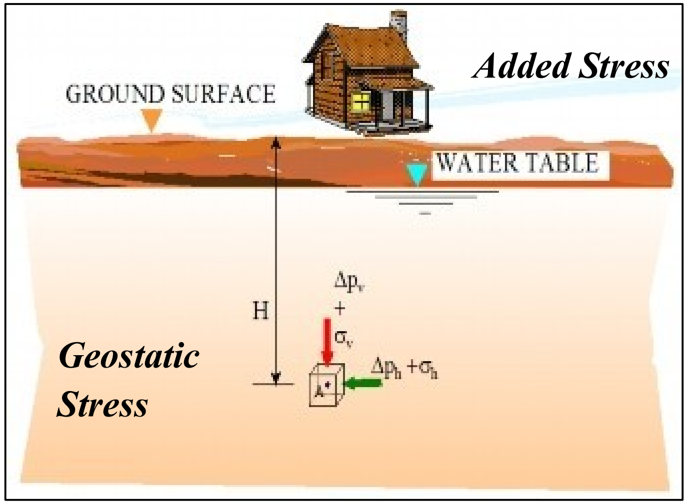
Stresses in a Soil Mass

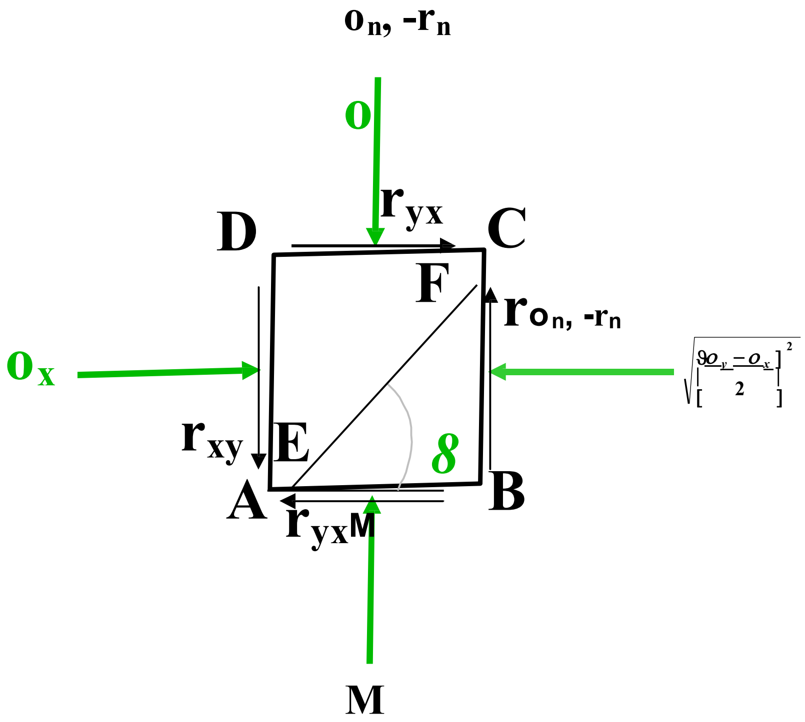
Topics

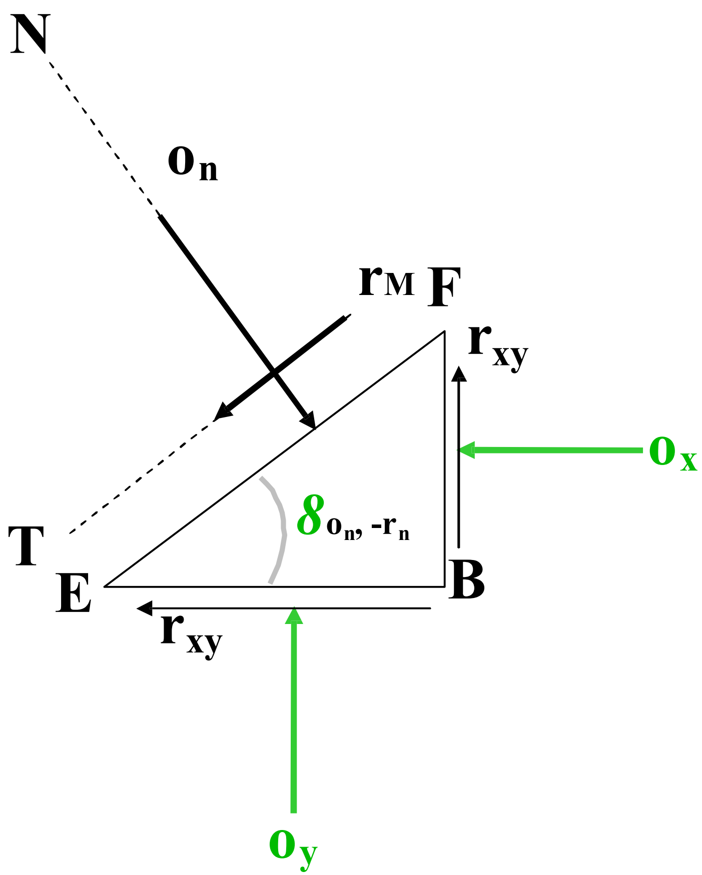


- **Normal and Shear Stresses on a Plane**
- **Stress distribution in soils**
- **Stress Caused by a Point Load**
- **Vertical Stress Caused by a Line Load**
- **Vertical Stress Caused by a Strip Load**
- **Vertical Stress Due to Embankment Loading**
- **Vertical Stress below the Center of a uniformly Loaded Circular Area**
- **Vertical Stress at any Point below a uniformly Loaded Circular Area**
- **Vertical Stress Caused by a Rectangularly Loaded Area**
- **Influence Chart for Vertical Pressure (Newmark Chart)**
- **Approximate methods**

• Normal and Shear Stresses on a Plane

$o_{Y} > o_{X}$





From geometry for the free body diagram EBF

$$\overline{EB} = \overline{EF}\cos\theta$$

$$\overline{FB} = \overline{EF}\sin\theta$$

Summing forces in N and T direction, we have

$$\sigma_n(\overline{EF}) = \sigma_x(\overline{EF})\sin^2\theta + \sigma_y(\overline{EF})\cos^2\theta + 2\tau_{xy}(\overline{EF})\sin\theta\cos\theta$$

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Again

$$\tau_n(\overline{EF}) = -\sigma_x(\overline{EF})\sin\theta\cos\theta + \sigma_y(\overline{EF})\sin\theta\cos\theta - \tau_{xy}(\overline{EF})\cos^2\theta + \tau_{xy}(\overline{EF})\sin^2\theta$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

If $\tau_n = 0$ then

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

This eq. gives 2 values of θ that are 90^{o} apart, this means that there are 2 planes that are right angles to each other on which shear stress = 0, such planes are called *principle planes* and the normal stress that act on the principle planes are to as *principle stresses*.

To find the principle stress substitute eq.3 into eq.1, we get

$$\sigma_n = \sigma_1 = \frac{\sigma_y + \sigma_x}{2} + \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2} \qquad \textbf{\textit{major principle stress}}$$

$$\sigma_n = \sigma_3 = \frac{\sigma_y + \sigma_x}{2} - \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2} \qquad \textbf{min}\textbf{\textit{or principle stress}}$$

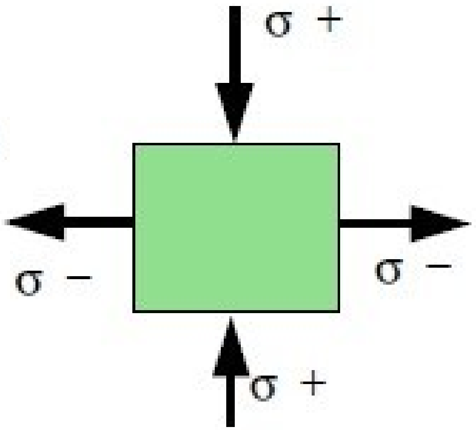
These stresses on any plane can be found using *Mohr's circle*

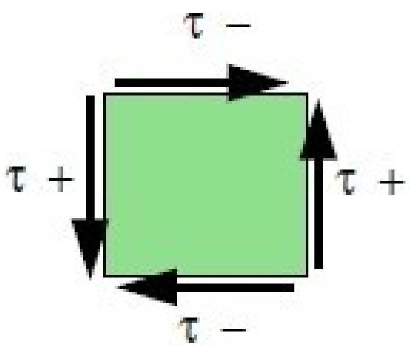
∅ ***Mohr's circle***

Mohr's Circle Sign Conventions:

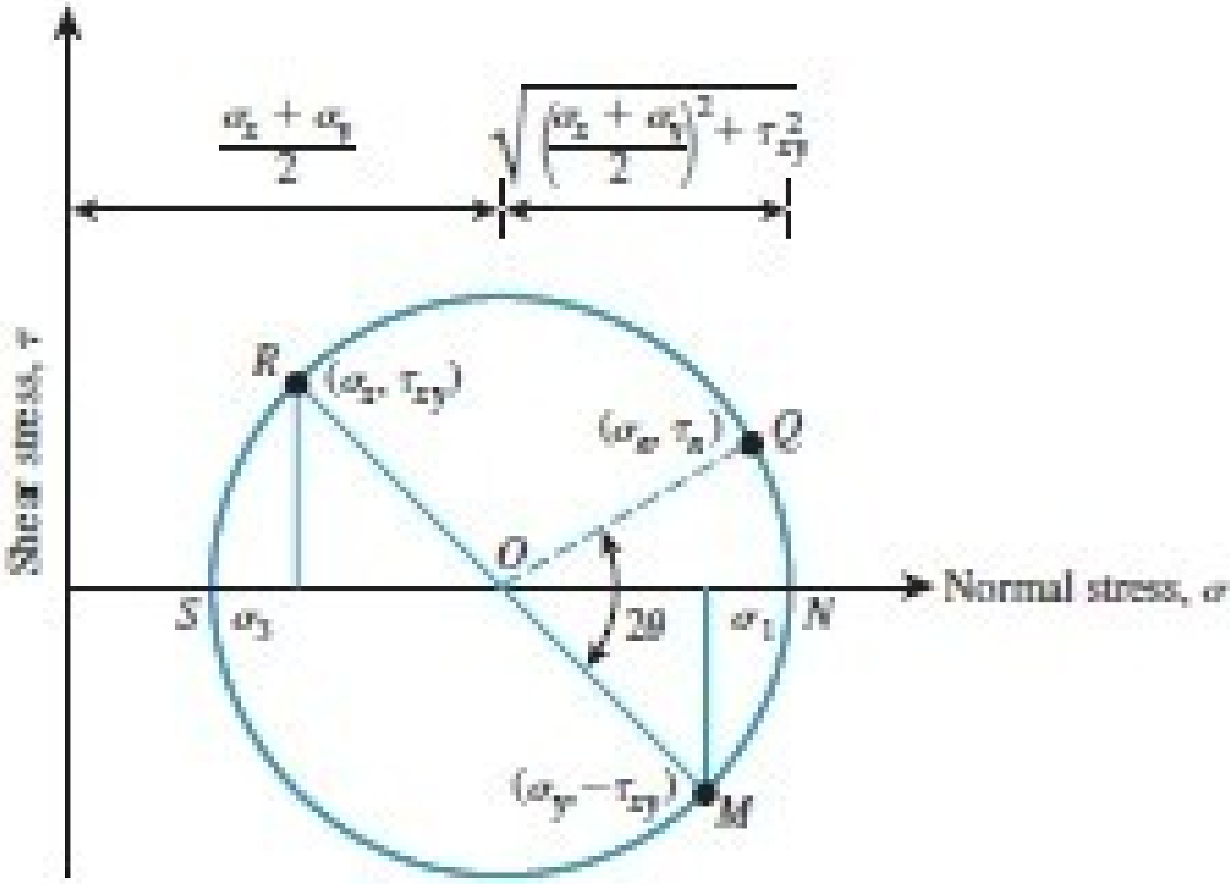
- Compressive normal stresses are positive

- Shear stresses are positive, if when they act on two opposing faces, they tend to produce a counterclockwise rotation.

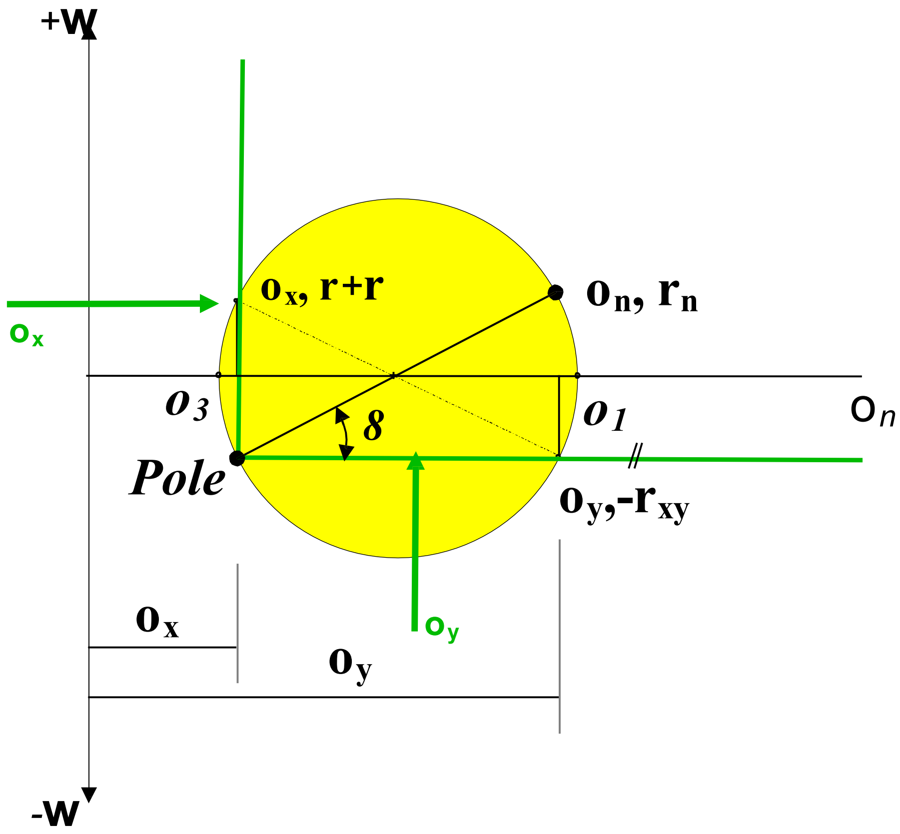




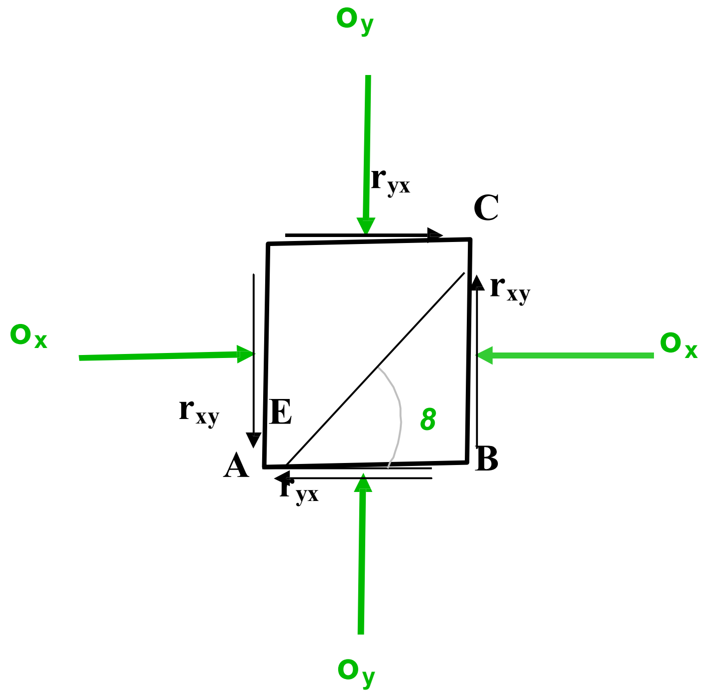
Refer to the element shown in Fig. above



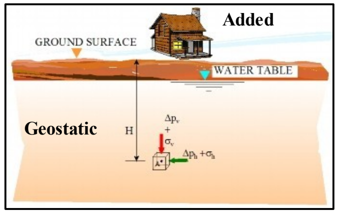
∅ ***Pole Method***



+w
ox, r+r
on, rn
ox
o3
o1
On
8
Pole
oy,-rxy
oy
ox
oy
-w



oy
ryx
C
rxy
ox
ox
rxy
E
8
B
A
ryx
oy

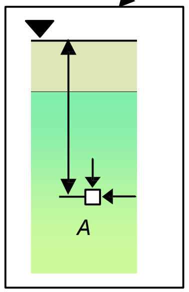


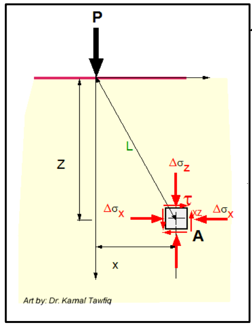
Stress Distribution in Soils

Geostatic stresses

Total Stress
Effective Stress
Pore Water Pressure

Total Stress= Effective Stress+ Pore Water





Added Stresses (Point, line, strip, triangular, circular, rectangular)

Bossinisque Equations

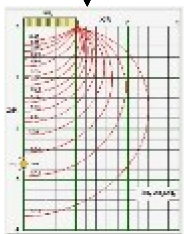
- Point Load
- Line Load
- Strip Load
- Triangular Load
- Circular Load
- Rectangular Load

σ_x
σ_y
τ_{xy}

Influence Charts

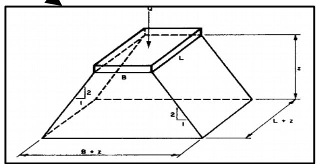
$$\Delta\sigma_z = I_\sigma \bullet q$$

Stress Bulbs

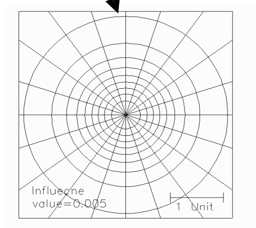


Westergaard's Method

Approximate Method

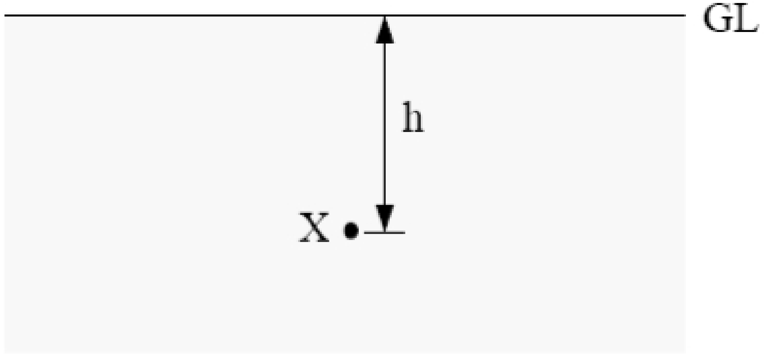


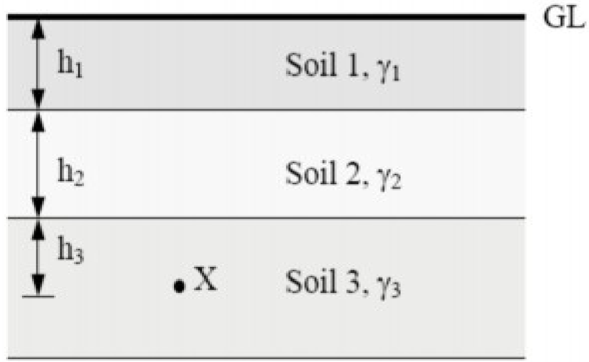
Newmark Charts



Geostatic stresses

The vertical geostatic stress at point X will be computed as following





$\sigma_V = \gamma h$ homogenous soils

$\sigma_V = \sum_{1}^{n} \gamma_i h_i$ stratified soils

$\sigma_v = \int_{0}^{h} \gamma \, dz$ density varies continuously with depth

The horizontal geostatic stress can be computed as following

$$\sigma_h = K\sigma_v$$

where K is the coefficient of lateral stress or lateral stress ratio

$$K = \frac{\sigma_h}{\sigma_v} \qquad 1 < K \,\square\, 1$$

- ∅ Geostatic stress are principle stresses (σ_1, σ_2 and σ_3 major, intermediate and minor principle stresses) and hence the horizontal and vertical planes through any point are principle planes.

$K < 1$ $\quad \sigma_v = \sigma_1 \quad \sigma_h = \sigma_3 \quad \sigma_2 = \sigma_3 = \sigma_h$

$K = 1$ $\quad \sigma_v = \sigma_h = \sigma_1 = \sigma_2 = \sigma_3$ *Isotropic*

$K > 1$ $\quad \sigma_h = \sigma_1 \quad \sigma_v = \sigma_3 \quad \sigma_2 = \sigma_1 = \sigma_h$

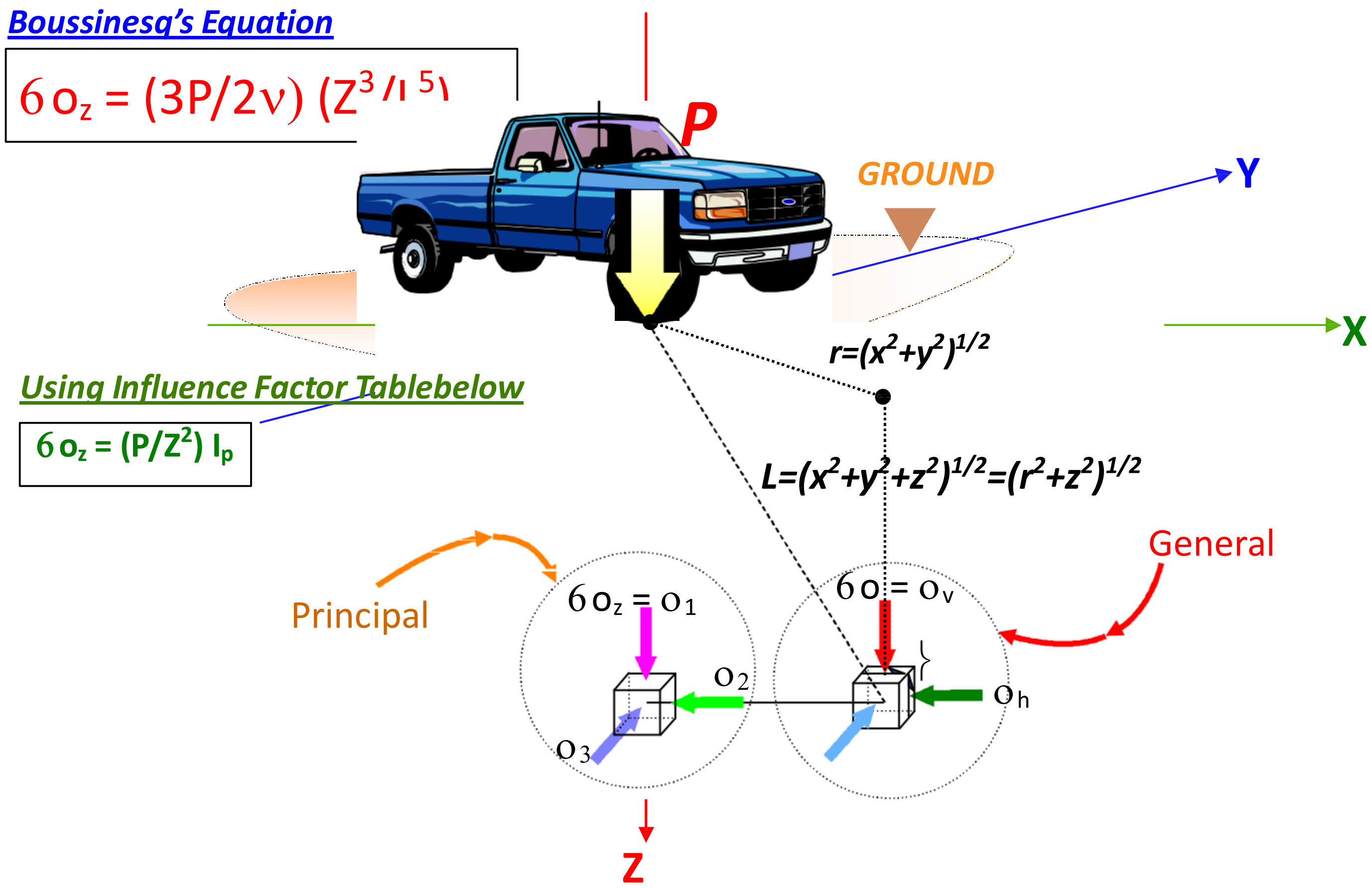
The largest shear stress will found on plane lying at 45^{o} to the horizontal

$K < 1$ $\quad \tau_{max} = \frac{\sigma_v}{2}(1 - K)$

$K = 1$ $\quad \tau_{max} = 0$

$K > 1$ $\quad \tau_{max} = \frac{\sigma_v}{2}(K - 1)$

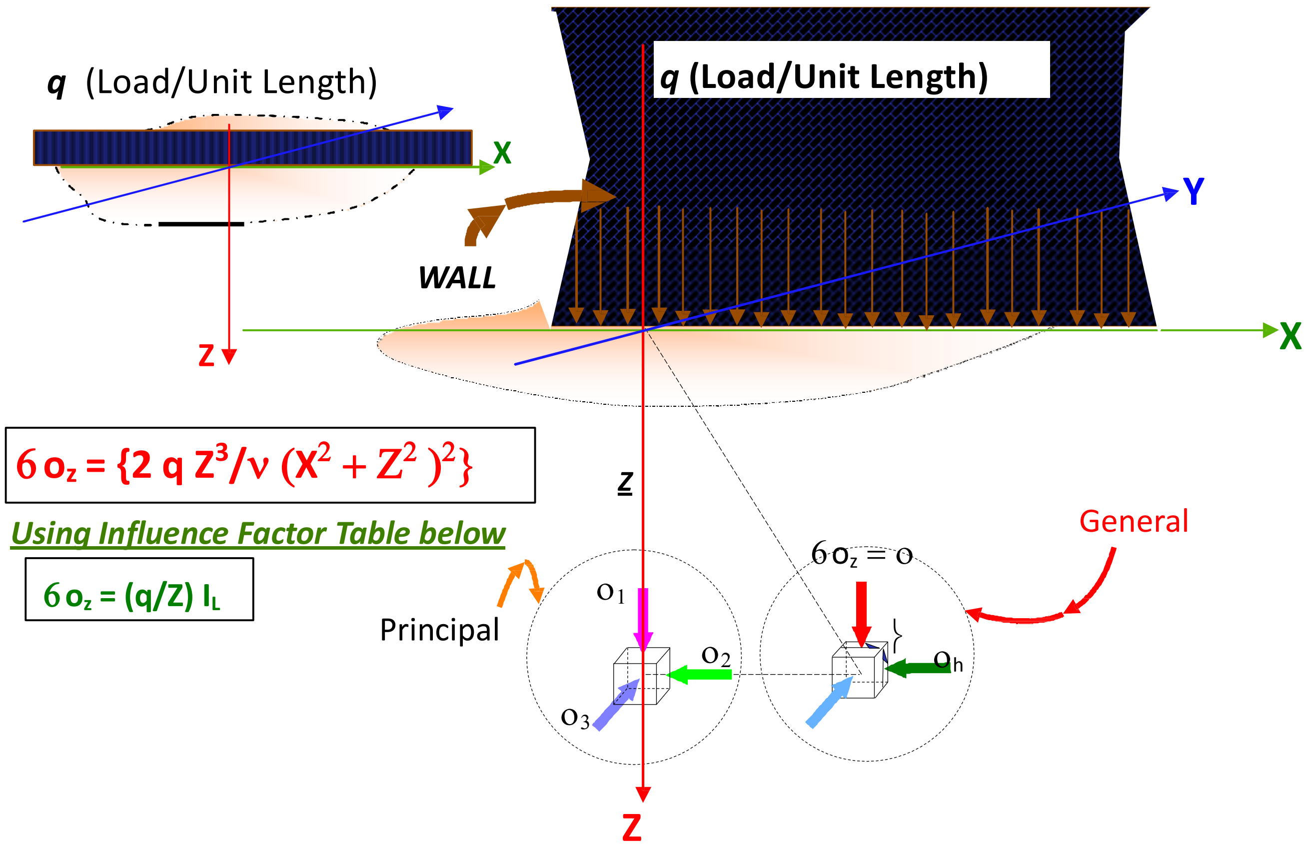
• Stress Caused by a Point Load



Variation of I_1 for Various Values of r/z [Eq. (10.14)]

r/z	I_1	r/z	I_1	r/z	I_1
0	0.4775	0.36	0.3521	1.80	0.0129
0.02	0.4770	0.38	0.3408	2.00	0.0085
0.04	0.4765	0.40	0.3294	2.20	0.0058
0.06	0.4723	0.45	0.3011	2.40	0.0040
0.08	0.4699	0.50	0.2733	2.60	0.0029
0.10	0.4657	0.55	0.2466	2.80	0.0021
0.12	0.4607	0.60	0.2214	3.00	0.0015
0.14	0.4548	0.65	0.1978	3.20	0.0011
0.16	0.4482	0.70	0.1762	3.40	0.00085
0.18	0.4409	0.75	0.1565	3.60	0.00066

● Vertical Stress Caused by a Line Load

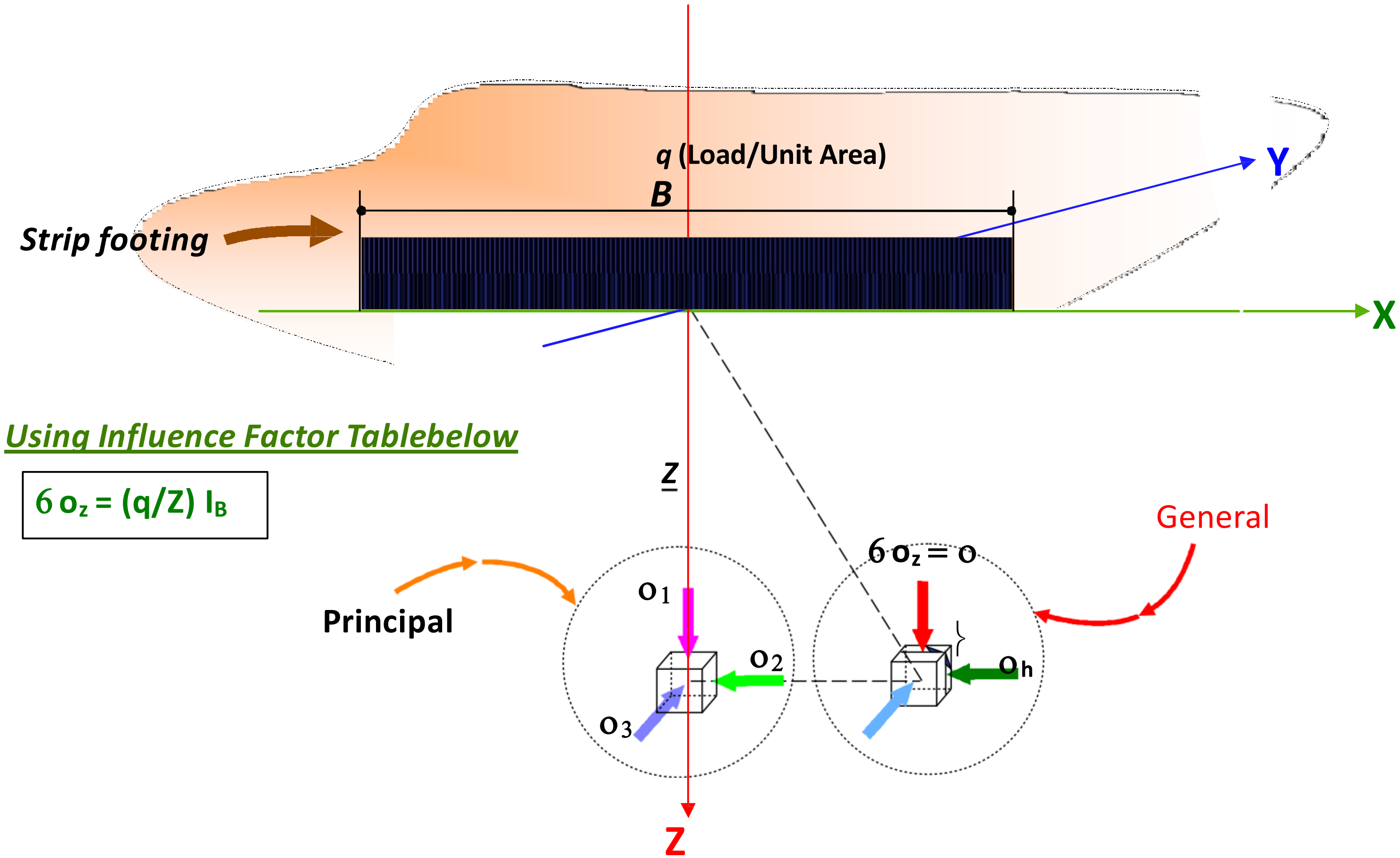


$$6\,o_z = \{2\,q\,Z^3/\nu\,(X^2 + Z^2)^2\}$$

Using Influence Factor Table below

$$6\,o_z = (q/Z)\,I_L$$

● Vertical Stress Caused by a Strip Load



● Vertical Stress Due to Embankment Loading

$6\, o_z = q/v[\{(B_1+B_2)/B_2\}(a_1+a_2)-(B_1/B_2)(a_2)]$

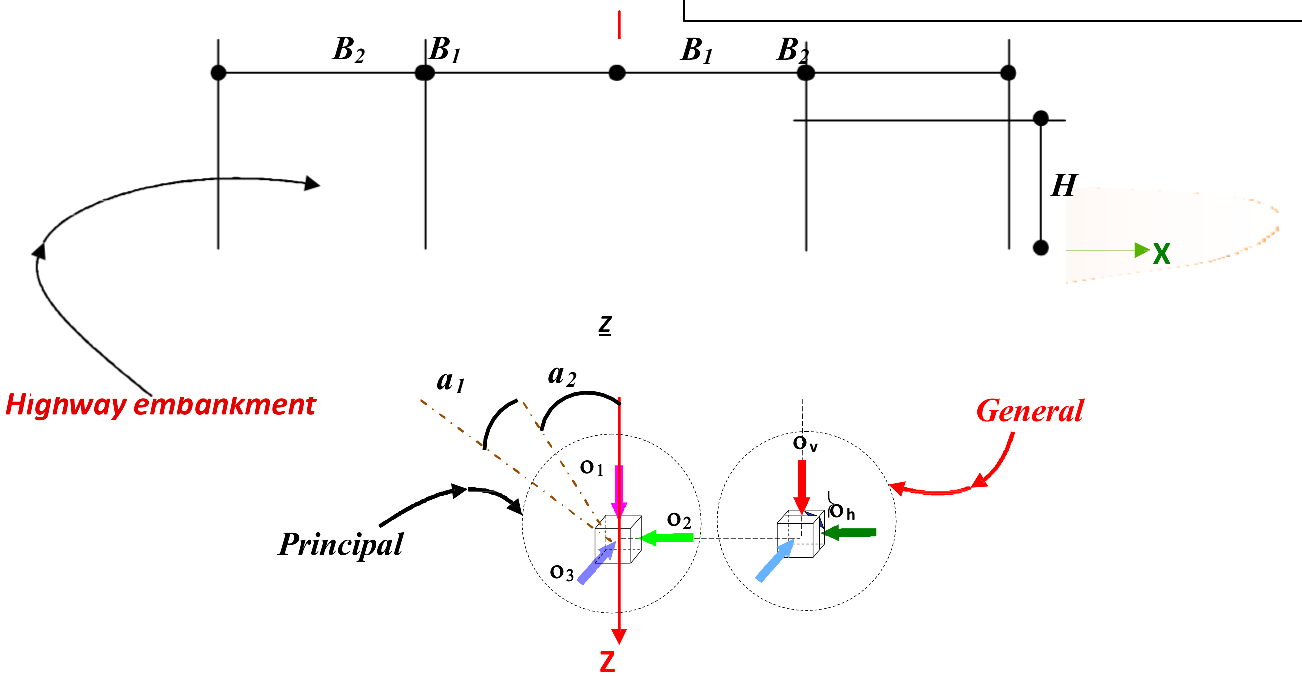
Using Influence Factor fig.(9.11-pp238)

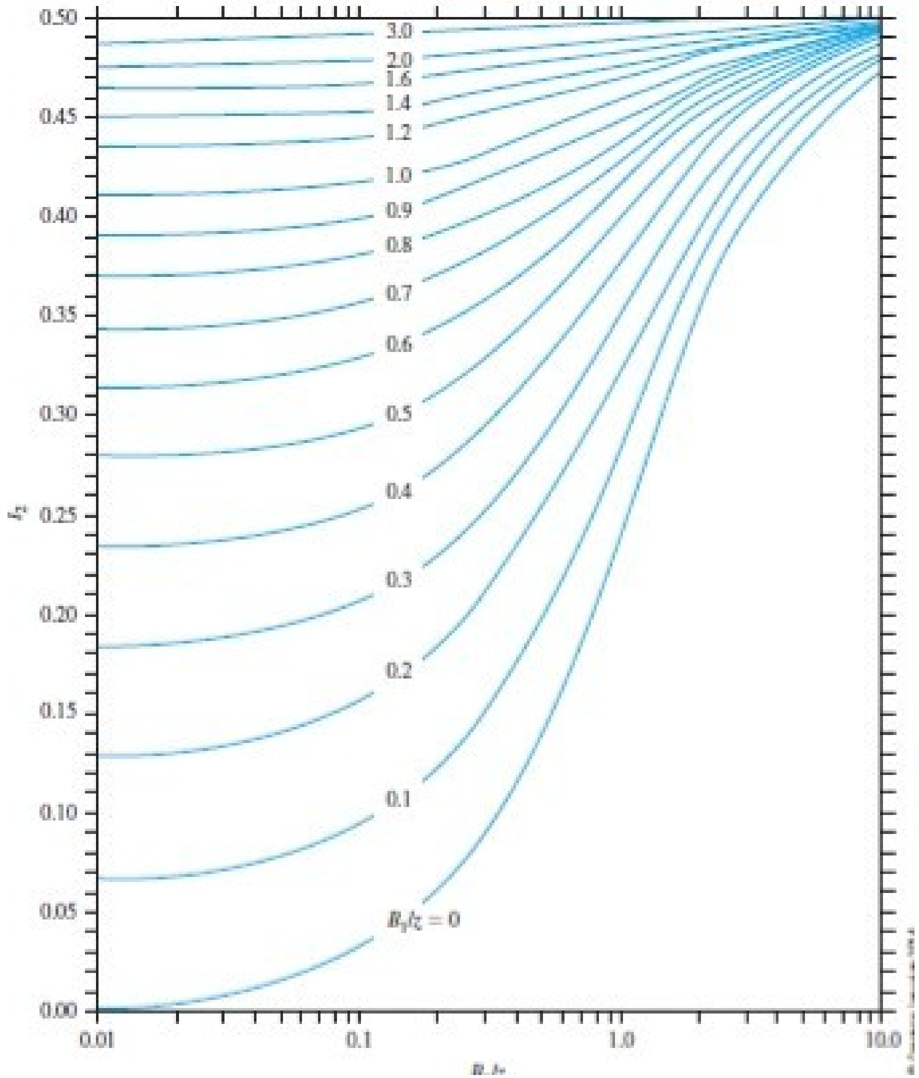
$6\, o_z = q I_2$

$q = yH$

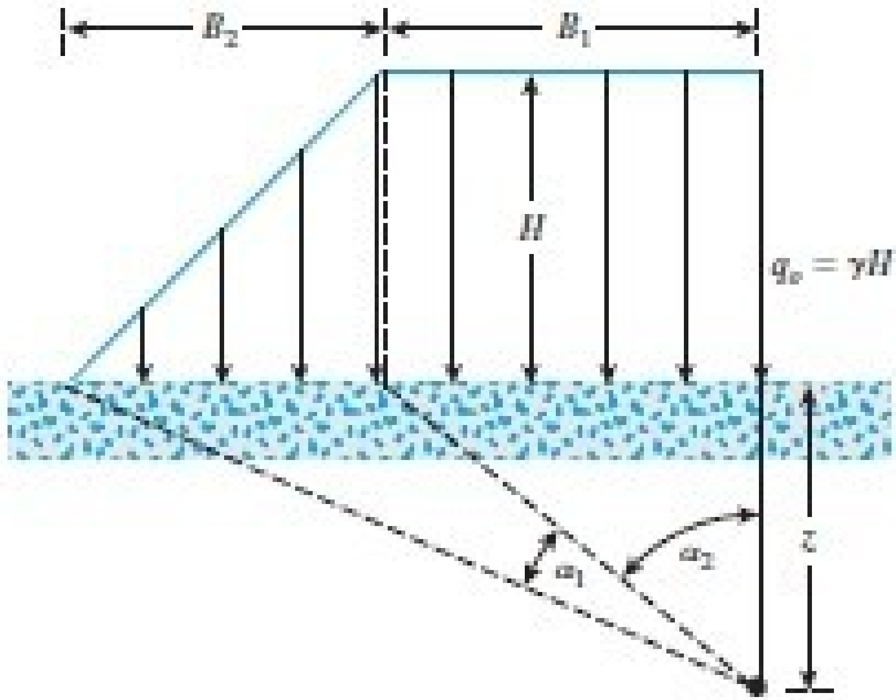
$a_1(radians) = \tan^{-1}\{(B_1+B_2)/z\} - \tan^{-1}(B_1/z)$

$a_2 = \tan^{-1}(B_1/z)$



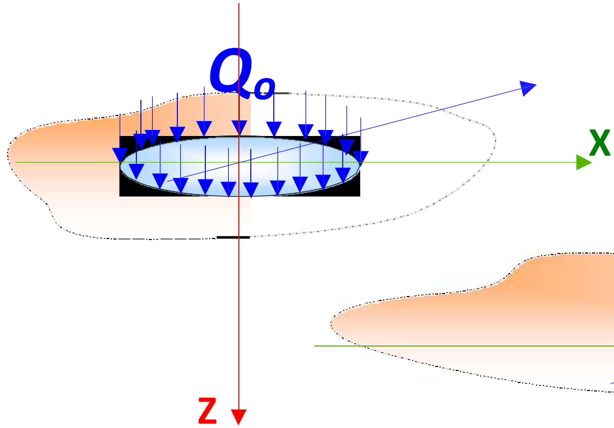


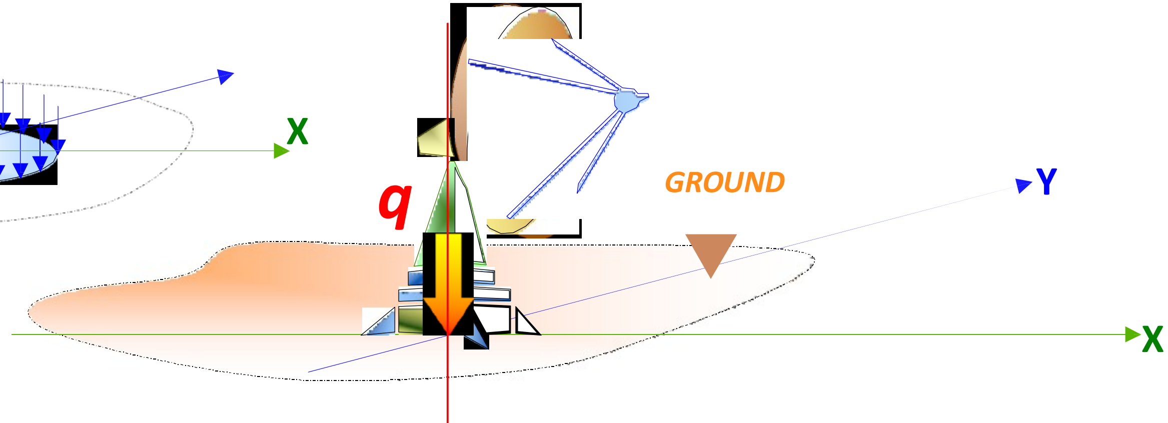
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
3.0
2.0
1.6
1.4
1.2
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
B1/z = 0
0.01
0.1
1.0
10.0



B2
B1
H
qv = γH
α1
α2
z

● Vertical Stress below the Center of a uniformly Loaded Circular Area

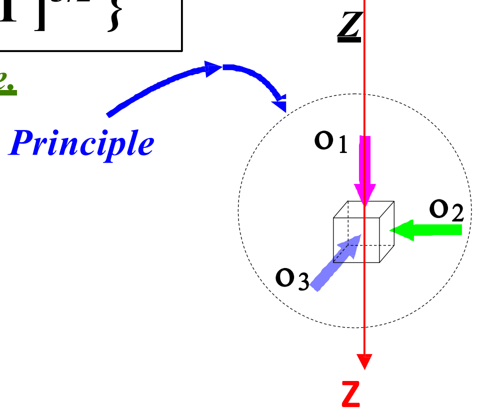




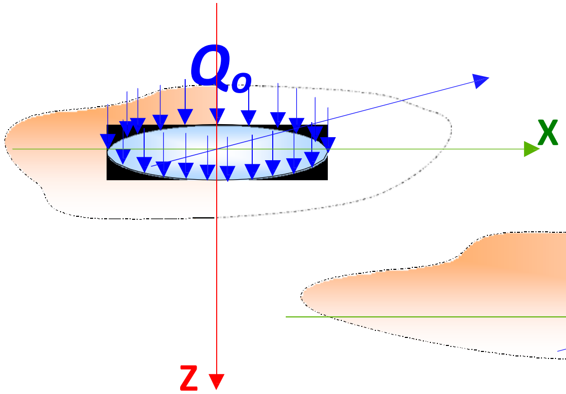
$$b o_z = q \{ 1 - 1/[(R/Z)^2 + 1]^{3/2} \}$$

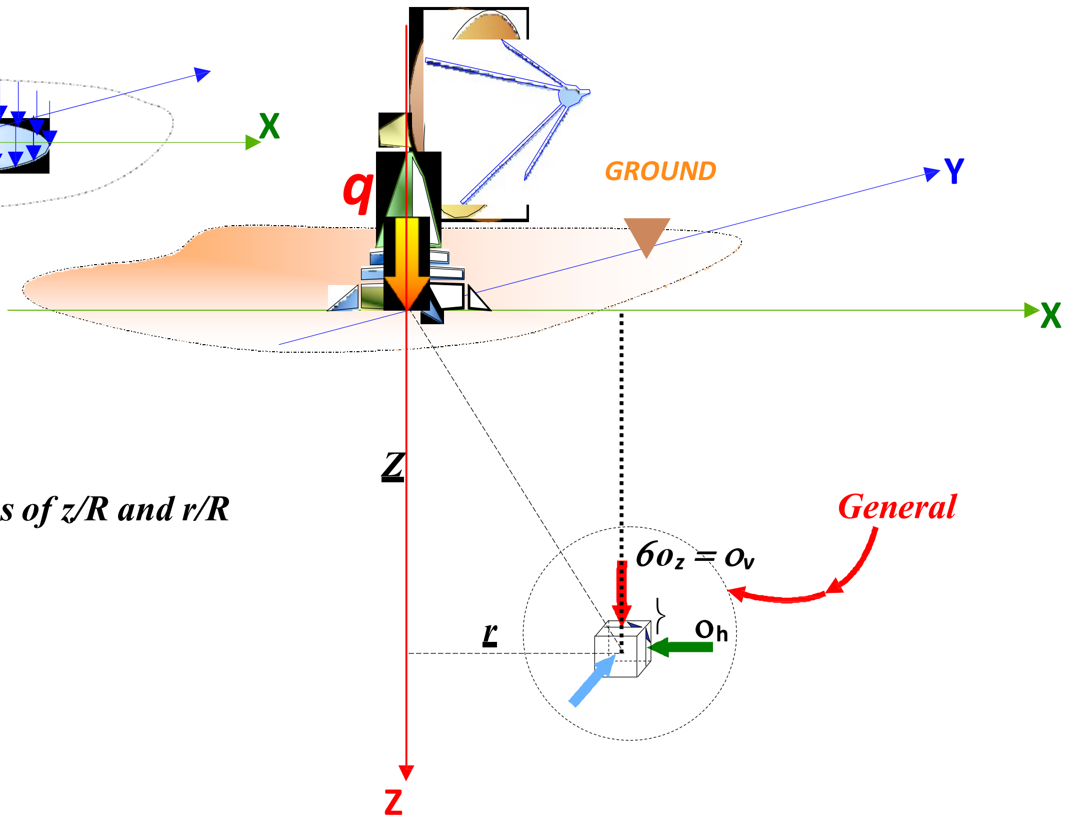
Using Influence Factorfrom table.

$6\ o_z = qI_c$



• Vertical Stress at any Point below a uniformly Loaded Circular Area



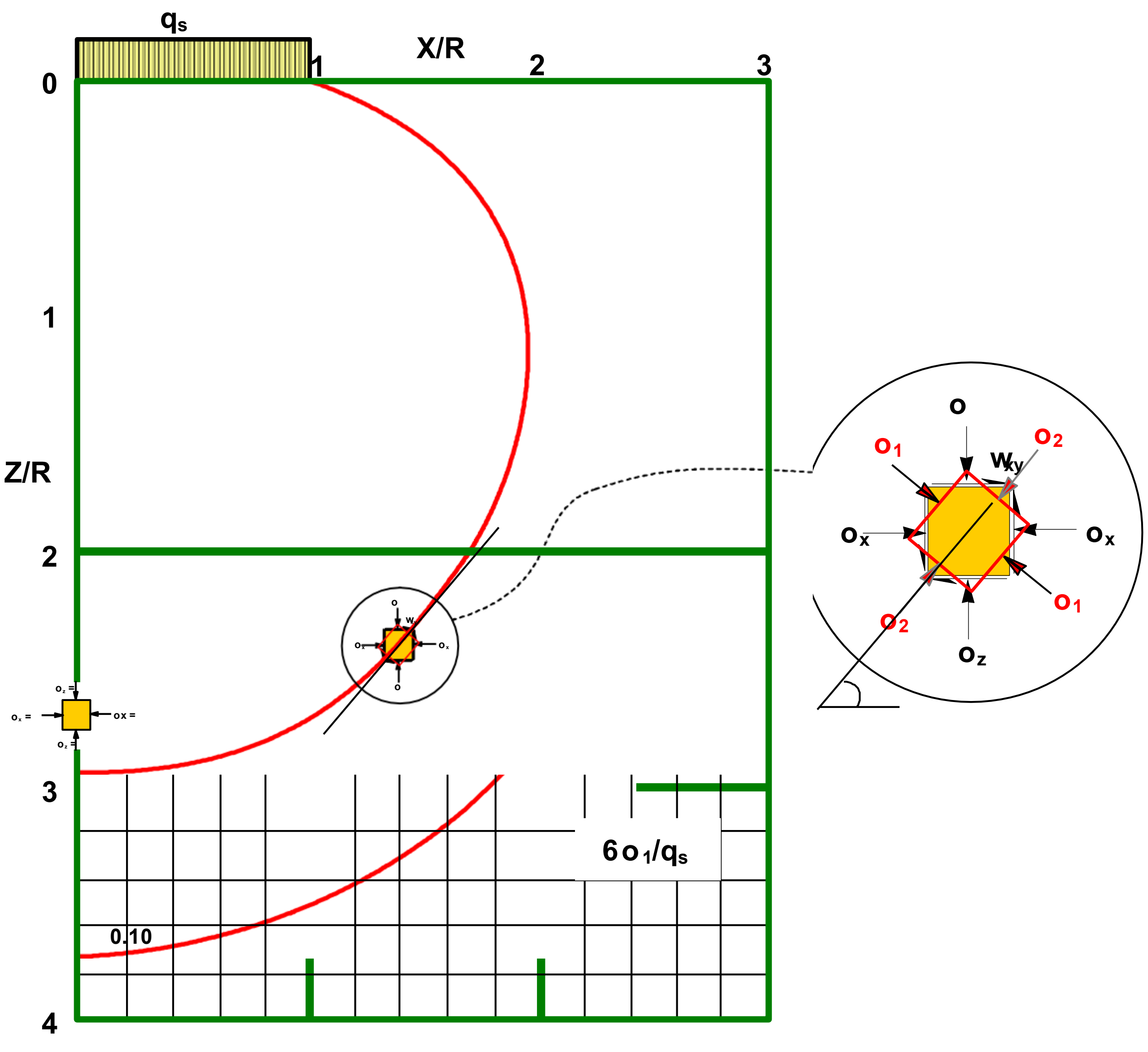


$$b\sigma_z = q\,(A' + B')$$

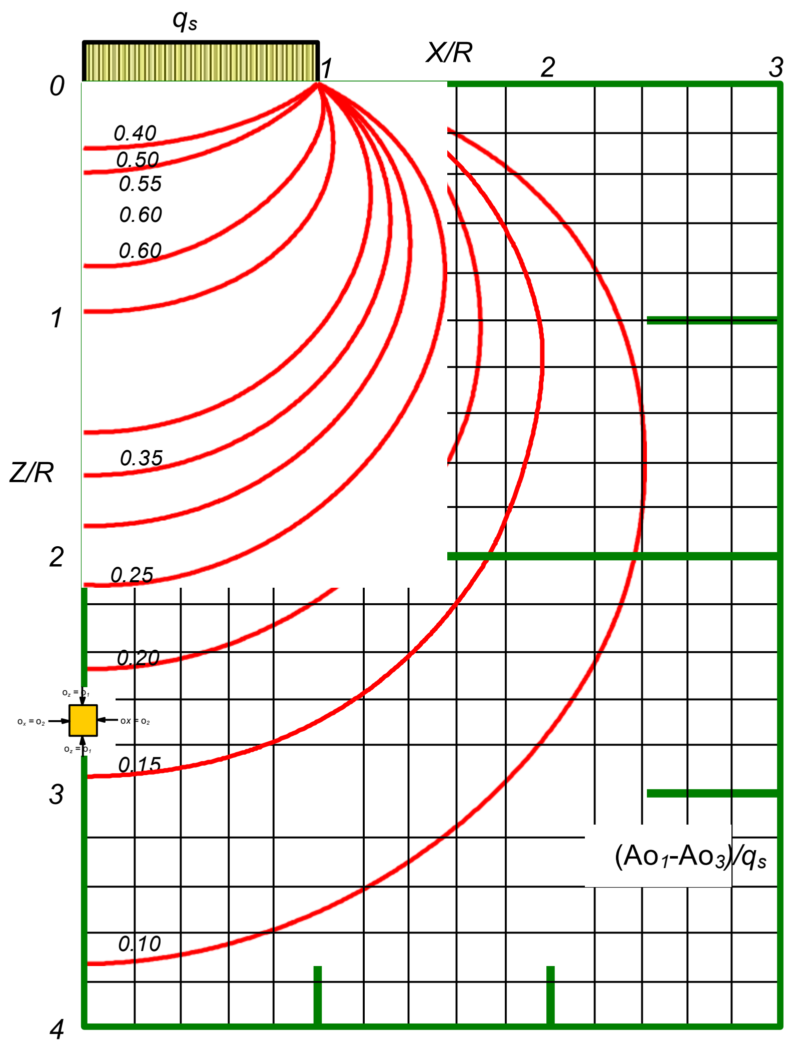
Where (A' & B') are functions of z/R and r/R

from tables

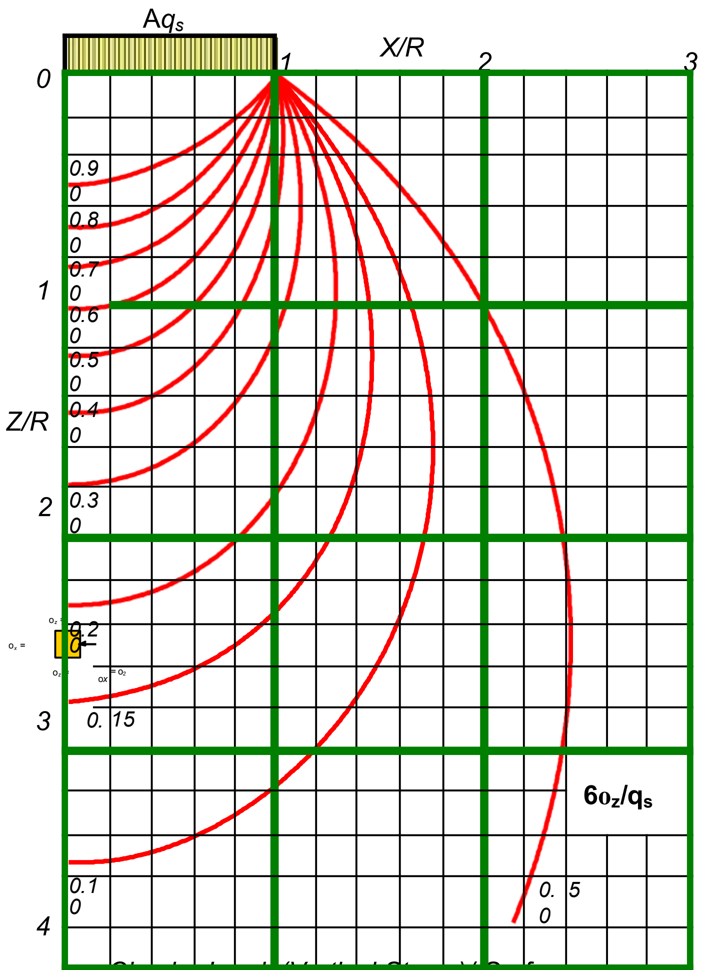
Or we can use the stress bulb charts



Circular Load: (Major Principal Stress)/(Surface Stress)



qs
X/R
0
1
2
3
0.40
0.50
0.55
0.60
0.60
0.35
0.25
0.20
0.15
0.10
Z/R
1
2
3
4
(Ao1-Ao3)/qs

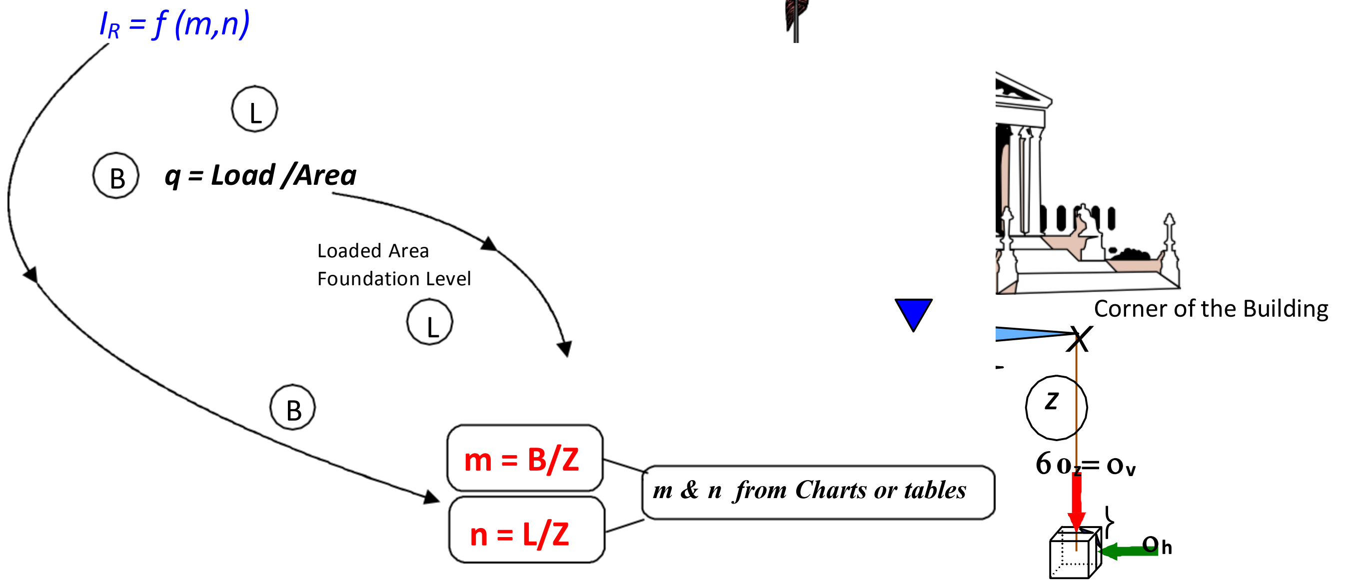


Circular Load: (Vertical Stress)/(Surface

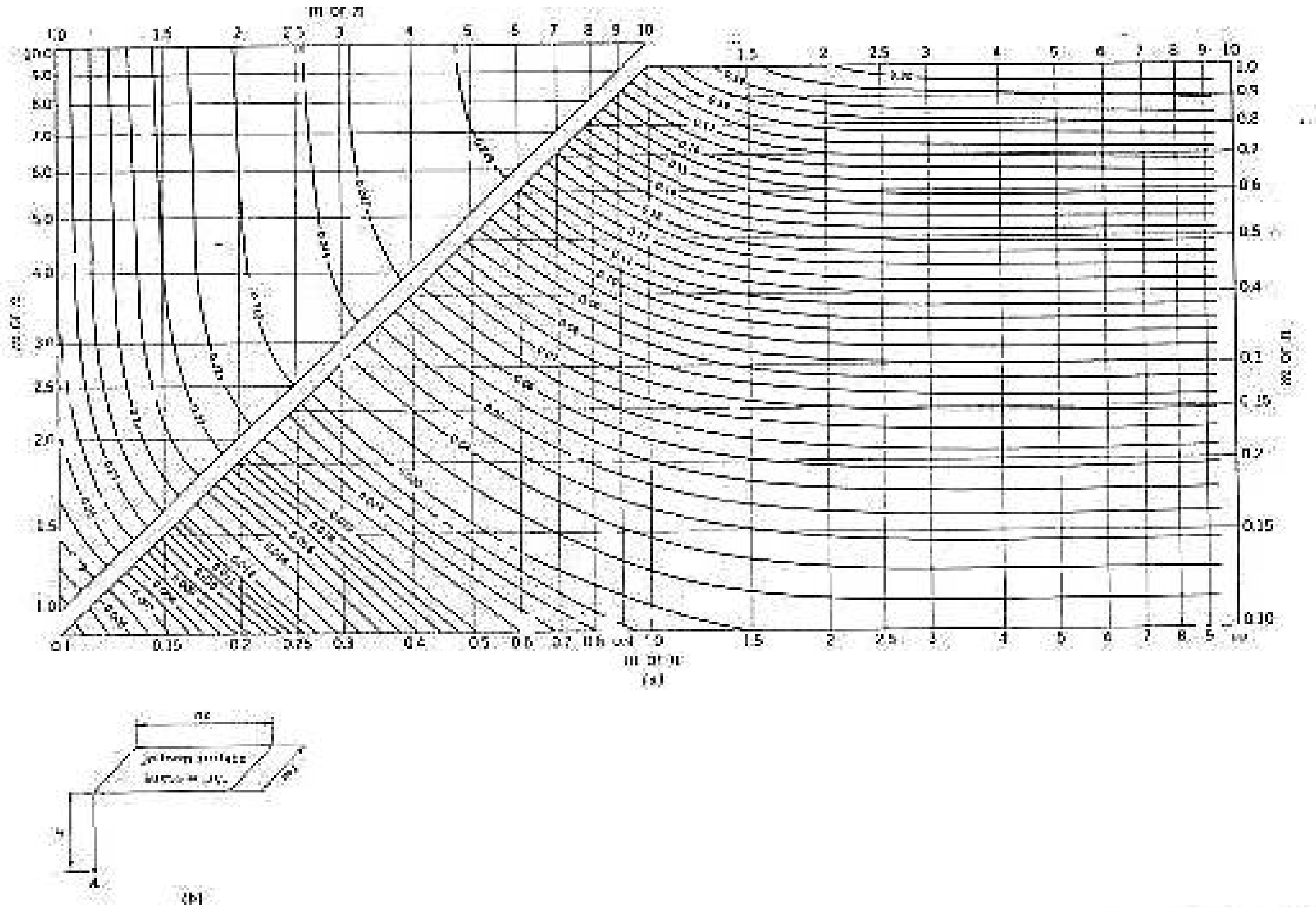
• Vertical Stress Caused by a Rectangularly Loaded Area

$$6P = q\, I_R$$

$$I_R = 1/4n \{[\,(2.\ m.n\ (m^2 + n^2 + 1)^{1/2}\,) / (m^2 + n^2 + m^2.\ n^2 + 1)\,]\ [(m^2 + n^2 + 2)/(m^2 + n^2 + 1)] + \tan^{-1} (2.m\ .n\ (m^2 + n^2 + 1)^{1/2} / (m^2 + n^2 - m^2\ .\ n^2 + 1)\}$$



from tables or one can use the charts below


m or n
m or n
m or n
(a)
A
(b)

∅ *Calculation of Stress below an interior point of the loaded area*

$$6 o_z = q[I(OXAY) + I(OYBZ) + I(OZCT) + I(OTDX))$$

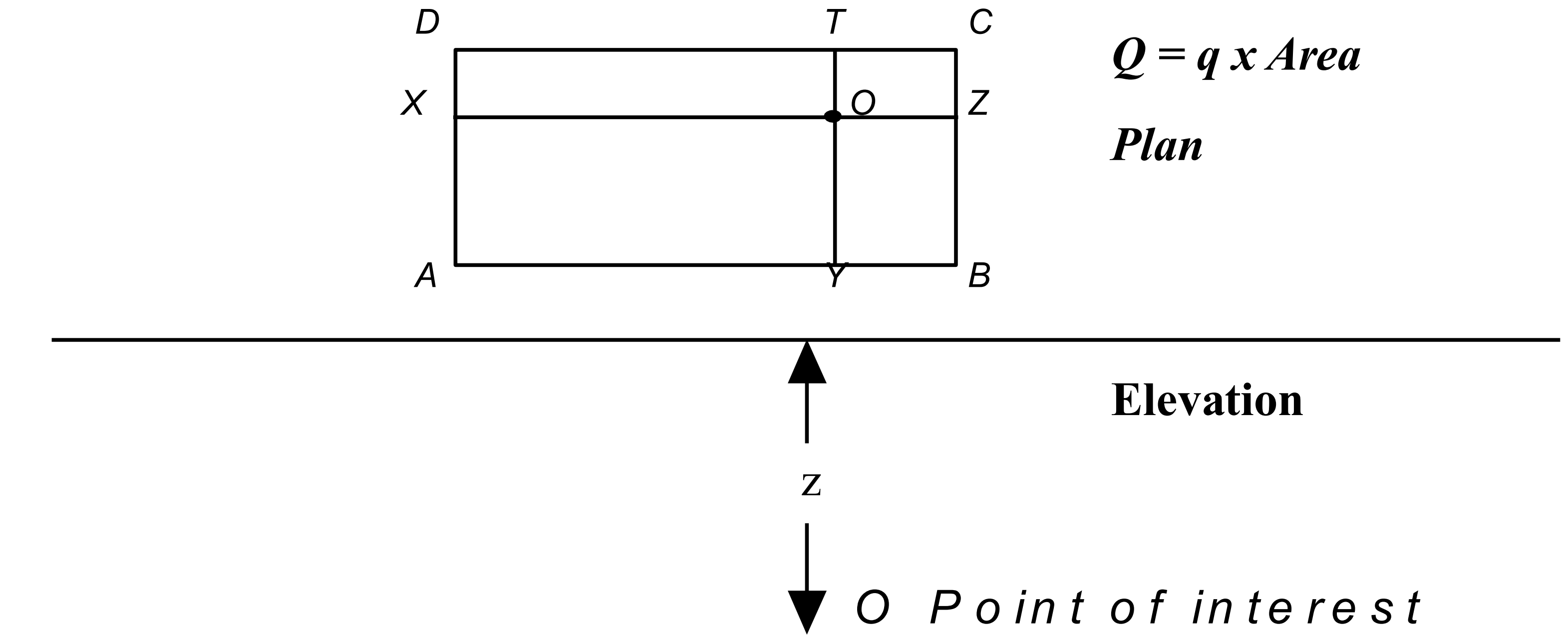
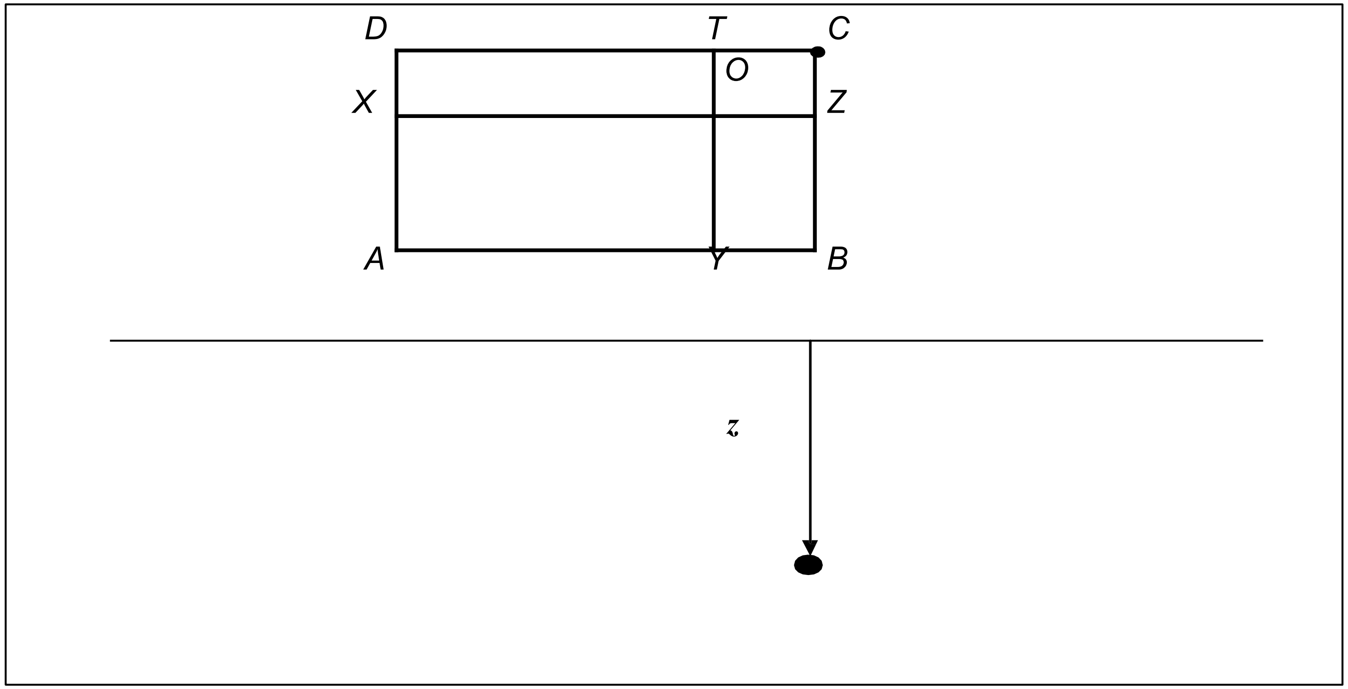


Fig. 10 stress increase at a point below a loaded rectangular region

∅ *Calculation of Stress below a point outside of the loaded area*

$$6 o_z = q[I(ABCD) + I(TYBZ) + I(XZCD) - I(OZCT))$$



● Influence Chart for Vertical Pressure (Newmark Chart)

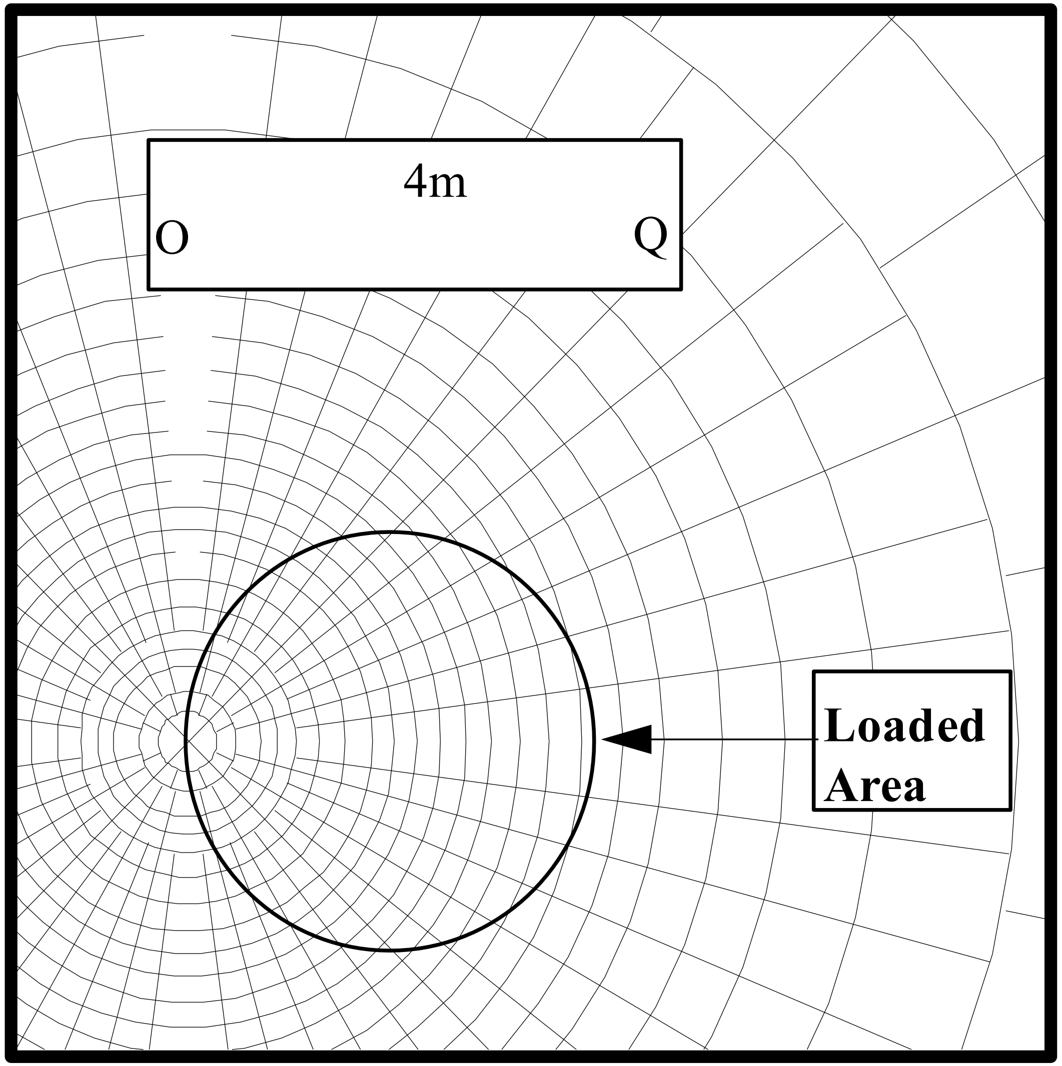
Stresses due to foundation loads of ***arbitrary shape*** *applied at the ground surface*

Newmark's chart provides a graphical method for calculating the stress increase due to a uniformly loaded region, of arbitrary shape resting on a deep homogeneous isotropic elastic region.

Newmark's chart is given in the data sheets and is reproduced in part in Fig 15. The procedure for its use is outlined below

1. The scale for this procedure is determined by the depth z at which the stress is to be evaluated, thus z is equal to the distance OQ shown on the chart.

2. Draw the loaded area to scale so that the point of interest (more correctly its vertical projection on the surface) is at the origin of the chart, the orientation of the drawing does not matter
3. Count the number of squares (N) within the loaded area, if more than half the square is in count the square otherwise neglect it.
4. The vertical stress increase $6o_z = N$ □[scale factor(0.001)] □ [surface stress (p)]


4m
O
Q
Loaded
Area

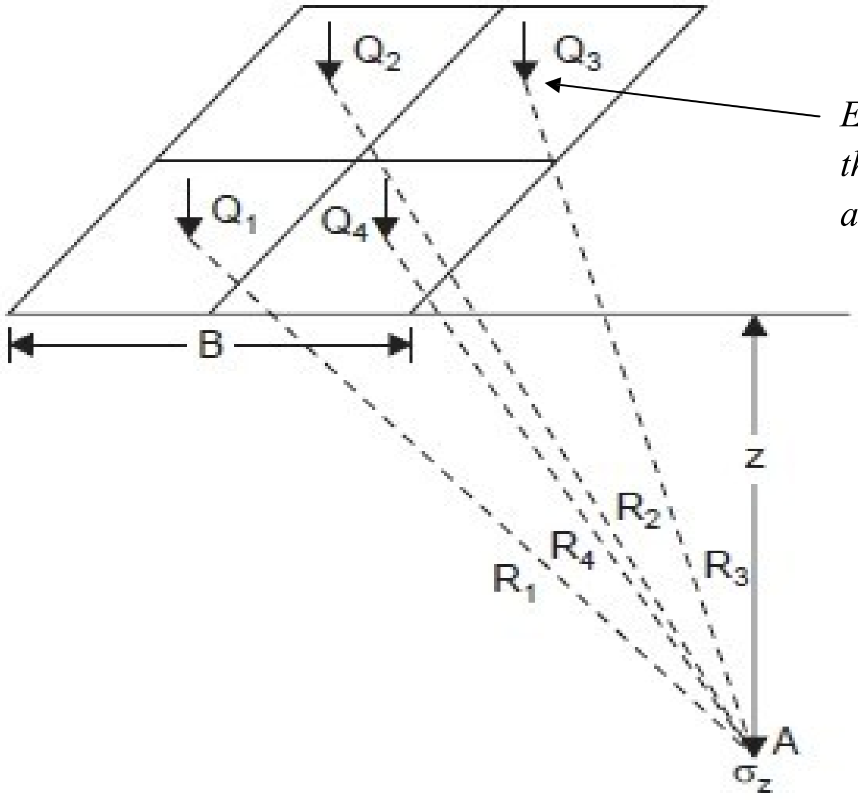
● Approximate Methods

∅ ***Equivalent Point Load Method***

In dividing the loaded area into smaller units, we have to remember to do it such that

z/*B* Ç 3; that is to say, in relation to the specified depth, the size of any unit area should not be greater than one-third of the depth.

$$6\,o_z =)\ \frac{Q_i}{z^2} I_{pi}$$



Each Q is the resultant of the uniform load on the unit area acts at the center of it and treated as a point load

Fig. 10.20 Equivalent point load method

∅ *2:1 Method*

$$6 o_z = \frac{Q}{(B+Z)(L+Z)}$$ Rectangula

$$6 o_z = \frac{Q}{(B+Z)^2}$$ Square are

$$6 o_z = \frac{Q}{\frac{v}{4}(D+Z)^2}$$ Circular ar

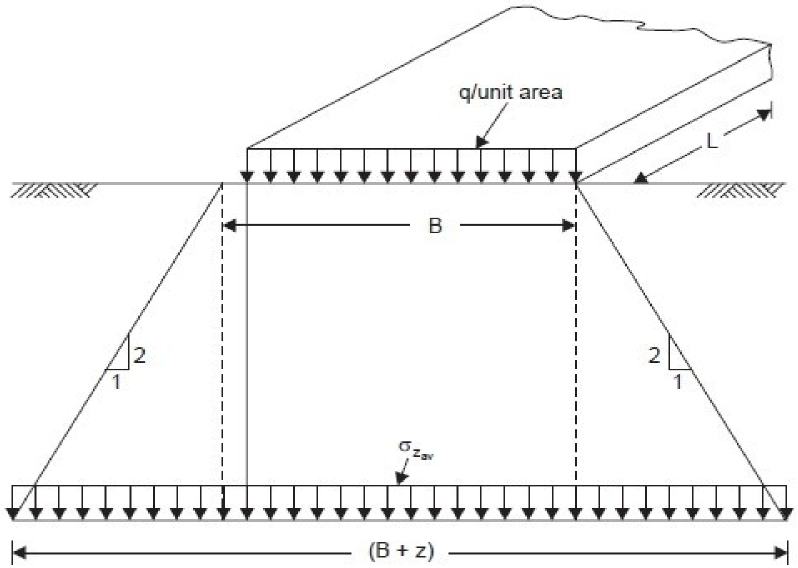
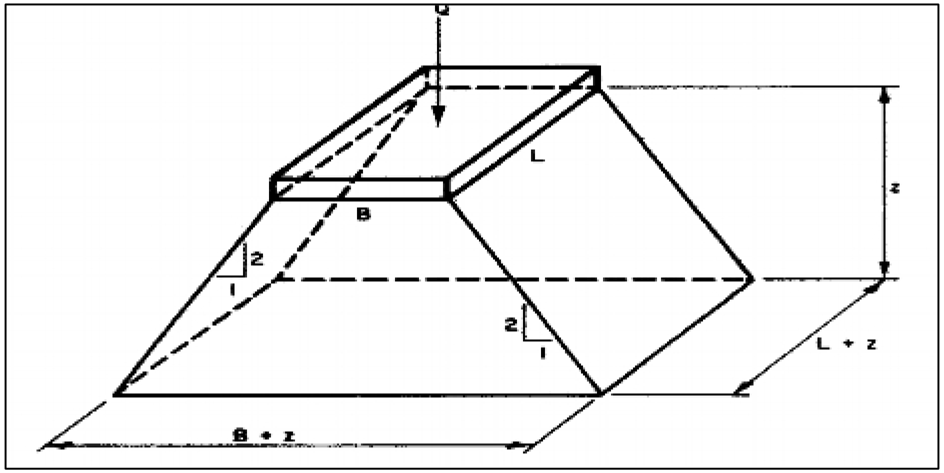


Fig. 10.21 Two is to one method



Examples (1-3)

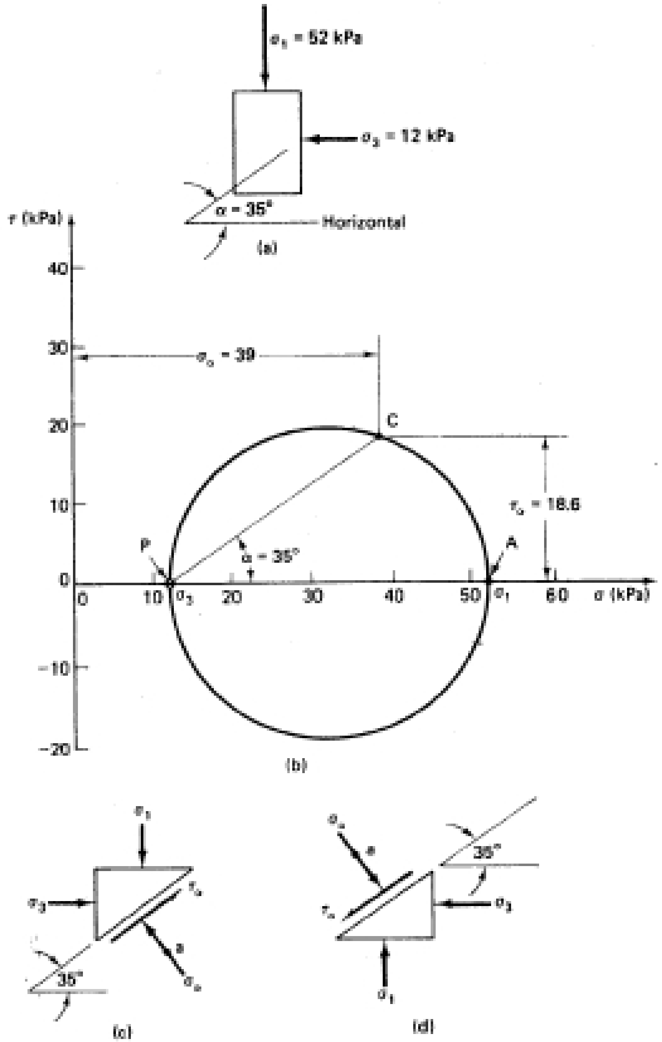
Given:

Stresses on an element as shown in Fig.

Plot the Mohr circle to some convenient

$$\text{center of circle} = \frac{\sigma_1 + \sigma_3}{2} = \frac{52 + 12}{2} = 32$$

$$\text{radius of circle} = \frac{\sigma_1 - \sigma_3}{2} = \frac{52 - 12}{2} = 20$$

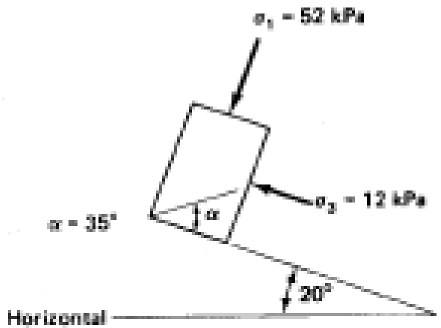


Given:

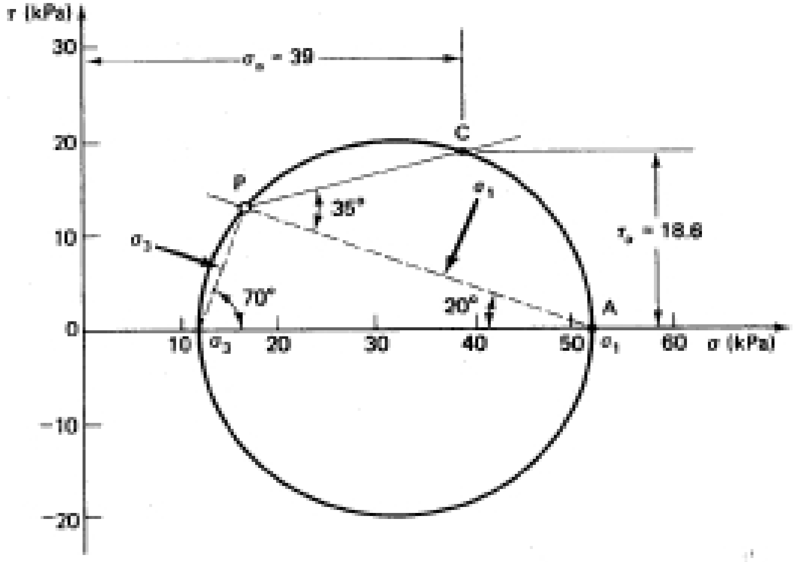
The same element and stresses as in Fig. Ex. 1 , except that the is rotated 20° from the horizontal, as shown in Fig.

Required:

As in Example 10.1, find the normal stress σ_α and the shear stress plane inclined at $\alpha = 35°$ from the base of the element.



(a)



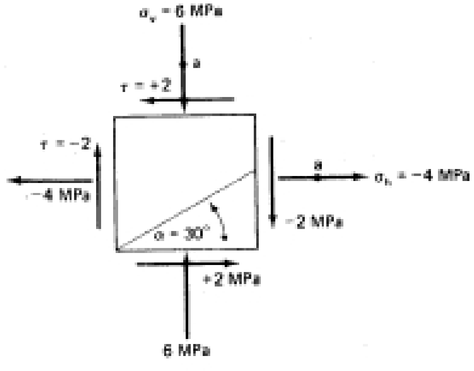
(b)

Given:

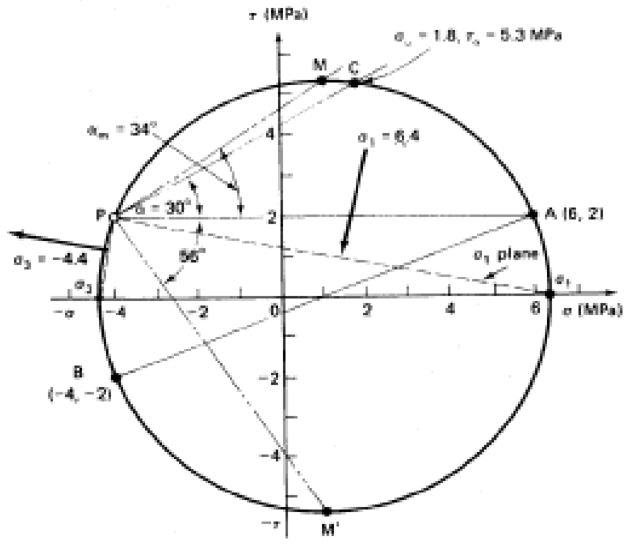
The stress shown on the element in Fig.

Required:

a. Evaluate σ_α and τ_α when $\alpha = 30°$.
b. Evaluate σ_1 and σ_3 when $\alpha = 30°$.



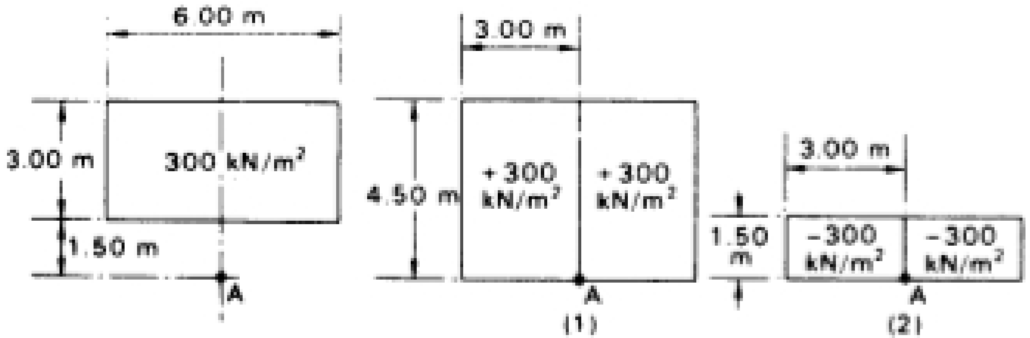
(a)



(b)

Example 4

A rectangular foundation 6 x 3m carries a uniform pressure of 300 kN/m^2 near the surface of a soil mass. Determine the vertical stress at a depth of 3m below a point (A) on the centre line 1.5m outside a long edge of the foundation using influence factors



m = 1

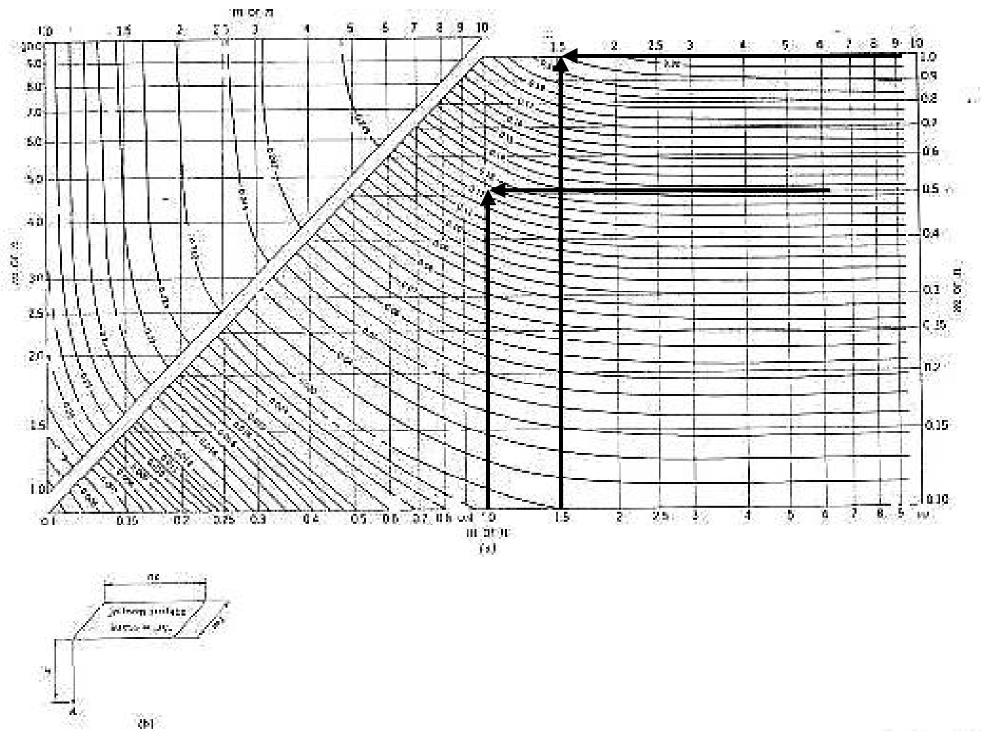
n =1.5

I = 0.193

m=1

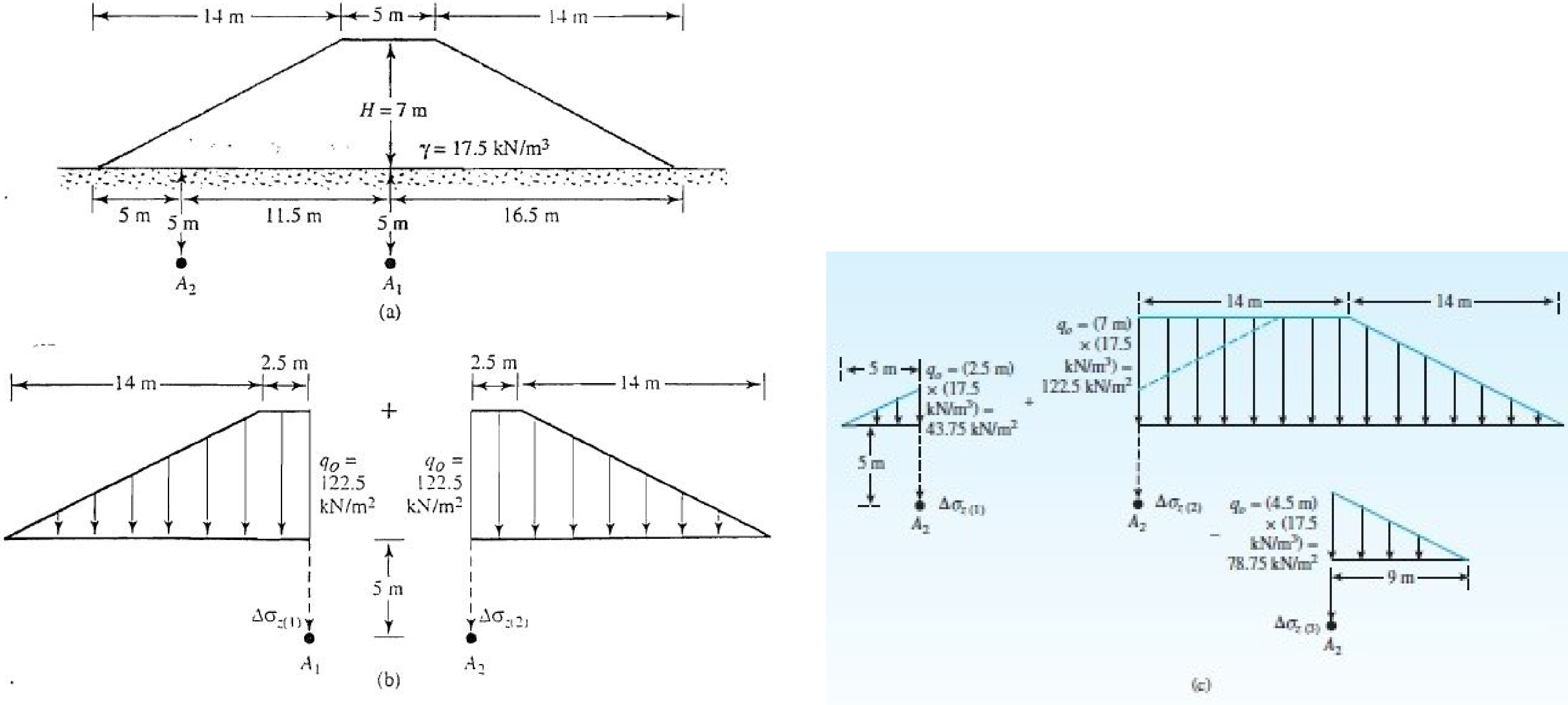
n=0.5

I=0.120



Example 5

Determine the stress increase under the embankment at points A_1 and A_2



Solution

$$\gamma H = (17.5)(7) = 122.5 \text{ kN/m}^2$$

Stress Increase at A_1

The left side of Figure a indicates that $B_1 = 2.5$ m and $B_2 = 14$ m. So

$$\frac{B_1}{z} = \frac{2.5}{5} = 0.5; \frac{B_2}{z} = \frac{14}{5} = 2.8$$

According to Figure b in this case, $I_2 = 0.445$. Because the two sides in Figure b are symmetrical, the value of I_2 for the right side will also be 0.445. So

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} = q_o[I_{2(\text{Left})} + I_{2(\text{Right})}]$$

$$= 122.5[0.445 + 0.445] = \mathbf{109.03\ kN/m^2}$$

Stress Increase at A_2

Refer to Figure c. For the left side, $B_2 = 5$ m and $B_1 = 0$. So

$$\frac{B_2}{z} = \frac{5}{5} = 1; \quad \frac{B_1}{z} = \frac{0}{5} = 0$$

According to osterberg chart for these values of B_2/z and B_1/z, $I_2 = 0.25$. So

$$\Delta\sigma_{z(1)} = 43.75(0.25) = 10.94 \text{ kN/m}^2$$

For the middle section,

$$\frac{B_2}{z} = \frac{14}{5} = 2.8; \quad \frac{B_1}{z} = \frac{14}{5} = 2.8$$

Thus, $I_2 = 0.495$. So

$$\Delta\sigma_{z(2)} = 0.495(122.5) = 60.64 \text{ kN/m}^2$$

For the right side

$$\frac{B_2}{z} = \frac{9}{5} = 1.8; \quad \frac{B_1}{z} = \frac{0}{5} = 0$$

and $I_2 = 0.335$. So

$$\Delta\sigma_{z(3)} = (78.75)(0.335) = 26.38 \text{ kN/m}^2$$

Total stress increase at point A_2 is

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} - \Delta\sigma_{z(3)} = 10.94 + 60.64 - 26.38 = \mathbf{45.2\ kN/m^2}$$