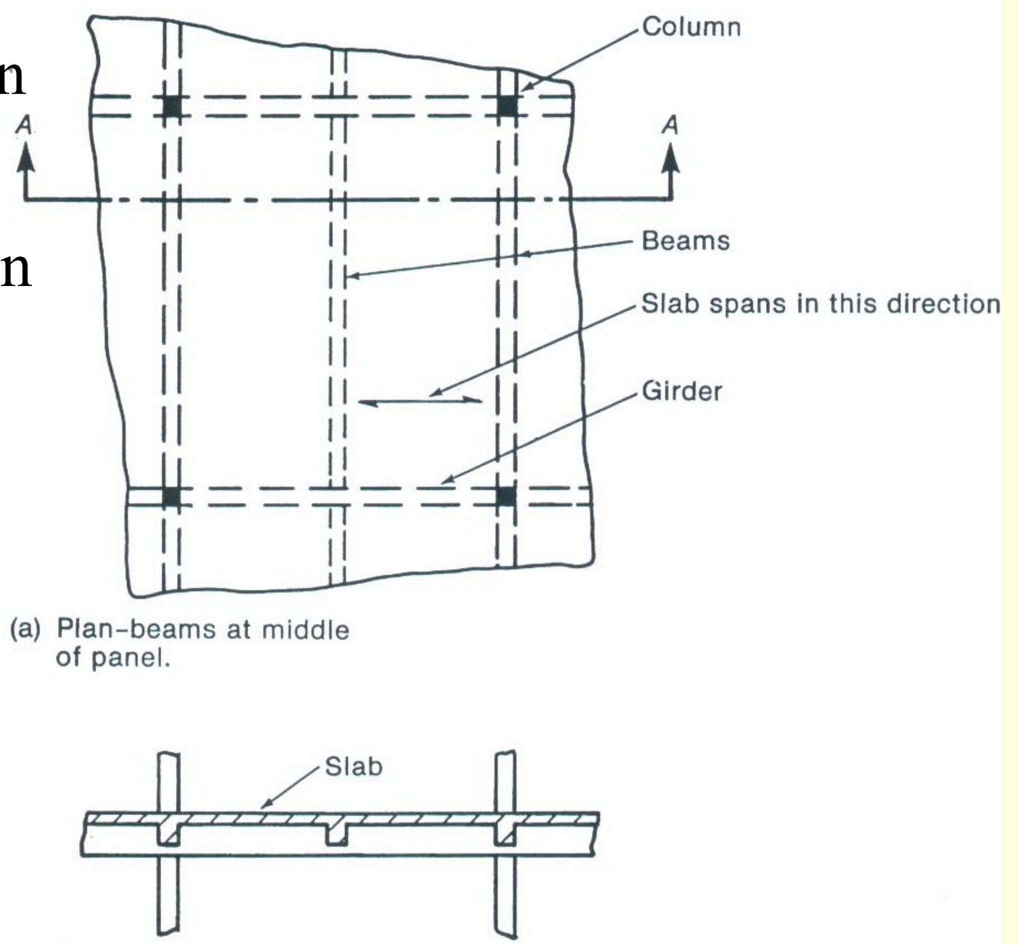
Two-way Slabs

Comparison of One-way and Two-way slab behavior

One-way slabs carry load in one direction.

Two-way slabs carry load in two directions.

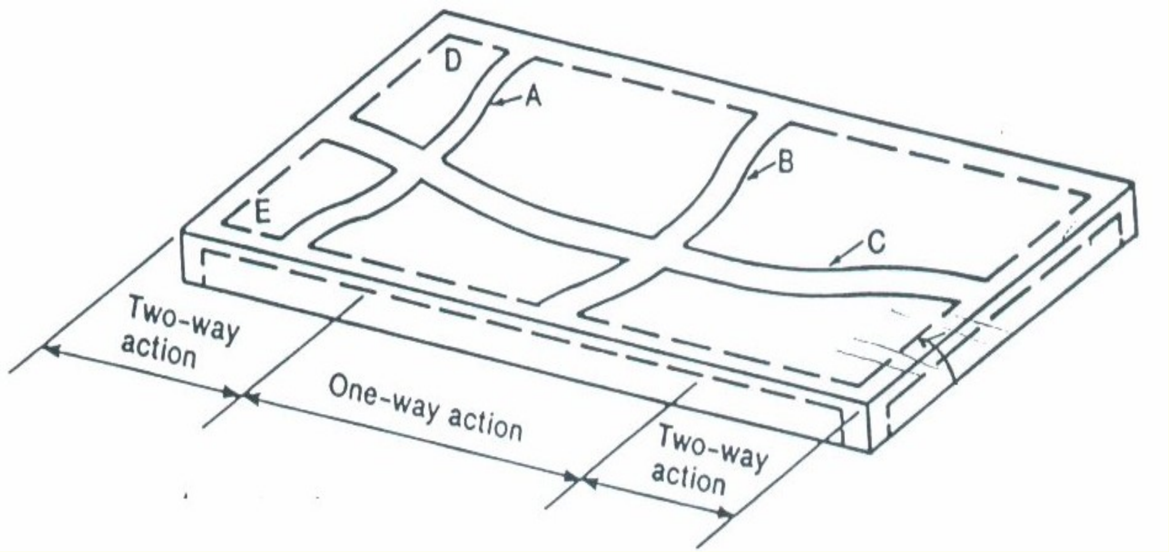


(a) Plan–beams at middle of panel.

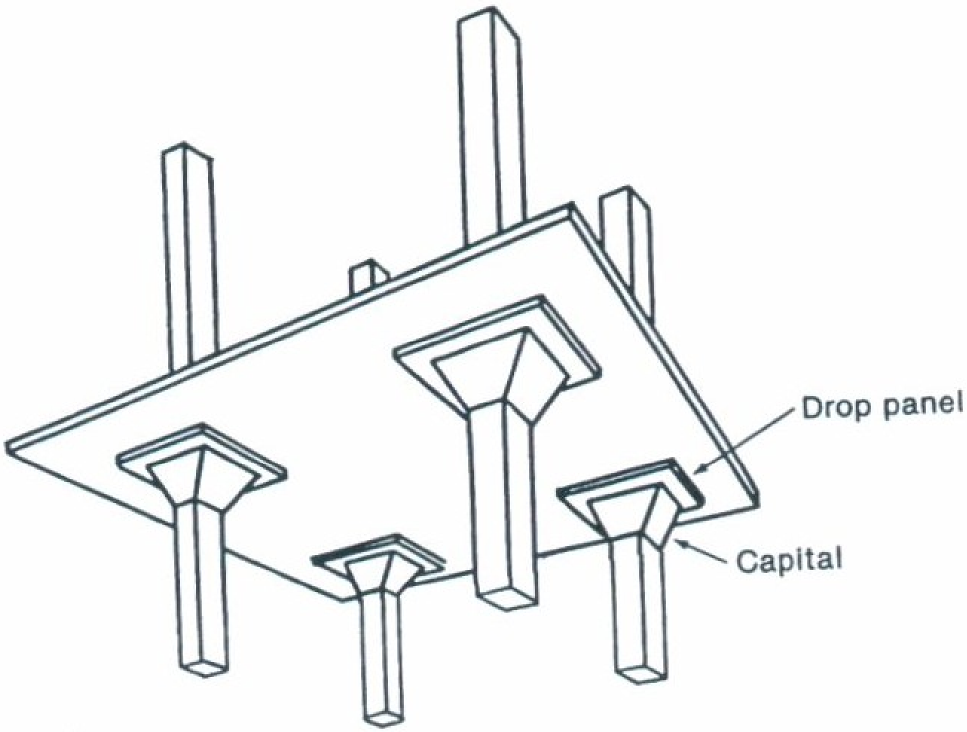
(b) Section A–A.

Comparison of One-way and Two-way slab behavior

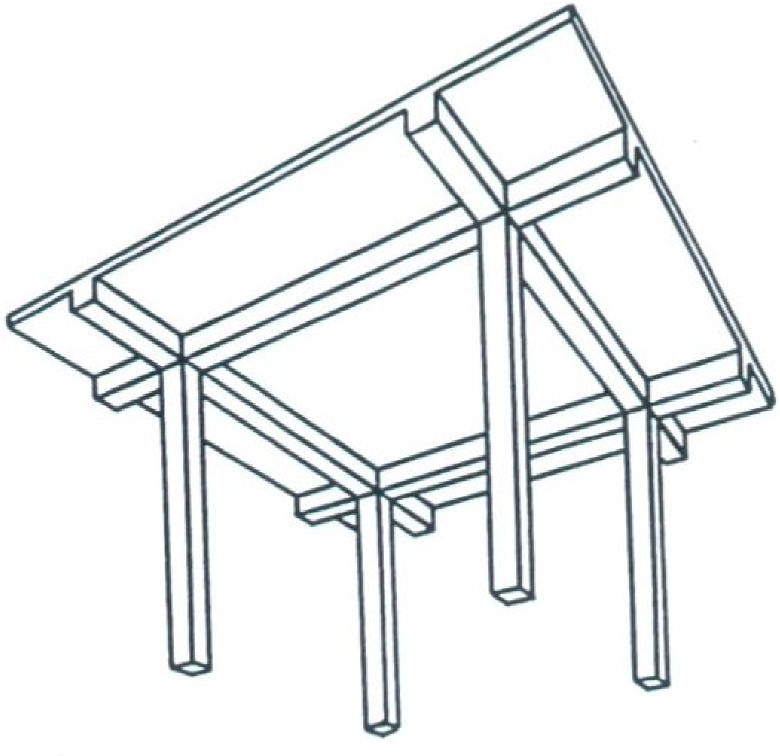
One-way and two-way slab action carry load in two directions.



Comparison of One-way and Two-way slab behavior



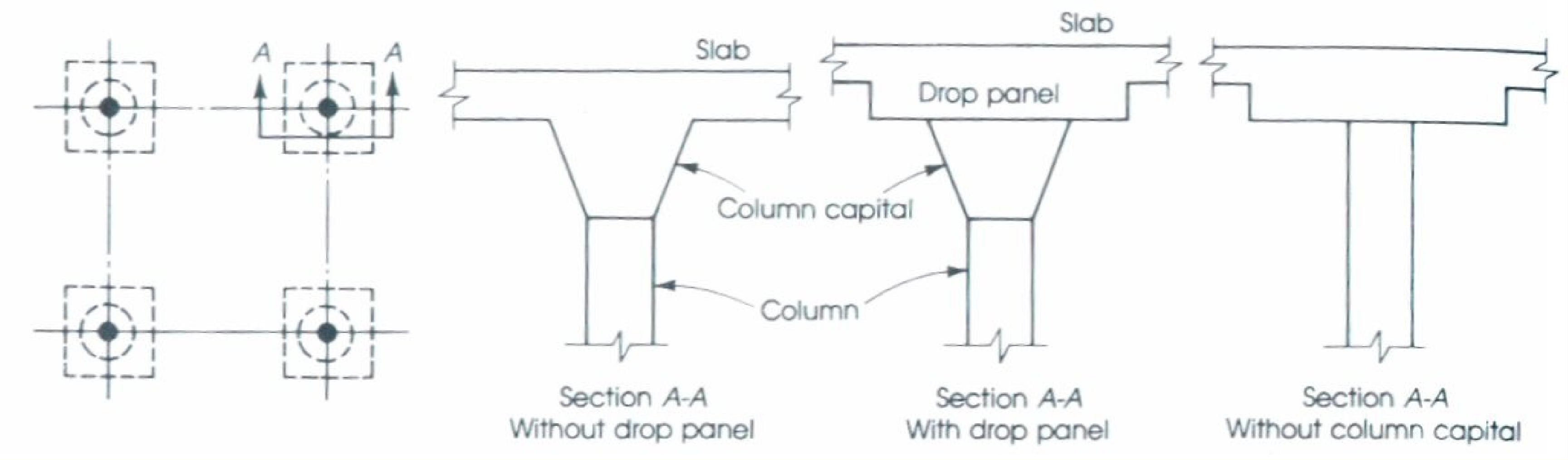
Flat slab



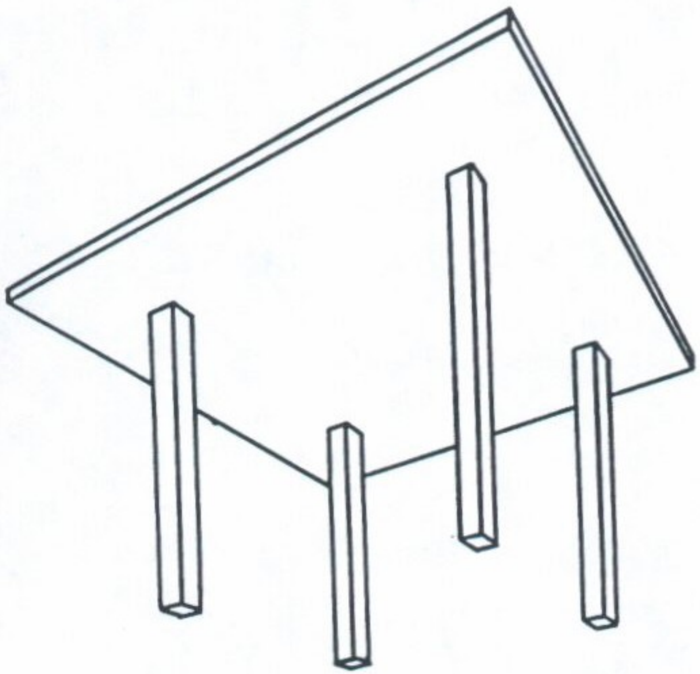
Two-way slab with beams

Comparison of One-way and Two-way slab behavior

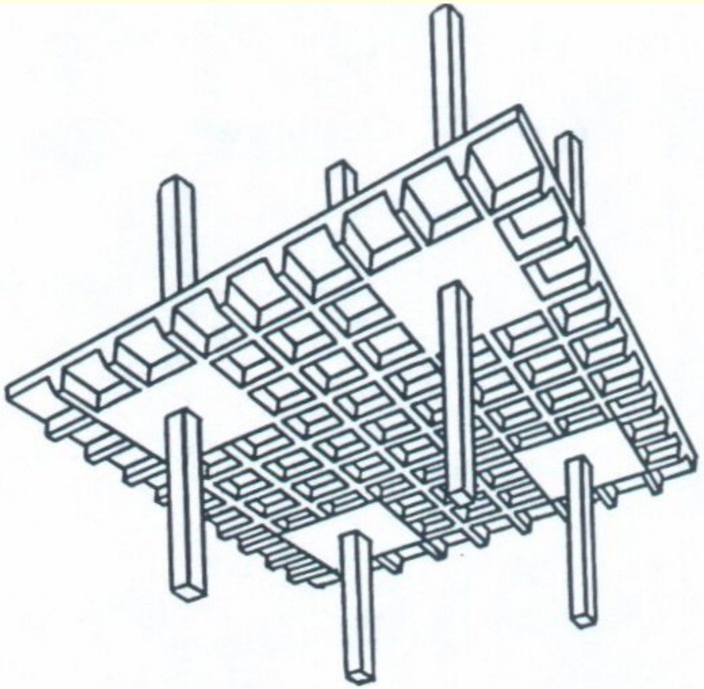
For flat plates and slabs the column connections can vary between:



Comparison of One-way and Two-way slab behavior

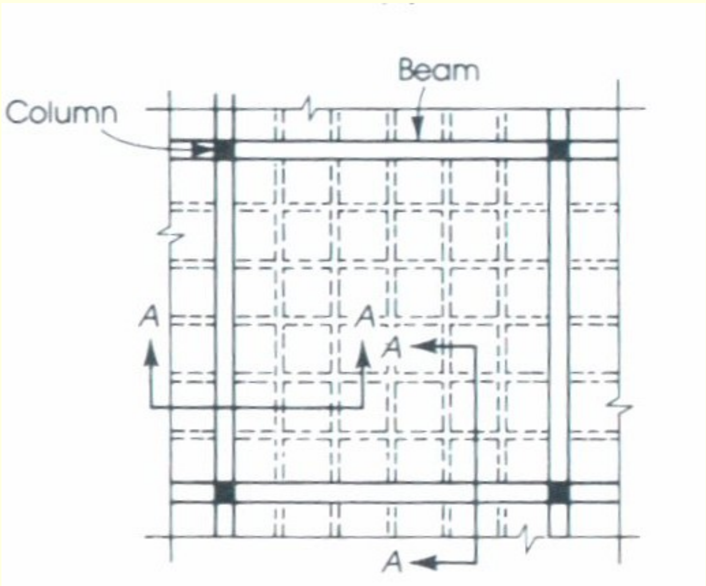


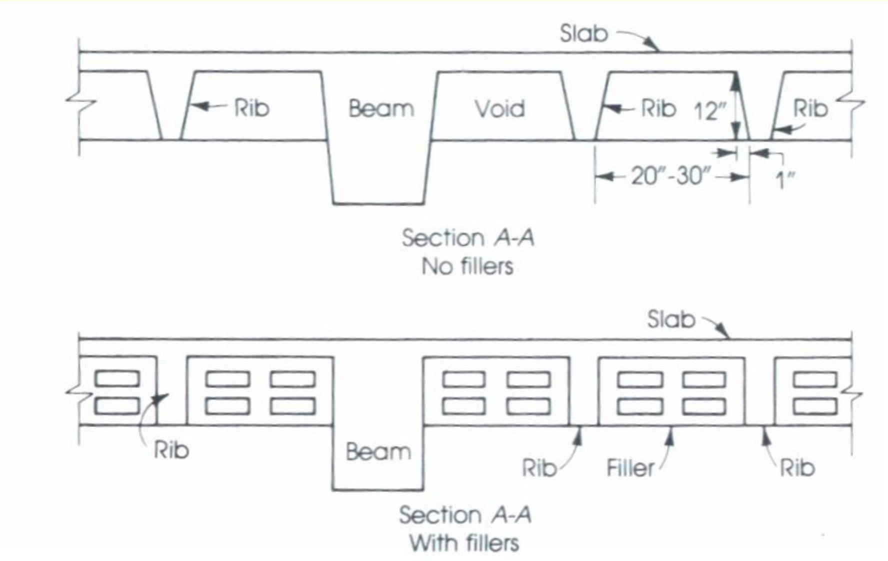
Flat Plate



Waffle slab

Comparison of One-way and Two-way slab behavior





The two-way ribbed slab and waffled slab system: General thickness of the slab is 5 to 10cm.

Comparison of One-way and Two-way slab behavior Economic Choices

- Flat Plate (for relatively light loads as in apartments or offices) suitable span 4.5m to 6.0m with LL= 3-5KN/m^2.

  Advantages

  - Low cost formwork
  - Exposed flat ceilings
  - Fast

  Disadvantages

  - Low shear capacity
  - Low Stiffness (notable deflection)

Comparison of One-way and Two-way slab behavior Economic Choices

- Flat Slab (for heavy industrial loads) suitable span 6 to 9m with LL= 5-7.5KN/m^2.

  Advantages

  - Low cost formwork
  - Exposed flat ceilings
  - Fast

  Disadvantages

  - Need more formwork for capital and panels

Comparison of One-way and Two-way slab behavior Economic Choices

- Waffle Slab (two-way joist system) suitable span 7.5m to 12m with LL= 4-7.5KN/m^2.

  Advantages

  - Carries heavy loads
  - Attractive exposed ceilings
  - Fast

  Disadvantages

  - Formwork with panels is expensive

Comparison of One-way and Two-way slab behavior Economic Choices

- One-way Slab on beams suitable span 3 to 6m with LL= 3-5KN/m^2.
  - Can be used for larger spans with relatively higher cost and higher deflections
- One-way joist system suitable span 6 to 9m with LL= 4-6KN/m^2.
  - Deep ribs, the concrete and steel quantities are relative low
  - Expensive formwork expected.

History of two-way slabs

- The feeling at start that only part of the load to be carried in each direction, so that statics somehow did not apply to slab construction.
- In 1914 Nichols used statics to compute the total moment in a slab panel. His analysis suggested that the current slab design (1914) underestimated the moments by 30 to 50%. He was subjected to attacks by scientists proportional to the amount of under-design in their favorite slab design systems (which were many at that time).
- Although Nichols analysis is correct, it was accepted as being correct in the mid 1920's and it was not until 1971 that the ACI code fully recognized it and required flat slabs to be designed for 100% of moments predicted from statics.

Behaviour of slabs loaded to failure p624-627

Comparison of One-way and Two-way slab behavior

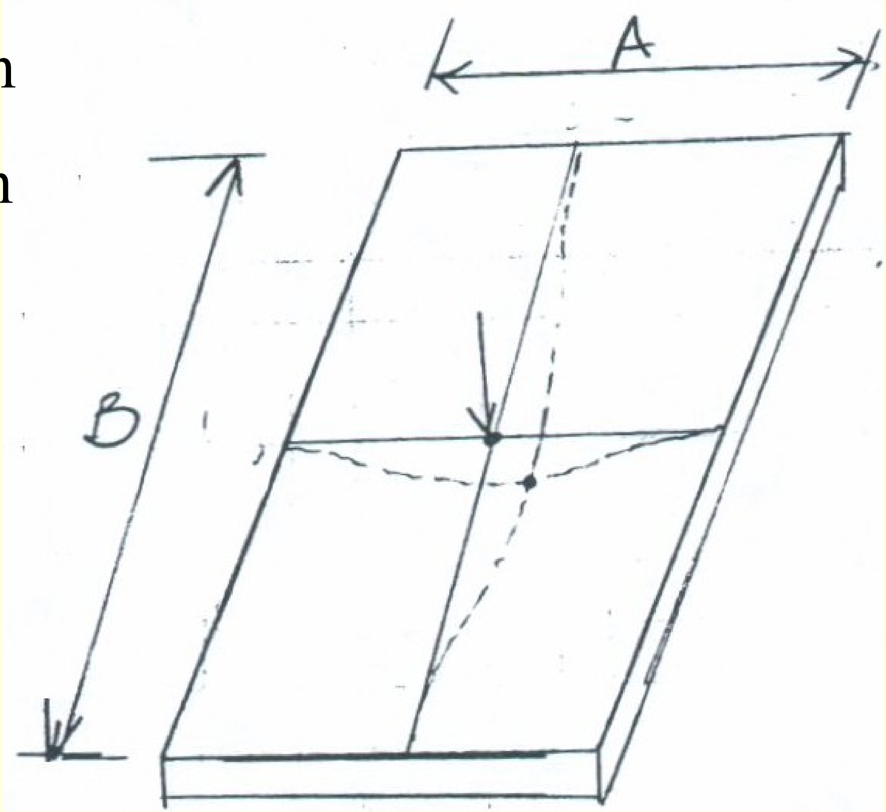
w_s =load taken by short direction

w_l = load taken by long direction

$$\delta_A = \delta_B$$

$$\frac{5 w_s A^4}{384 EI} = \frac{5 w_l B^4}{384 EI}$$

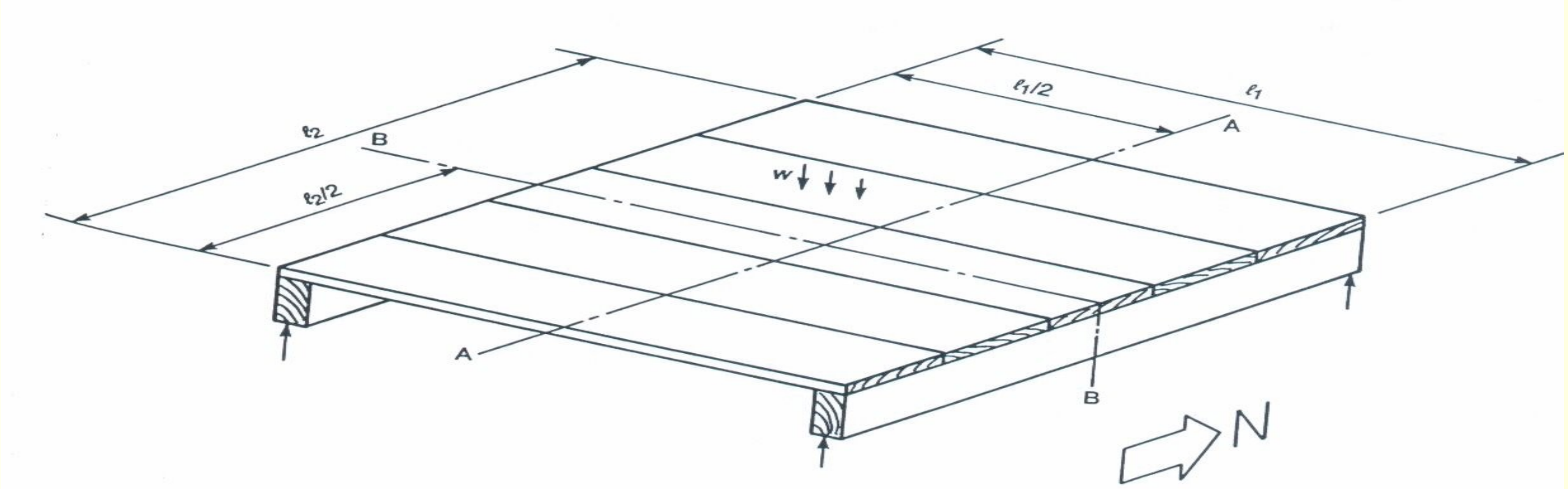
$$\frac{w_s}{w_l} = \frac{B^4}{A^4} \quad \text{For B} = 2\text{A} \Rightarrow w_s = 16 w_l$$



<u>Rule of Thumb</u>: For B/A > 2, design as one-way slab

Two-Way Slab Design

Static Equilibrium of Two-Way Slabs



Analogy of two-way slab to plank and beam floor

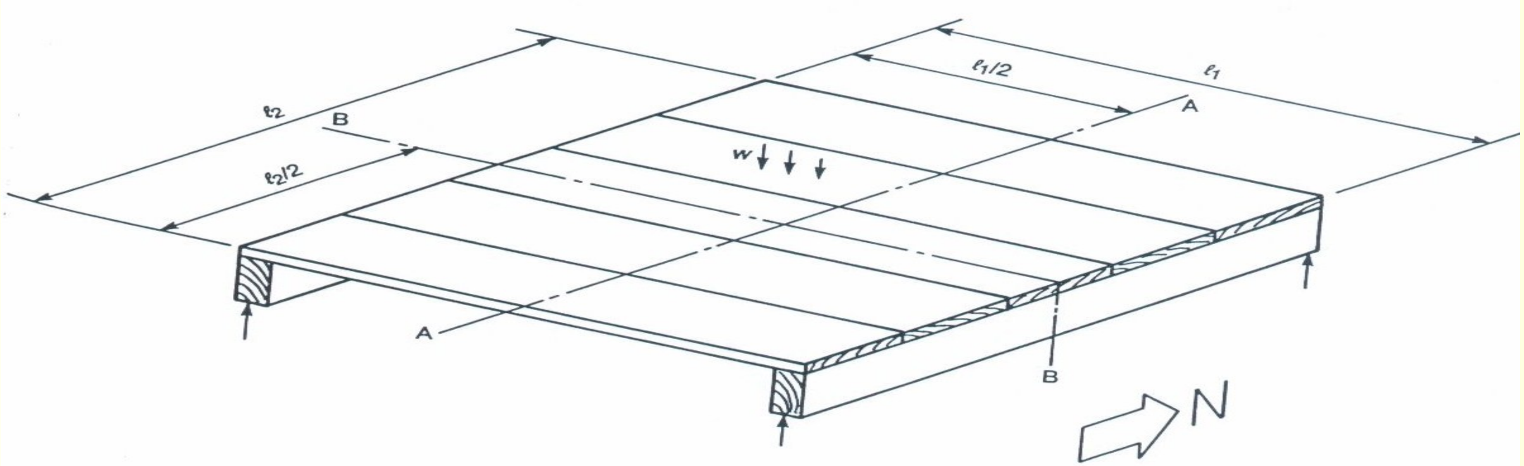
Section A-A:

Moment per m width in planks $\Rightarrow m = \dfrac{wl_1^2}{8}$ kN-m/m

Total Moment $\Rightarrow M_{\text{A-A}} = \left(wl_2\right)\dfrac{l_1^2}{8}$ kN-m

Two-Way Slab Design

Static Equilibrium of Two-Way Slabs



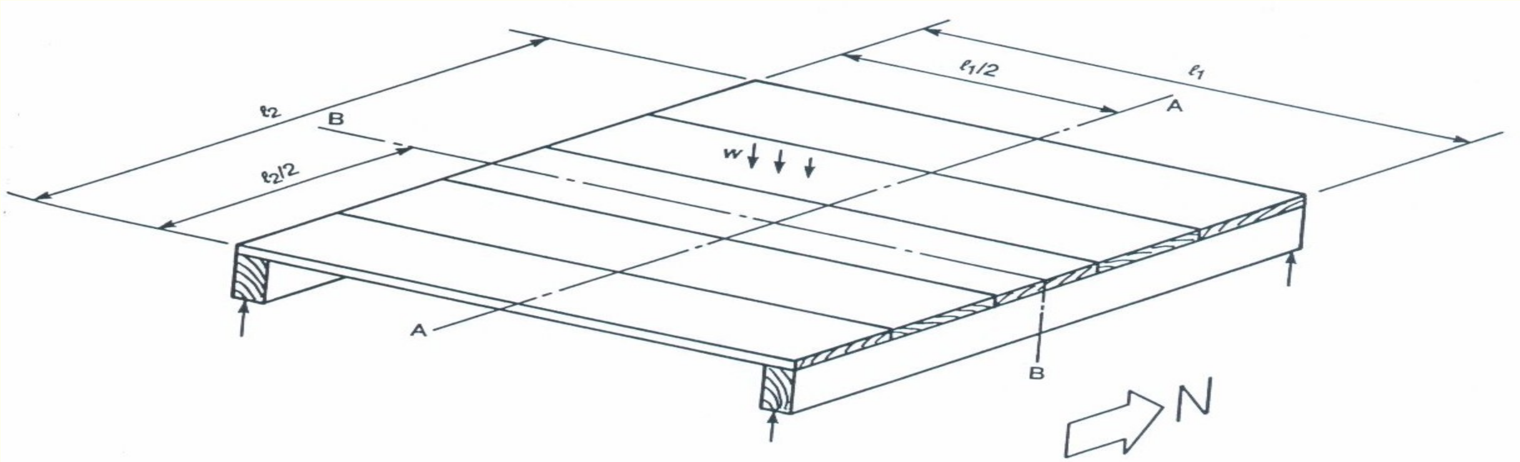
Analogy of two-way slab to plank and beam floor

Uniform load on each beam $\Rightarrow \dfrac{wl_1}{2}$ kN/m

Moment in one beam (Sec: B-B) $\Rightarrow M_{\text{lb}} = \left(\dfrac{wl_1}{2}\right)\dfrac{l_2^2}{8}$ kN.m

Two-Way Slab Design

Static Equilibrium of Two-Way Slabs



Total Moment in both beams

Full load was transferred east-west by the planks and then was transferred north-south by the beams; $\Rightarrow M_{B-B} = (wl_1)\dfrac{l_2^2}{8}$ kN.m

The same is true for a two-way slab or any other floor system where:

$$M_{\text{A-A}} = (wl_2)\frac{l_1^2}{8}\ \text{kN-m},\ M_{B-B} = (wl_1)\frac{l_2^2}{8}\ \text{kN.m}$$

Distribution of moments in slabs

- Read p631-637

General Design Concepts

13.5.1 — A slab system shall be designed by any procedure satisfying conditions of equilibrium and geometric compatibility, if shown that the design strength at every section is at least equal to the required strength set forth in 9.2 and 9.3, and that all serviceability conditions, including limits on deflections, are met.

General Design Concepts

(1) Direct Design Method (DDM)

Limited to slab systems to uniformly distributed loads and supported on equally spaced columns. Method uses a set of coefficients to determine the design moment at critical sections.

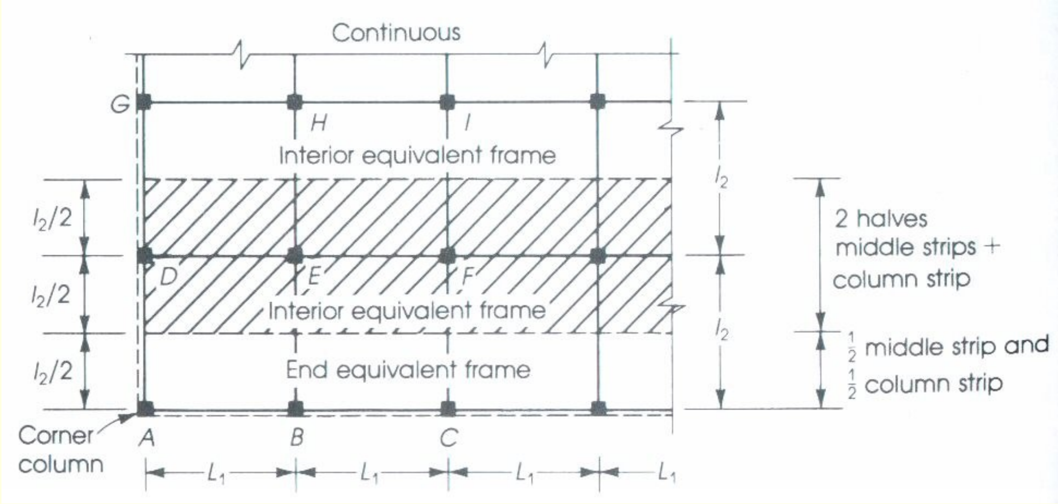
(2) Equivalent Frame Method (EFM)

A 3D building is divided into a series of 2D equivalent frames by cutting the building along lines midway between columns. The resulting frames are considered separately in the longitudinal and transverse directions of the building and treated floor by floor.

Equivalent Frames

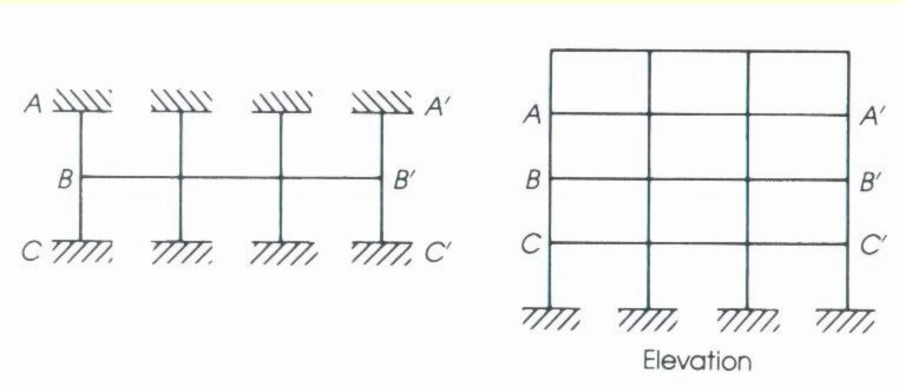
l_1= length of span in direction moments are being determined.

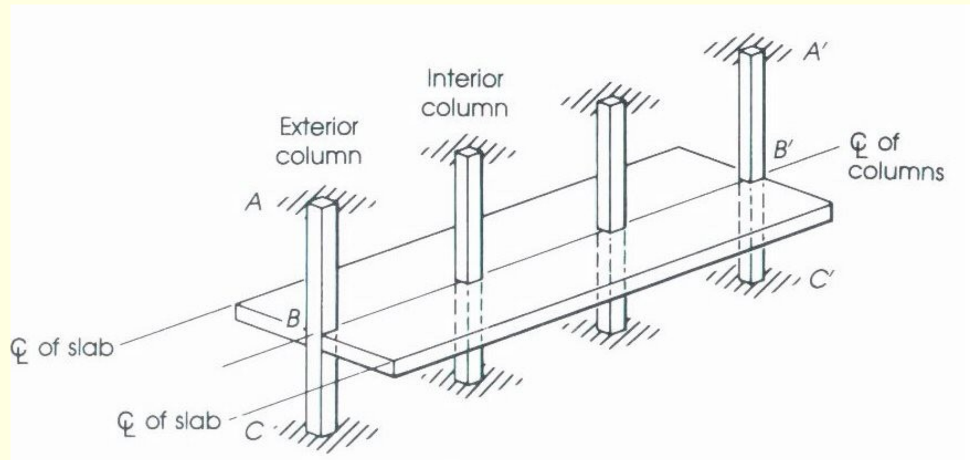
l_2= length of span transverse to l_1



Transverse equivalent frame

Equivalent Frame Method (EFM)





Elevation of the frame

Perspective view

Methods of Analysis

(1) Elastic Analysis

Concrete slab may be treated as an elastic plate. Use Timoshenko's method of analyzing the structure.
Finite element analysis

Methods of Analysis

(2) Plastic Analysis

The ***yield method*** used to determine the limit state of slab by considering the yield lines that occur in the slab as a collapse mechanism.

The ***strip method***, where slab is divided into strips and the load on the slab is distributed in two orthogonal directions and the strips are analyzed as beams.

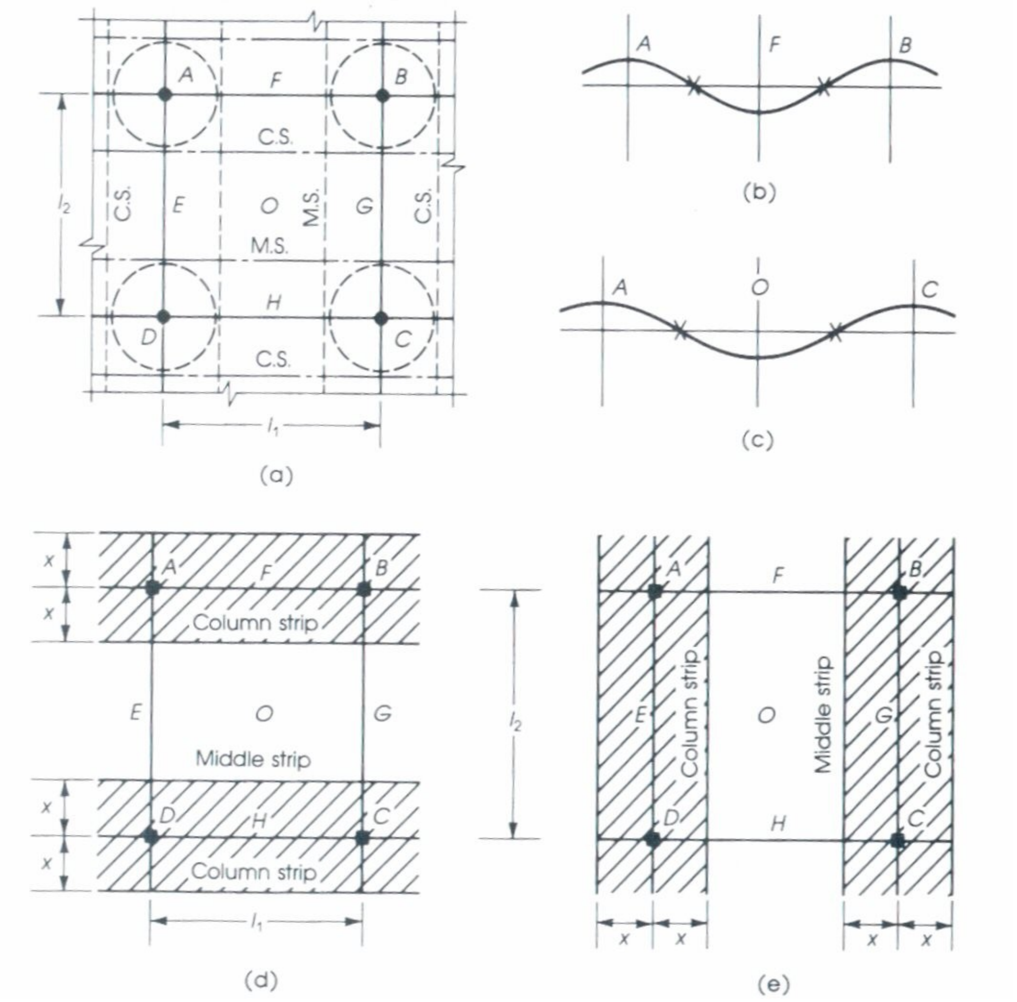
Methods of Analysis

(3) Nonlinear analysis

Simulates the true load-deformation characteristics of a reinforced concrete slab with finite-element method takes into consideration of nonlinearities of the stress-strain relationship of the individual members.

Column and Middle Strips

The slab is broken up into column and middle strips for analysis



Basic Steps in Two-way Slab Design

1. Choose layout and type of slab. Type of slab is strongly affected by architectural and construction considerations.
2. Choose slab thickness to control deflection. Also, check if thickness is adequate for shear.
3. Choose Design method
   - Equivalent Frame Method- use elastic frame analysis to compute positive and negative moments
   - Direct Design Method - uses coefficients to compute positive and negative slab moments

Basic Steps in Two-way Slab Design

4. Calculate positive and negative moments in the slab.
5. Determine distribution of moments across the width of the slab. Based on geometry and beam stiffness.
6. Assign a portion of moment to beams, if present.
7. Design reinforcement for moments from steps 5 and 6. Steps 3-7 need to be done for both principal directions.
8. Check shear strengths at the columns

Definition of Beam-to-Slab Stiffness Ratio, α

Accounts for stiffness effect of beams located along slab edge $\longrightarrow$ reduces deflections of panel adjacent to beams.

$$\alpha_f = \frac{\text{flexural stiffness of beam}}{\text{flexural stiffness of slab}}$$

Definition of Beam-to-Slab Stiffness Ratio, α

$$\alpha_f = \frac{4E_{cb}I_b / l}{4E_{cs}I_s / l} = \frac{E_{cb}I_b}{E_{cs}I_s}$$

E_{cb} = Modulus of elasticity of beam concrete

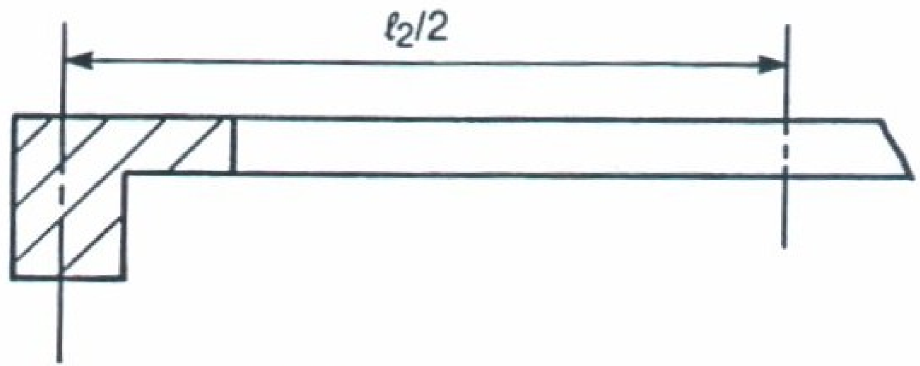
E_{sb} = Modulus of elasticity of slab concrete

I_b = Moment of inertia of uncracked beam

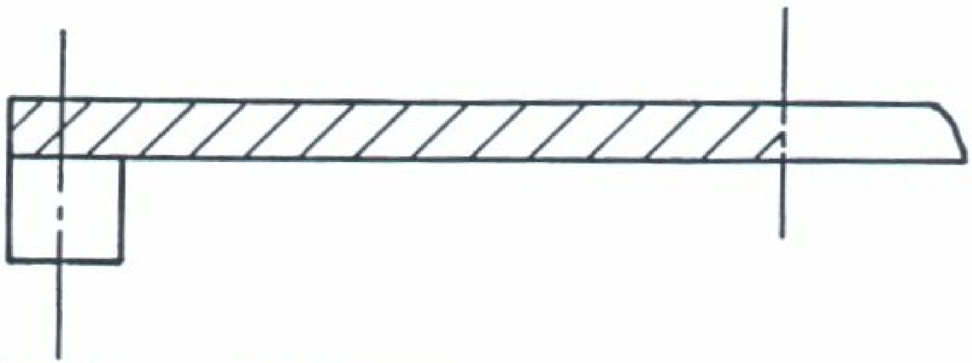
I_s = Moment of inertia of uncracked slab

With width bounded laterally by centerline of adjacent panels on each side of the beam.

Beam and Slab Sections for calculation of α

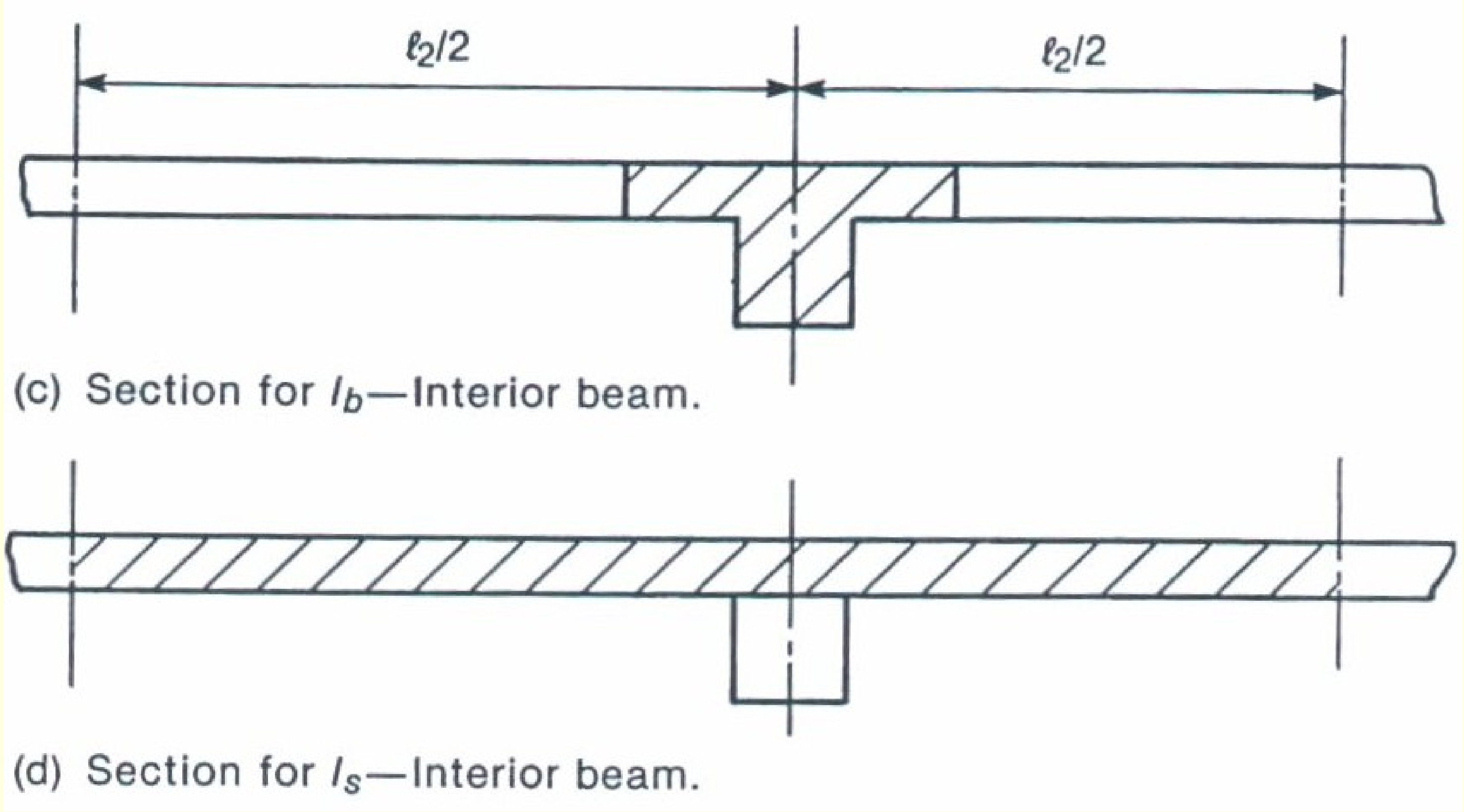


(a) Section for I_b—Edge beam.



(b) Section for I_s—Edge beam.

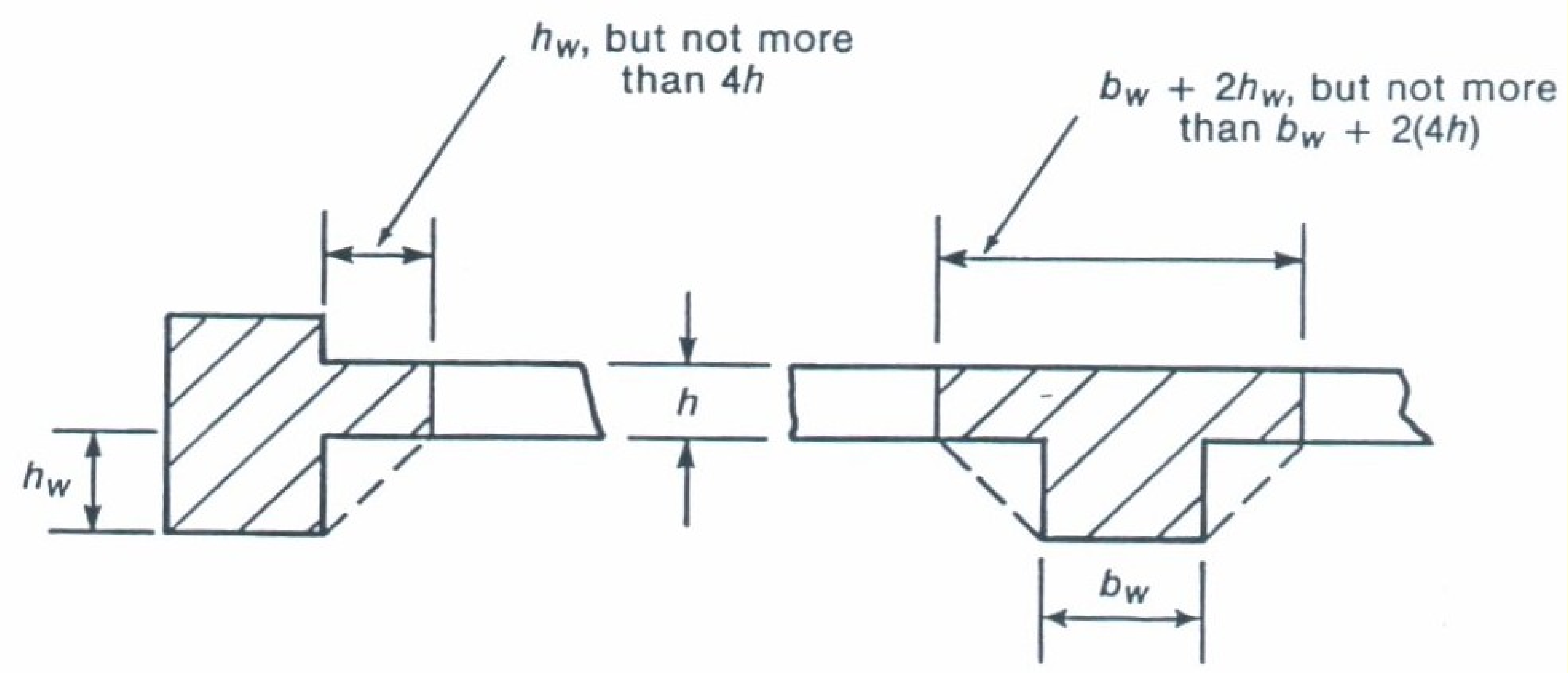
Beam and Slab Sections for calculation of α



(c) Section for I_b—Interior beam.

(d) Section for I_s—Interior beam.

Beam and Slab Sections for calculation of α



Definition of beam cross-section

Charts may be used to calculate α

Example 13-1

Minimum Slab Thickness for Two-way Construction

The ACI Code 9.5.3 specifies a minimum slab thickness to control deflection. There are three empirical limitations for calculating the slab thickness (h), which are based on experimental research. If these limitations are not met, it will be necessary to compute deflection.

Minimum Slab Thickness for Two-way Construction

(a) For $0.2 \le \alpha_f \le 2$

$$h = \frac{l_n \left(0.8 + \frac{f_y}{1400} \right)}{36 + 5\beta\left(\alpha_{fm} - 0.2\right)}$$

f_y in MPa. But not less than 12.5cm

Minimum Slab Thickness for Two-way Construction

(b) For $2 < \alpha_{\mathrm{fm}}$

$$h = \frac{l_{\mathrm{n}}\left(0.8 + \dfrac{f_{\mathrm{y}}}{1400}\right)}{36 + 9\beta}$$

f_{y} in MPa. But not less than 9cm.

Minimum Slab Thickness for 2-way Construction

(c) For $\alpha_{fm} < 0.2$

Use the following table 9.5(c)

Slabs without drop panels $t_{min} = 12.5cm$

Slabs with drop panels $t_{min} = 10cm$

TABLE 9.5(c)—MINIMUM THICKNESS OF SLABS WITHOUT INTERIOR BEAMS*

	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
f_y, MPa†	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

*For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.

†For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.

‡Drop panels as defined in 13.2.5.

§Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

Minimum Slab Thickness for two-way construction

The definitions of the terms are:

h = Minimum slab thickness without interior beams

l_n =Clear span in the long direction measured face to face of column

β = the ratio of the long to short clear span

α_{fm}=the average value of α for all beams on the sides of the panel.

Direct Design Method for Two-way Slab

Method of dividing total static moment M_o into positive and negative moments.

Limitations on use of Direct Design method ACI 13.6.1

1. Minimum of 3 continuous spans in each direction. (3 x 3 panel)
2. Rectangular panels with long span/short span $\leq$ 2

Direct Design Method for Two-way Slab

Limitations on use of Direct Design method

3. Successive span in each direction shall not differ by more than 1/3 the longer span.

4. Columns may be offset from the basic rectangular grid of the building by up to 0.1 times the span parallel to the offset.

5. All loads must be due to gravity only (N/A to unbraced laterally loaded frames, foundation mats or pre-stressed slabs)

6. Service (unfactored) live load $\leq$ 2 service dead load

Direct Design Method for Two-way Slab

Limitations on use of Direct Design method

7. For panels with beams between supports on all sides, relative stiffness of the beams in the 2 perpendicular directions.

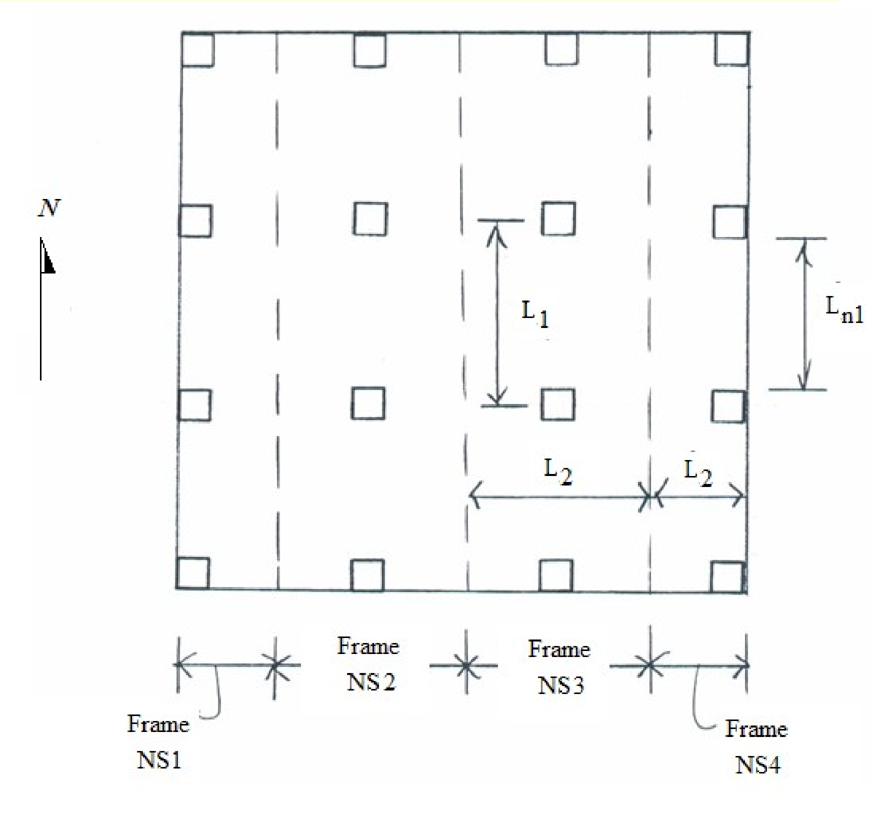
$$\frac{\alpha_{f1} l_2^2}{\alpha_{f2} l_1^2}$$

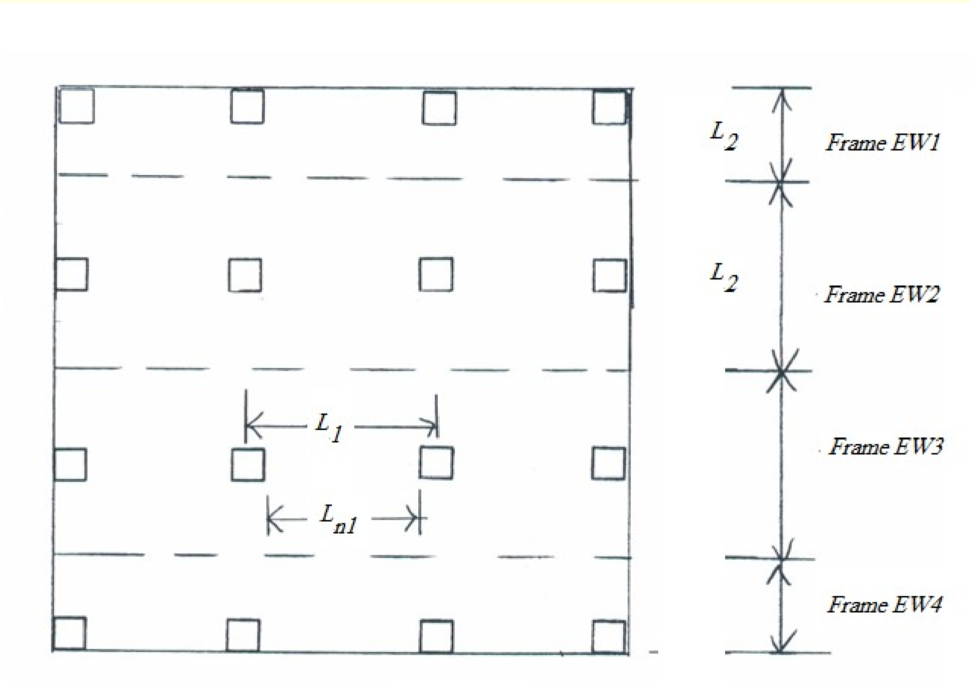
Shall not be less than 0.2 nor greater than 5.0

Limitations 2 and 7 do not allow use of DDM for slab panels that transmit loads as one way slabs.

Distribution of Moments

Slab is considered to be a series of frames in two directions:





Distribution of Moments

Total static Moment, M_o

$$M_0 = \frac{w_u l_2 l_n^2}{8} \quad (\text{ACI } 13\text{-}3)$$

where

w_u = factored load per unit area

l_2 = transverse width of the strip

l_n = clear span between columns

(for circular columns, calc. l_n using $h = 0.886 d_c$)

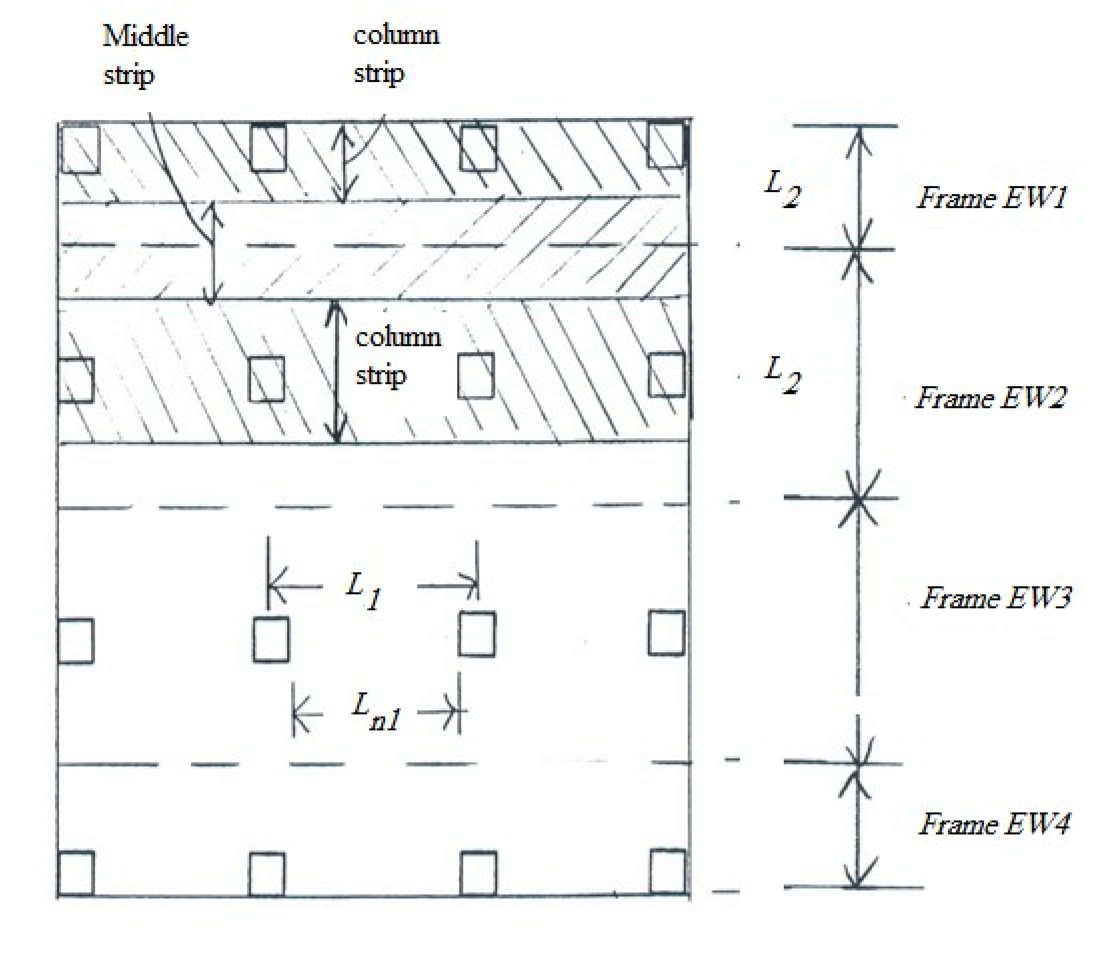
Example 13.2

Column Strips and Middle Strips

Moments vary across width of slab panel

∴ Design moments are averaged over the width of column strips over the columns & middle strips between column strips.

Column Strips and Middle Strips



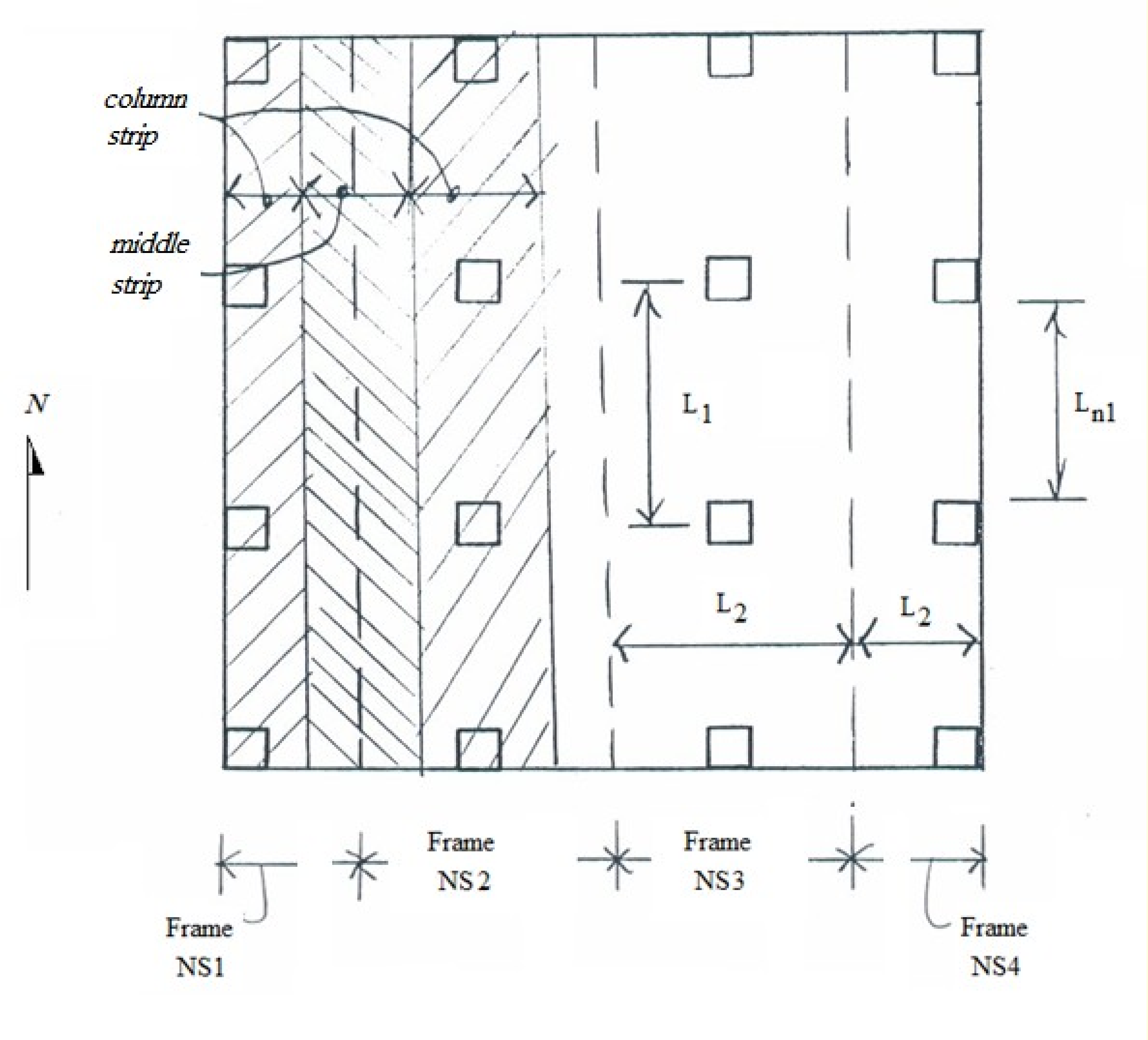
Column strips Design width on either side of a column centerline equal to smaller of $\begin{cases} 0.25\ l_2 \\ 0.25\ l_1 \end{cases}$

l_1= length of span in direction moments are being determined.

l_2= length of span transverse to l_1

Column Strips and Middle Strips

Middle strips: Design strip bounded by two column strips.

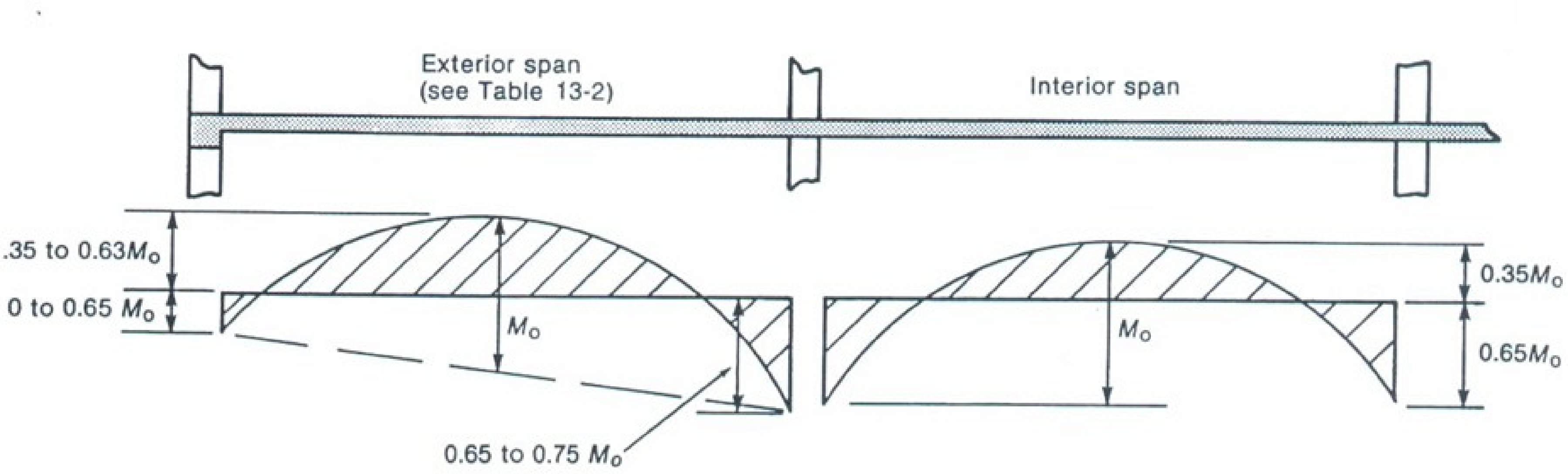


Positive and Negative Moments in Panels

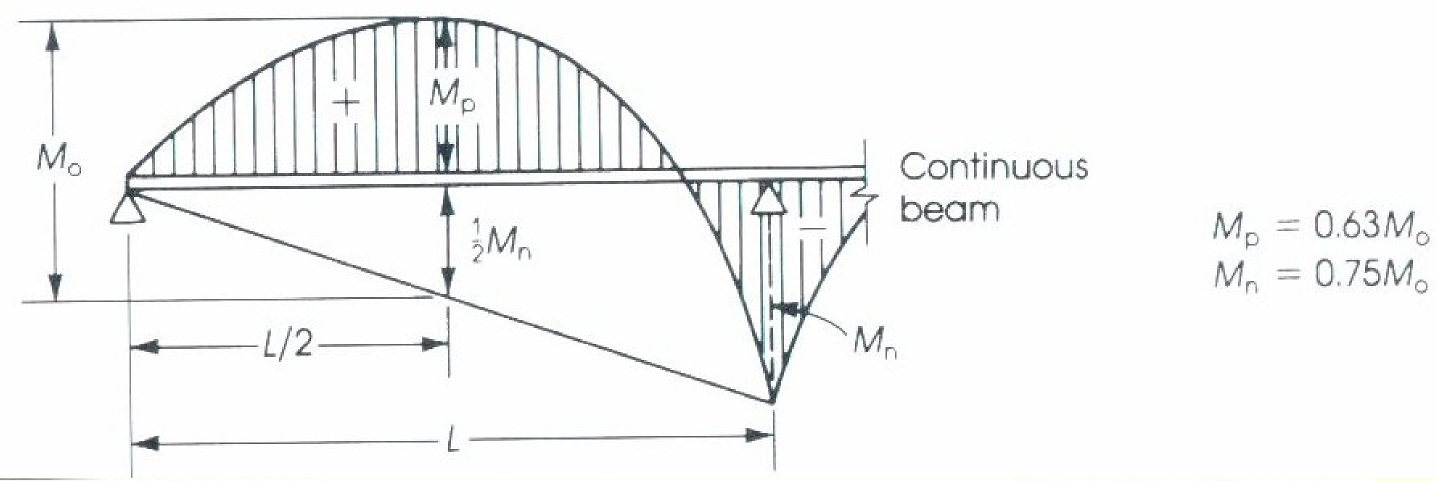
M_0 is divided into + M and -M Rules given in ACI 13.6.3

For a typical interior panel, the total static moment is divided into positive moment 0.35 M_o and negative moment of 0.65 M_o.

For an exterior panel, the total static moment division is dependent on the type of end conditions at the outside edge.



Moment Distribution



	Exterior Edge		Slab with Beams Between All Supports	Slab without Beams Between Interior Supports	
	Unrestrained (1)	***Fully Restrained (2)***	***(3)***	***With Edge Beam (4)***	***Without Edge Beam (5)***
Exterior negative factored moment	0	0.65	0.16	0.30	0.26
Positive factored moment	0.63	0.35	0.57	0.50	0.52
Interior negative factored moment	0.75	0.65	0.70	0.70	0.70

Distribution of M_0

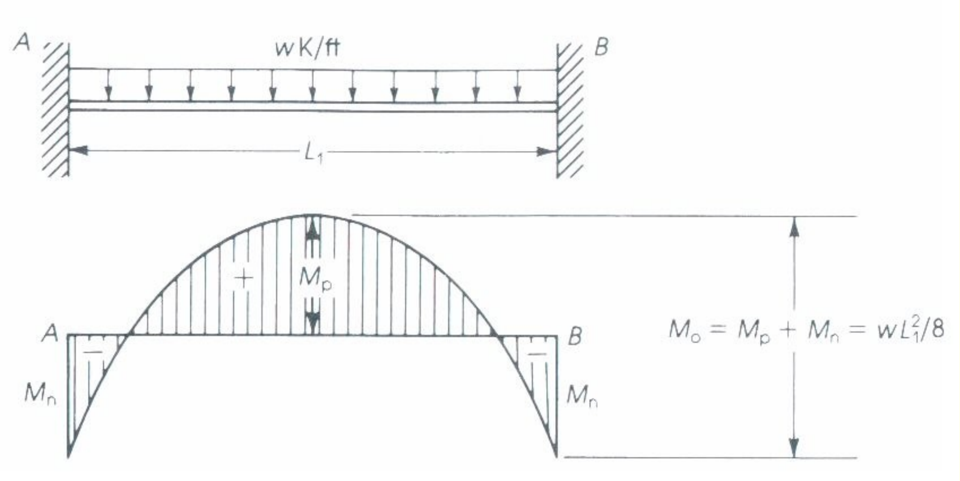
13.6.3.3 — In an end span, total factored static moment M_o shall be distributed as follows:

	(1)	(2)	(3)	(4)	(5)
	Exterior edge unre-strained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0	0.16	0.26	0.30	0.65

Positive and Negative Moments in Panels

M_0 is divided into + M and -M Rules given in ACI sec. 13.6.3

$$\left| \oplus M_u + \left| -M_{u(avg)} \right| \right| \geq M_0 = \frac{w_u l_2 l_n^2}{8}$$



Transverse Distribution of Moments

The longitudinal moment values mentioned are for the entire width of the equivalent building frame. The width of two half column strips and two half-middle stripes of adjacent panels.

Transverse Distribution of Moments

Transverse distribution of the longitudinal moments to middle and column strips is a function of the ratio of length $l_2/l_1, \alpha_{f1}$, and b_t.

$$\alpha_{f1} = \frac{E_{cb} I_b}{E_{cs} I_s} \qquad \beta_t = \frac{E_{cb} C}{2 E_{cs} I_s}$$

$$C = \sum \left(1 - \frac{0.63x}{y}\right)\left(\frac{x^3 y}{3}\right)$$ torsional constant

Distribution of M_0

ACI Sec 13.6.3.4

For spans framing into a common support negative moment sections shall be designed to resist the larger of the 2 interior M_u's

ACI Sec. 13.6.3.5

Edge beams or edges of slab shall be proportioned to resist in torsion their share of exterior negative factored moments

Factored Moment in Column Strip

13.6.4.1 — Column strips shall be proportioned to resist the following portions in percent of interior negative factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

13.6.4.4 — Column strips shall be proportioned to resist the following portions in percent of positive factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

Factored Moment in Column Strip

β_t= Ratio of torsional stiffness of edge beam to flexural stiffness of slab(width= to beam length)

13.6.4.2 — Column strips shall be proportioned to resist the following portions in percent of exterior negative factored moments:

ℓ_2/ℓ_1		0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

Factored Moments

Factored Moments in beams (ACI Sec. 13.6.3): resist a percentage of column strip moment plus moments due to loads applied directly to beams.

13.6.5 — Factored moments in beams

13.6.5.1 — Beams between supports shall be proportioned to resist 85 percent of column strip moments if $\alpha_{f1}\ell_2/\ell_1$ is equal to or greater than 1.0.

13.6.5.2 — For values of $\alpha_{f1}\ell_2/\ell_1$ between 1.0 and zero, proportion of column strip moments resisted by beams shall be obtained by linear interpolation between 85 and zero percent.

13.6.5.3 — In addition to moments calculated for uniform loads according to 13.6.2.2, 13.6.5.1, and 13.6.5.2, beams shall be proportioned to resist all moments caused by concentrated or linear loads applied directly to beams, including weight of projecting beam stem above or below the slab.

Factored Moments

Factored Moments in Middle strips (ACI Sec. 13.6.3)

The portion of the + M_u and - M_u not resisted by column strips shall be proportionately assigned to corresponding half middle strips.

Each middle strip shall be proportioned to resist the sum of the moments assigned to its 2 half middle strips.

Example 13-3 and 13-4 page 650

ACI Provisions for Effects of Pattern Loads

13.7.6.2 — When the unfactored live load is variable but does not exceed three-quarters of the unfactored dead load, or the nature of live load is such that all panels will be loaded simultaneously, it shall be permitted to assume that maximum factored moments occur at all sections with full factored live load on entire slab system.

Transfer of moments to columns

Exterior columns shall be designed for $0.3M_o$ assumed to be about the centroid of the shear perimeter, and should be divided between columns above and below the slab in proportion to their stiffnesses.

13.6.3.6 — The gravity load moment to be transferred between slab and edge column in accordance with 13.5.3.1 shall be $\mathbf{0.3}\boldsymbol{M_o}$.

Interior Columns: refer to textbook page 657

Shear Strength of Slabs

In two-way floor systems, the slab must have adequate thickness to resist both bending moments and shear forces at critical section. There are three cases to look at for shear.

1. Two-way Slabs supported on beams
2. Two-Way Slabs without beams
3. Shear Reinforcement in two-way slabs without beams.

Shear Strength of Slabs

1. Two-way slabs supported on beams

The critical location is found at d distance from the column, where

$$\phi V_c = \phi\left(\sqrt{f_c}\, bd / 6\right)$$

The supporting beams are stiff $(\alpha_{f1} \ell_2 / \ell_1) \geq 1.0$

and are capable of transmitting floor loads to the columns.

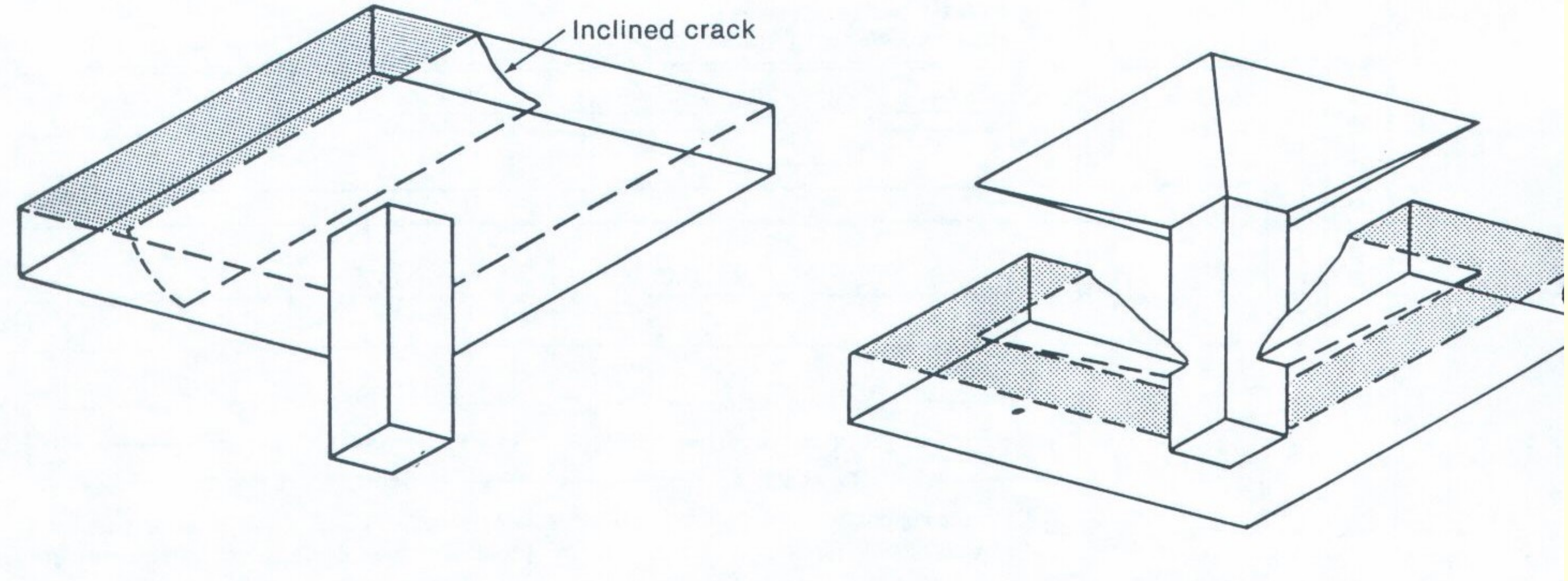
Shear Strength of Slabs

2. Two-Way Slabs without beams

There are two types of shear that need to be addressed

1. One-way shear or beam shear at distance d from the column

2. Two-way or punch out shear which occurs along a truncated cone.



(a) One-way shear.

(b) Two-way shear.

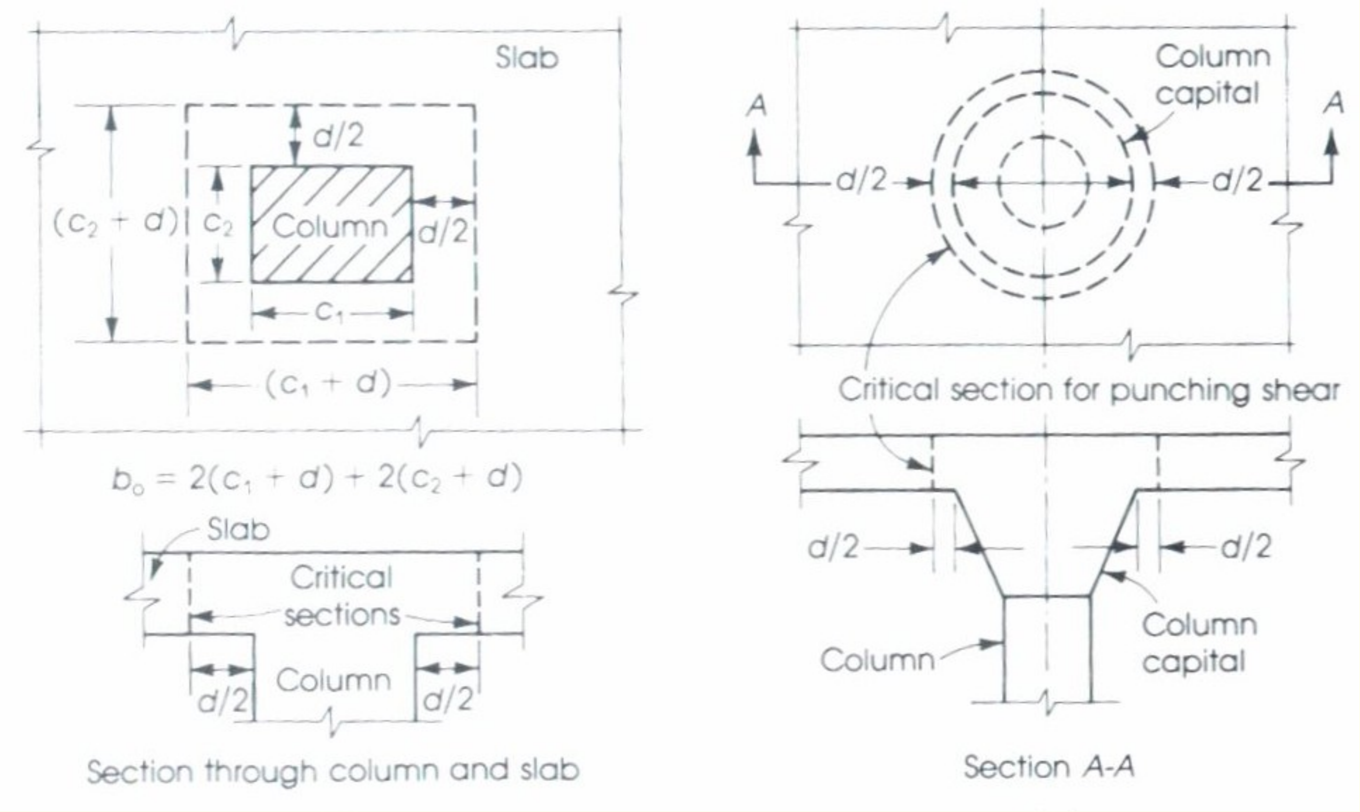
Shear Strength of Slabs

One-way shear considers critical section a distance d from the column and the slab is considered as a wide beam spanning between supports.

$$V_{\mathrm{ud}} \leq \phi\, V_{\mathrm{c}} = \phi\left(\sqrt{f_{\mathrm{c}}'}\; bd / 6\right)$$

Shear Strength of Slabs

Two-way shear fails along a a truncated cone or pyramid around the column. The critical section is located d/2 from the column face, column capital, or drop panel.



Shear Strength of Slabs

If shear reinforcement is not provided, the shear strength of concrete is the smaller of:

1. $$\phi V_{c} = \phi 0.17\left(1+\frac{2}{\beta_{c}}\right)\sqrt{f_{c}}\; b_{o}d \leq \phi\left(0.33\sqrt{f_{c}}\; b_{o}d\right)$$

b_o = perimeter of the critical section

β_c =ratio of long side of column to short side

2. $$\phi V_{c} = \phi 0.083\left(\frac{\alpha_{s}d}{b_{o}}+2\right)\sqrt{f_{c}}\; b_{o}d$$

α_s is 40 for interior columns, 30 for edge columns, and 20 for corner columns.

Shear Strength of Slabs

3. Shear Reinforcement in two-way slabs without beams.

For plates and flat slabs, which do not meet the condition for shear, one can either

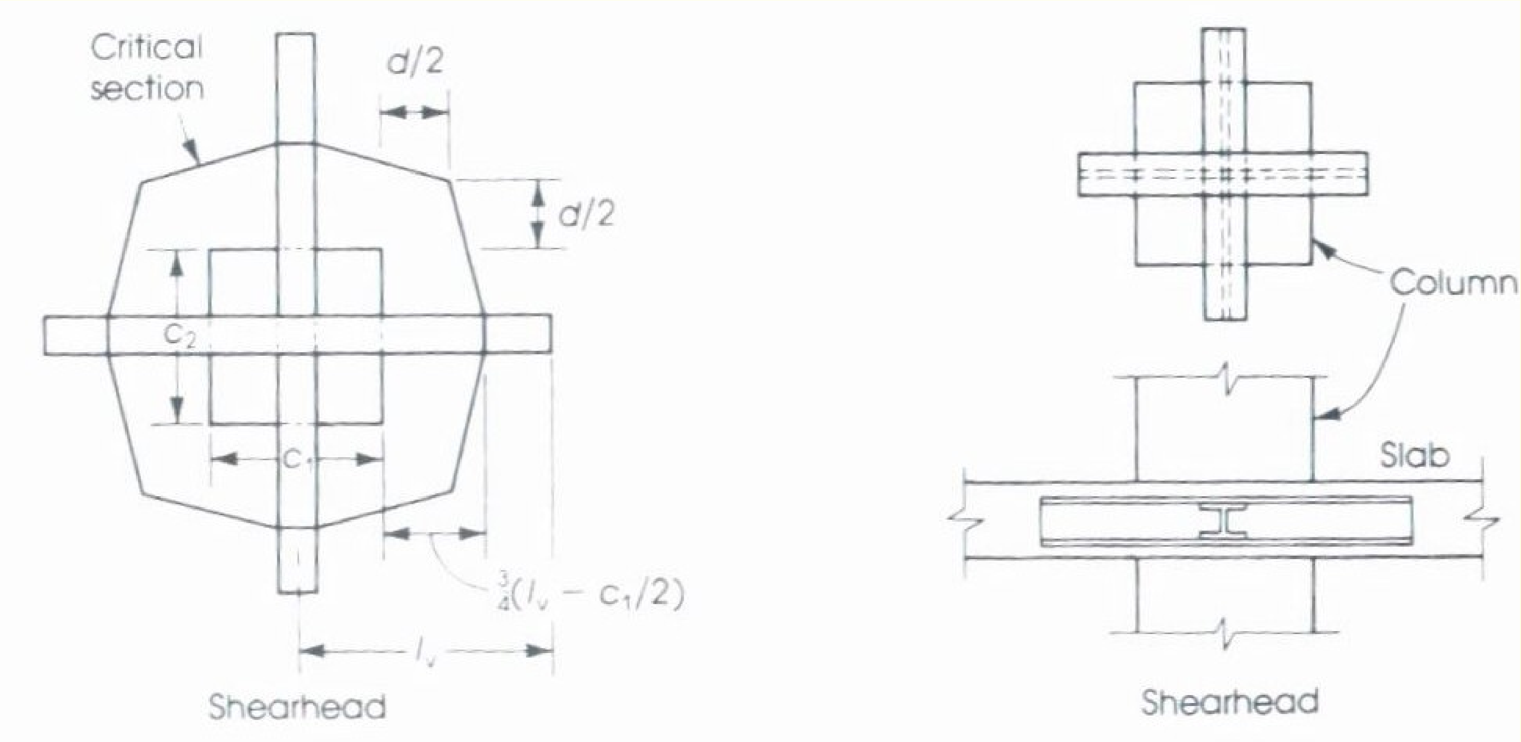
-Increase slab thickness

- Add reinforcement

Reinforcement can be done by shearheads, anchor bars, conventional stirrup cages and studded steel strips (see ACI 11.11.4.

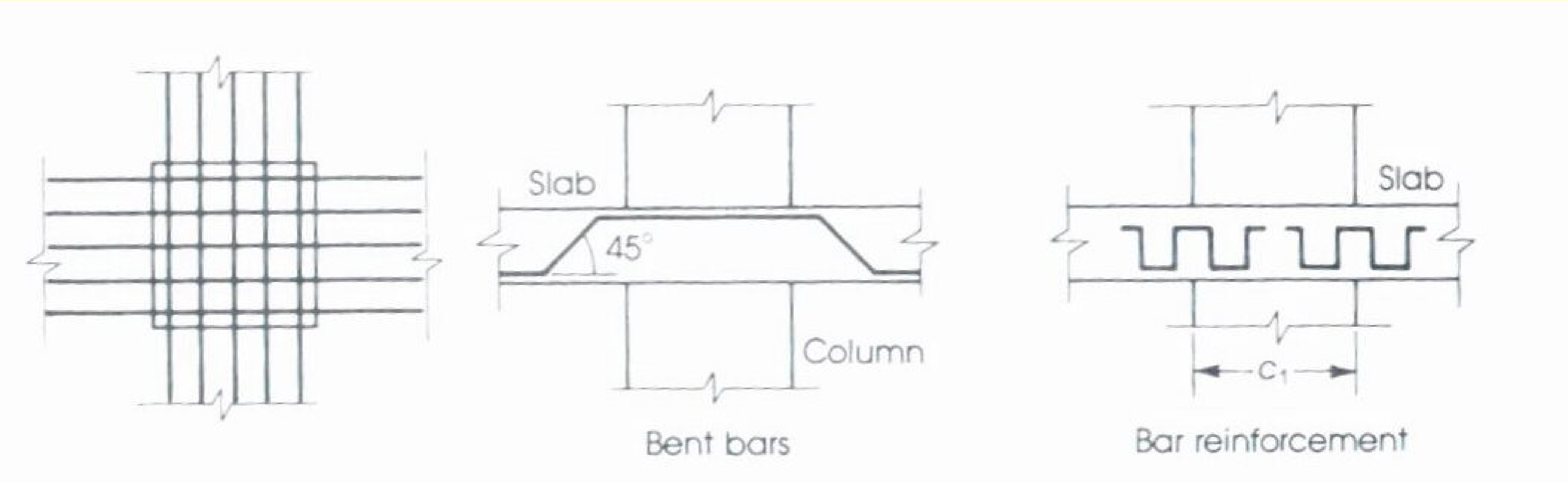
Shear Strength of Slabs

Shearhead consists of steel I-beams or channel welded into four cross arms to be placed in slab above a column. Does not apply to external columns due to lateral loads and torsion.



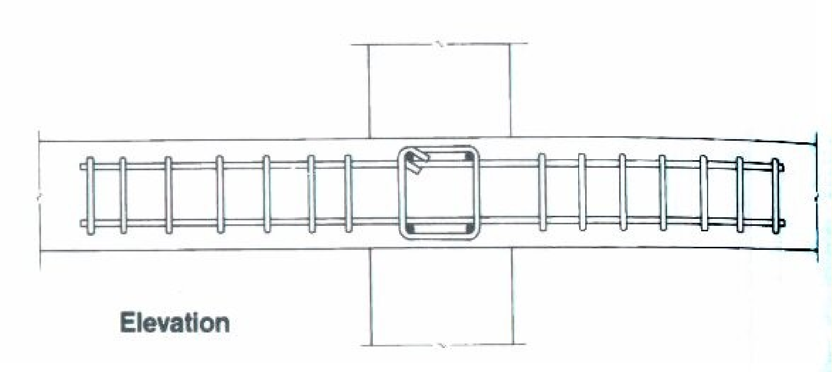
Shear Strength of Slabs

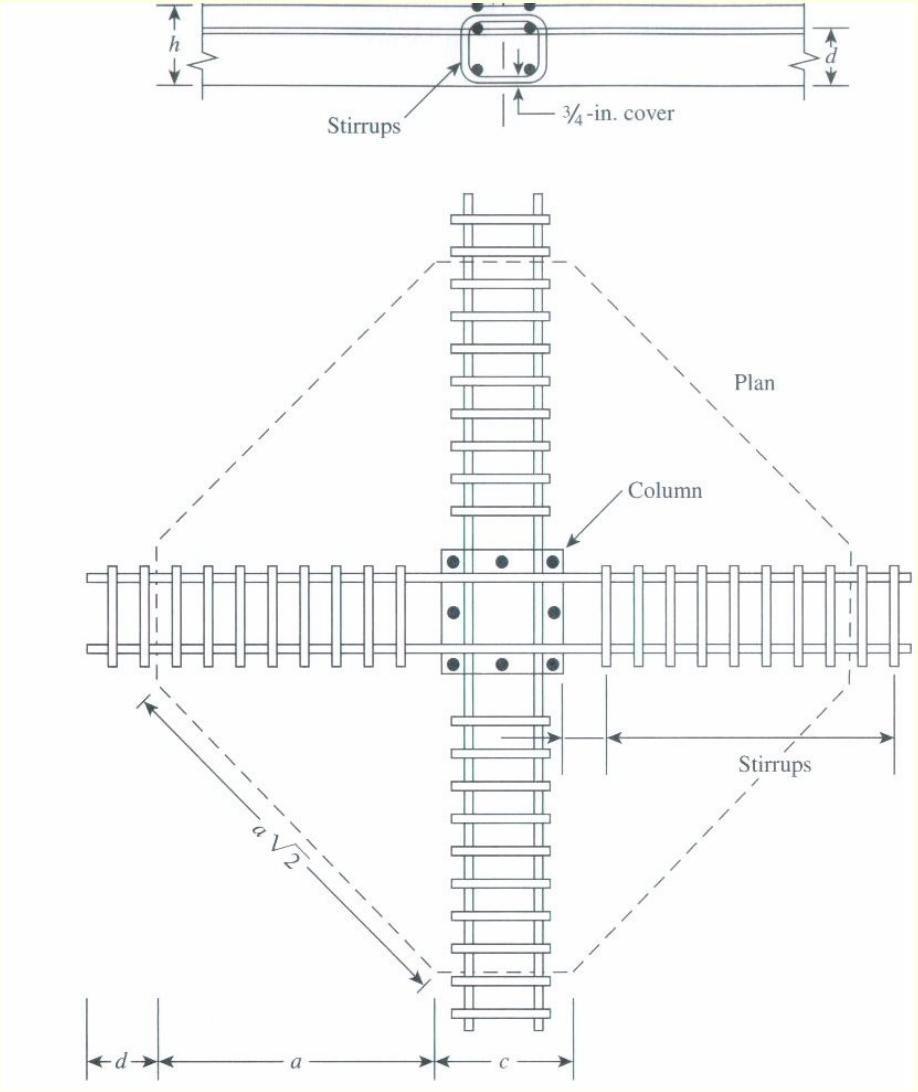
Anchor bars consists of steel reinforcement rods or bent bar reinforcement



Shear Strength of Slabs

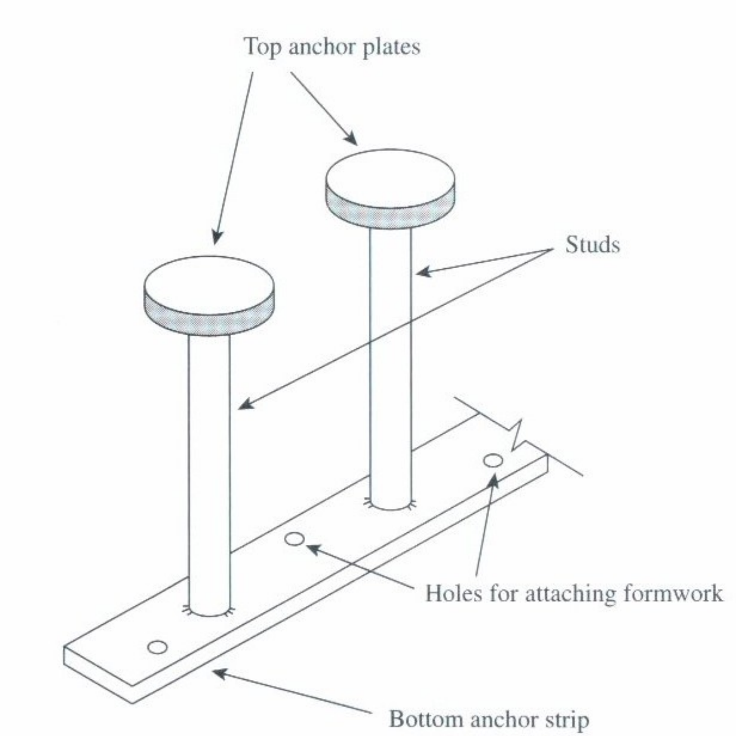
Conventional stirrup cages

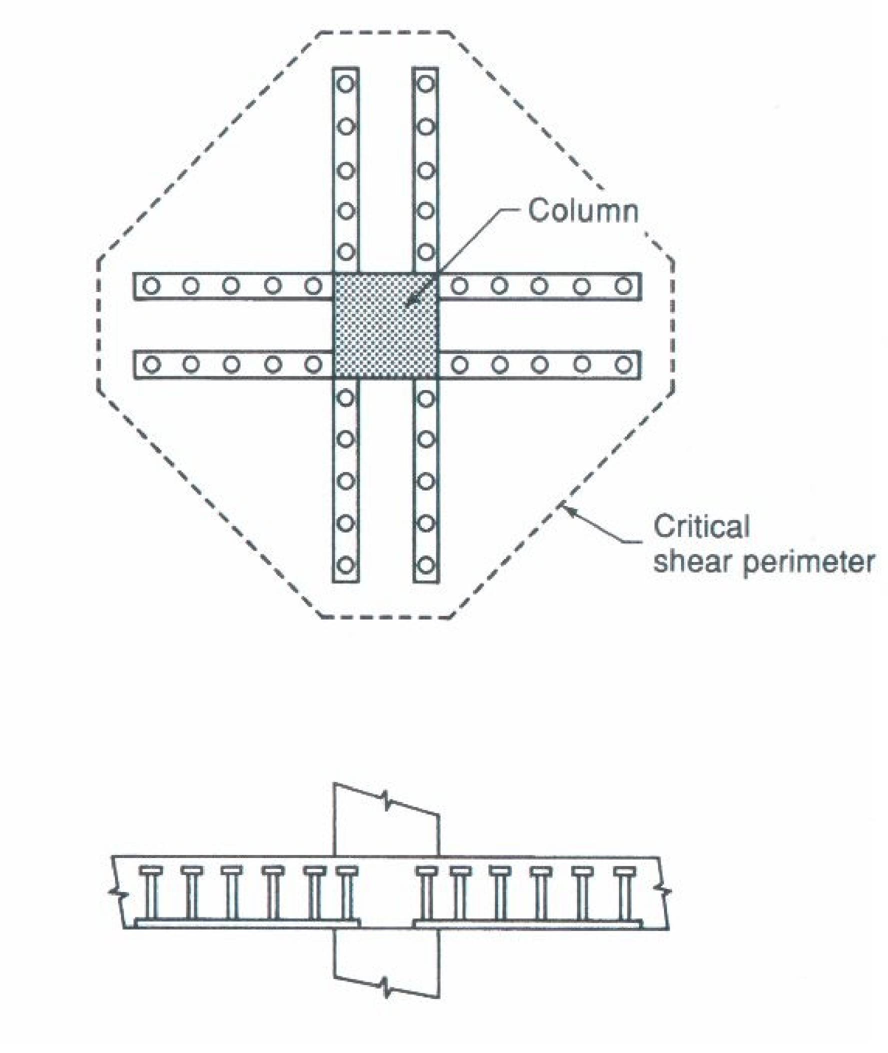




Shear Strength of Slabs

Studded steel strips





Reinforcement Details Loads

After all percentages of the static moments in the column and middle strip are determined, the steel reinforcement can be calculated for negative and positive moments in each strip.

Maximum Spacing of Reinforcement

At points of max. +/- M: $s \leq 2t \quad (\text{ACI } 13.3.2)$

and $s \leq 45cm \quad (\text{ACI } 7.12.3)$

Min Reinforcement Requirements

$$A_{\text{s(min)}} = A_{\text{s(T\&S)}} \text{ from ACI 7.12 } (\text{ACI } 13.3.1)$$

Minimum extension for reinforcement in slabs without beams(Fig. 13.3.8)