uniformly to give the same compression force actually developed in compression zone of width $b_{(actual)}$



ACI Code Provisions for Estimating b_{eff}

From ACI 318, Section 8.10.2

T Beam Flange:

$$
\begin{aligned}
b_{\mathrm{eff}} &\leq \frac{L}{4} \\
&\leq 16h_{\mathrm{f}} + b_{\mathrm{w}} \\
&\leq b_{\mathrm{actual}}
\end{aligned}
$$

ACI Code Provisions for Estimating b_{eff}

From ACI 318, Section 8.10.3

Inverted L Shape Flange

$$b_{\text{eff}} \leq \frac{L}{12} + b_{\text{w}}$$

$$\leq 6h_{\text{f}} + b_{\text{w}}$$

$$\leq b_{\text{actual}} = b_{\text{w}} + 0.5*(\text{clear distance to next web})$$

ACI Code Provisions for Estimating b_{eff}

From ACI 318, Section 8.10

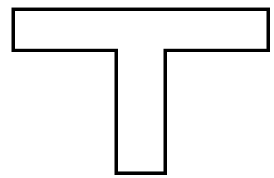
Isolated T-Beams

$$h_f \geq \frac{b_w}{2}$$

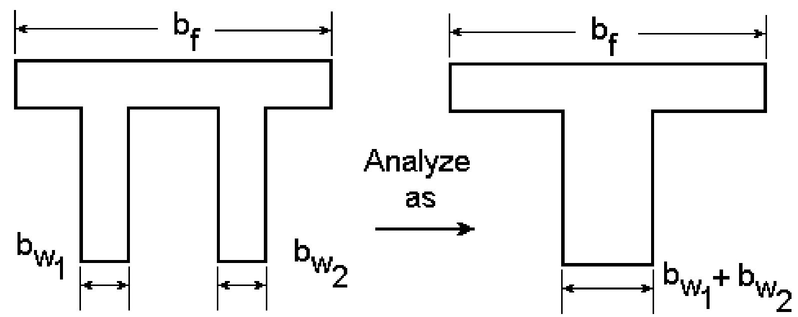
$$b_{eff} \leq 4b_w$$

Various Possible Geometries of T-Beams

Single Tee



Twin Tee



Box