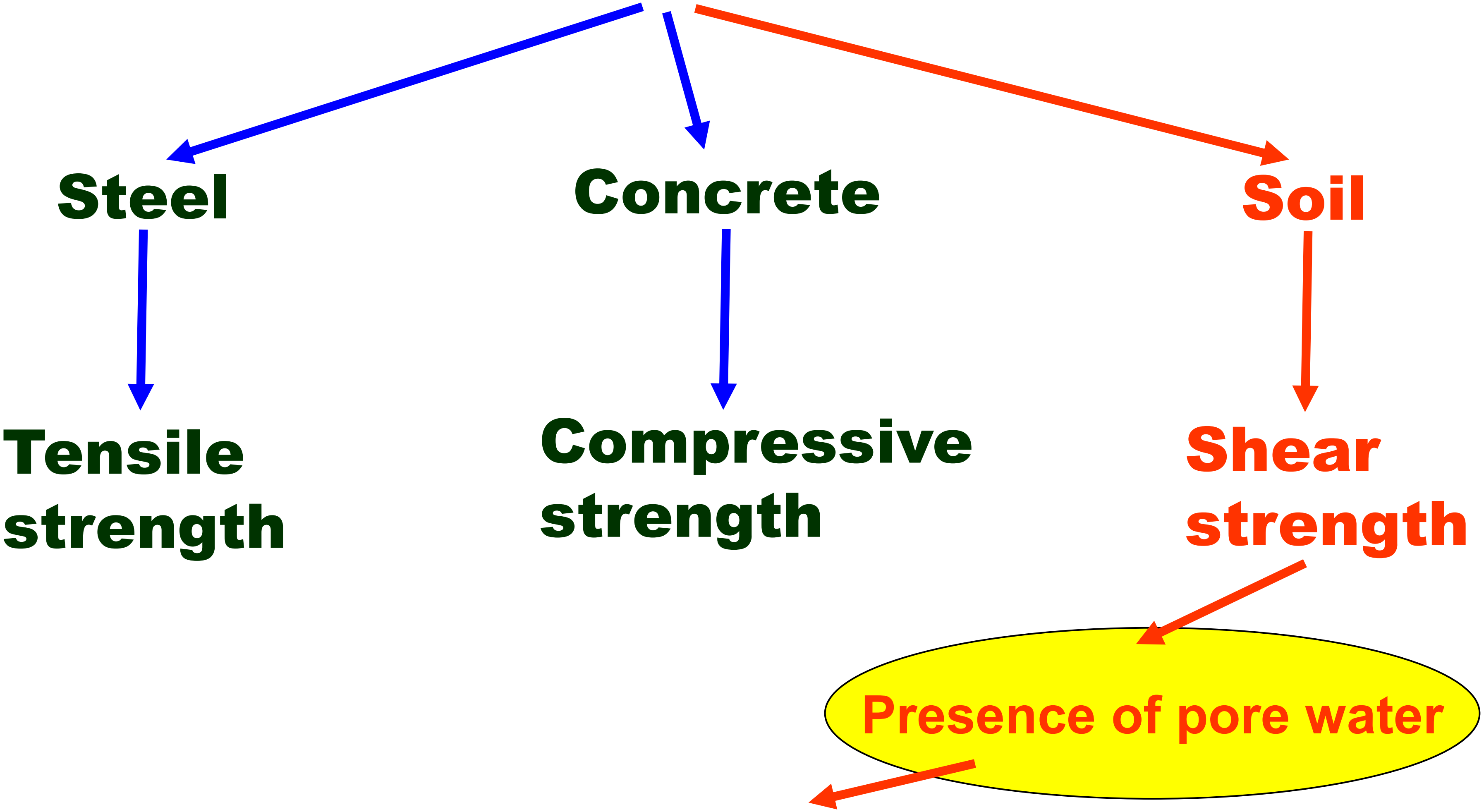
Shear Strength of Soils

Strength of different materials



Steel

Concrete

Soil

Tensile strength

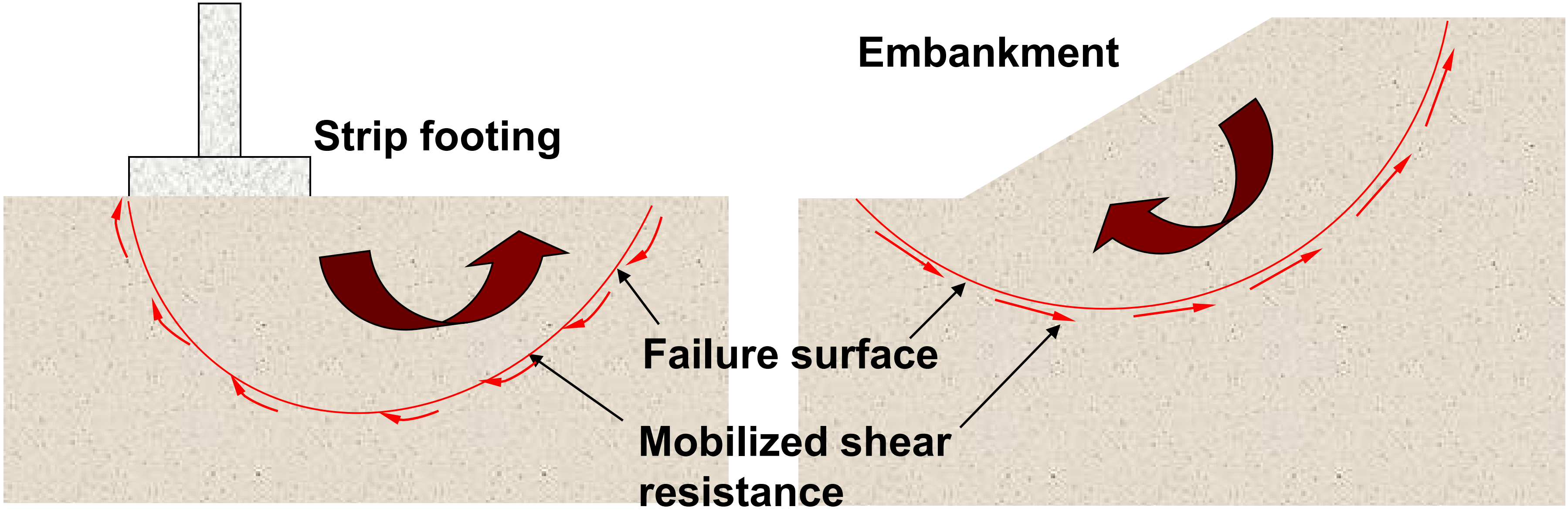
Compressive strength

Shear strength

Presence of pore water

Shear failure of soils

Soils generally fail in shear



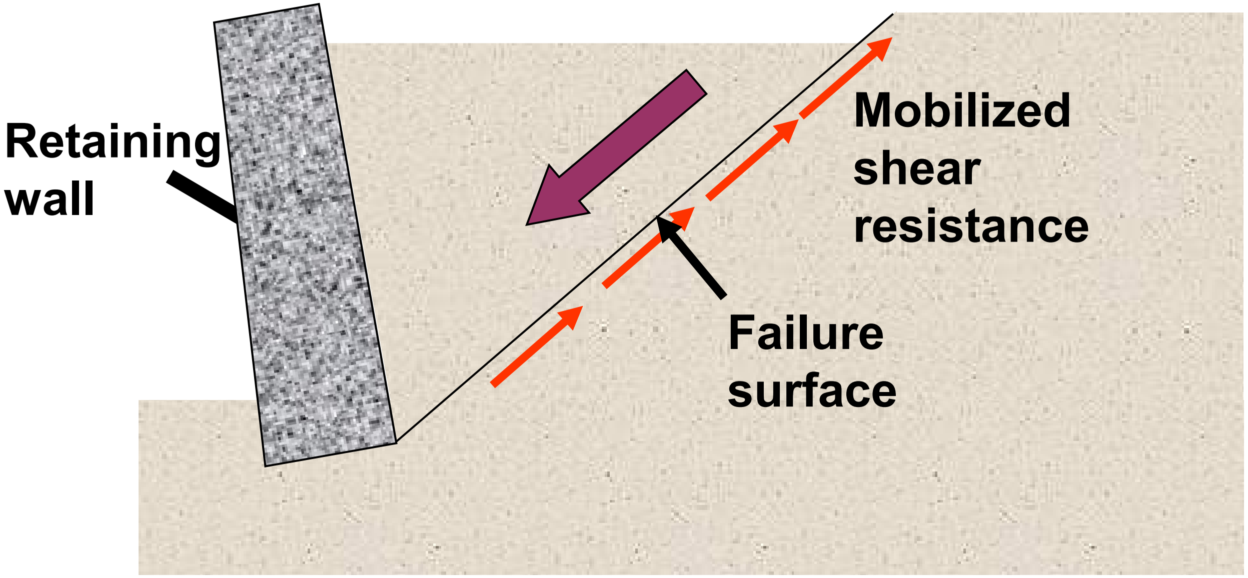
At failure, shear stress along the failure surface (mobilized shear resistance) reaches the shear strength.

Shear failure of soils

Soils generally fail in <u>shear</u>

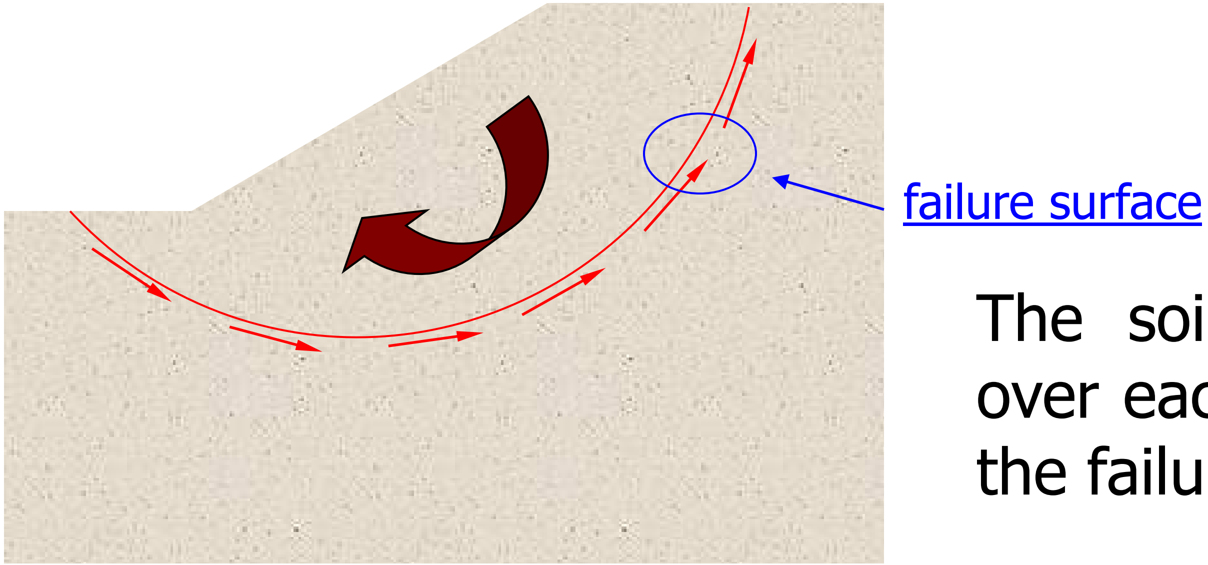
Shear failure of soils

Soils generally fail in <u>shear</u>



At failure, shear stress along the failure surface (mobilized shear resistance) reaches the shear strength.

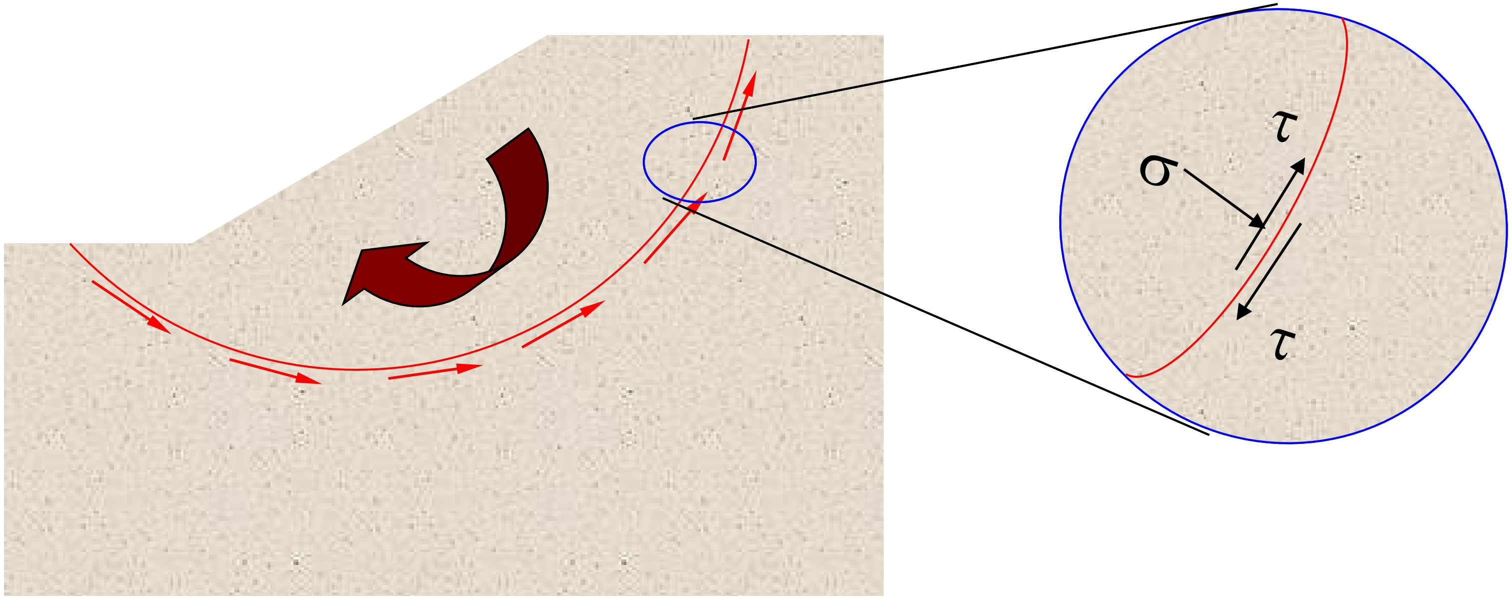
Shear failure mechanism



The soil grains slide over each other along the failure surface.

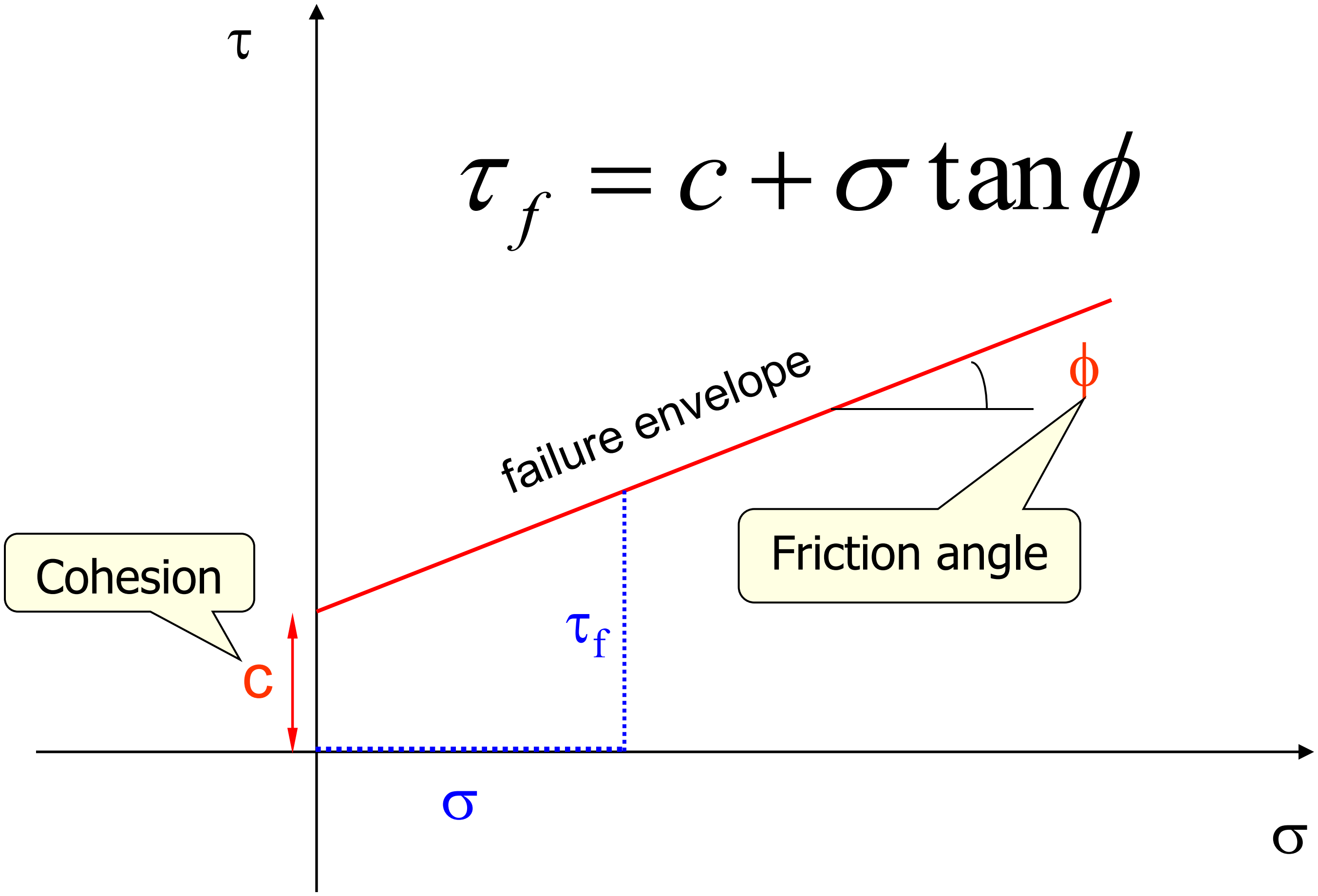
No crushing of individual grains.

Shear failure mechanism



At failure, shear stress along the failure surface (τ) reaches the shear strength (τ_f).

Mohr-Coulomb Failure Criterion (in terms of **total stresses**)

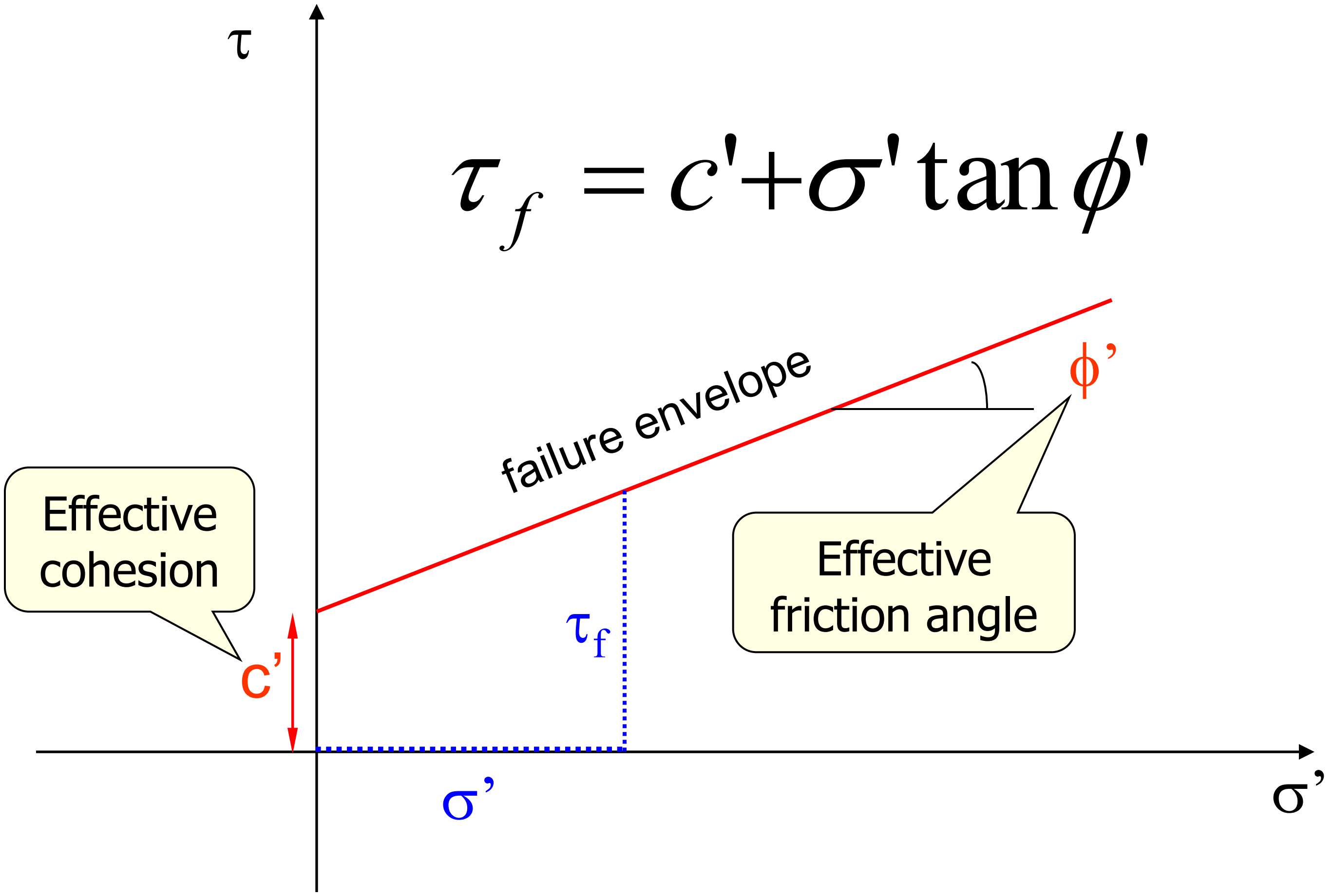


$$\tau_f = c + \sigma \tan\phi$$

τ_f is the maximum shear stress the soil can take without failure, under normal stress of σ.

Mohr-Coulomb Failure Criterion

(in terms of effective stresses)



$$\tau_f = c' + \sigma' \tan \phi'$$

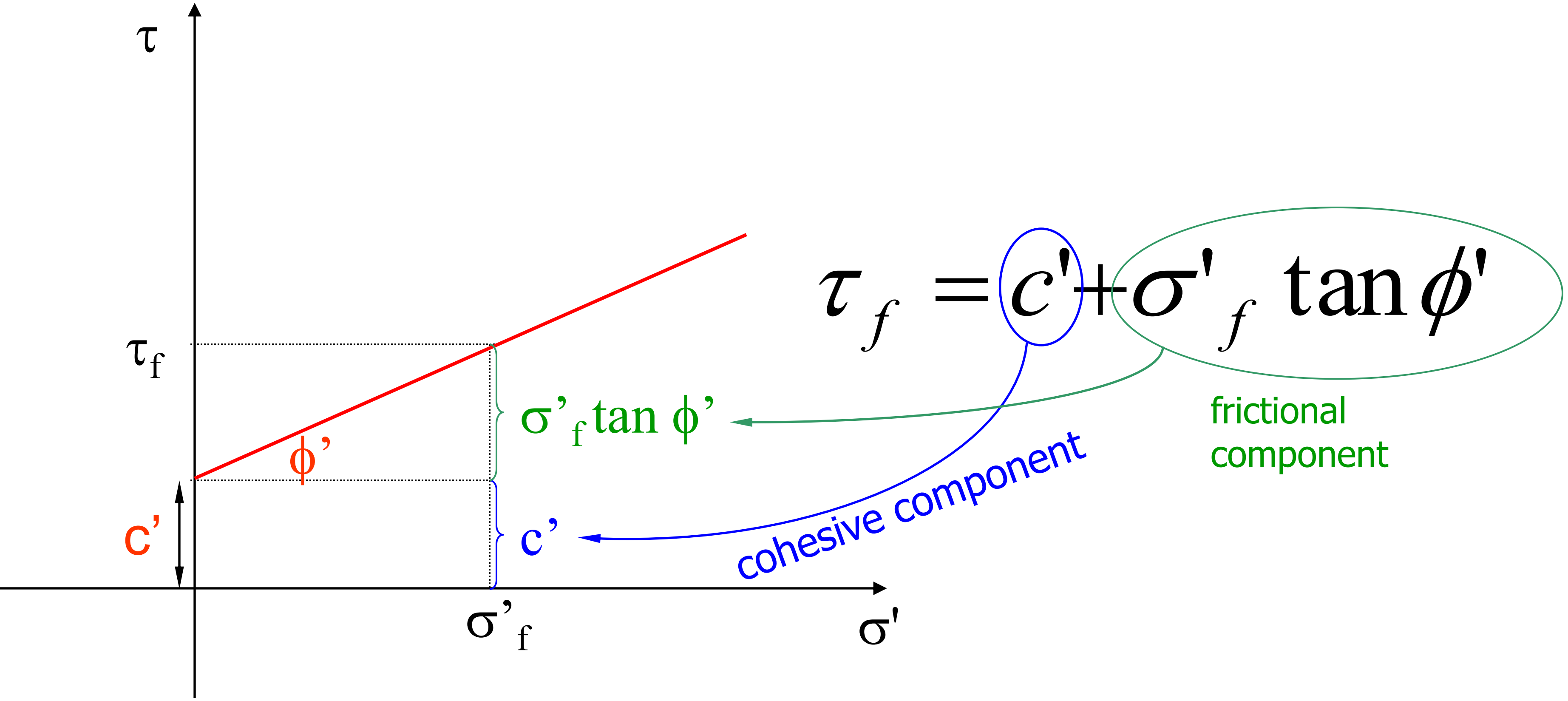
$$\sigma' = \sigma - u$$

u = pore water pressure

τ_f is the maximum shear stress the soil can take without failure, under normal effective stress of σ'.

Mohr-Coulomb Failure Criterion

Shear strength consists of two components: cohesive and frictional.

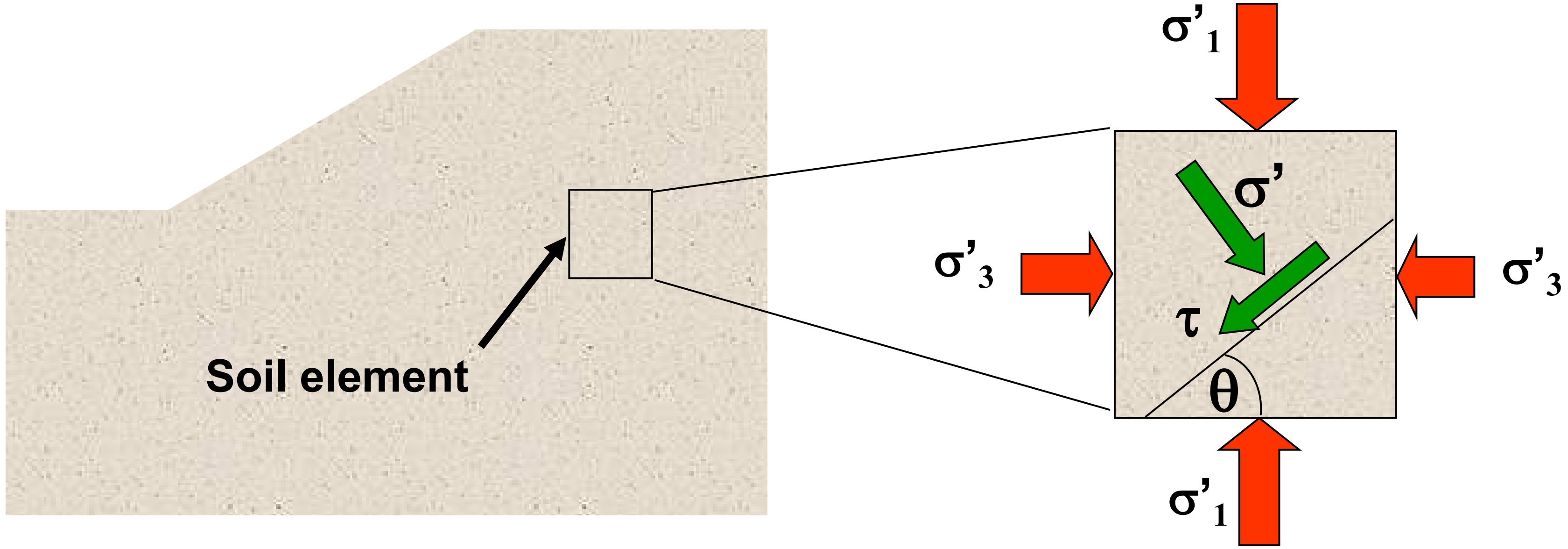


$$\tau_f = c' + \sigma'_f \tan\phi'$$

c and ϕ are measures of shear strength.

Higher the values, higher the shear strength.

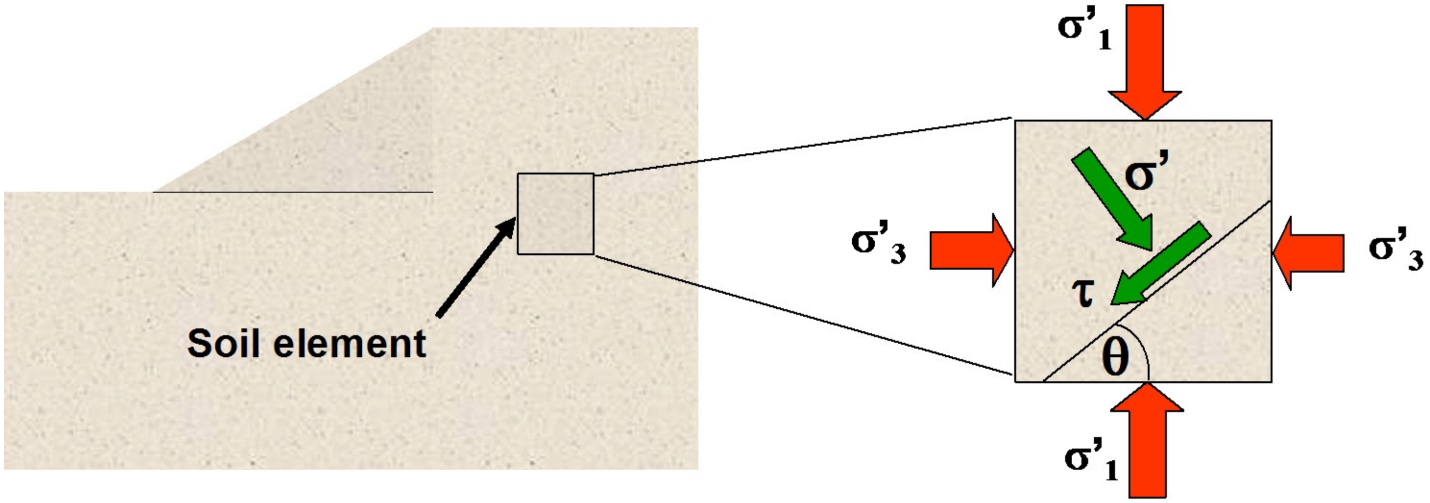
Mohr Circle of stress



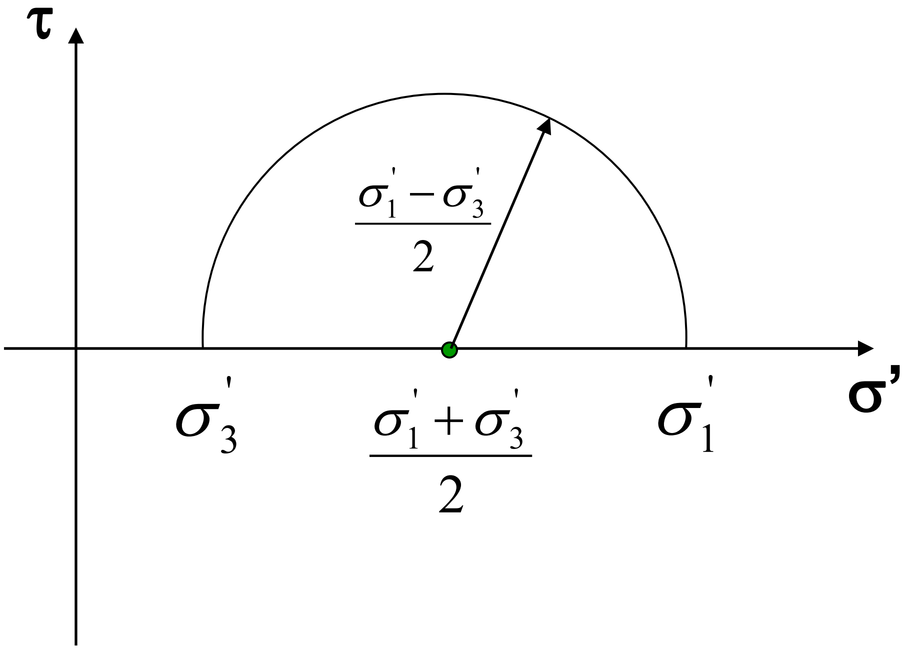
Resolving forces in σ and τ directions,

$$\left.\begin{aligned}\tau &= \frac{\sigma_1^{'} - \sigma_3^{'}}{2} Sin2\theta \\ \sigma^{'} &= \frac{\sigma_1^{'} + \sigma_3^{'}}{2} + \frac{\sigma_1^{'} - \sigma_3^{'}}{2} Cos2\theta\end{aligned}\right\} \quad \tau^2 + \left(\sigma^{'} - \frac{\sigma_1^{'} + \sigma_3^{'}}{2}\right)^2 = \left(\frac{\sigma_1^{'} - \sigma_3^{'}}{2}\right)^2$$

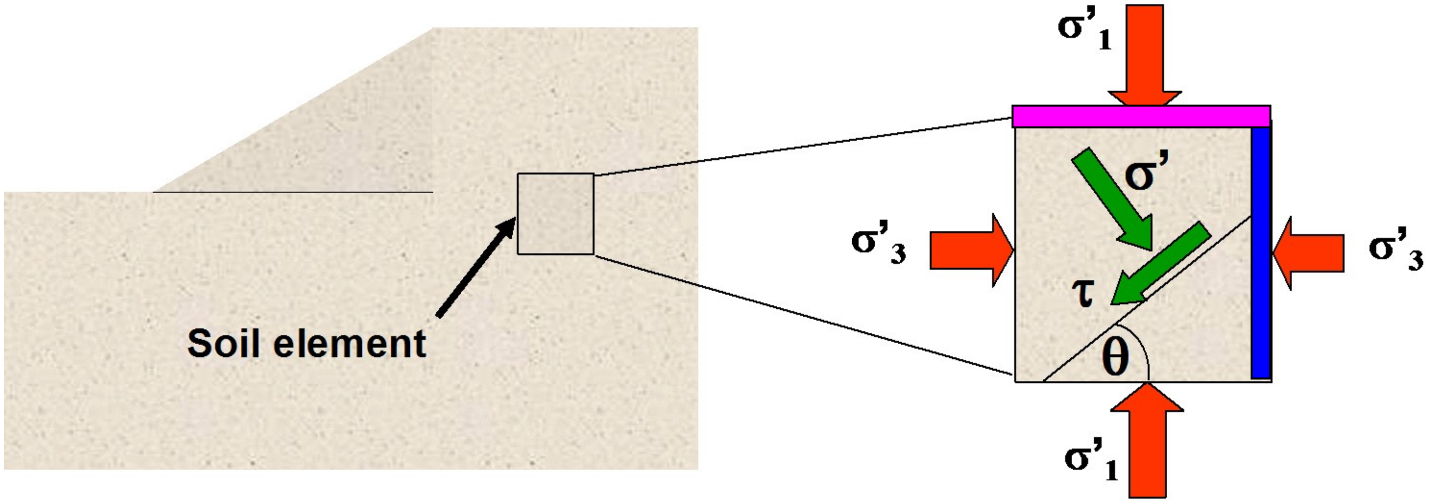
Mohr Circle of stress



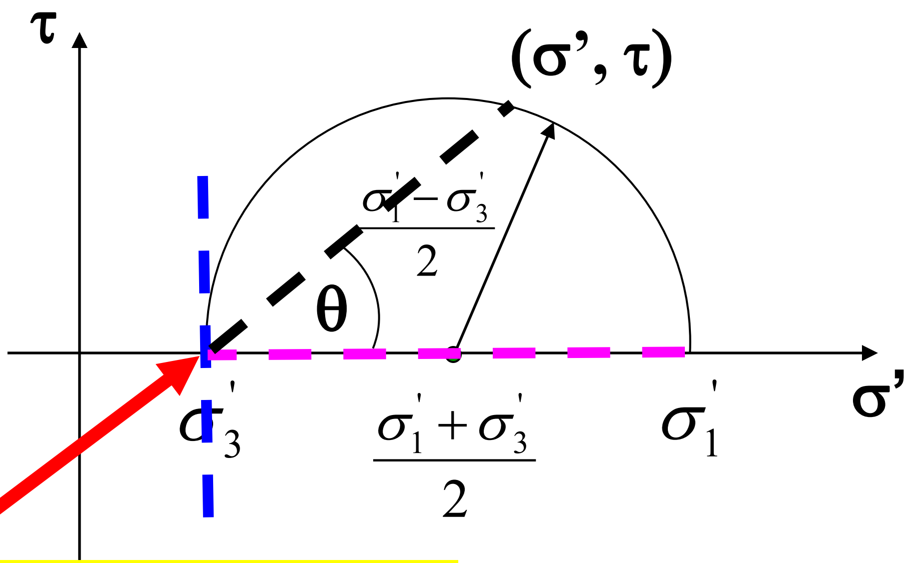
$$\tau^2 + \left(\sigma' - \frac{\sigma_1' + \sigma_3'}{2}\right)^2 = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)^2$$



Mohr Circle of stress

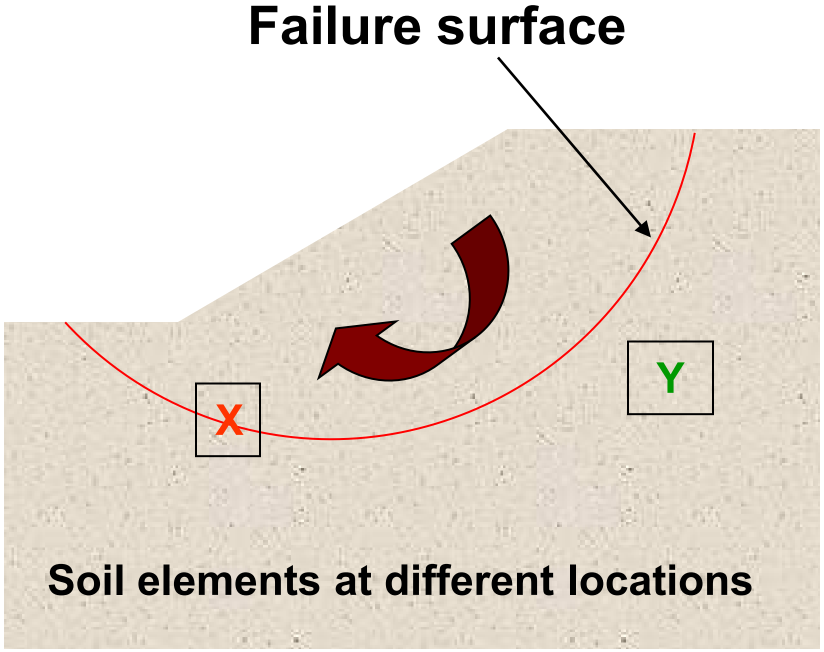


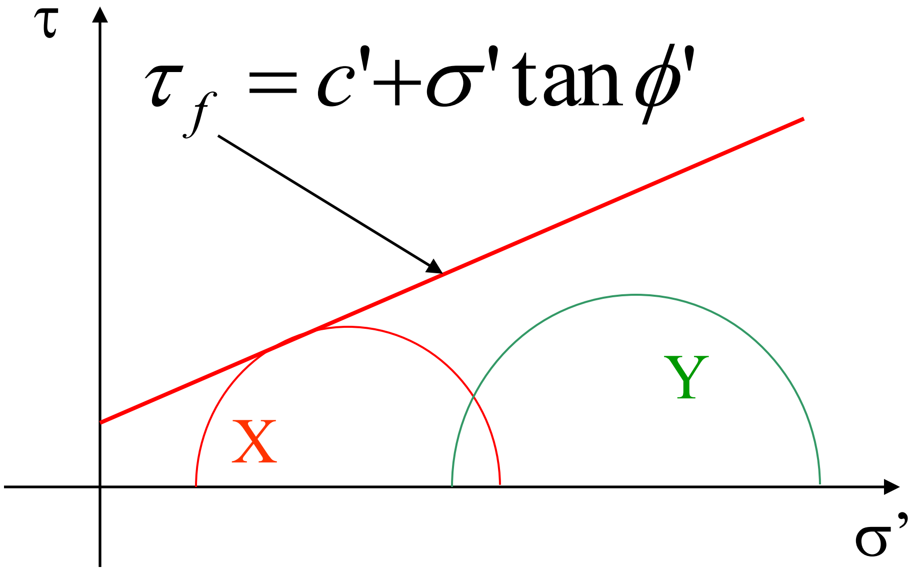
$$\tau^2 + \left(\sigma' - \frac{\sigma_1' + \sigma_3'}{2}\right)^2 = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)^2$$



P_D = Pole w.r.t. plane

Mohr Circles & Failure Envelope

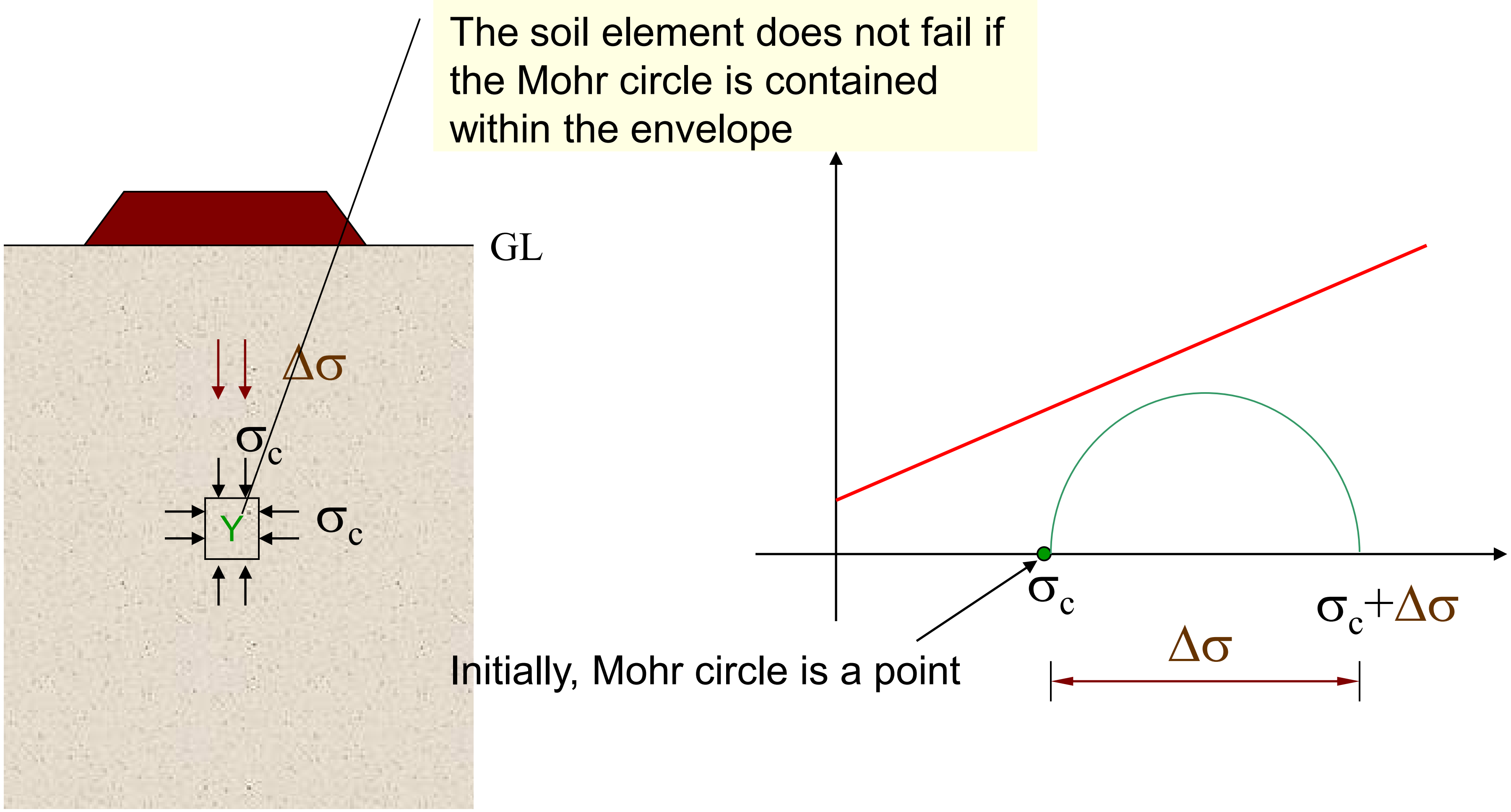




Y ~ stable

X ~ failure

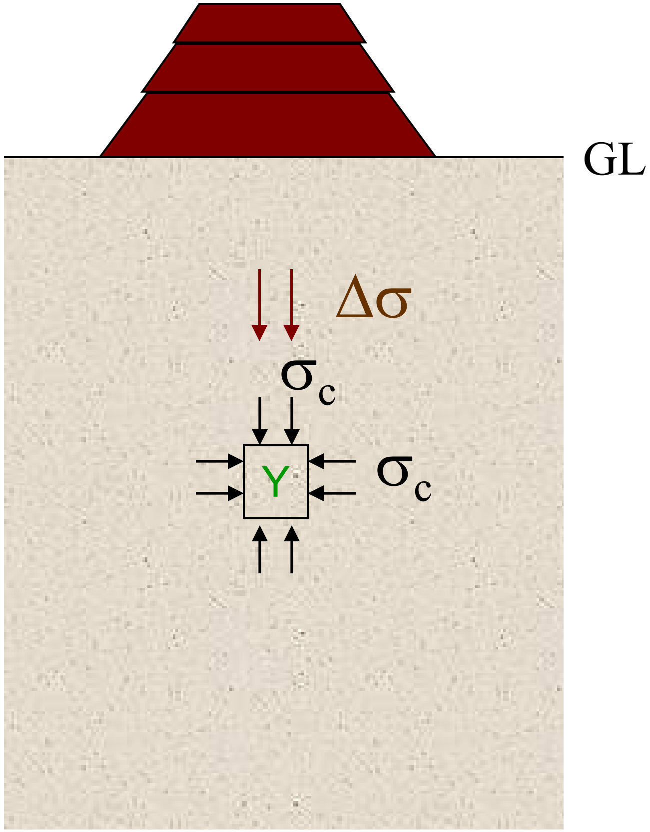
Mohr Circles & Failure Envelope



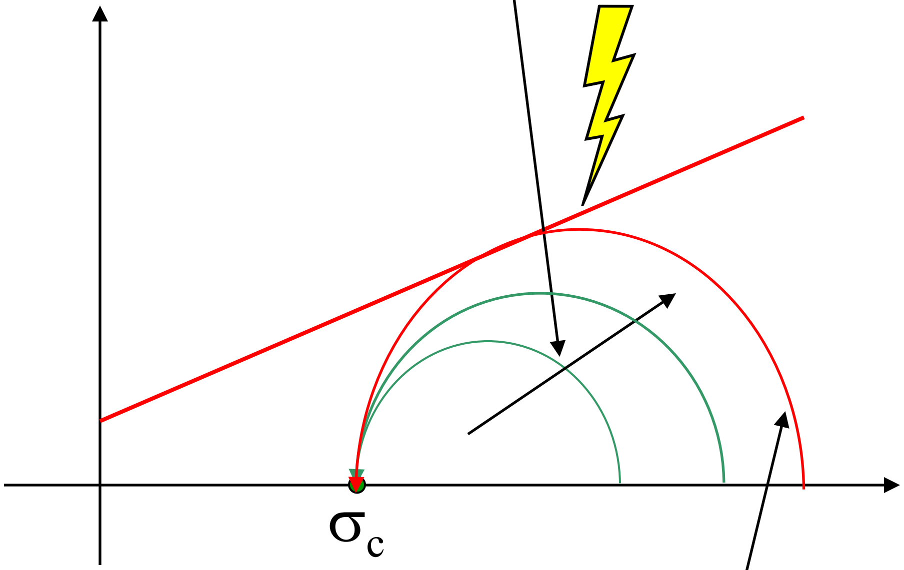
The soil element does not fail if the Mohr circle is contained within the envelope

Initially, Mohr circle is a point

Mohr Circles & Failure Envelope

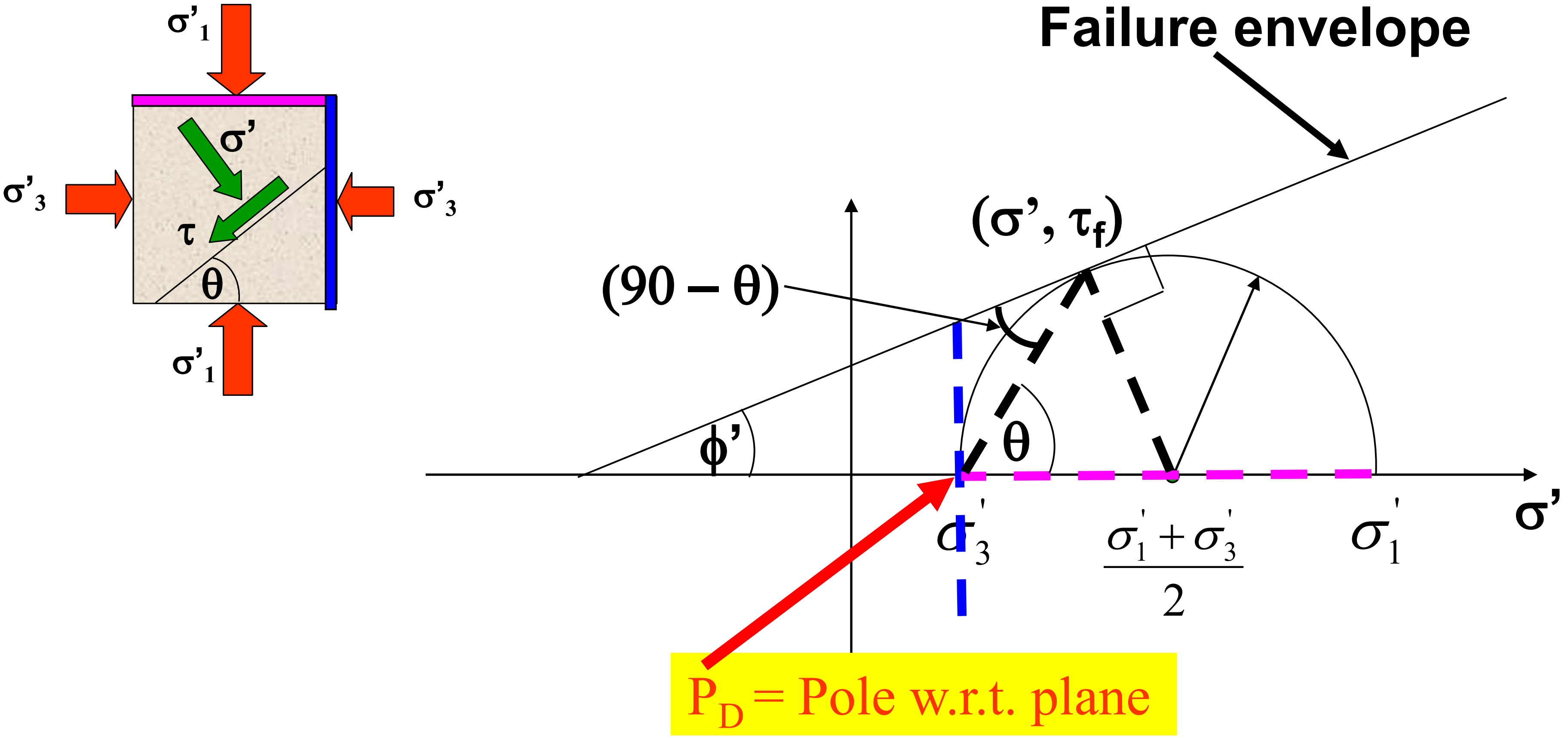


As loading progresses, Mohr circle becomes larger…



.. and finally failure occurs when Mohr circle touches the envelope

Orientation of Failure Plane



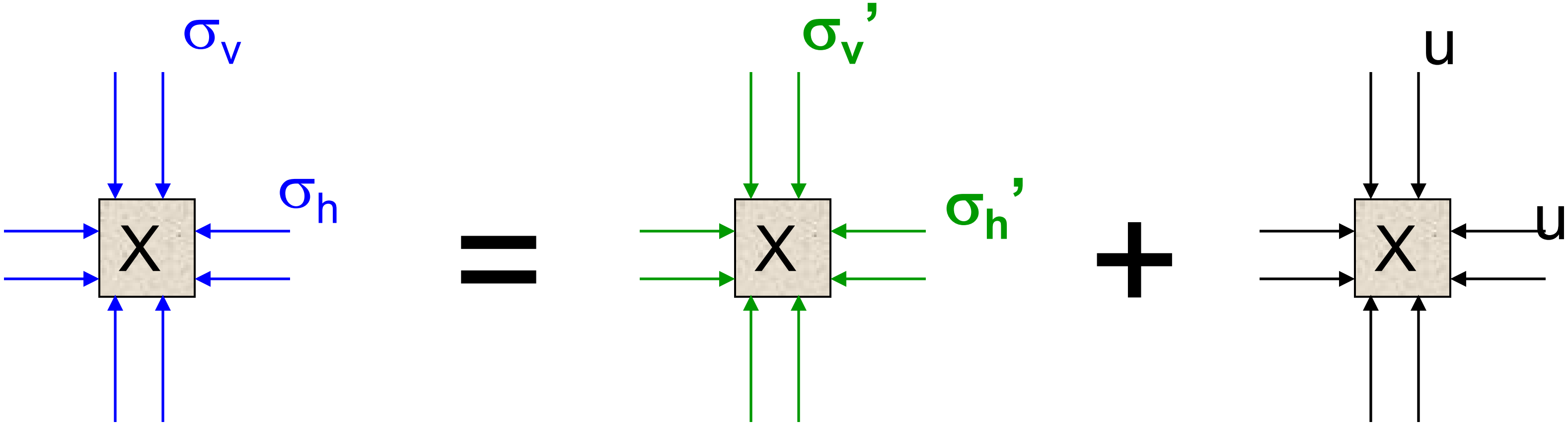
P_D = Pole w.r.t. plane

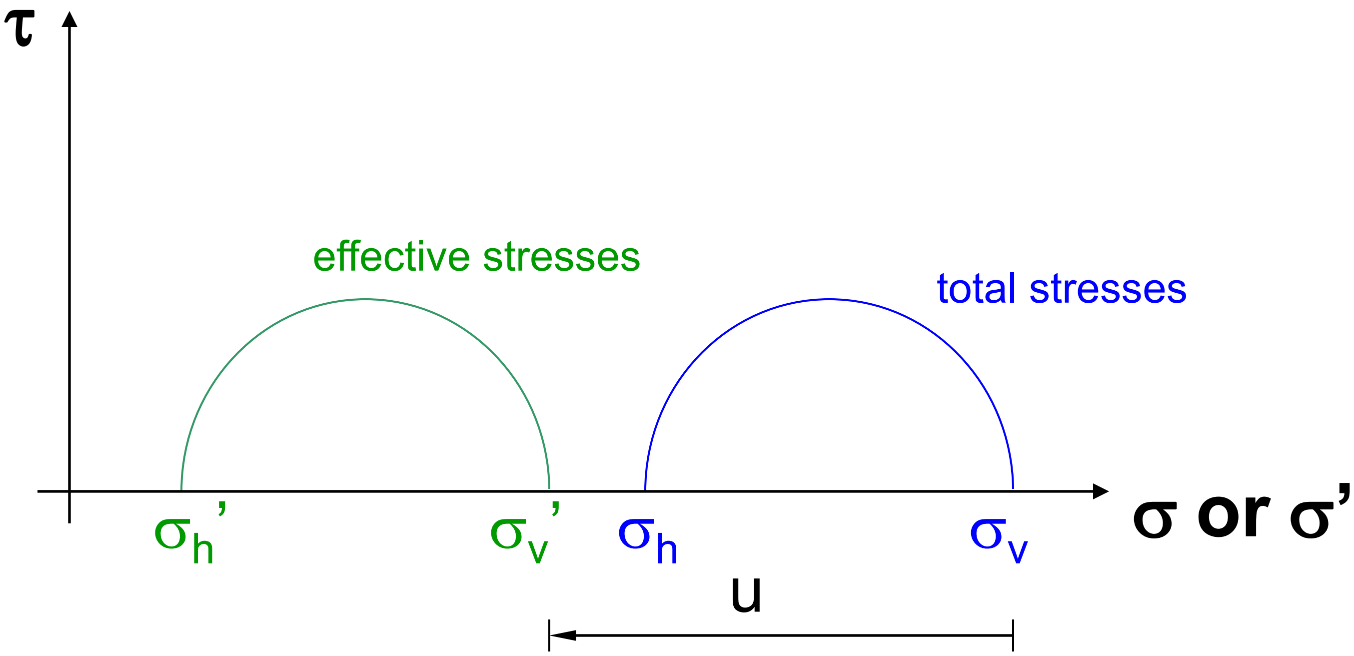
Therefore,

$90 - \theta + \phi' = \theta$

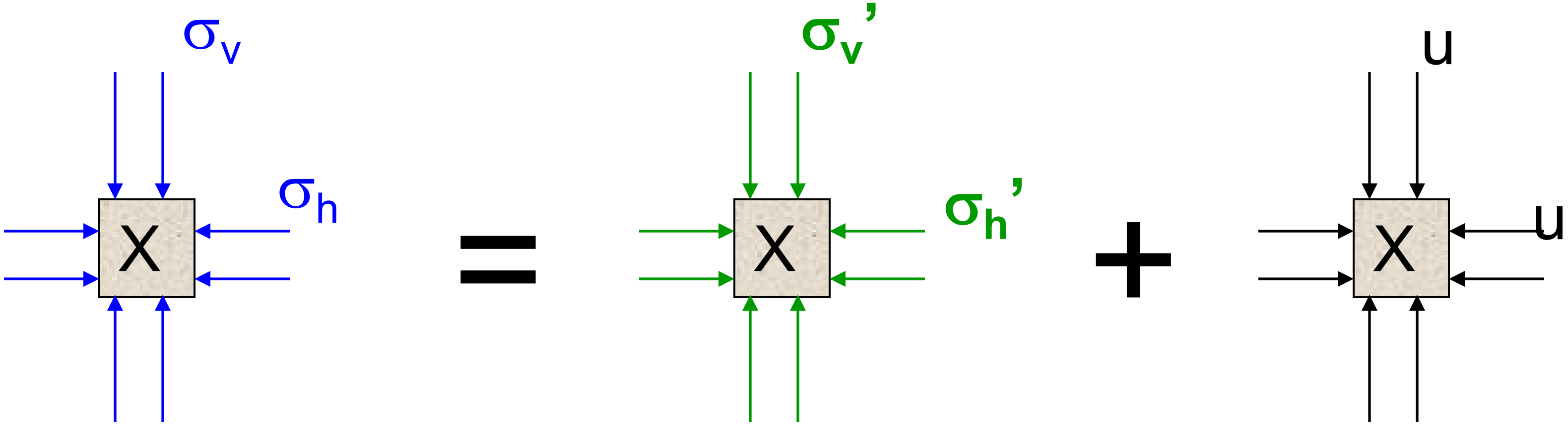
$\theta = 45 + \phi'/2$

Mohr circles in terms of total & effective stresses

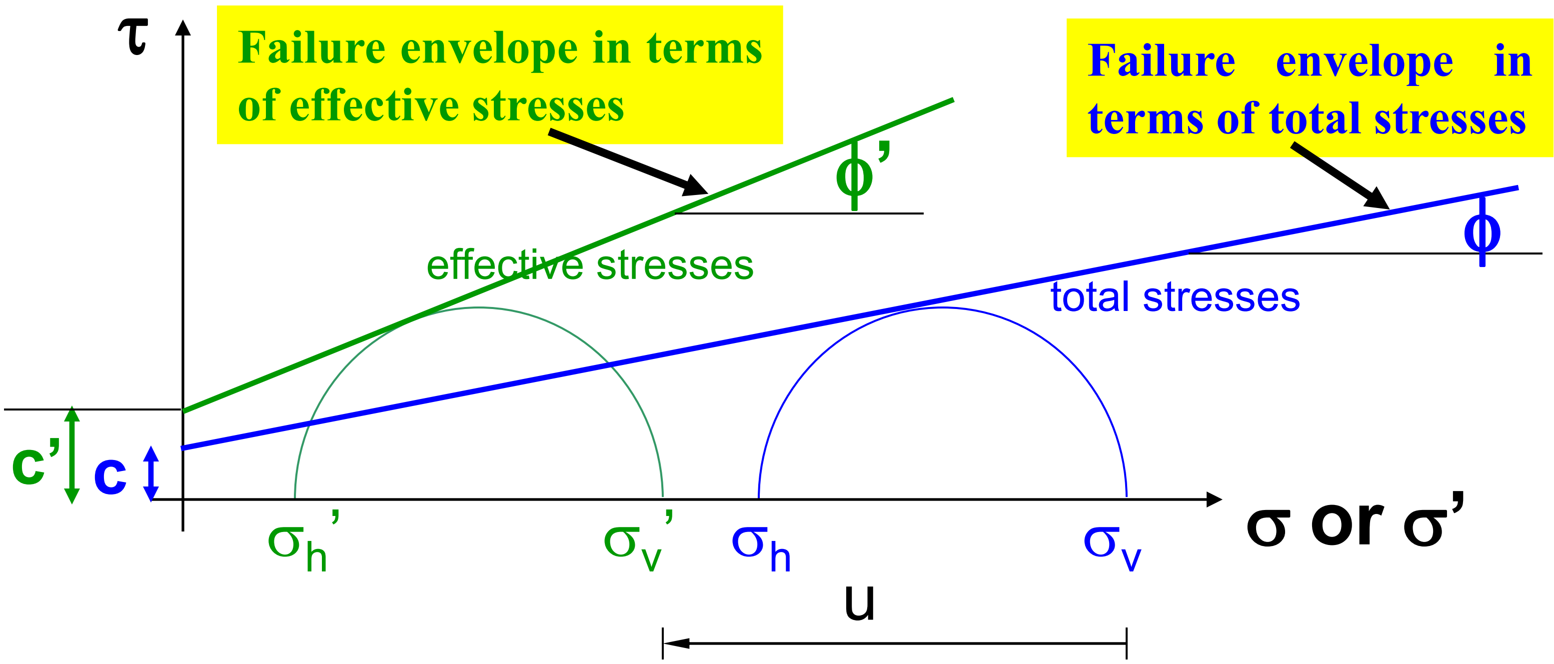




Failure envelopes in terms of total & effective stresses

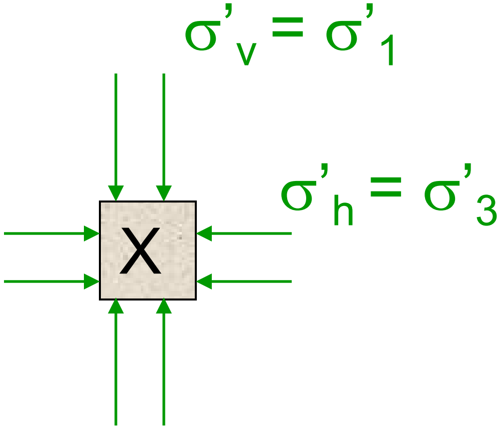


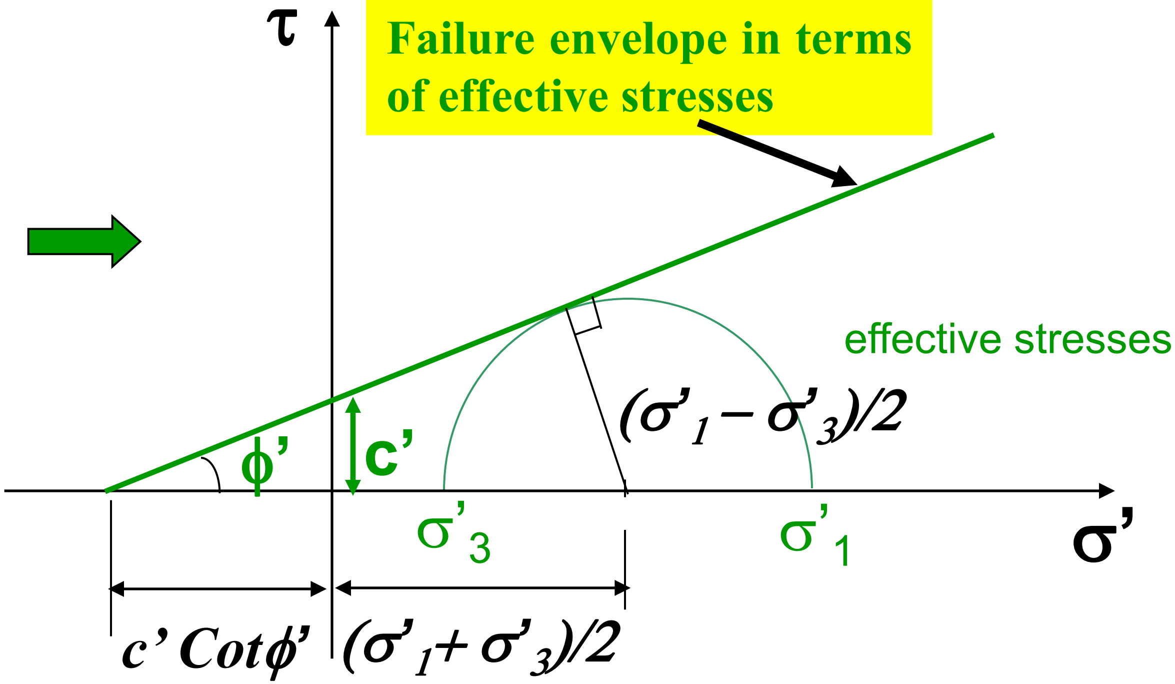
If X is on failure


Failure envelope in terms of effective stresses

Failure envelope in terms of total stresses

Mohr Coulomb failure criterion with Mohr circle of stress





X is on failure

Therefore,

$$\left[c'Cot\phi' + \left(\frac{\sigma_1' + \sigma_3'}{2}\right)\right] Sin\phi' = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)$$

Mohr Coulomb failure criterion with Mohr circle of stress

$$\left[c'\,Cot\,\phi' + \left(\frac{\sigma_1' + \sigma_3'}{2}\right)\right] Sin\,\phi' = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)$$

$$\left(\sigma_1' - \sigma_3'\right) = \left(\sigma_1' + \sigma_3'\right) Sin\,\phi' + 2c'\,Cos\,\phi'$$

$$\sigma_1'\left(1 - Sin\,\phi'\right) = \sigma_3'\left(1 + Sin\,\phi'\right) + 2c'\,Cos\,\phi'$$

$$\sigma_1' = \sigma_3' \frac{\left(1 + Sin\,\phi'\right)}{\left(1 - Sin\,\phi'\right)} + 2c' \frac{Cos\,\phi'}{\left(1 - Sin\,\phi'\right)}$$

$$\sigma_1' = \sigma_3' Tan^2\left(45 + \frac{\phi'}{2}\right) + 2c'\,Tan\left(45 + \frac{\phi'}{2}\right)$$

Determination of shear strength parameters of soils (c, ϕ or c', ϕ')

Laboratory tests on specimens taken from representative undisturbed samples

Most common laboratory tests to determine the shear strength parameters are,

1. Direct shear test
2. Triaxial shear test

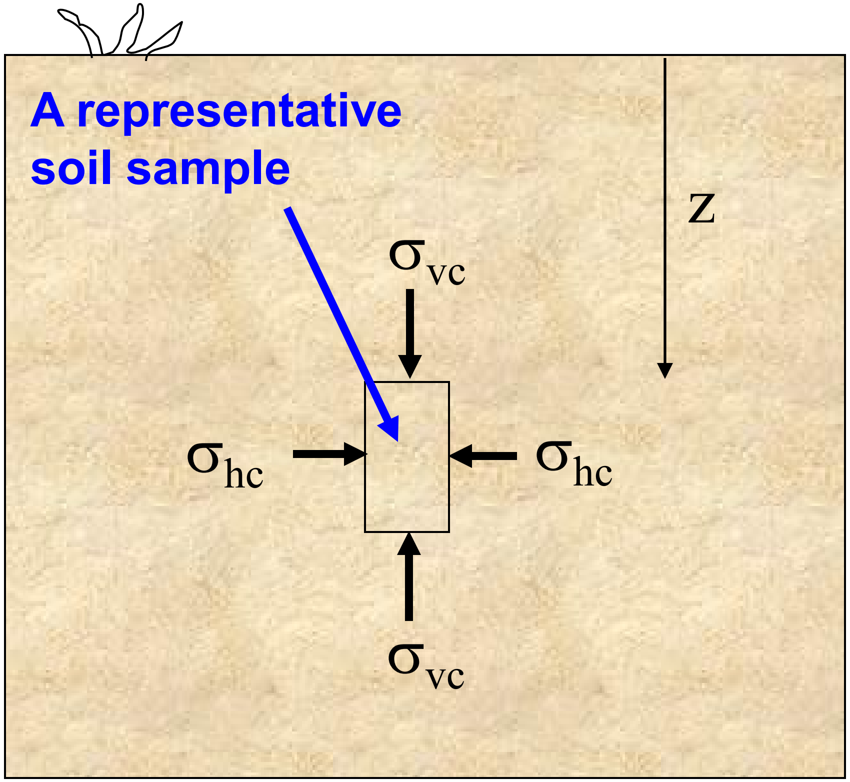
Other laboratory tests include, Direct simple shear test, torsional ring shear test, plane strain triaxial test, laboratory vane shear test, laboratory fall cone test

Field tests

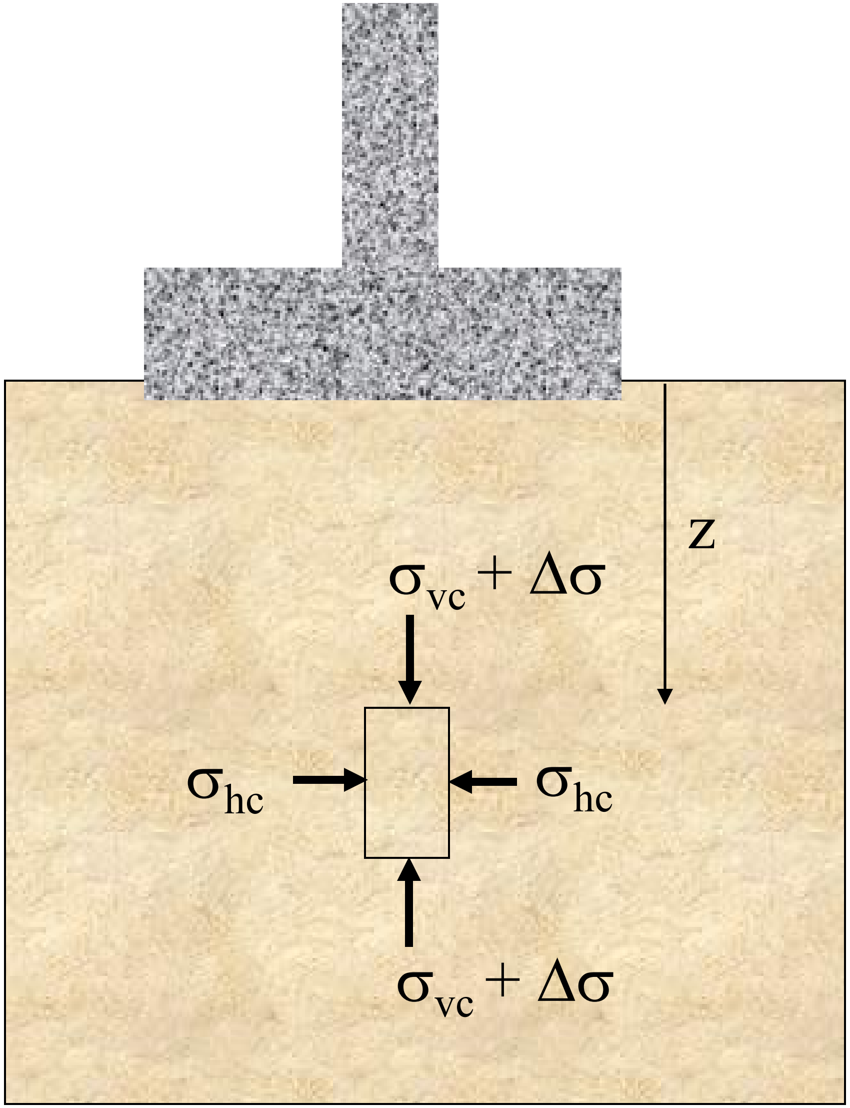
1. Vane shear test
2. Torvane
3. Pocket penetrometer
4. Fall cone
5. Pressuremeter
6. Static cone penetrometer
7. Standard penetration test

Laboratory tests

Field conditions



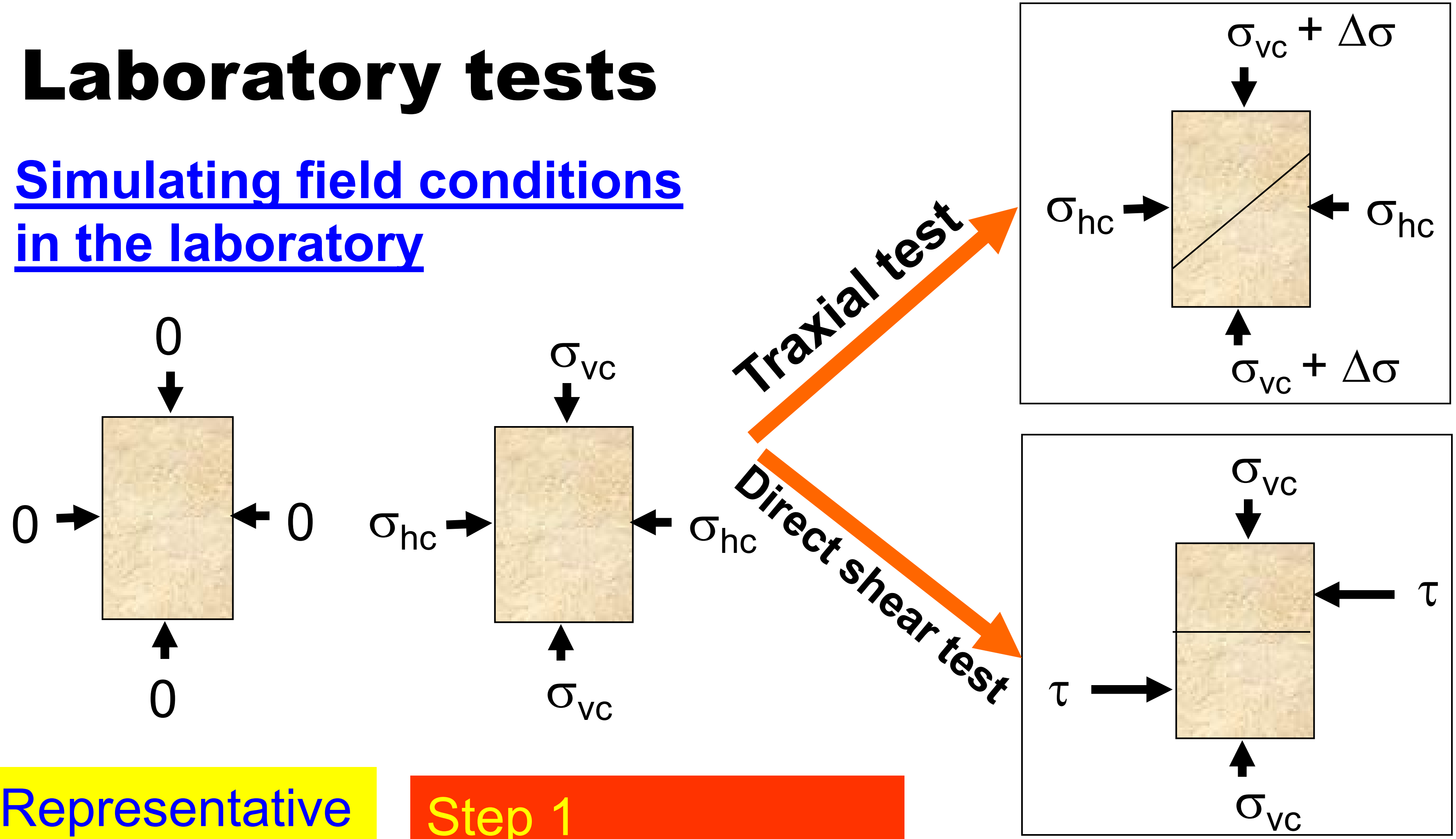
Before construction



After and during construction

Laboratory tests

Simulating field conditions in the laboratory



Representative soil sample taken from the site

Step 1

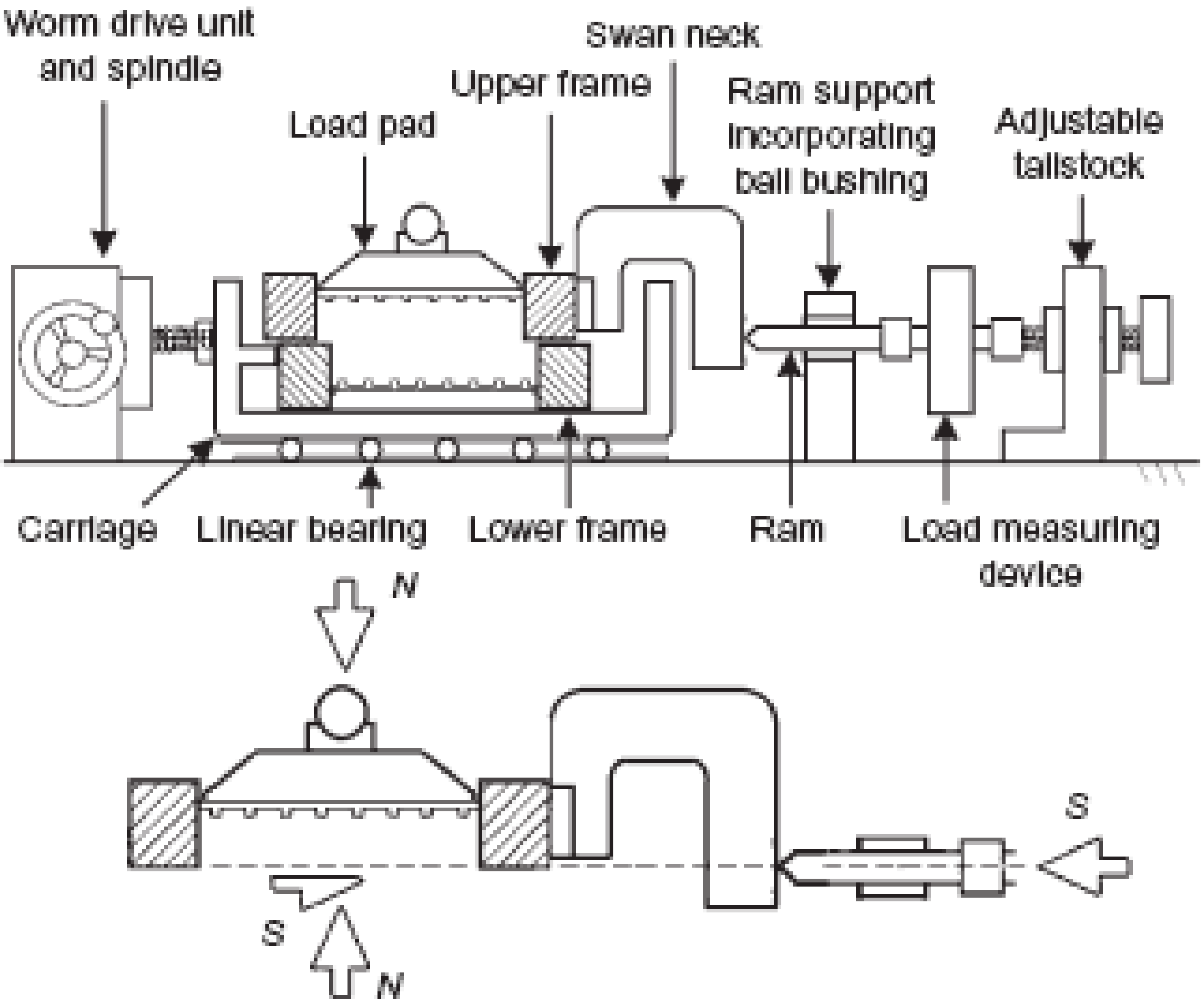
Set the specimen in the apparatus and apply the initial stress condition

Step 2

Apply the corresponding field stress conditions

Direct shear test

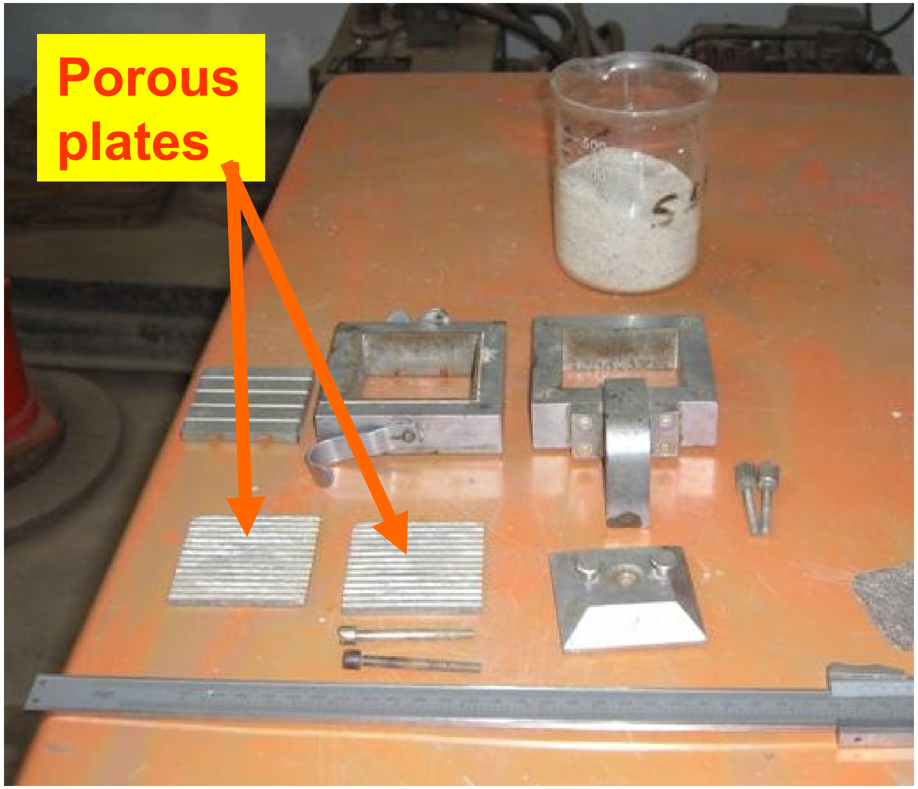
Schematic diagram of the direct shear apparatus



Direct shear test

Direct shear test is most suitable for consolidated drained tests specially on granular soils (e.g.: sand) or stiff clays

Preparation of a sand specimen



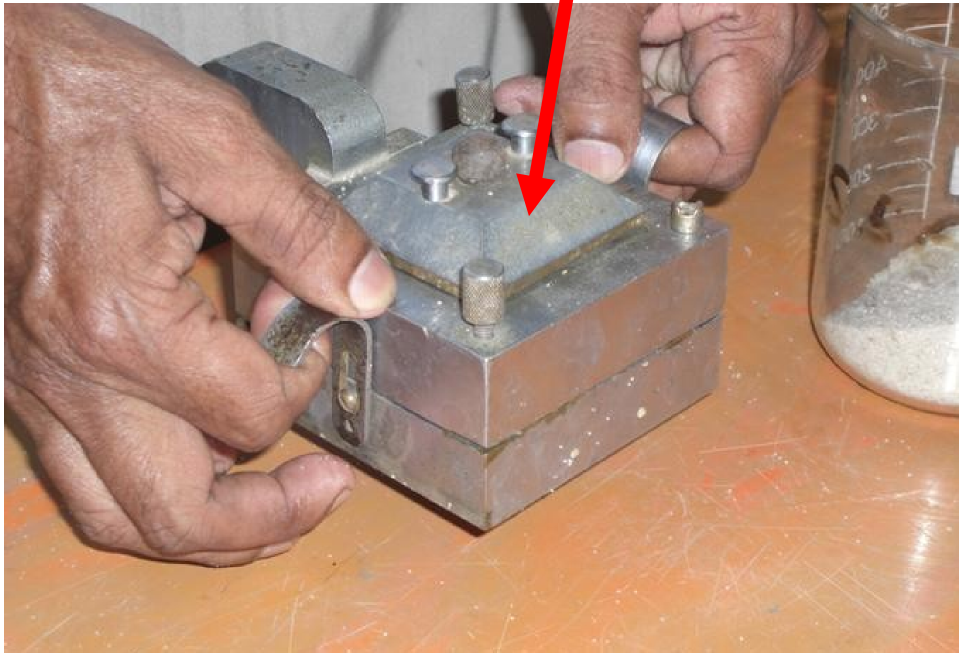
Components of the shear box

Preparation of a sand specimen

Direct shear test

Preparation of a sand specimen

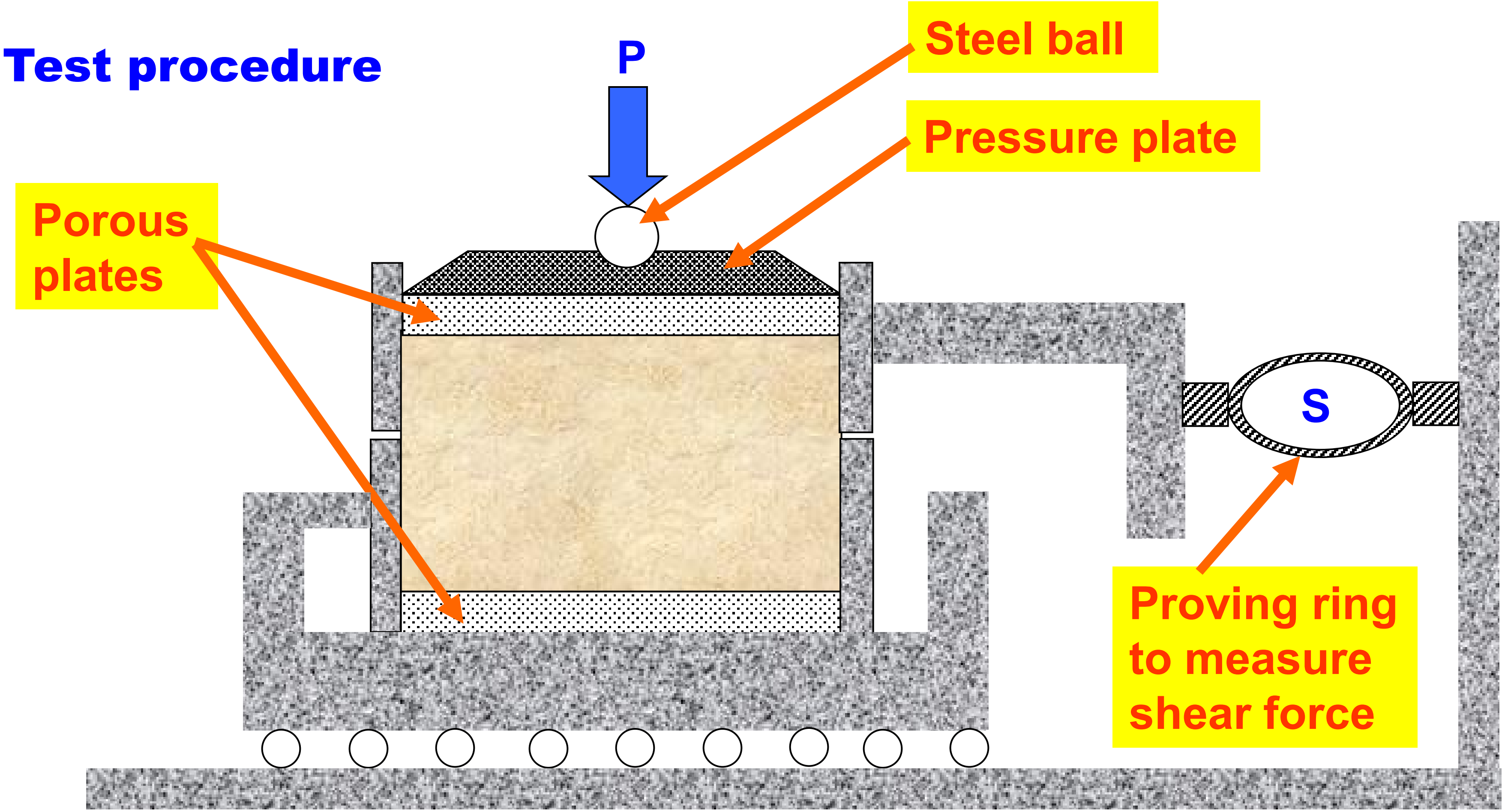
Leveling the top surface of specimen


Pressure plate

Specimen preparation completed

Direct shear test

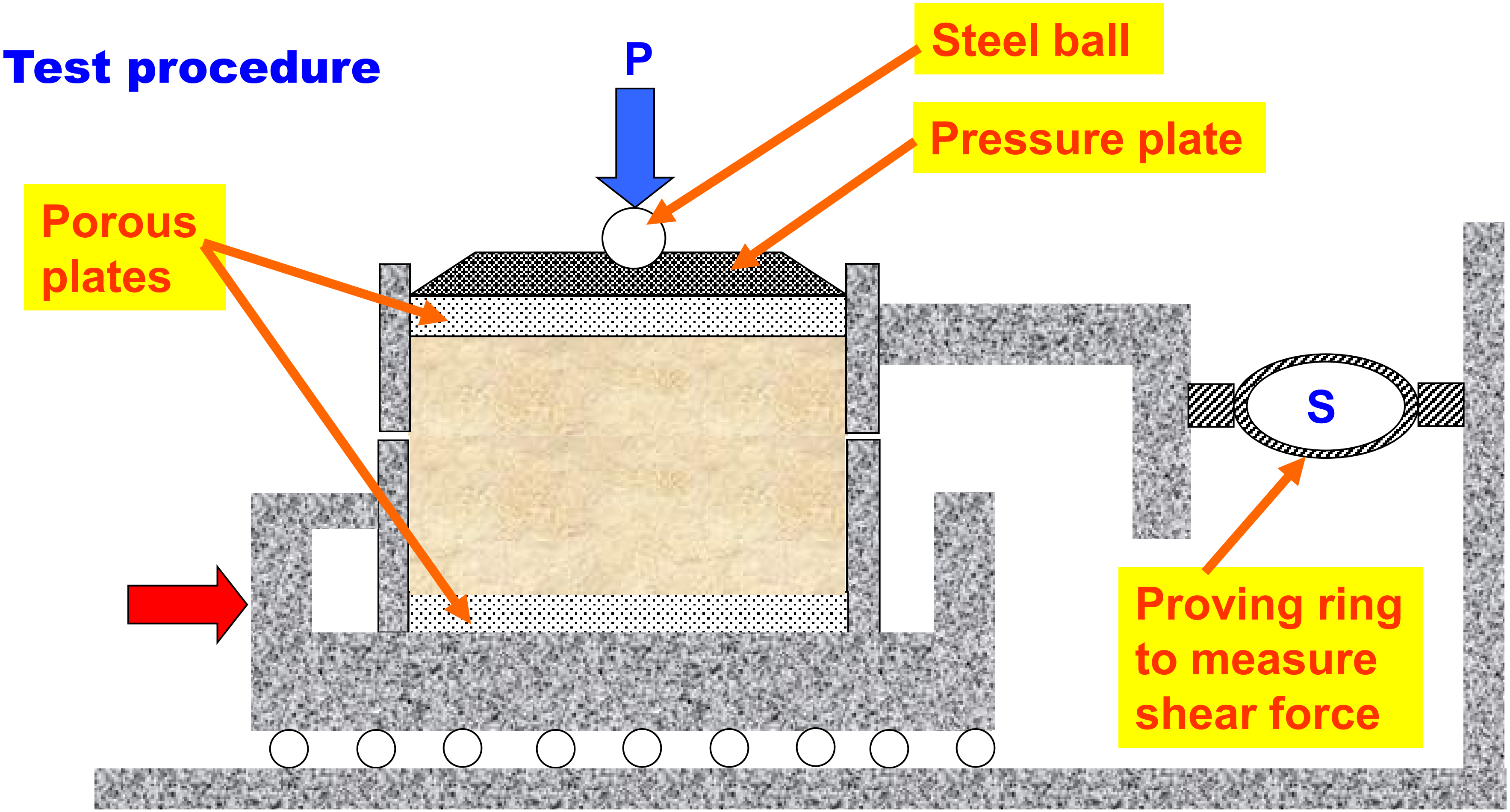
Test procedure



Step 1: Apply a vertical load to the specimen and wait for consolidation

Direct shear test

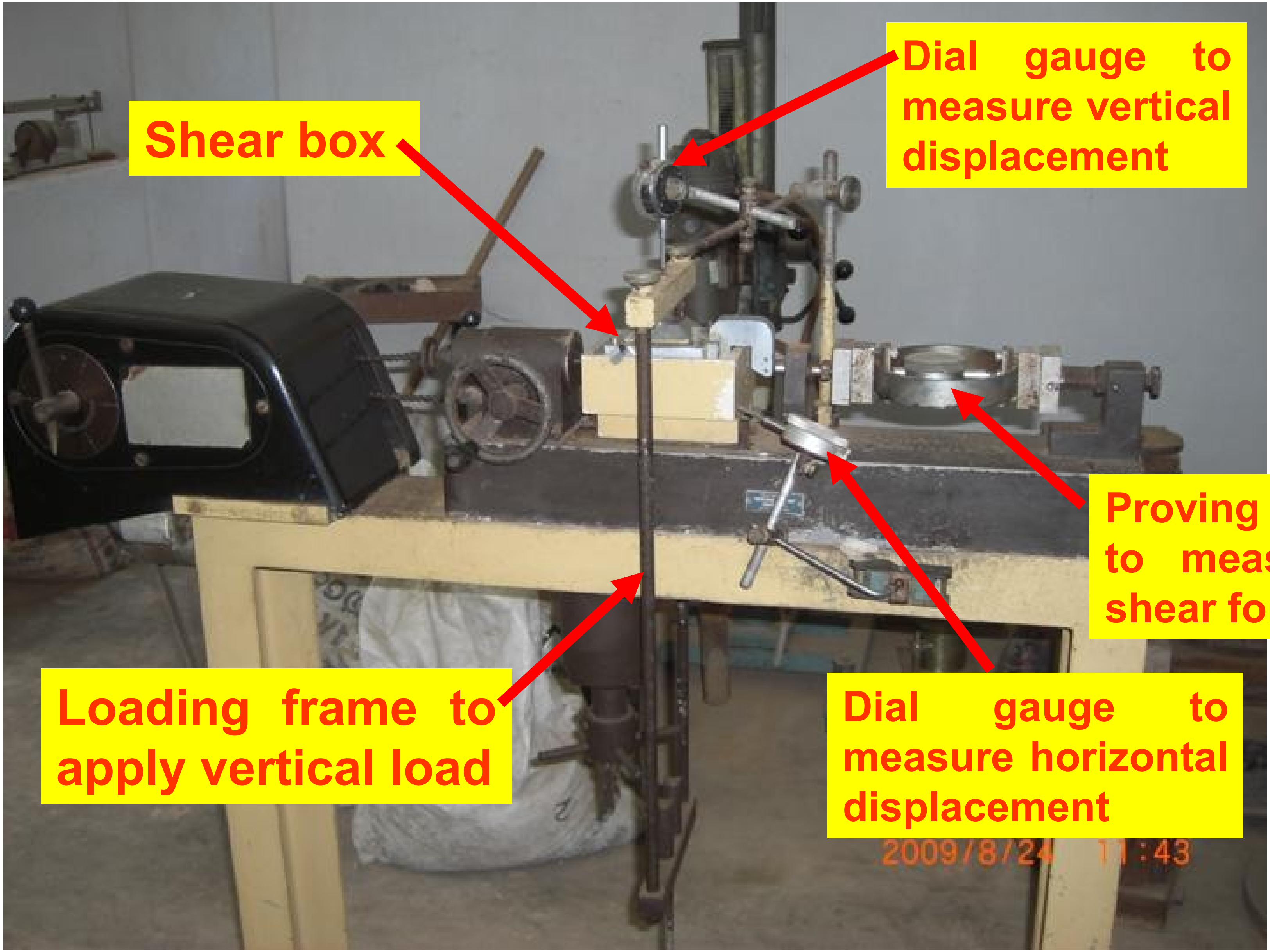
Test procedure



Step 1: Apply a vertical load to the specimen and wait for consolidation

Step 2: Lower box is subjected to a horizontal displacement at a constant rate

Direct shear test



Shear box

Dial gauge to measure vertical displacement

Proving ring to measure shear force

Loading frame to apply vertical load

Dial gauge to measure horizontal displacement

2009/8/24 11:43

Direct shear test

Analysis of test results

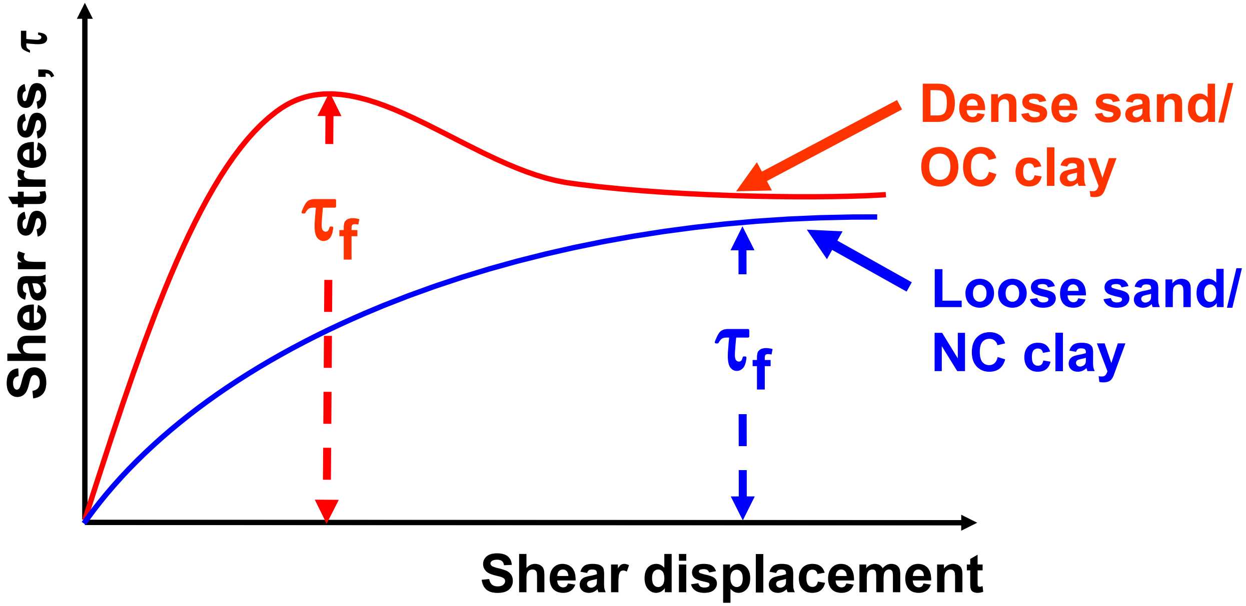
$$\sigma = \text{Normal stress} = \frac{\text{Normal force (P)}}{\text{Area of cross section of the sample}}$$

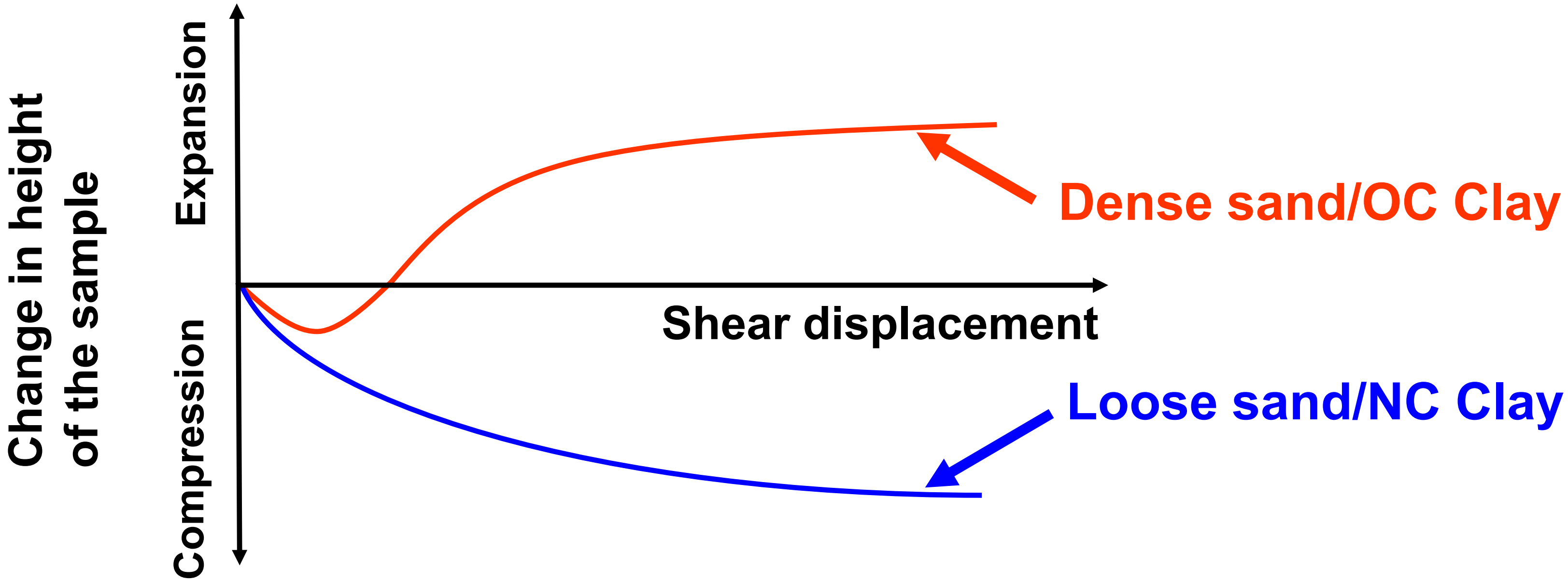
$$\tau = \text{Shear stress} = \frac{\text{Shear resistance developed at the sliding surface (S)}}{\text{Area of cross section of the sample}}$$

Note: Cross-sectional area of the sample changes with the horizontal displacement

Direct shear tests on sands

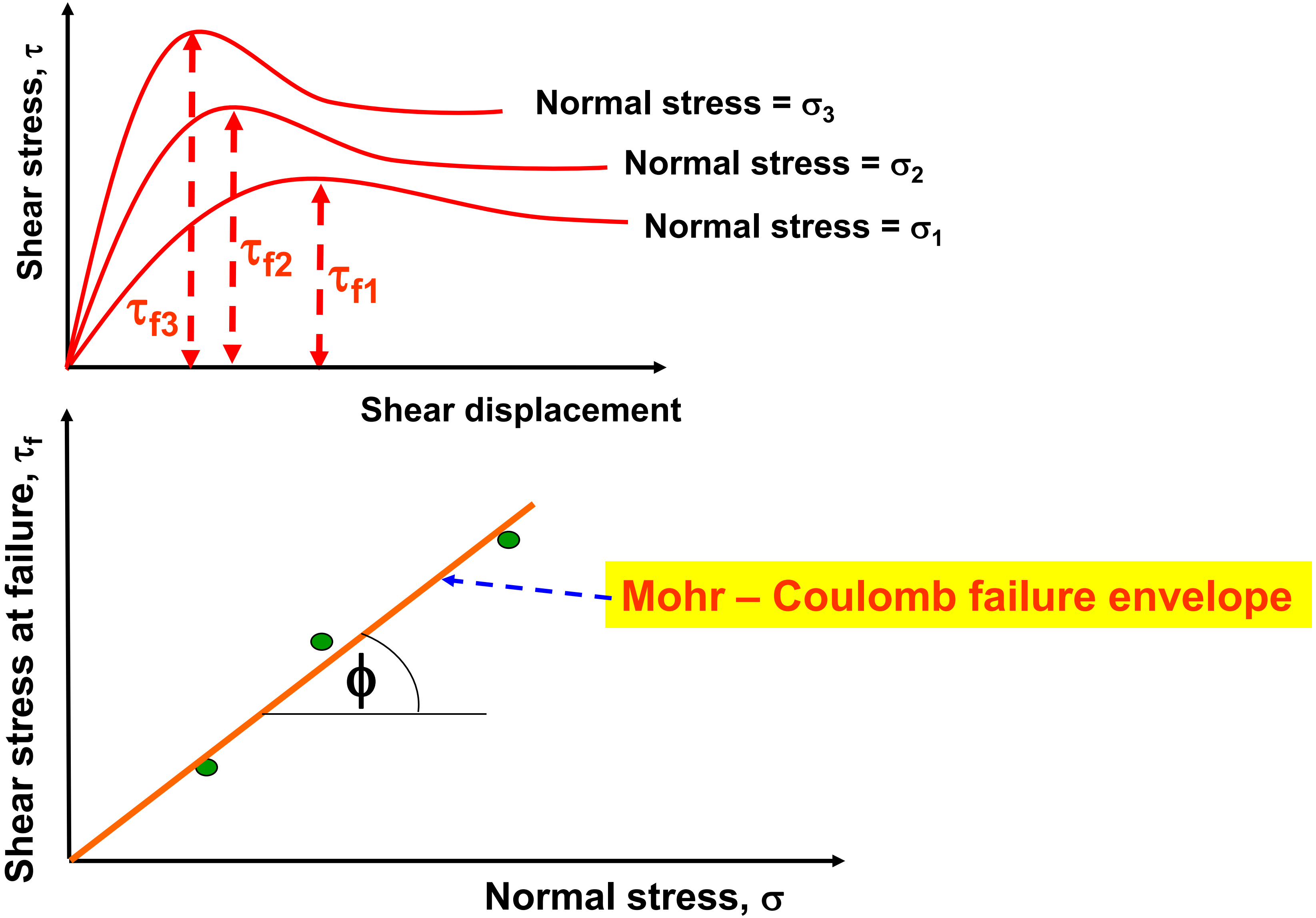
Stress-strain relationship





Direct shear tests on sands

How to determine strength parameters c and ϕ



Mohr – Coulomb failure envelope

Direct shear tests on sands

Some important facts on strength parameters c and ϕ of sand

Sand is cohesionless hence $c = 0$

Direct shear tests are drained and pore water pressures are dissipated, hence $u = 0$

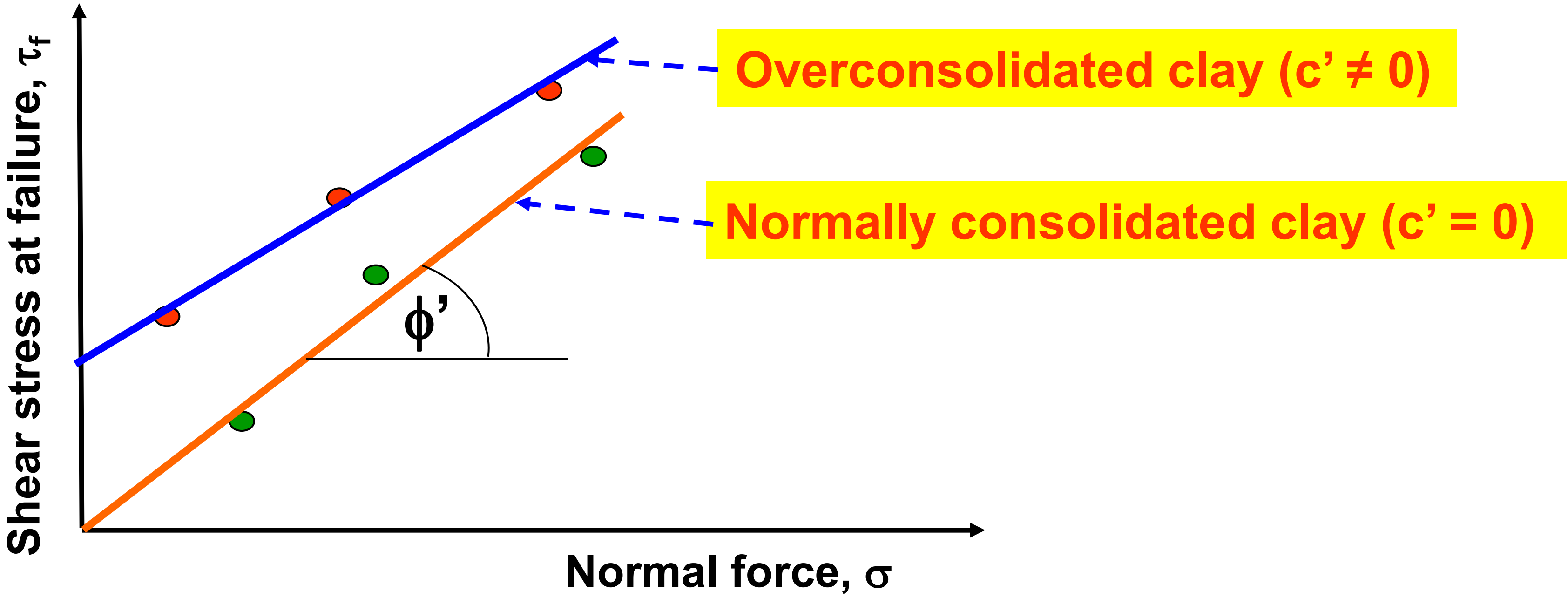
Therefore,

$\phi' = \phi$ and $c' = c = 0$

Direct shear tests on clays

In case of clay, horizontal displacement should be applied at a very slow rate to allow dissipation of pore water pressure (therefore, one test would take several days to finish)

Failure envelopes for clay from drained direct shear tests

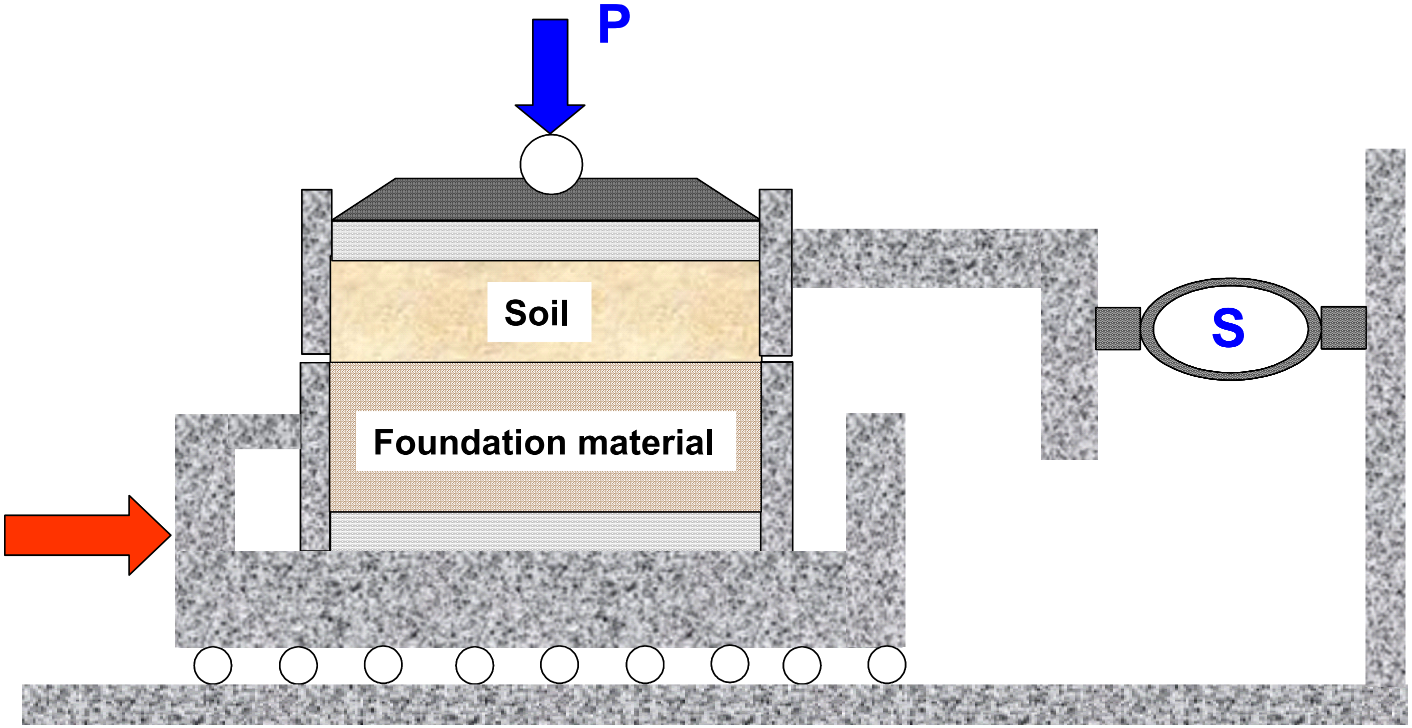


Overconsolidated clay (c’ ≠ 0)

Normally consolidated clay (c’ = 0)

Interface tests on direct shear apparatus

In many foundation design problems and retaining wall problems, it is required to determine the angle of internal friction between soil and the structural material (concrete, steel or wood)



$$\tau_f = c_a + \sigma' \tan\delta$$

Where,

c_a = adhesion,

δ = angle of internal friction

Advantages of direct shear apparatus

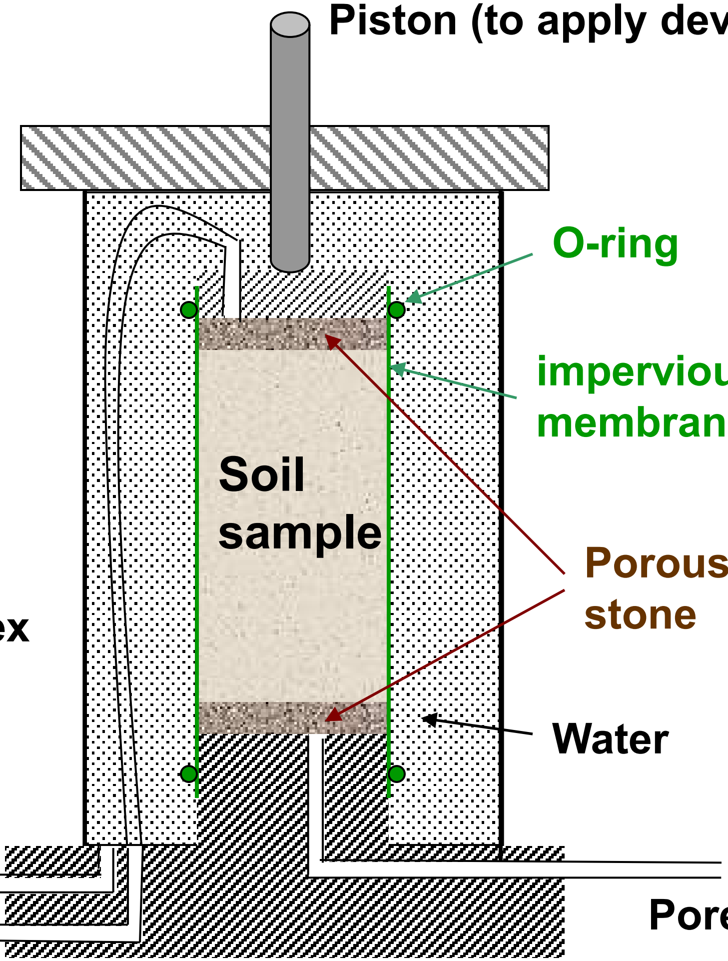
- ☐ Due to the smaller thickness of the sample, rapid drainage can be achieved
- ☐ Can be used to determine interface strength parameters
- ☐ Clay samples can be oriented along the plane of weakness or an identified failure plane

Disadvantages of direct shear apparatus

- ☐ Failure occurs along a predetermined failure plane
- ☐ Area of the sliding surface changes as the test progresses
- ☐ Non-uniform distribution of shear stress along the failure surface

Triaxial Shear Test

Soil sample at failure



Triaxial Shear Test

Specimen preparation (undisturbed sample)

Sampling tubes

Sample extruder

Triaxial Shear Test

Specimen preparation (undisturbed sample)

Edges of the sample are carefully trimmed

Setting up the sample in the triaxial cell

Triaxial Shear Test

Specimen preparation (undisturbed sample)

Sample is covered with a rubber membrane and sealed

Cell is completely filled with water

Triaxial Shear Test

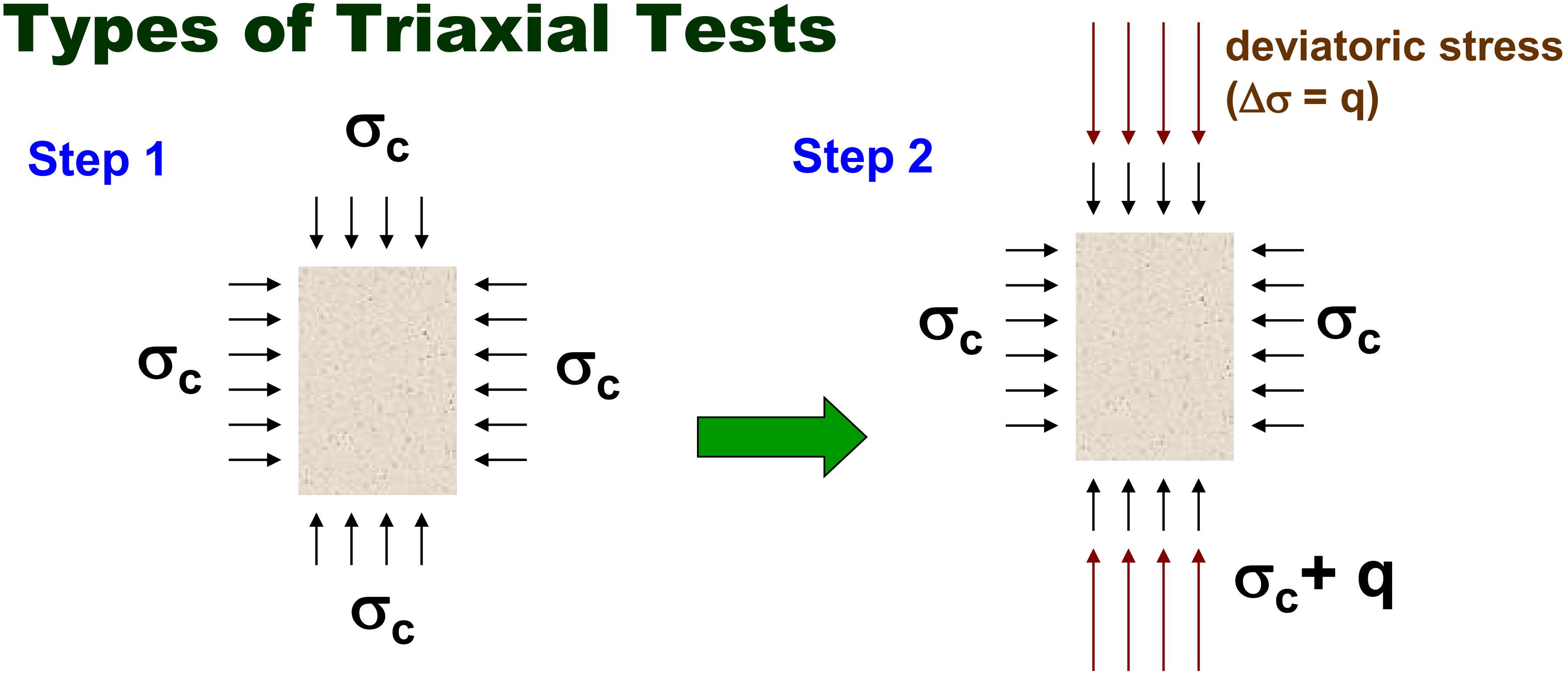
Specimen preparation (undisturbed sample)

Proving ring to measure the deviator load

Dial gauge to measure vertical displacement

In some tests

Types of Triaxial Tests



Under all-around cell pressure σ_c

Is the drainage valve open?

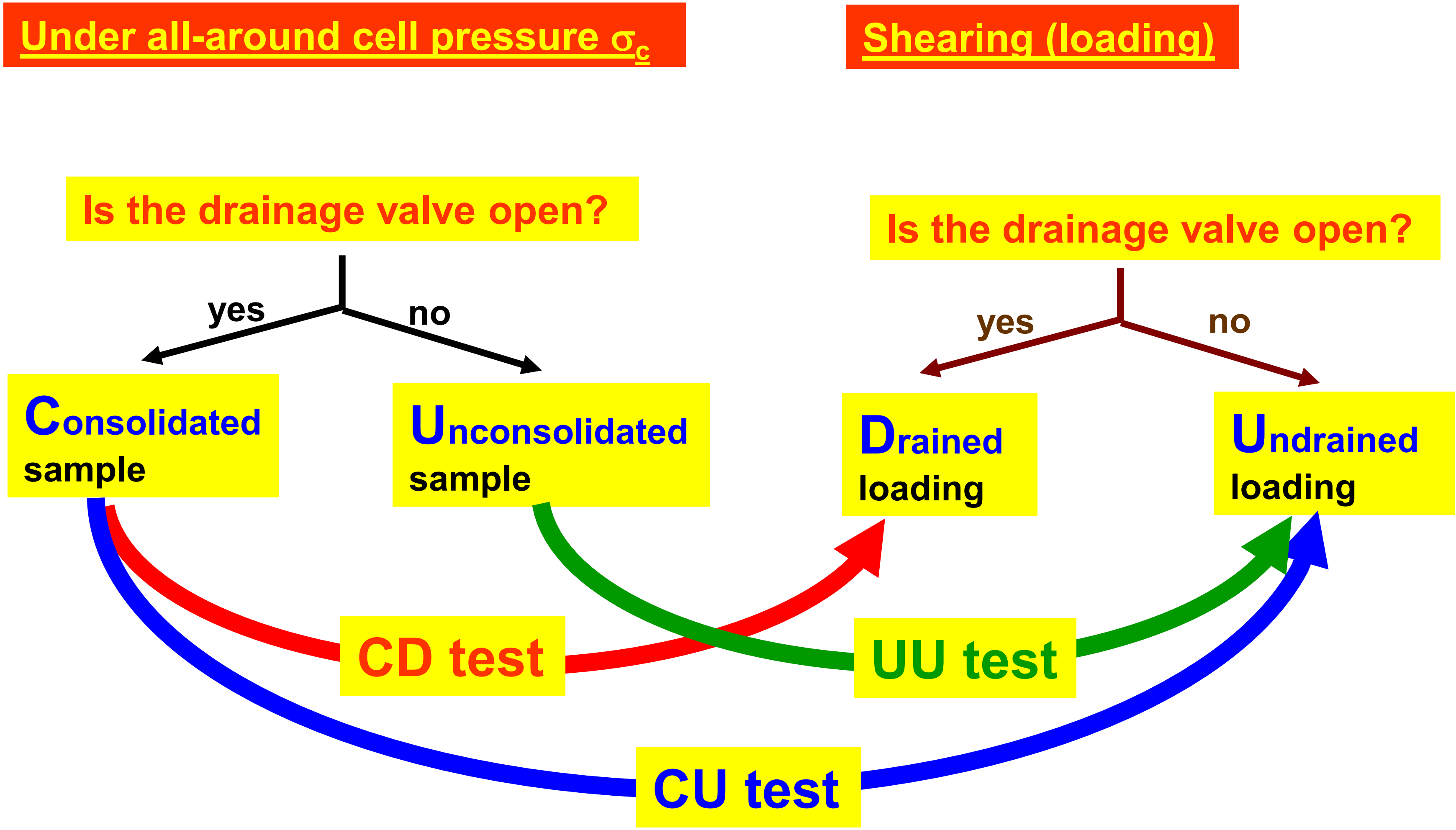
- yes → **Consolidated** sample
- no → **Unconsolidated** sample

Shearing (loading)

Is the drainage valve open?

- yes → **Drained** loading
- no → **Undrained** loading

Types of Triaxial Tests



Step 1

Under all-around cell pressure σ_c

Is the drainage valve open?

yes

no

Consolidated sample

Unconsolidated sample

Step 2

Shearing (loading)

Is the drainage valve open?

yes

no

Drained loading

Undrained loading

CD test

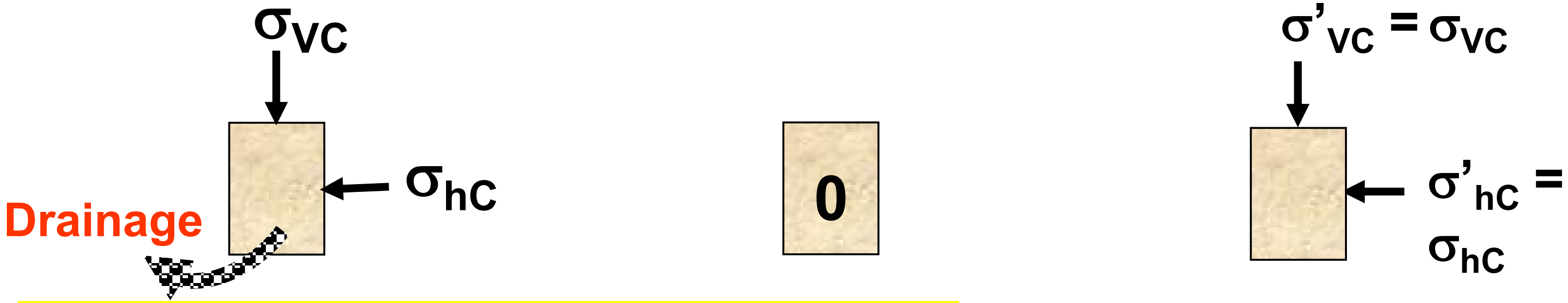
UU test

CU test

Consolidated- drained test (CD Test)

Total, σ = **Neutral, u** + **Effective, σ'**

Step 1: At the end of consolidation



σ_{VC}

σ_{hC}

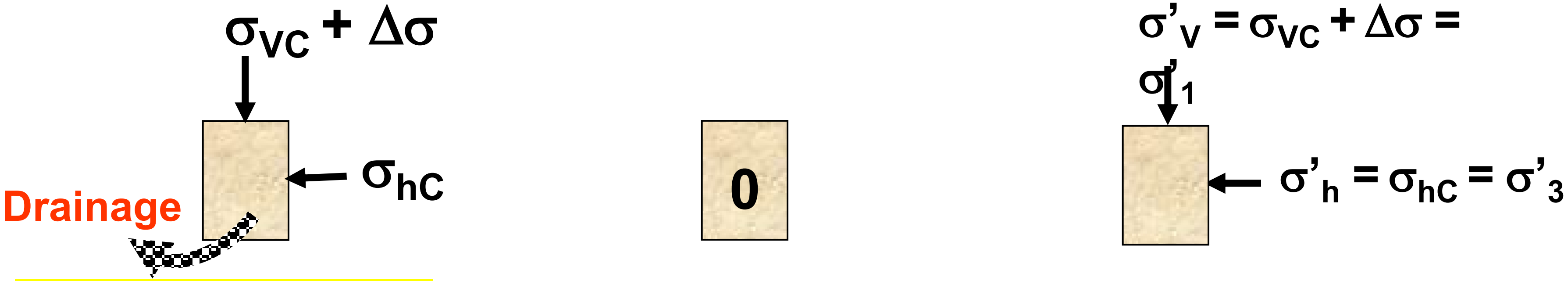
Drainage

0

$\sigma'_{VC} = \sigma_{VC}$

$\sigma'_{hC} = \sigma_{hC}$

Step 2: During axial stress increase



$\sigma_{VC} + \Delta\sigma$

σ_{hC}

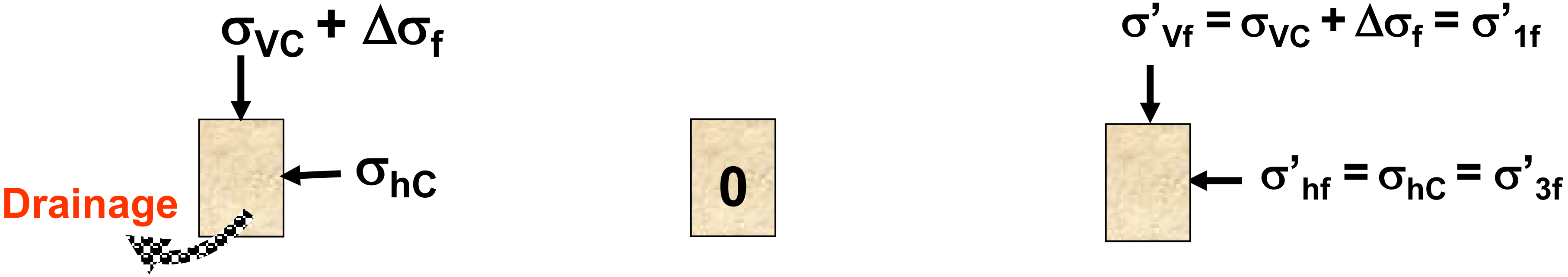
Drainage

0

$\sigma'_{V} = \sigma_{VC} + \Delta\sigma = \sigma'_{1}$

$\sigma'_{h} = \sigma_{hC} = \sigma'_{3}$

Step 3: At failure



$\sigma_{VC} + \Delta\sigma_{f}$

σ_{hC}

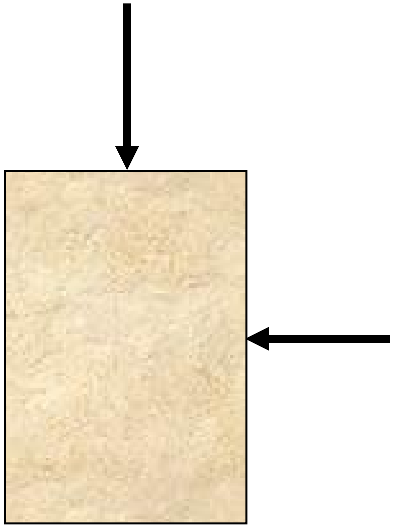
Drainage

0

$\sigma'_{Vf} = \sigma_{VC} + \Delta\sigma_{f} = \sigma'_{1f}$

$\sigma'_{hf} = \sigma_{hC} = \sigma'_{3f}$

Consolidated- drained test (CD Test)



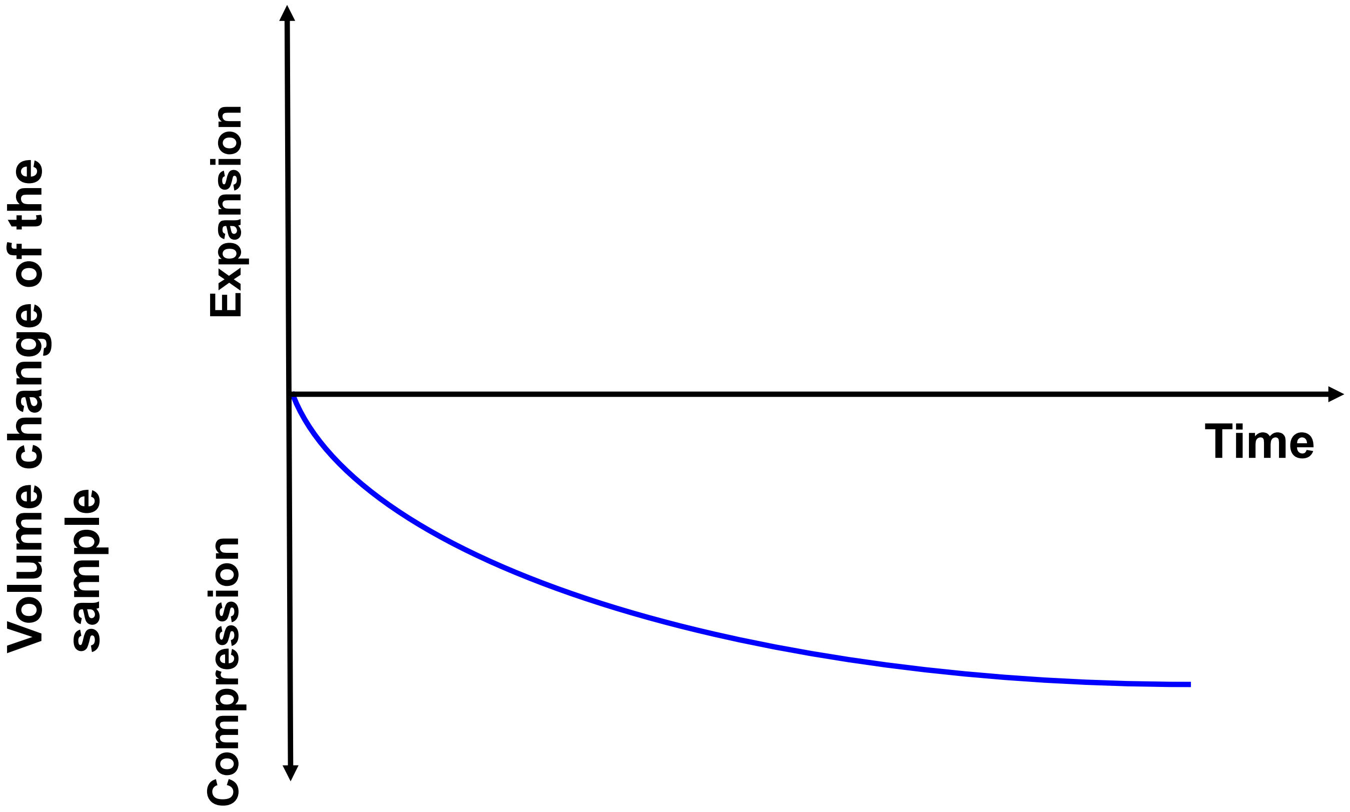
$\sigma_1 = \sigma_{VC} + \Delta\sigma$

$\sigma_3 = \sigma_{hC}$

Deviator stress (q or $\Delta\sigma_d$) = $\sigma_1 - \sigma_3$

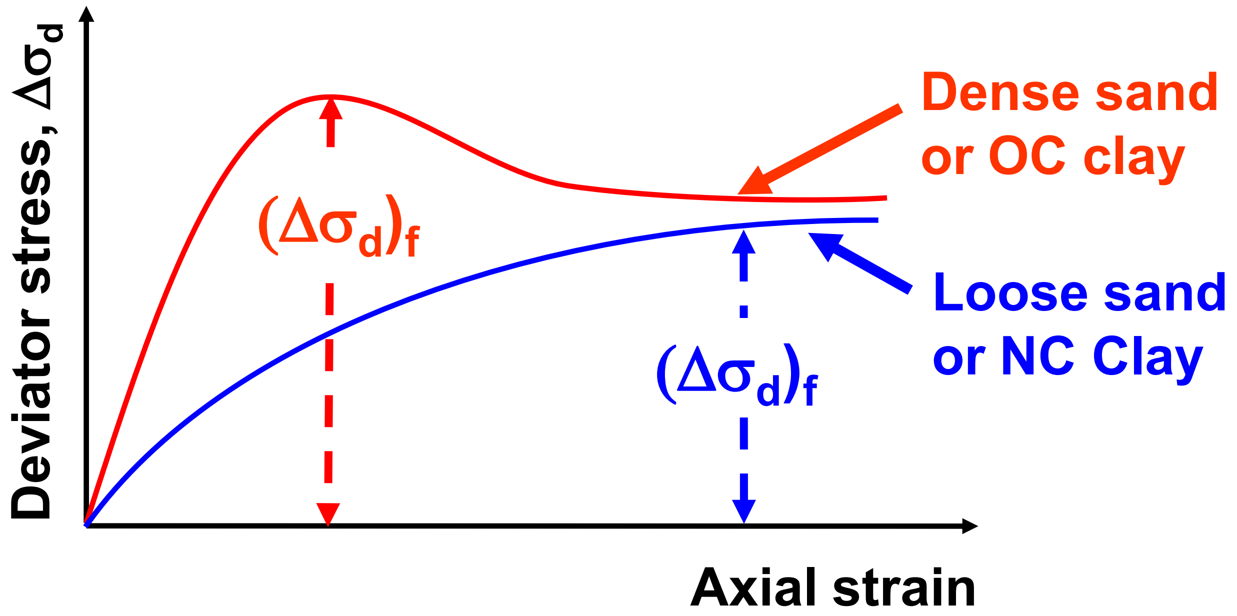
Consolidated- drained test (CD Test)

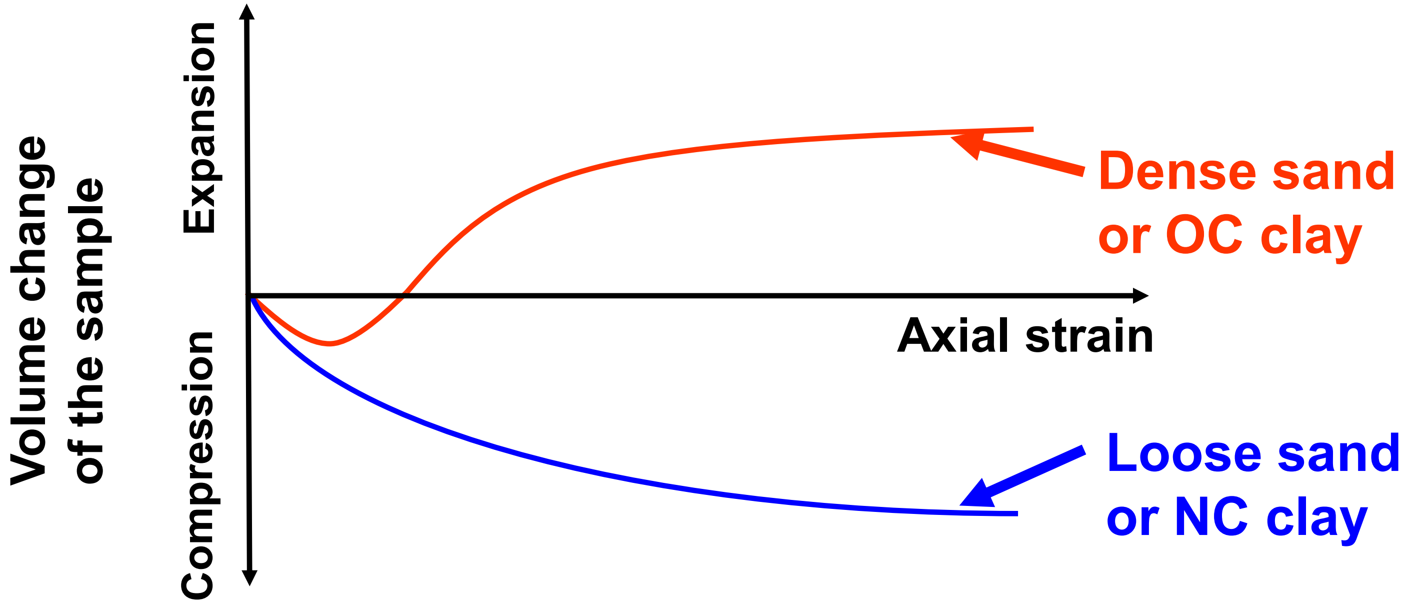
Volume change of sample during consolidation



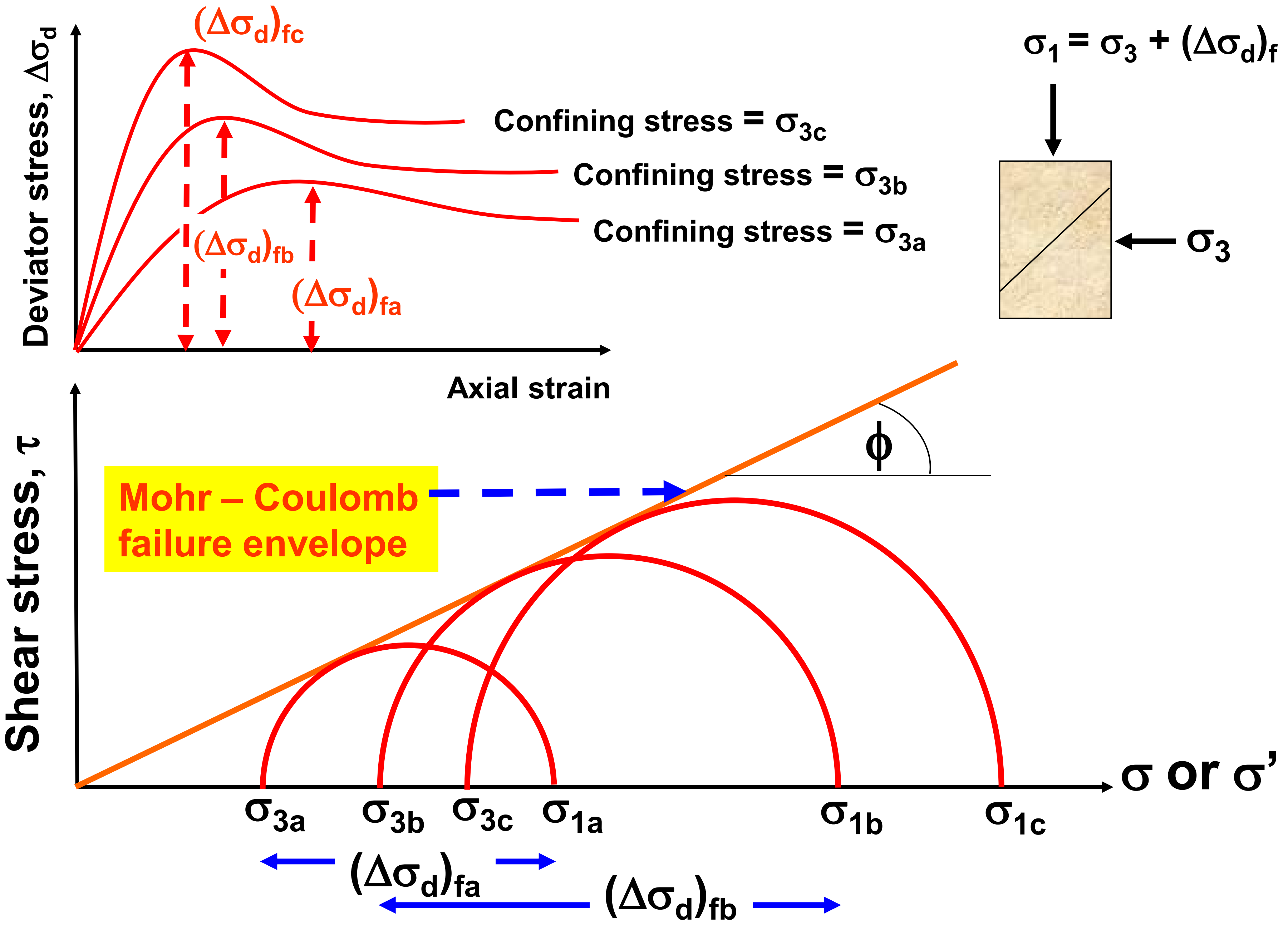
Consolidated- drained test (CD Test)

Stress-strain relationship during shearing





CD tests How to determine strength parameters c and ϕ



CD tests

Strength parameters c and ϕ obtained from CD tests

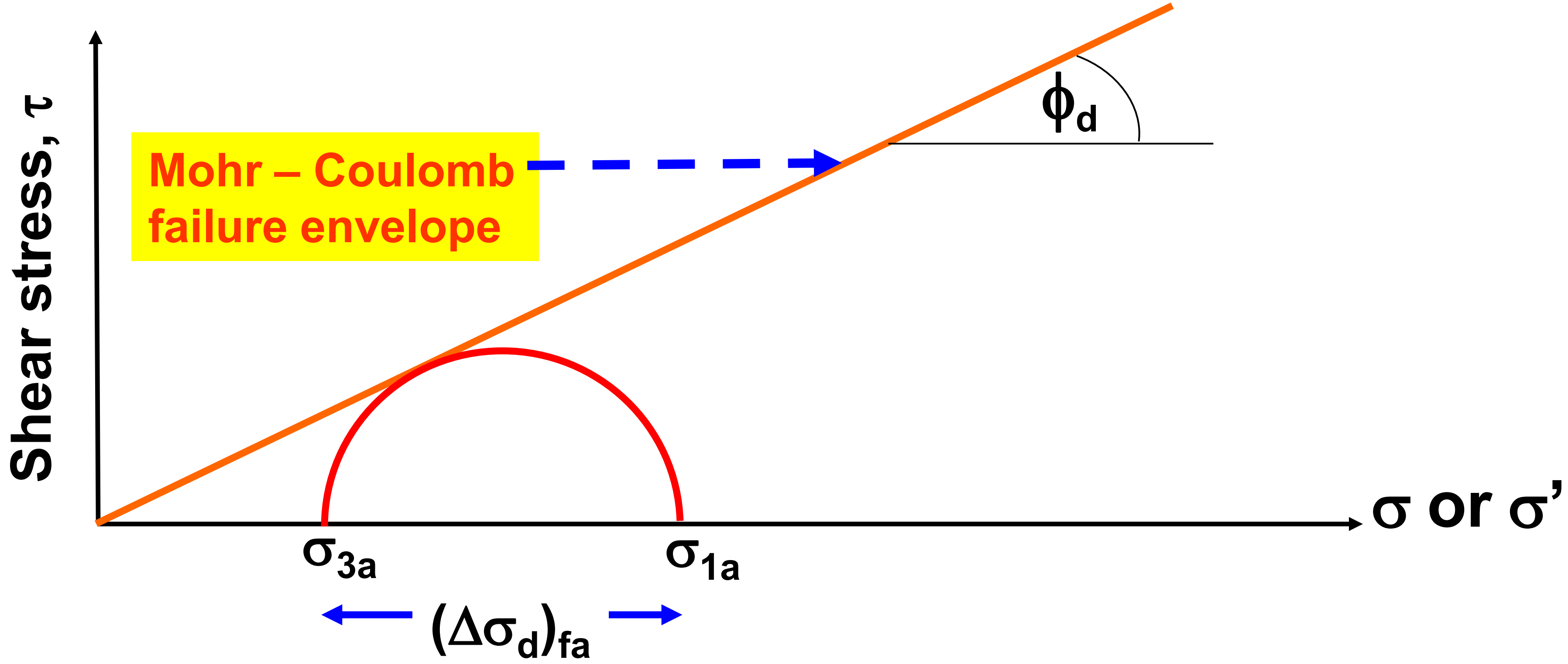
Since u = 0 in CD tests, $\sigma = \sigma'$

Therefore, $c = c'$ and $\phi = \phi'$

c_d and ϕ_d are used to denote them

CD tests Failure envelopes

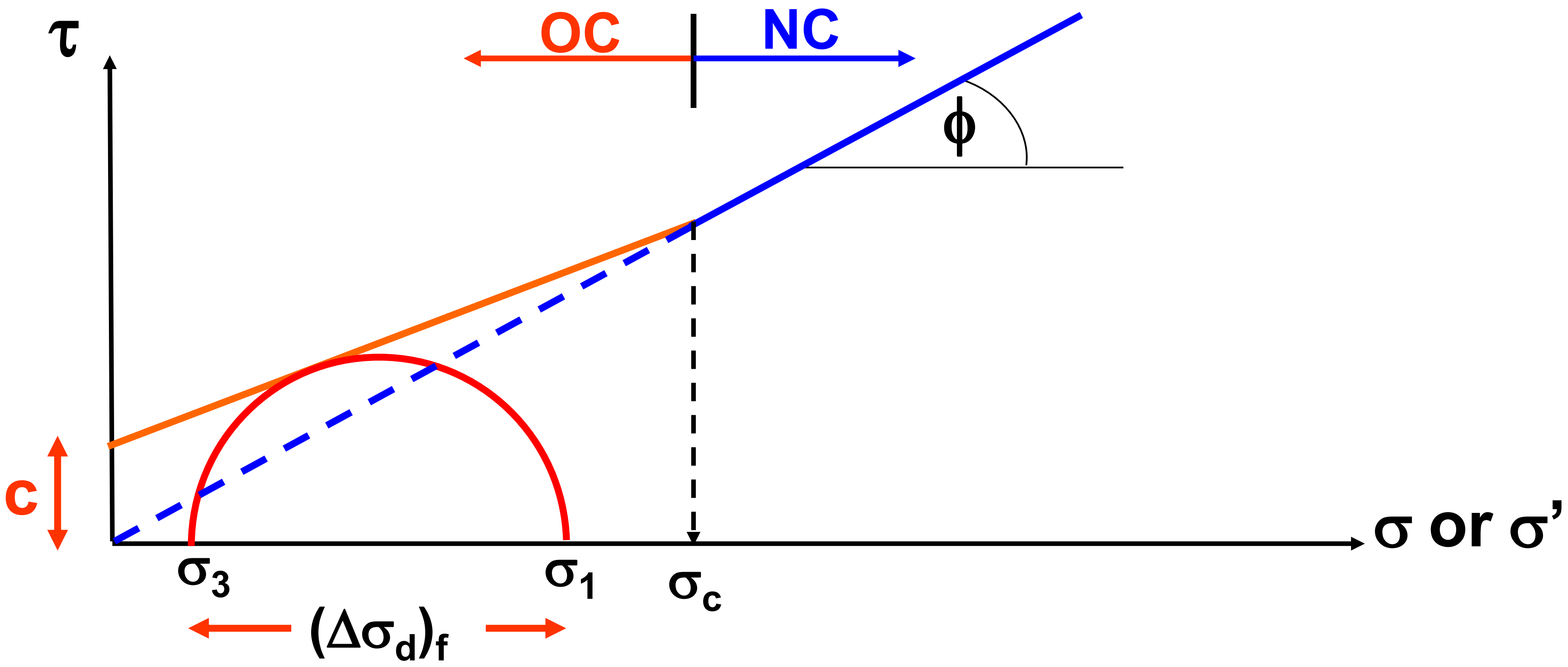
For sand and NC Clay, $c_d = 0$



Therefore, one CD test would be sufficient to determine ϕ_d of sand or NC clay

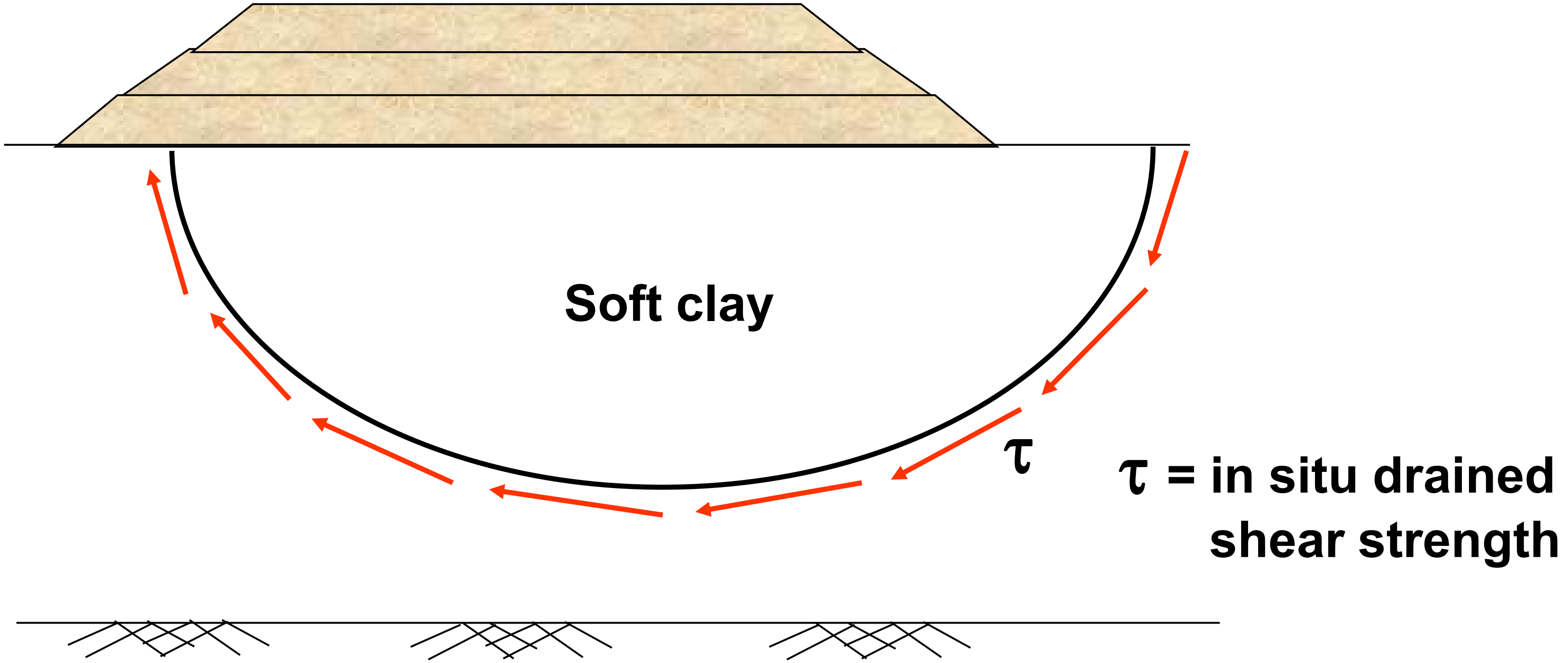
CD tests Failure envelopes

For OC Clay, $c_d \neq 0$



Some practical applications of CD analysis for clays

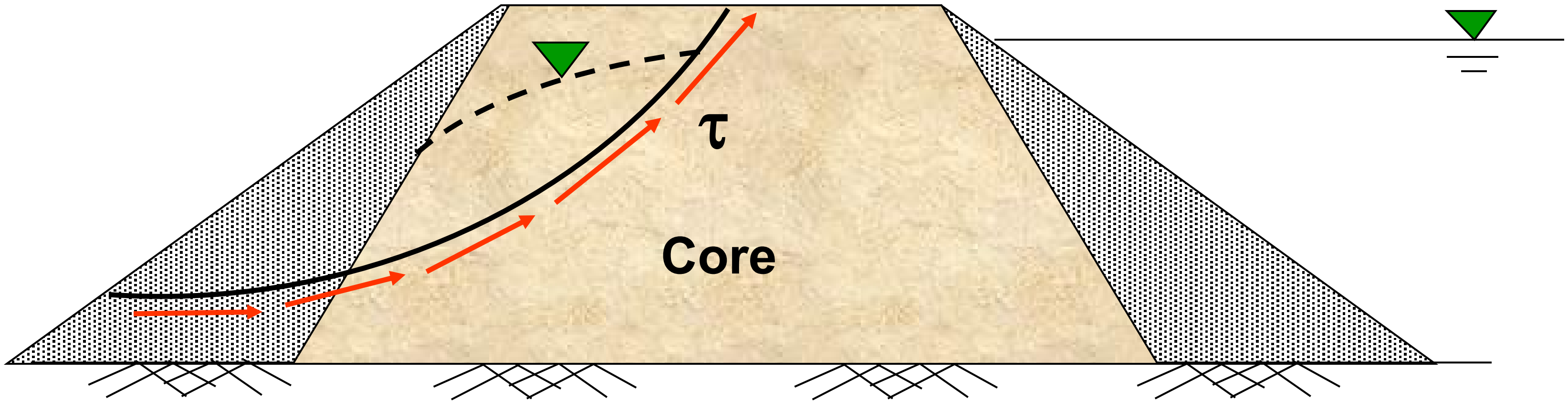
1. Embankment constructed very slowly, in layers over a soft clay deposit



τ = in situ drained shear strength

Some practical applications of CD analysis for clays

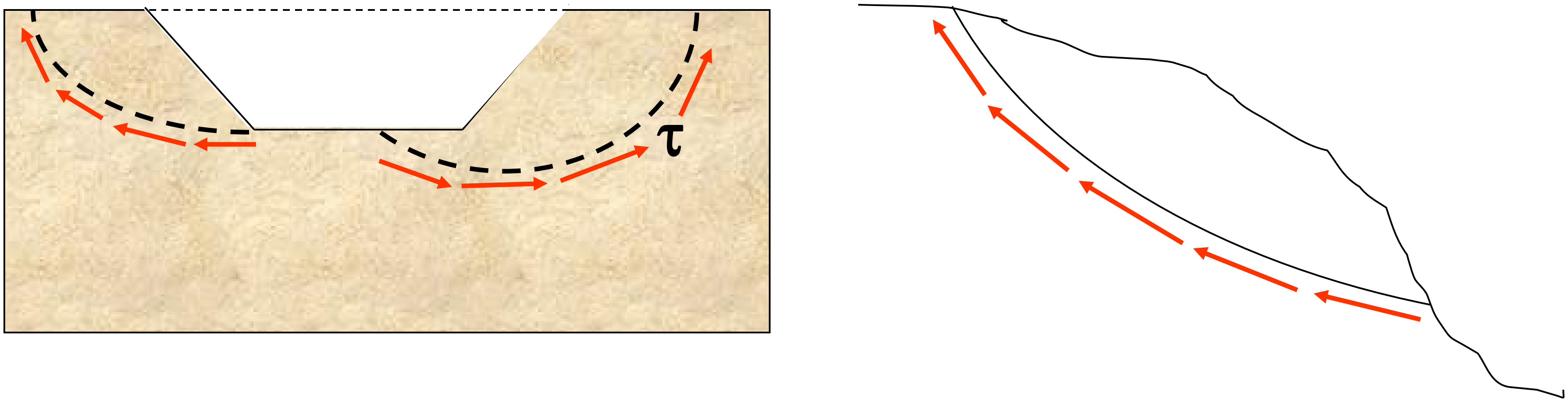
2. Earth dam with steady state seepage



τ = drained shear strength of clay core

Some practical applications of CD analysis for clays

3. Excavation or natural slope in clay

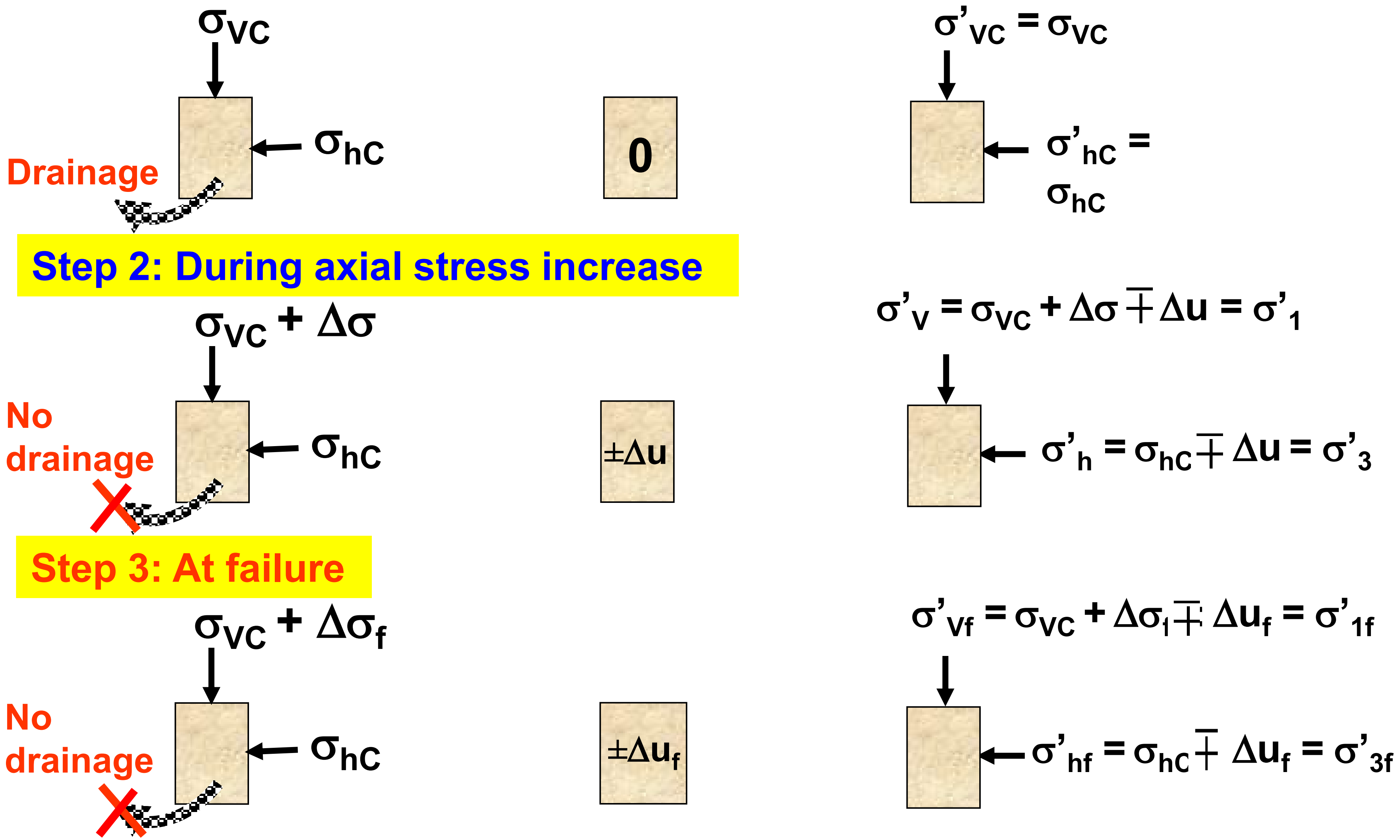


τ = In situ drained shear strength

Note: CD test simulates the <u>long term condition</u> in the field. Thus, c_d and ϕ_d should be used to evaluate the long term behavior of soils

Consolidated- Undrained test (CU Test)

Total, σ = **Neutral, u** + **Effective, σ'**



Step 1: At the end of consolidation

σ_{VC}

σ_{hC}

Drainage

0

$\sigma'_{VC} = \sigma_{VC}$

$\sigma'_{hC} = \sigma_{hC}$

Step 2: During axial stress increase

$\sigma_{VC} + \Delta\sigma$

σ_{hC}

No drainage

$\pm\Delta u$

$\sigma'_{V} = \sigma_{VC} + \Delta\sigma \mp \Delta u = \sigma'_{1}$

$\sigma'_{h} = \sigma_{hC} \mp \Delta u = \sigma'_{3}$

Step 3: At failure

$\sigma_{VC} + \Delta\sigma_{f}$

σ_{hC}

No drainage

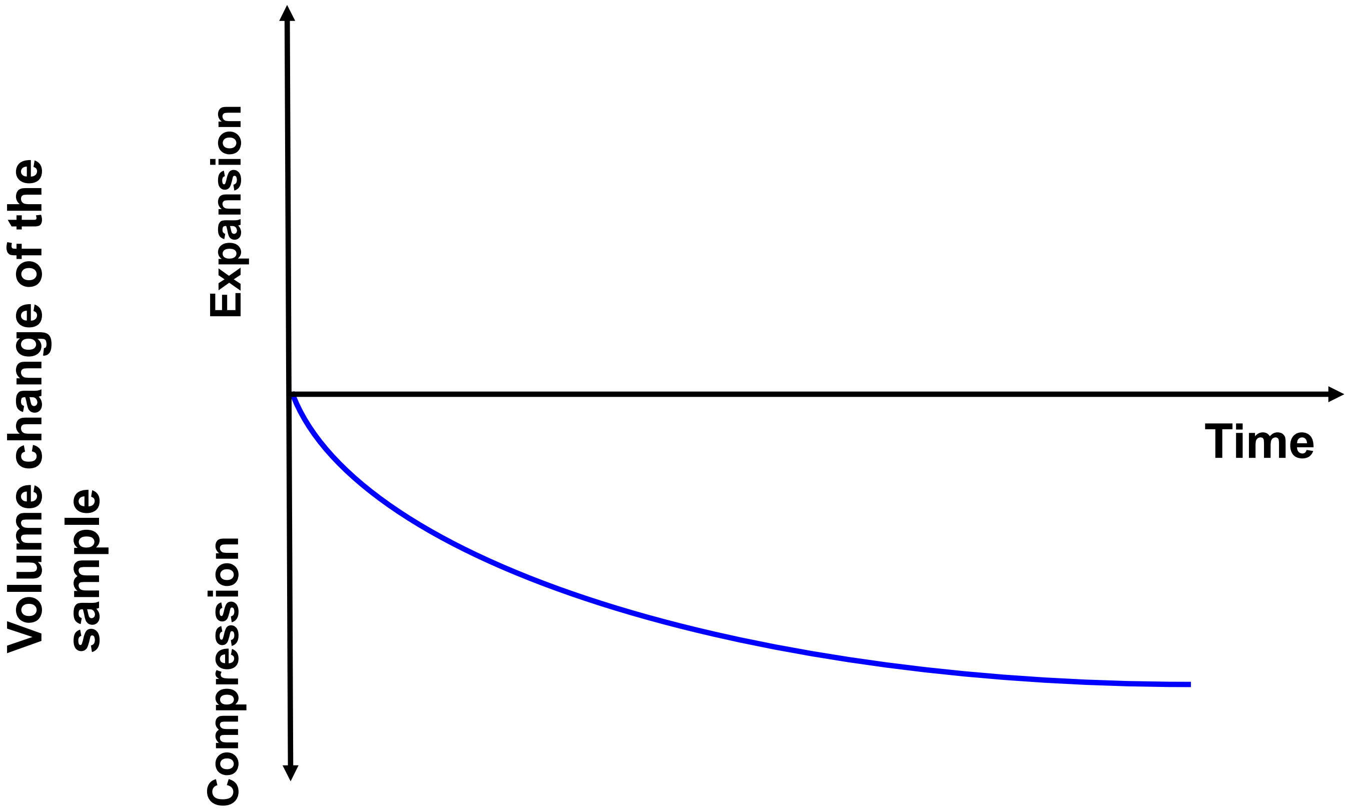
$\pm\Delta u_{f}$

$\sigma'_{Vf} = \sigma_{VC} + \Delta\sigma_{f} \mp \Delta u_{f} = \sigma'_{1f}$

$\sigma'_{hf} = \sigma_{hC} \mp \Delta u_{f} = \sigma'_{3f}$

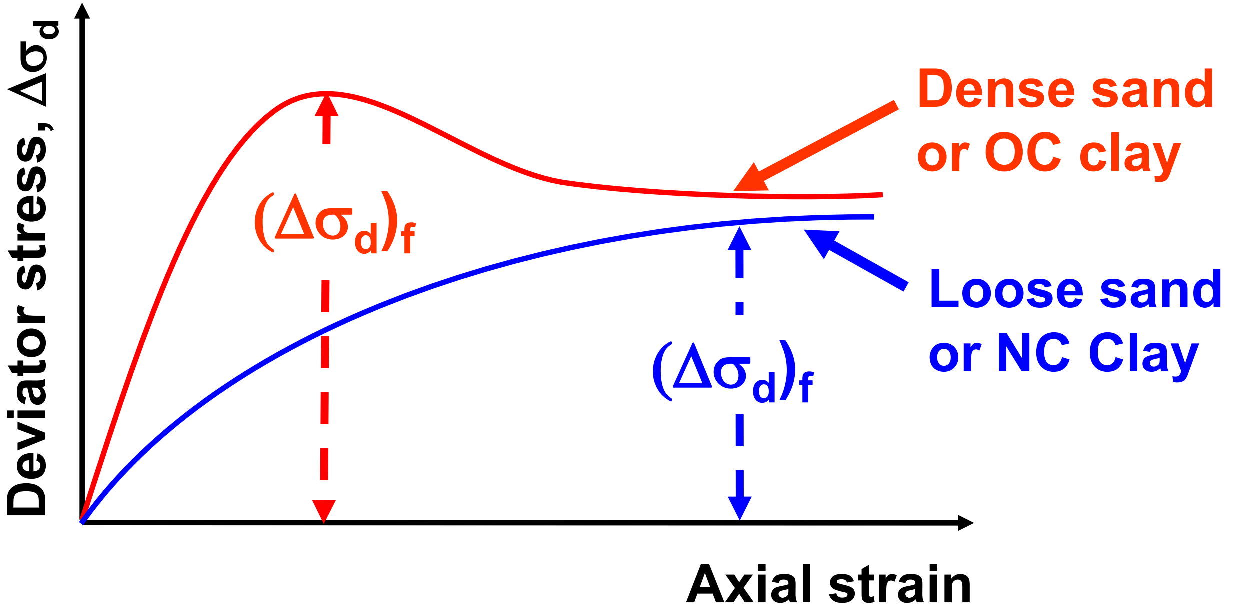
Consolidated- Undrained test (CU Test)

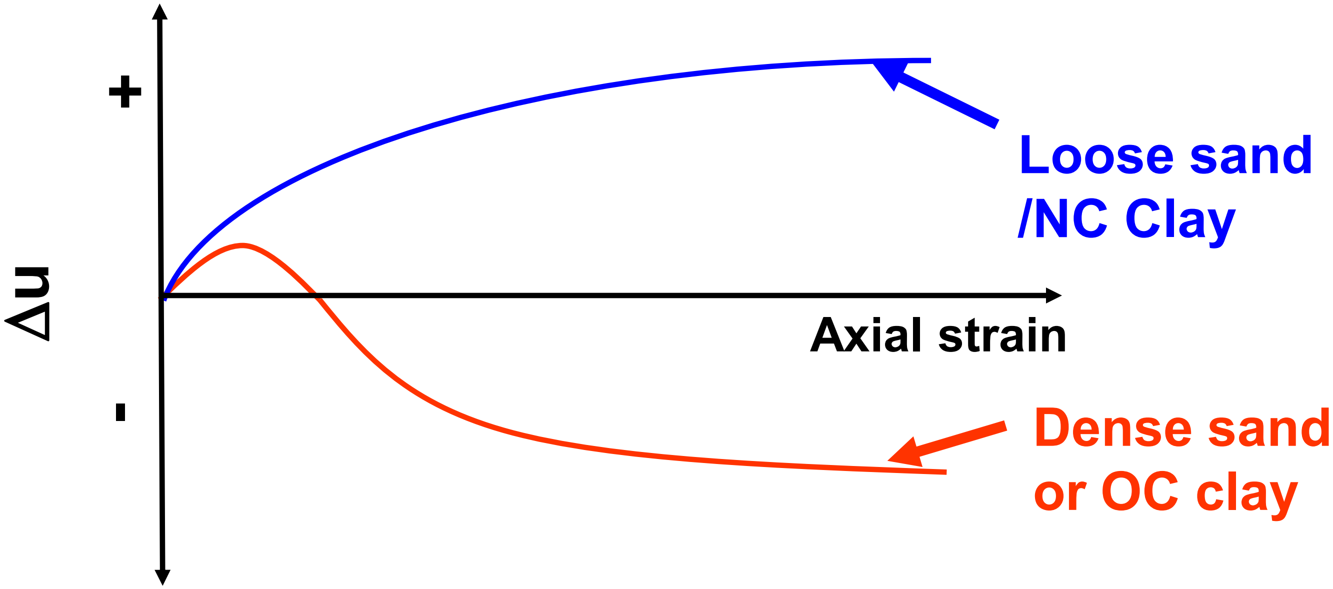
Volume change of sample during consolidation



Consolidated- Undrained test (CU Test)

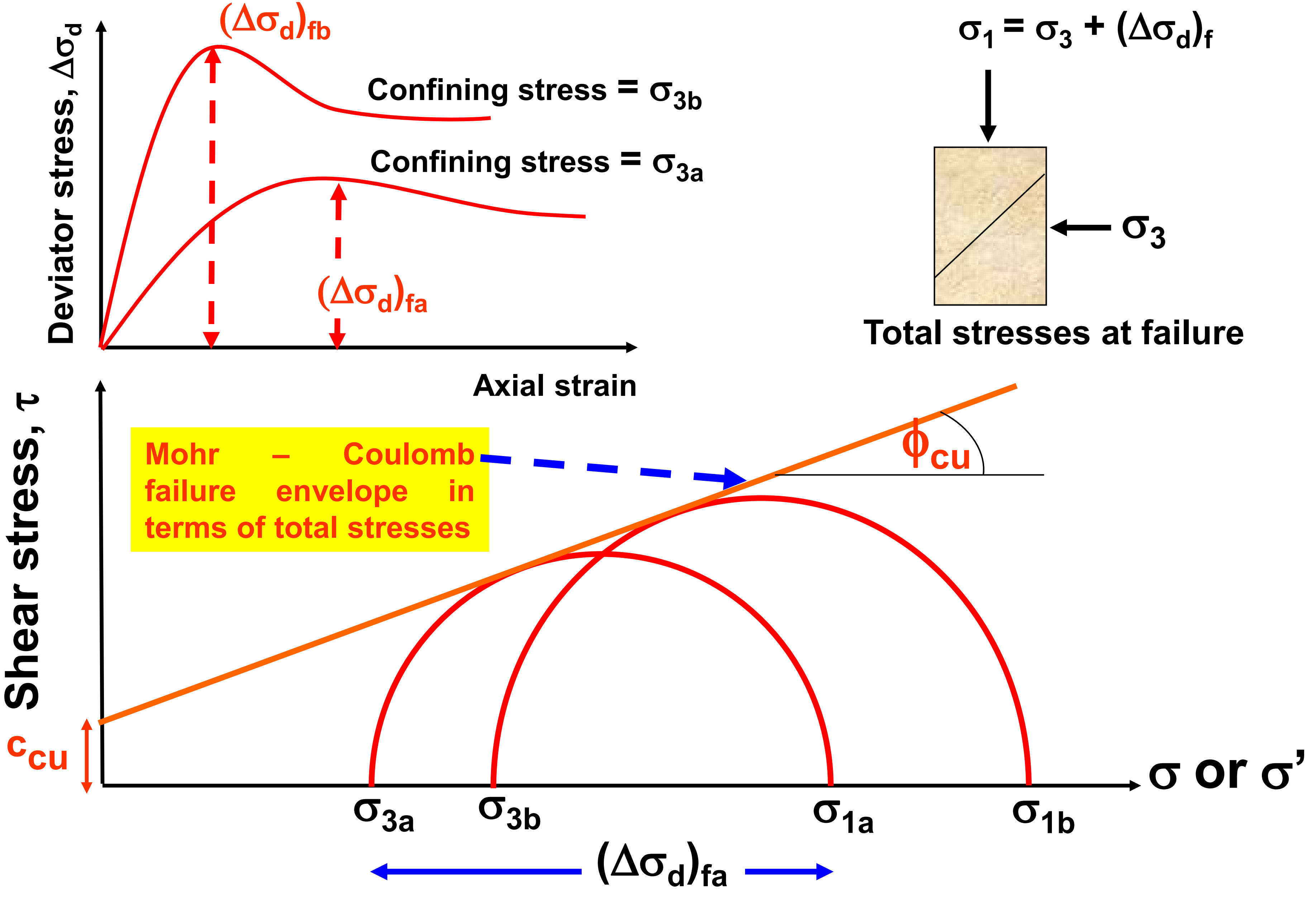
Stress-strain relationship during shearing





CU tests

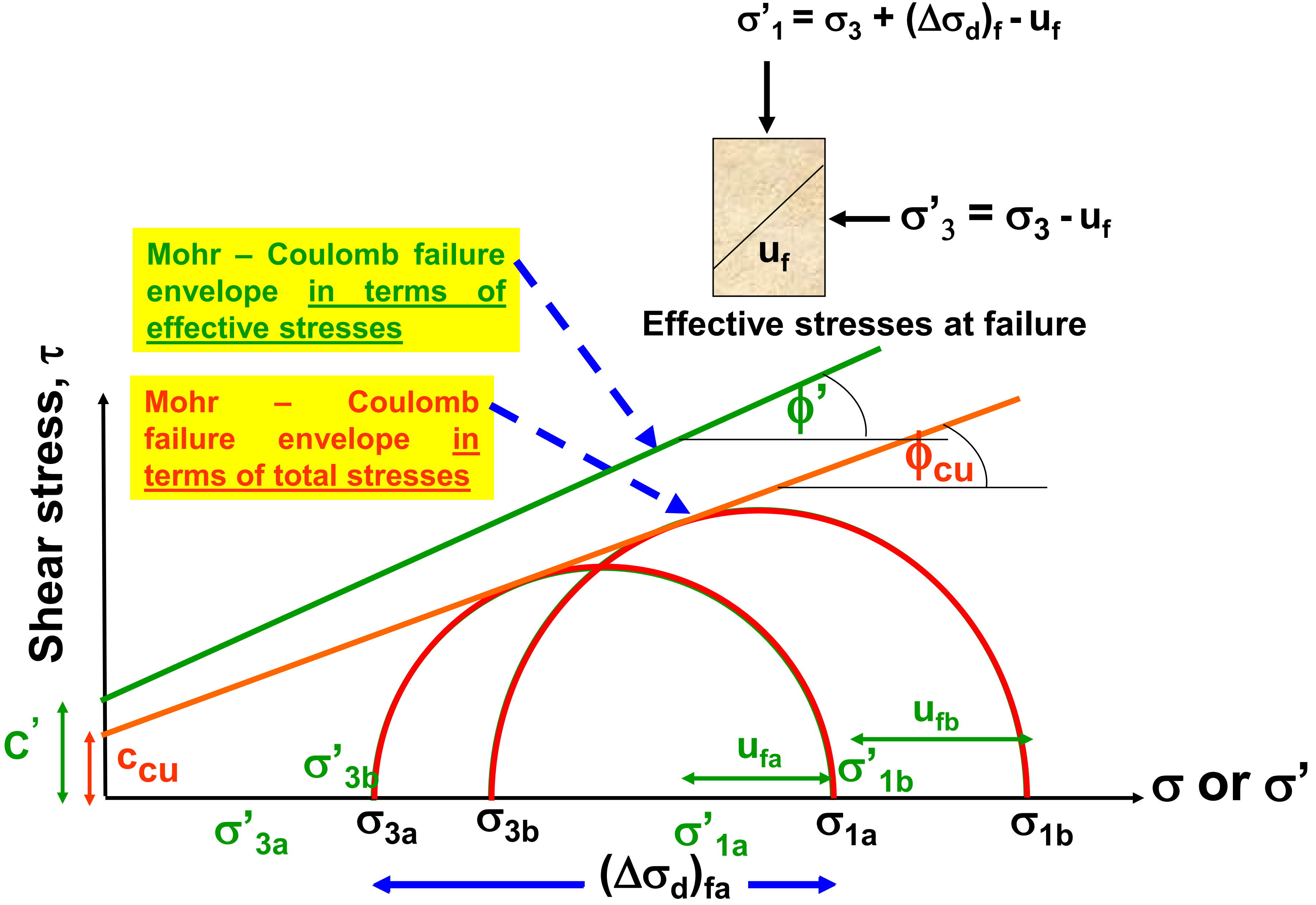
How to determine strength parameters c and ϕ



Total stresses at failure

Mohr – Coulomb failure envelope in terms of total stresses

CU tests How to determine strength parameters c and ϕ



$\sigma'_1 = \sigma_3 + (\Delta\sigma_d)_f - u_f$

$\sigma'_3 = \sigma_3 - u_f$

Effective stresses at failure

Mohr – Coulomb failure envelope in terms of effective stresses

Mohr – Coulomb failure envelope in terms of total stresses

CU tests

Strength parameters c and ϕ obtained from CD tests

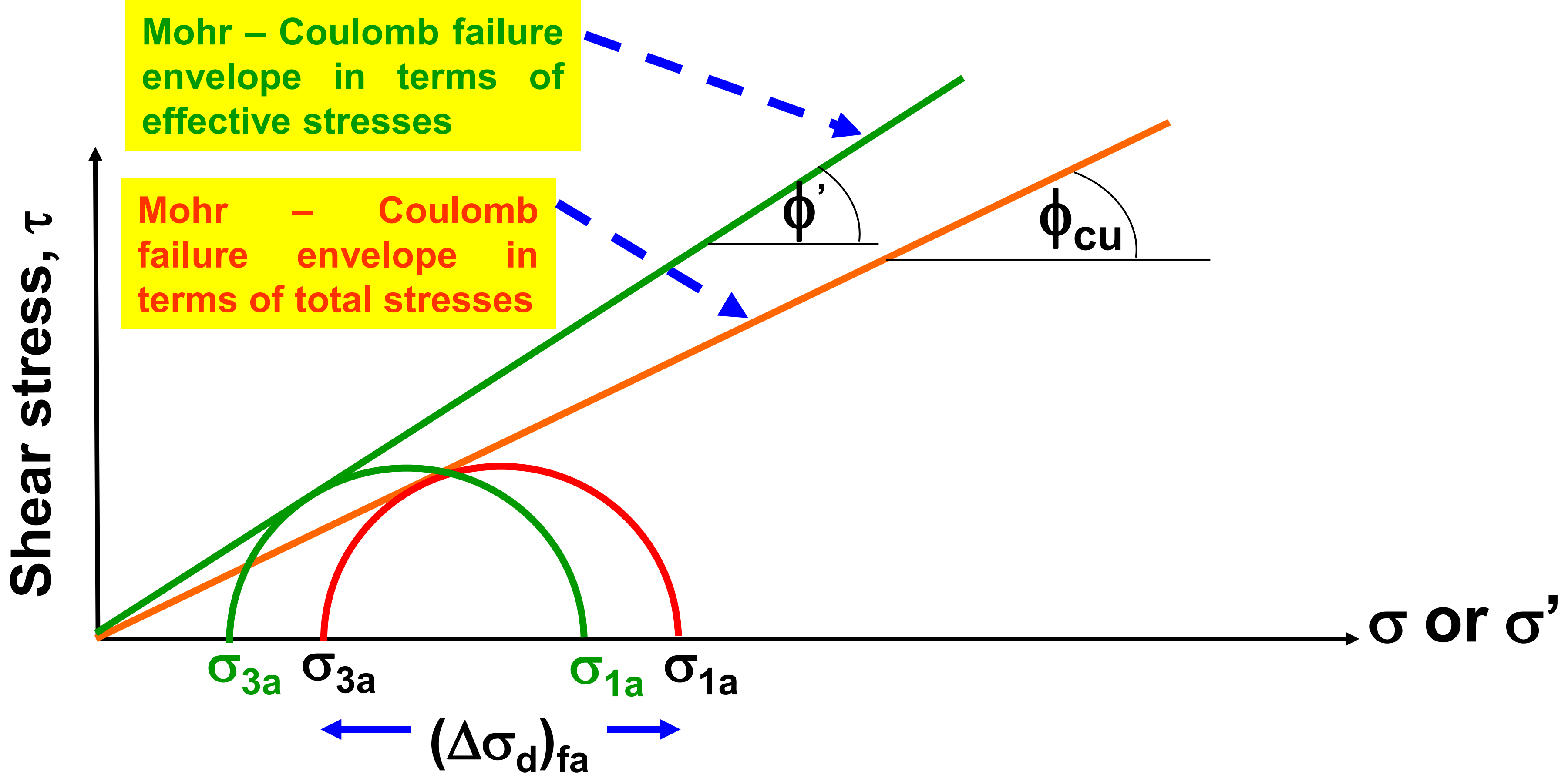
Shear strength parameters in terms of total stresses are c_{cu} and ϕ_{cu}

Shear strength parameters in terms of effective stresses are c' and ϕ'

$c' = c_d$ and $\phi' = \phi_d$

CU tests Failure envelopes

For sand and NC Clay, c_{cu} and $c' = 0$

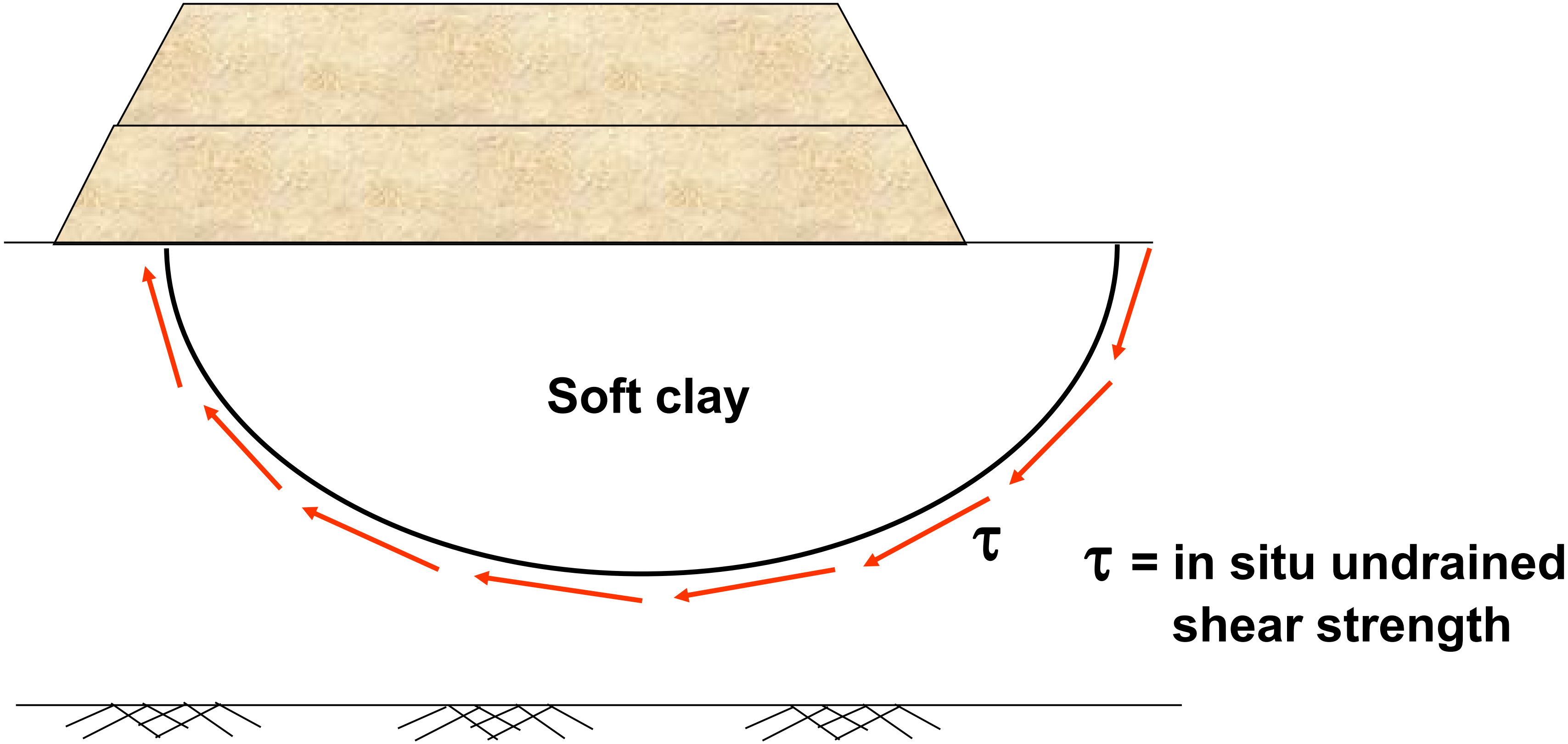

Mohr – Coulomb failure envelope in terms of effective stresses

Mohr – Coulomb failure envelope in terms of total stresses

Therefore, one CU test would be sufficient to determine ϕ_{cu} and $\phi'(= \phi_d)$ of sand or NC clay

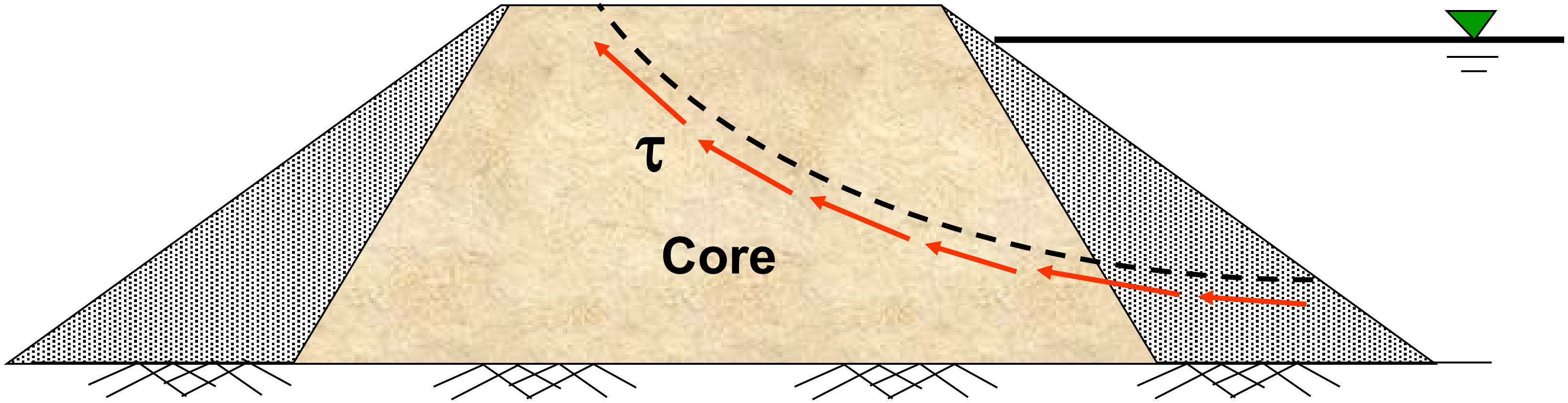
Some practical applications of CU analysis for clays

1. Embankment constructed rapidly over a soft clay deposit



Some practical applications of CU analysis for clays

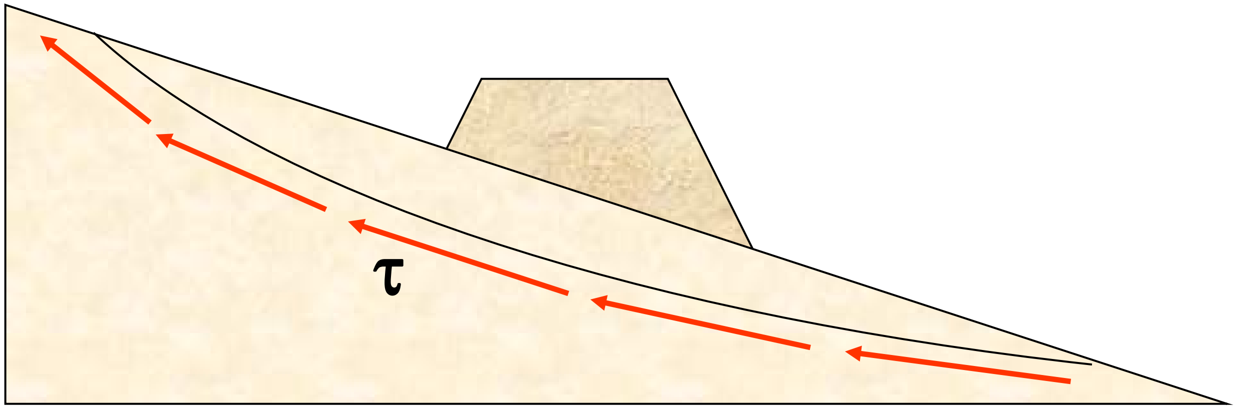
2. Rapid drawdown behind an earth dam



τ = Undrained shear strength of clay core

Some practical applications of CU analysis for clays

3. Rapid construction of an embankment on a natural slope



τ = In situ undrained shear strength

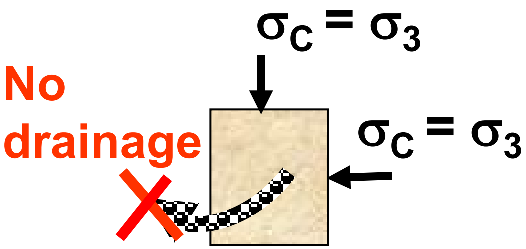
Note: Total stress parameters from CU test (c_{cu} and ϕ_{cu}) can be used for stability problems where,

Soil have become fully consolidated and are at equilibrium with the existing stress state; Then for some reason additional stresses are applied quickly with no drainage occurring

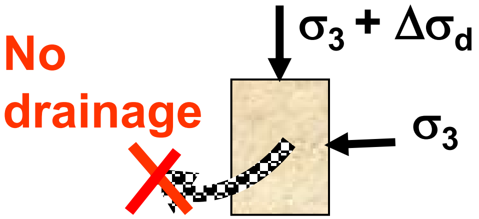
Unconsolidated- Undrained test (UU Test)

Data analysis

Initial specimen condition



Specimen condition during shearing



Initial volume of the sample = $A_0 \times H_0$

Volume of the sample during shearing = $A \times H$

Since the test is conducted under undrained condition,

$A \times H = A_0 \times H_0$

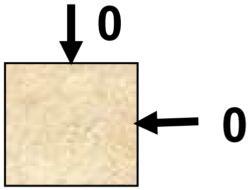
$A \times (H_0 – \Delta H) = A_0 \times H_0$

$A \times (1 – \Delta H/H_0) = A_0$

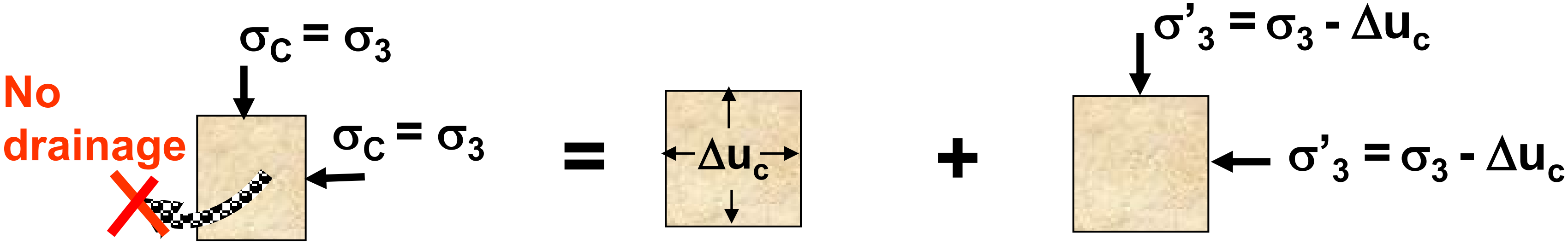
$$A = \frac{A_0}{1 - \varepsilon_z}$$

Unconsolidated- Undrained test (UU Test)

Step 1: Immediately after sampling



Step 2: After application of hydrostatic cell pressure



$$\Delta u_c = B \Delta \sigma_3$$

Increase of pwp due to increase of cell pressure

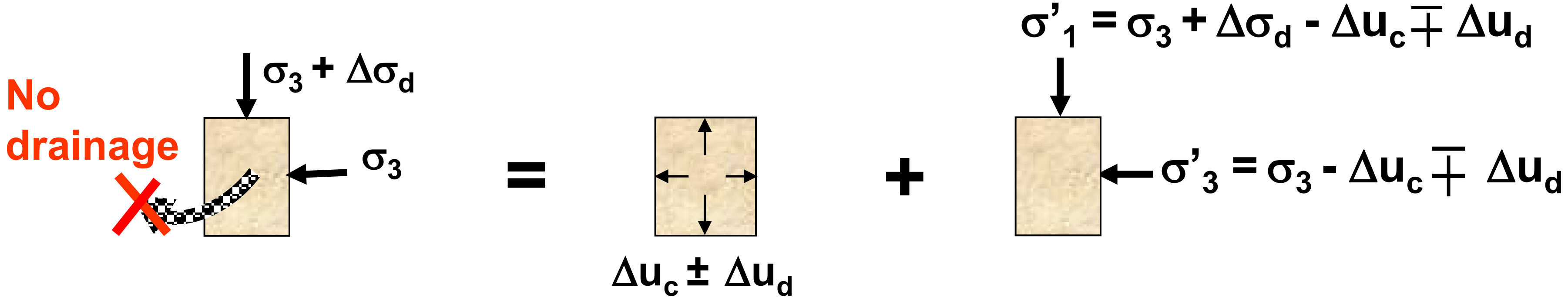
Skempton's pore water pressure parameter, B

Increase of cell pressure

Note: If soil is fully saturated, then B = 1 (hence, $\Delta u_c = \Delta \sigma_3$)

Unconsolidated- Undrained test (UU Test)

Step 3: During application of axial load



$\sigma'_1 = \sigma_3 + \Delta\sigma_d - \Delta u_c \mp \Delta u_d$

$\sigma'_3 = \sigma_3 - \Delta u_c \mp \Delta u_d$

$$\Delta u_d = AB\Delta\sigma_d$$

Increase of pwp due to increase of deviator stress

Increase of deviator stress

Skempton's pore water pressure parameter, A

Unconsolidated- Undrained test (UU Test)

Combining steps 2 and 3,

$$\Delta u_c = B\ \Delta\sigma_3 \qquad \Delta u_d = AB\Delta\sigma_d$$

Total pore water pressure <u>increment</u> at any stage, Δu

$$\Delta u = \Delta u_c + \Delta u_d$$

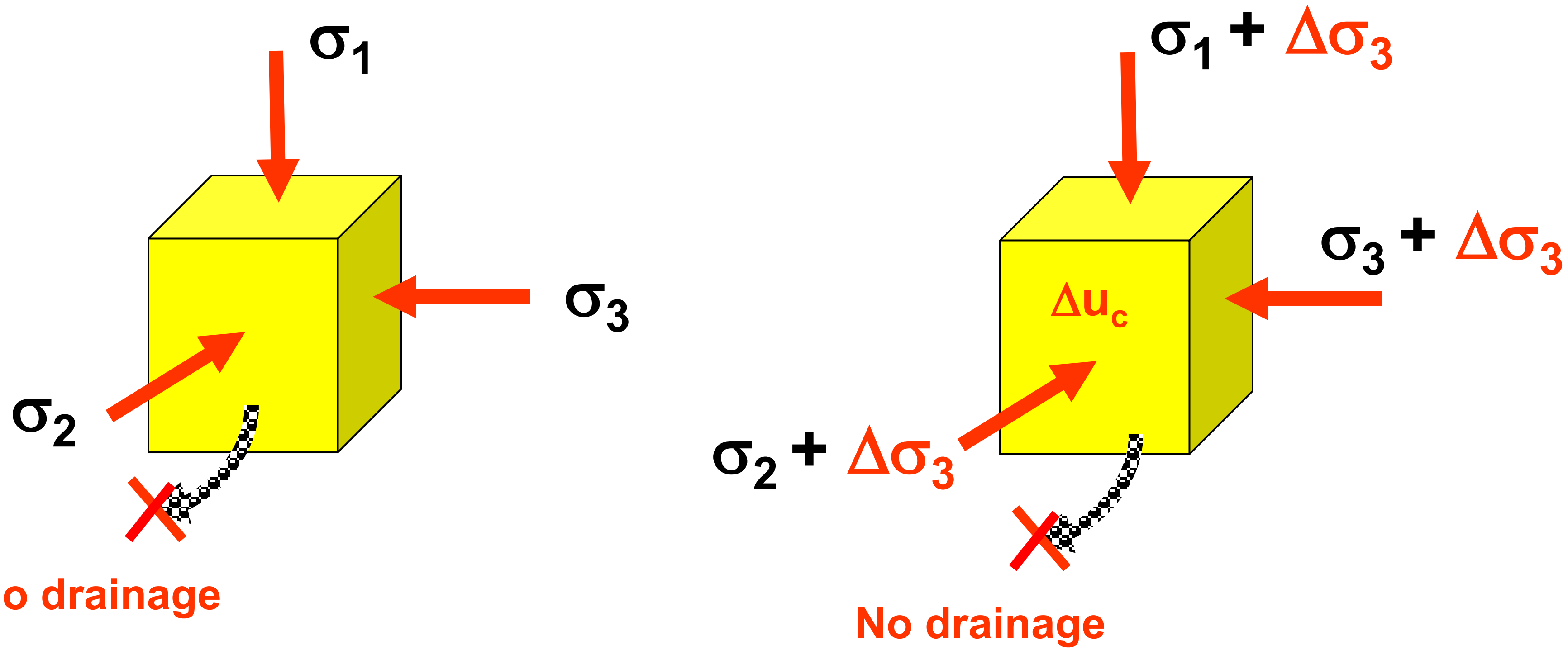
$$\Delta u = B\ [\Delta\sigma_3 + A\Delta\sigma_d]$$

$$\Delta u = B\ [\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3]$$

Skempton's pore water pressure equation

Derivation of Skempton’s pore water pressure equation

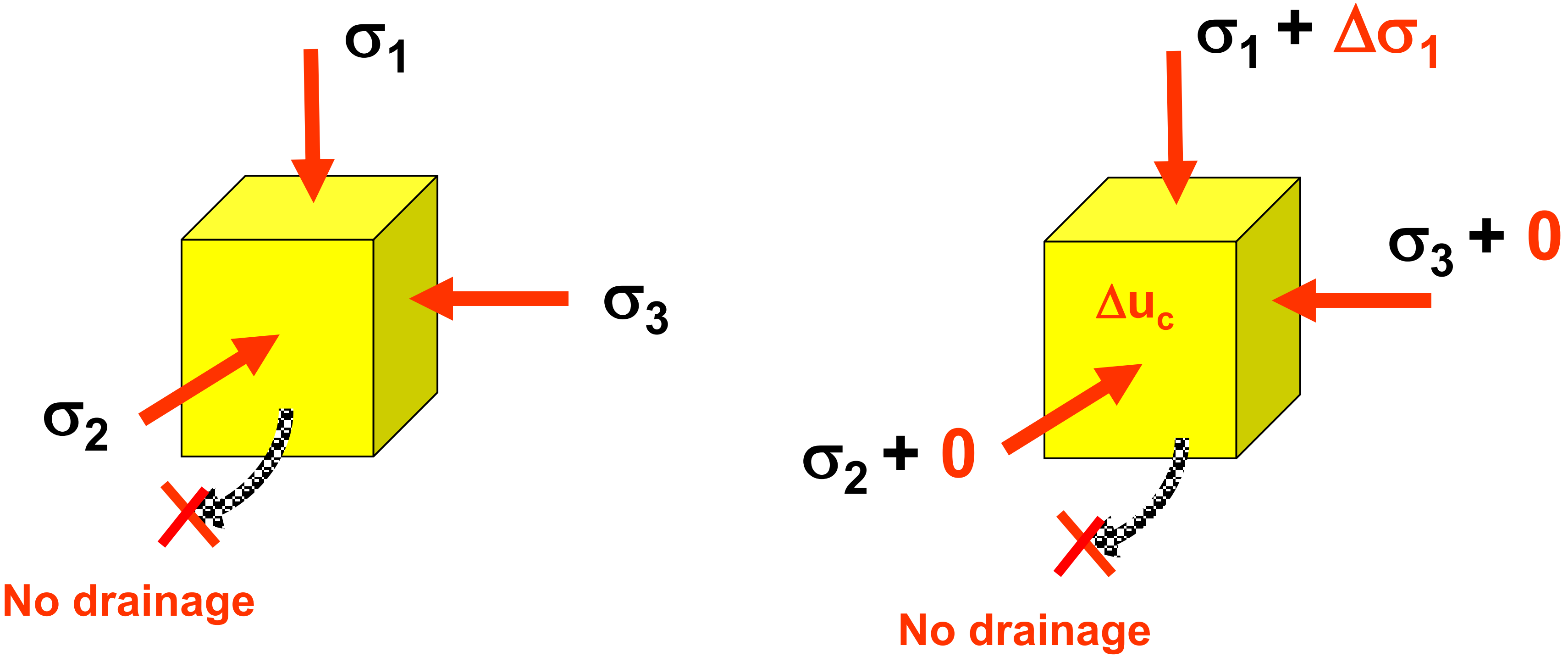
Step 1 :Increment of isotropic stress



Increase in effective stress in each direction = $\Delta\sigma_3 - \Delta u_c$

Derivation of Skempton’s pore water pressure equation

Step 2 :Increment of major principal stress

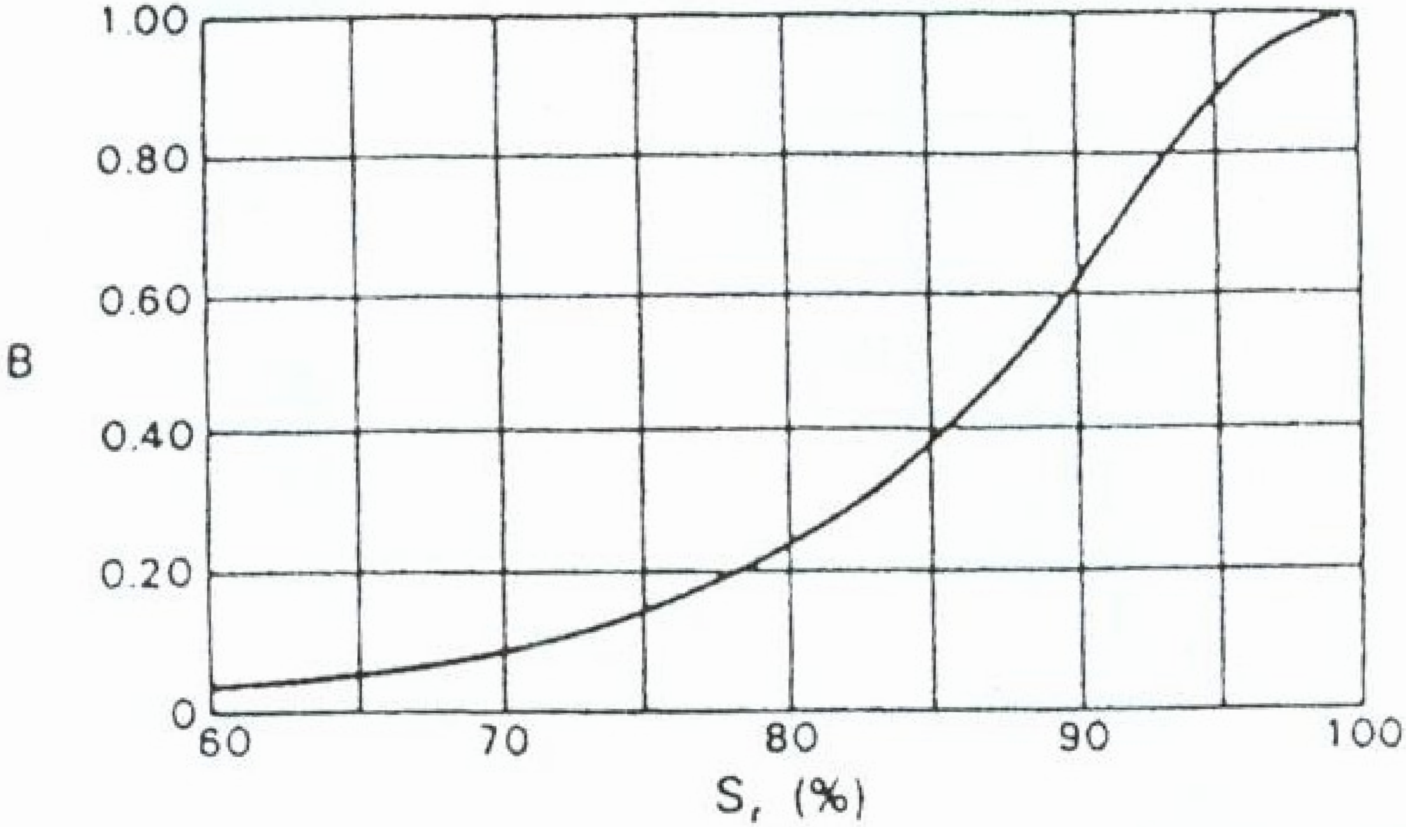


Increase in effective stress in σ_1 direction = $\Delta\sigma_1 - \Delta u_d$

Increase in effective stress in σ_2 and σ_3 directions = $0 - \Delta u_d$

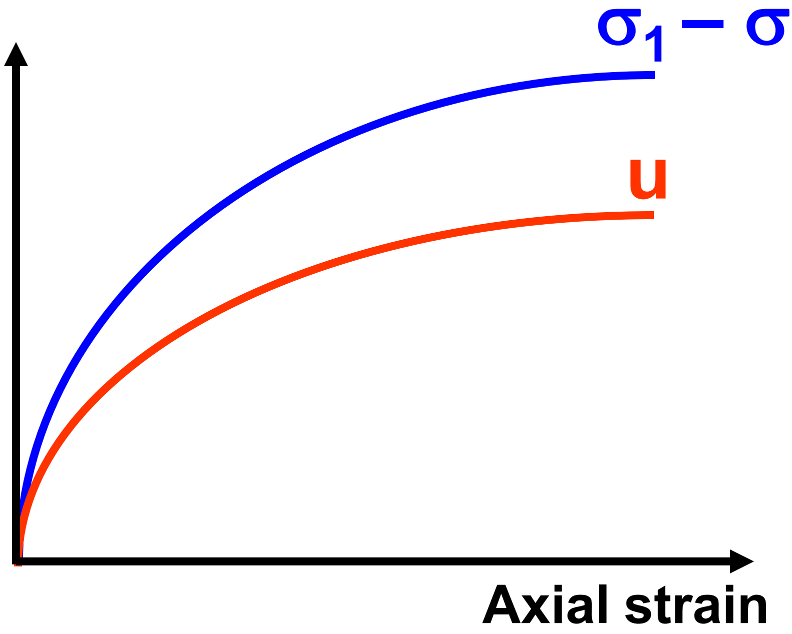
Average Increase in effective stress = $(\Delta\sigma_1 - \Delta u_d - \Delta u_d – \Delta u_d)/3$

Typical values for parameter B



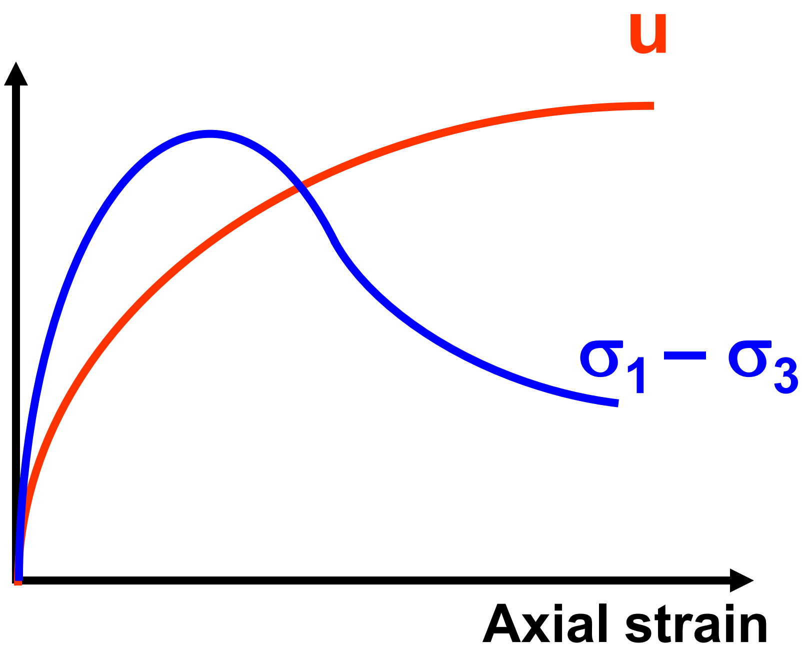
Typical relationship between *B* and degree of saturation.

Typical values for parameter A



NC Clay (low sensitivity)

(A = 0.5 – 1.0)

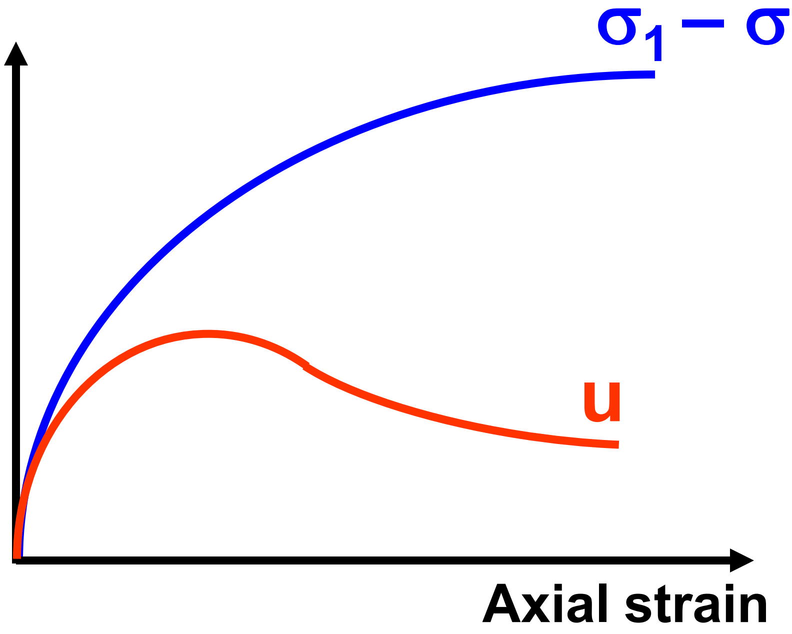


NC Clay (High sensitivity)

(A > 1.0)

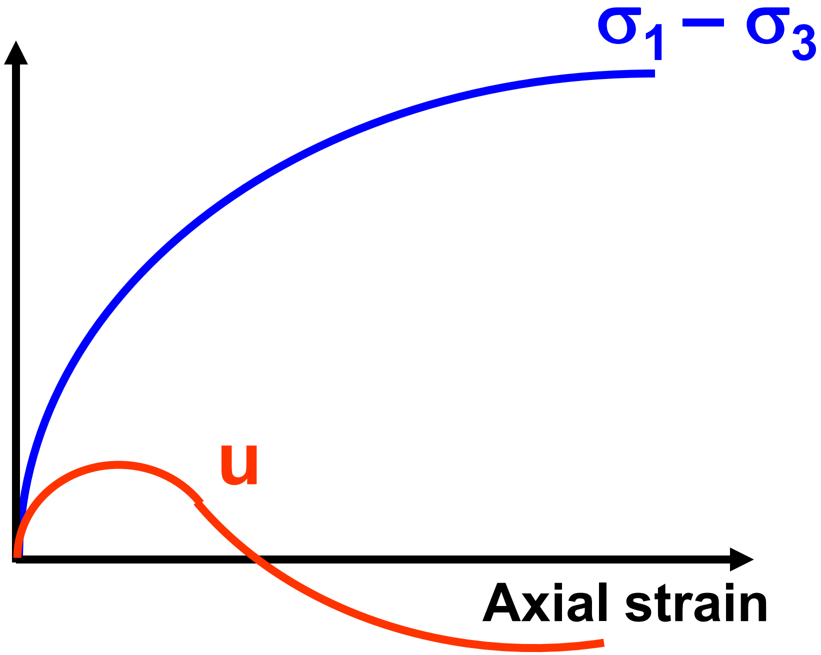
Collapse of soil structure may occur in high sensitivity clays due to very high pore water pressure generation

Typical values for parameter A



OC Clay (Lightly overconsolidated)

(A = 0.0 – 0.5)

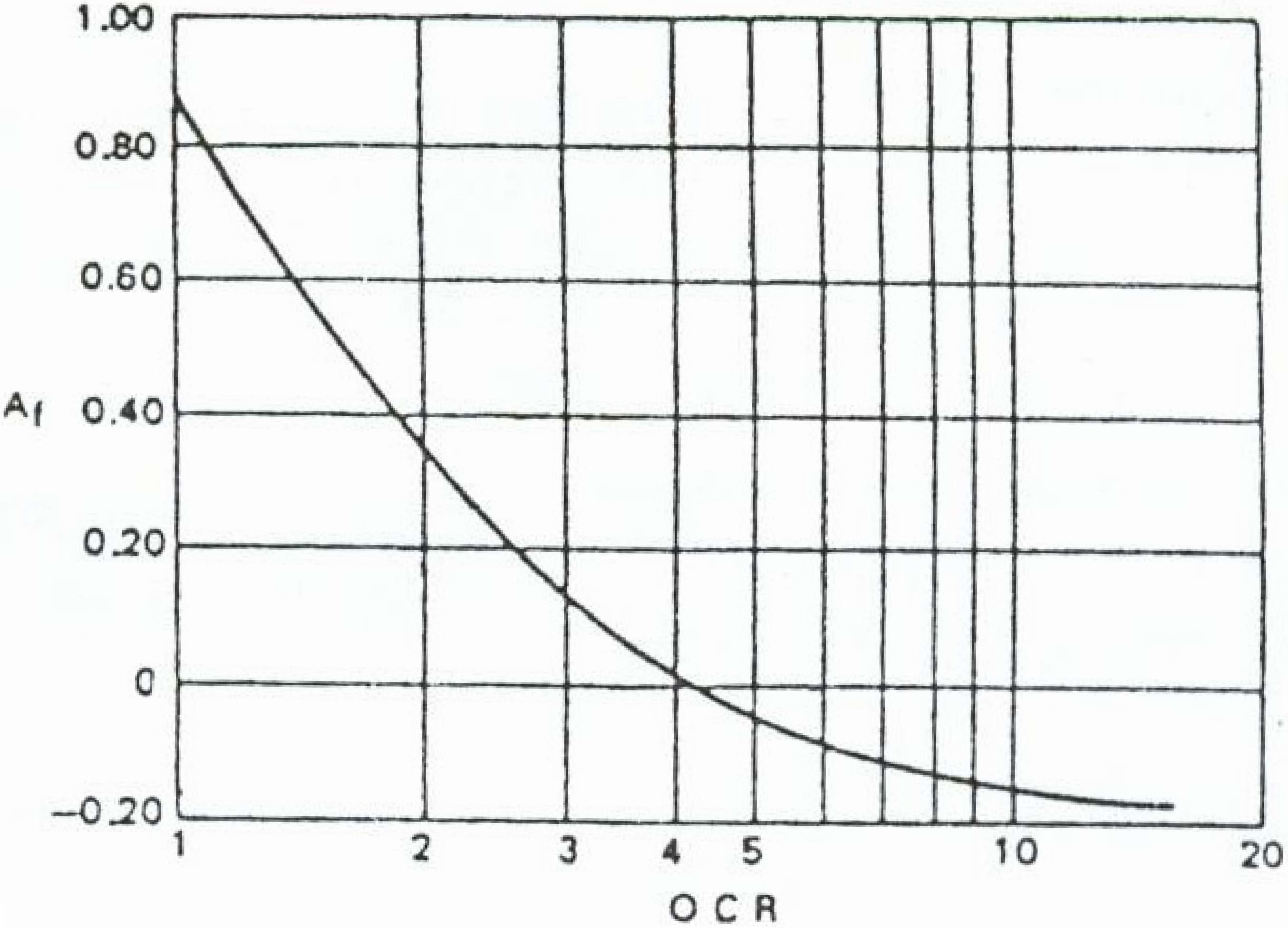


OC Clay (Heavily overconsolidated)

(A = -0.5 - 0.0)

During the increase of major principal stress pore water pressure can become negative in heavily overconsolidated clays due to dilation of specimen

Typical values for parameter A

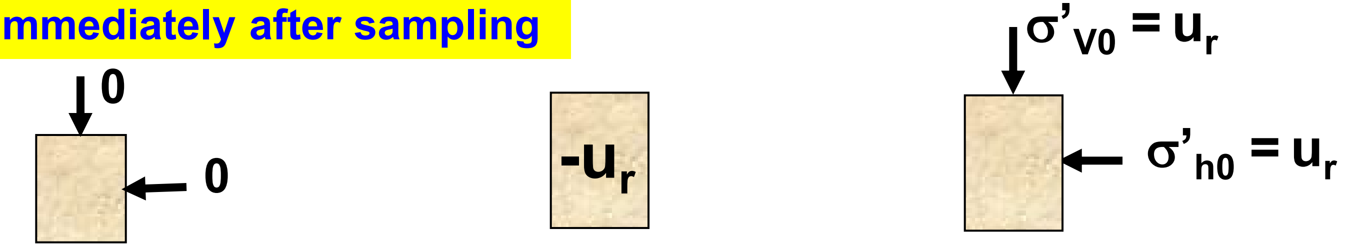


Typical relationship between A at failure and overconsolidation

Unconsolidated- Undrained test (UU Test)

Total, σ = **Neutral, u** + **Effective, σ'**

Step 1: Immediately after sampling

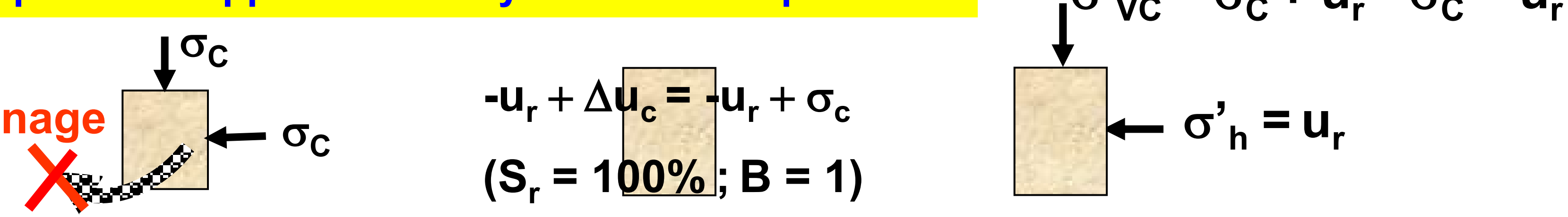


$\sigma'_{V0} = u_r$

$\sigma'_{h0} = u_r$

Step 2: After application of hydrostatic cell pressure

No drainage



$-u_r + \Delta u_c = -u_r + \sigma_c$

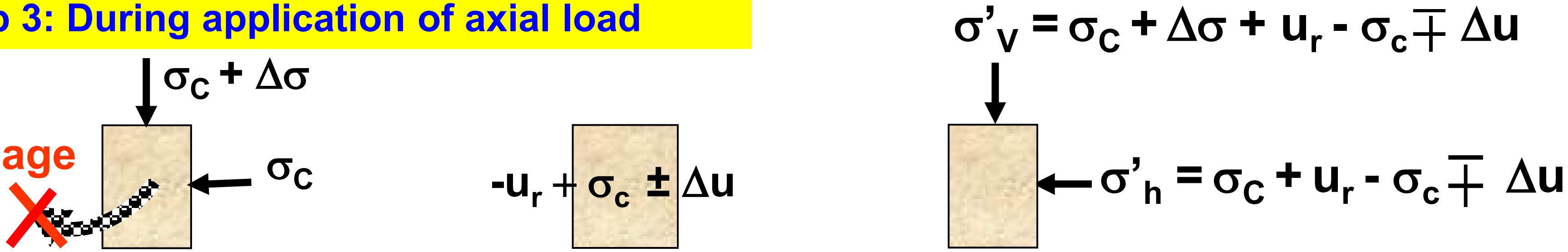
$(S_r = 100\%\ ;\ B = 1)$

$\sigma'_{VC} = \sigma_C + u_r - \sigma_C = u_r$

$\sigma'_h = u_r$

Step 3: During application of axial load

No drainage



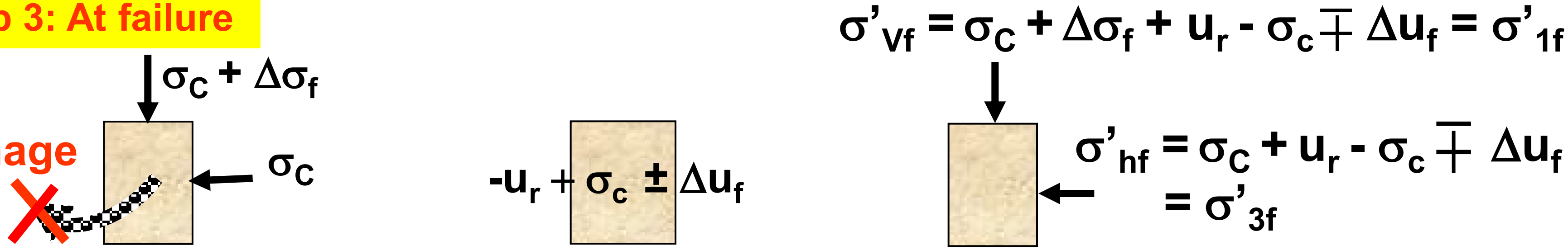
$-u_r + \sigma_c \pm \Delta u$

$\sigma'_V = \sigma_C + \Delta\sigma + u_r - \sigma_c \mp \Delta u$

$\sigma'_h = \sigma_C + u_r - \sigma_c \mp \Delta u$

Step 3: At failure

No drainage



$-u_r + \sigma_c \pm \Delta u_f$

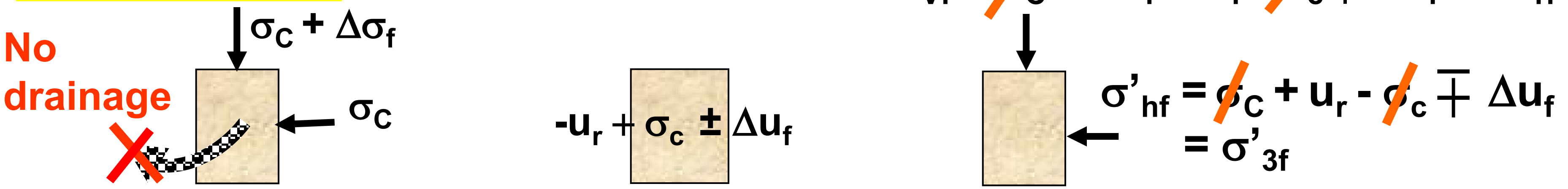
$\sigma'_{Vf} = \sigma_C + \Delta\sigma_f + u_r - \sigma_c \mp \Delta u_f = \sigma'_{1f}$

$\sigma'_{hf} = \sigma_C + u_r - \sigma_c \mp \Delta u_f$
$= \sigma'_{3f}$

Unconsolidated- Undrained test (UU Test)

Total, σ = **Neutral, u** + **Effective, σ'**

Step 3: At failure



$\sigma_C + \Delta\sigma_f$

σ_C

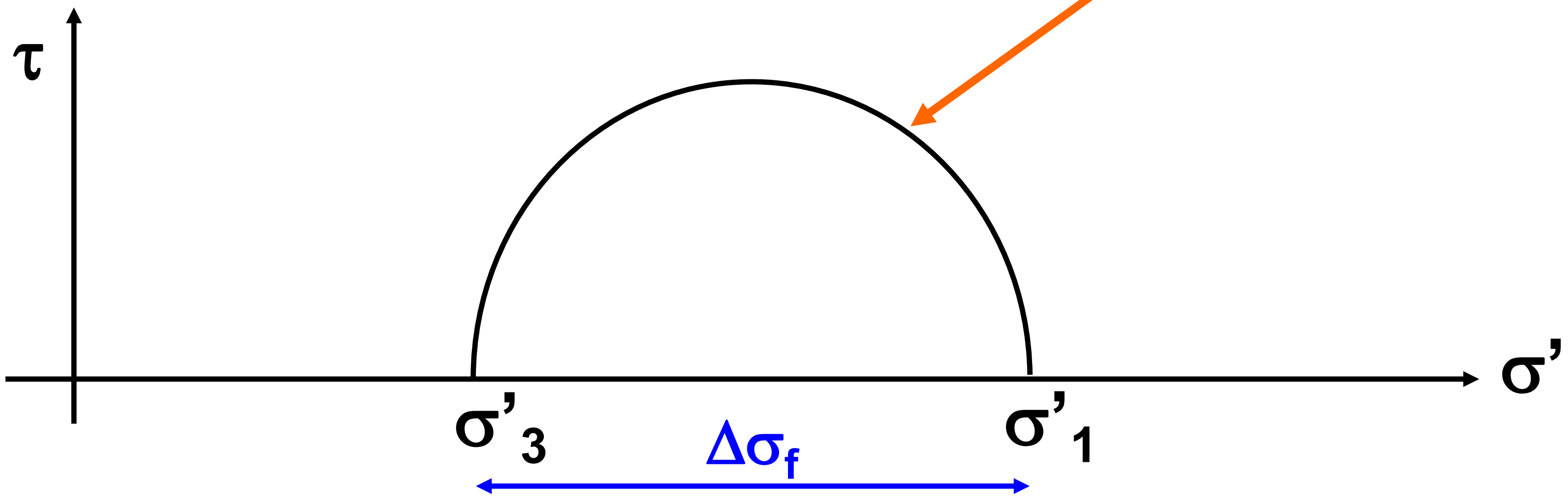
$-u_r + \sigma_c \pm \Delta u_f$

$\sigma'_{Vf} = \cancel{\sigma_C} + \Delta\sigma_f + u_r - \cancel{\sigma_c} \mp \Delta u_f = \sigma'_{1f}$

$\sigma'_{hf} = \cancel{\sigma_C} + u_r - \cancel{\sigma_c} \mp \Delta u_f = \sigma'_{3f}$

Mohr circle in terms of effective stresses do not depend on the cell pressure.

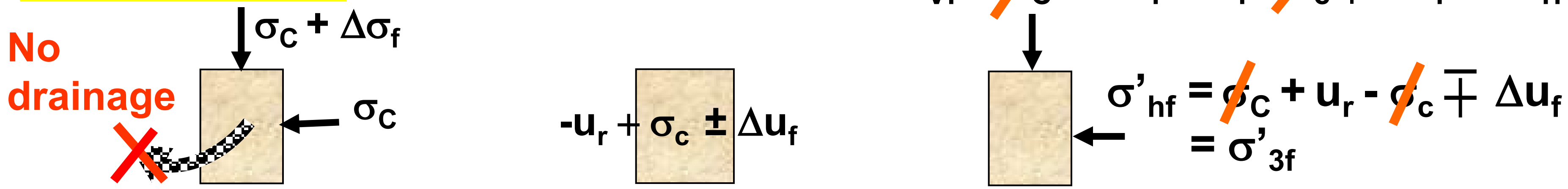
Therefore, we get only one Mohr circle in terms of effective stress for different cell pressures



Unconsolidated- Undrained test (UU Test)

Total, σ = **Neutral, u** + **Effective, σ'**

Step 3: At failure

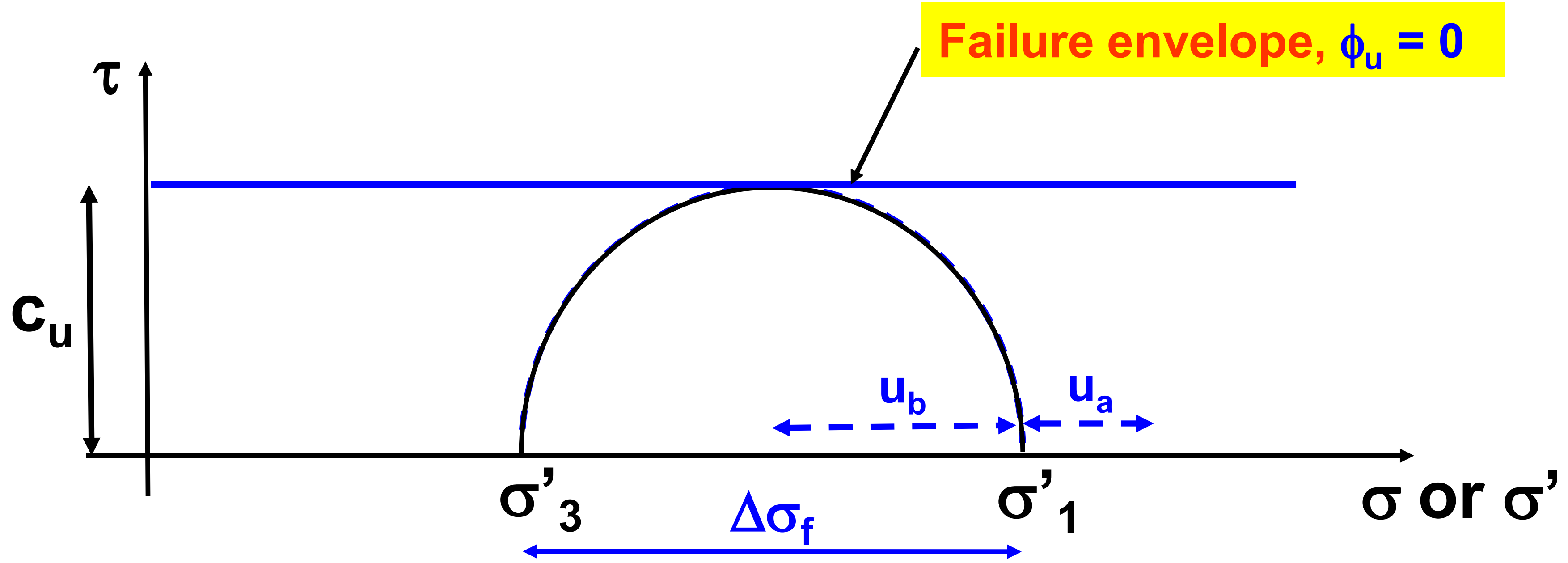


$-u_r + \sigma_c \pm \Delta u_f$

$\sigma'_{Vf} = \cancel{\sigma_C} + \Delta\sigma_f + u_r - \cancel{\sigma_c} \mp \Delta u_f = \sigma'_{1f}$

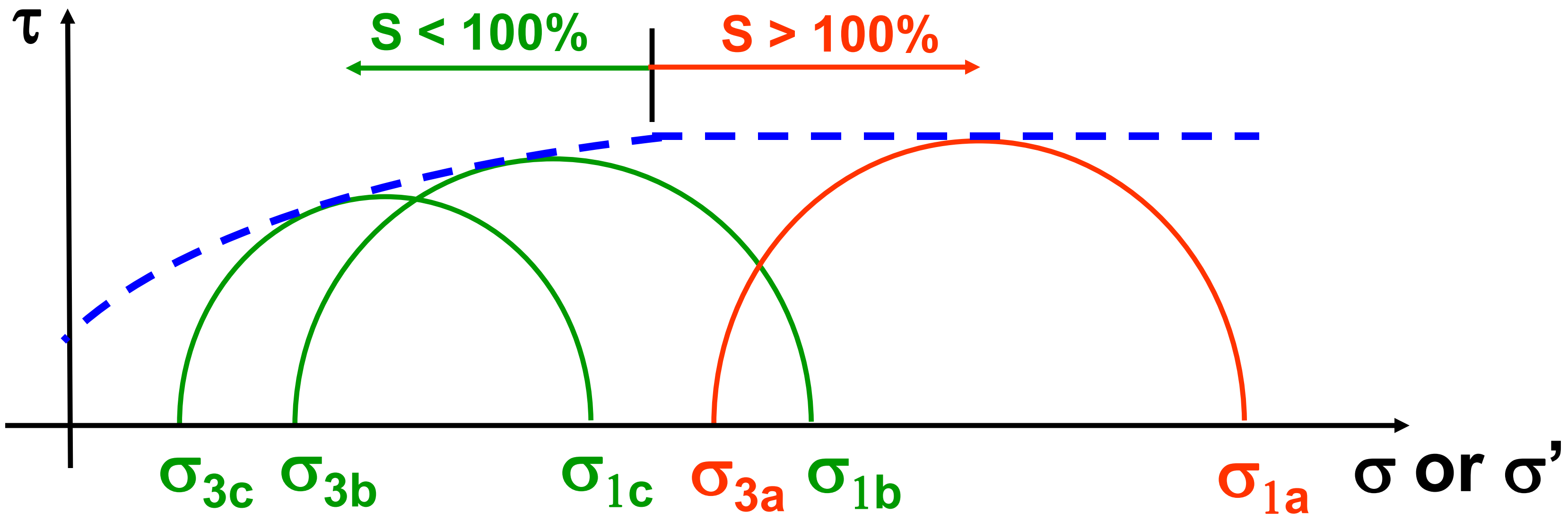
$\sigma'_{hf} = \cancel{\sigma_C} + u_r - \cancel{\sigma_c} \mp \Delta u_f$
$= \sigma'_{3f}$

Mohr circles in terms of total stresses



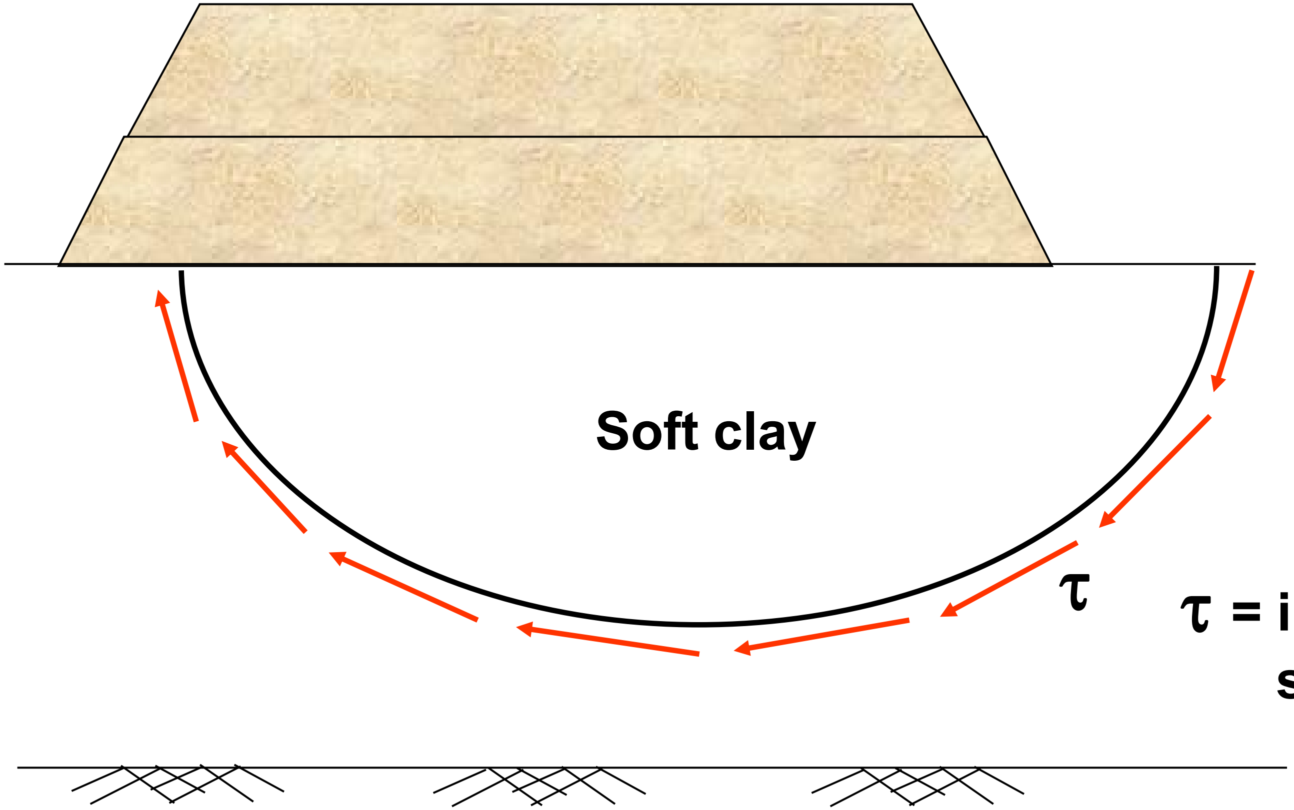
Unconsolidated- Undrained test (UU Test)

Effect of degree of saturation on failure envelope



Some practical applications of UU analysis for clays

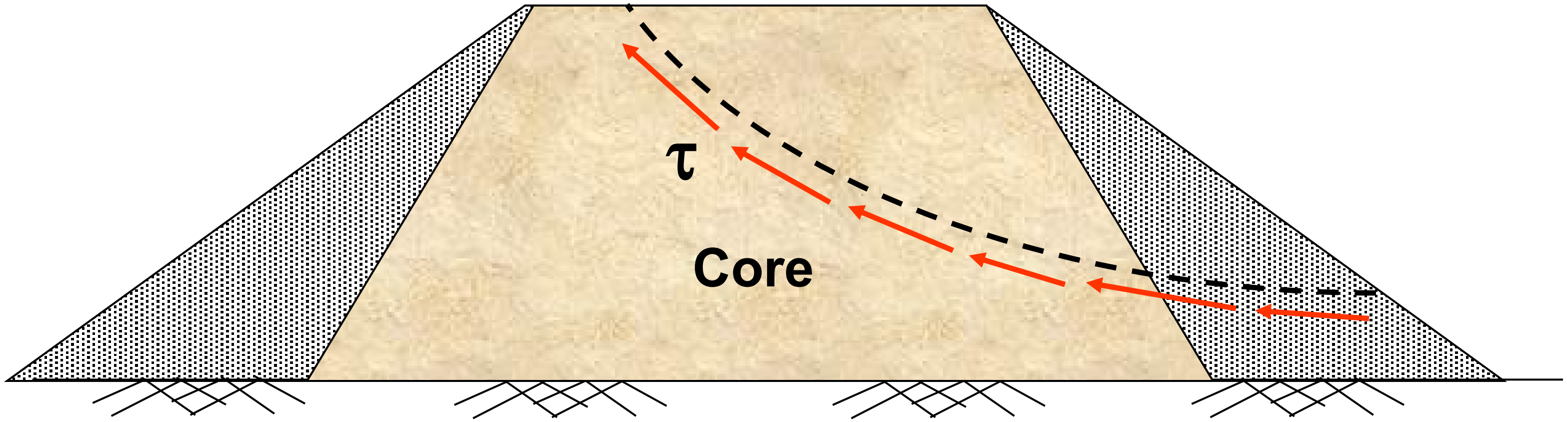
1. Embankment constructed rapidly over a soft clay deposit



τ = in situ undrained shear strength

Some practical applications of UU analysis for clays

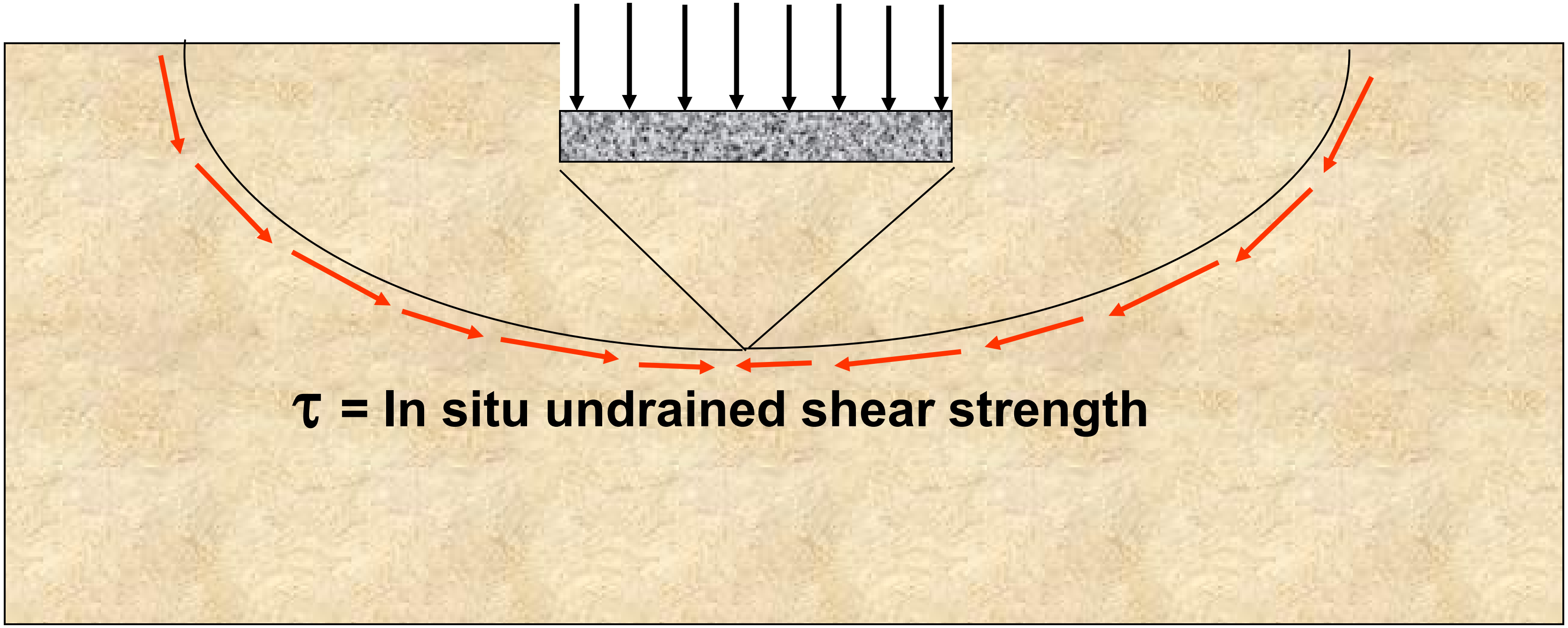
2. Large earth dam constructed rapidly with no change in water content of soft clay



τ = Undrained shear strength of clay core

Some practical applications of UU analysis for clays

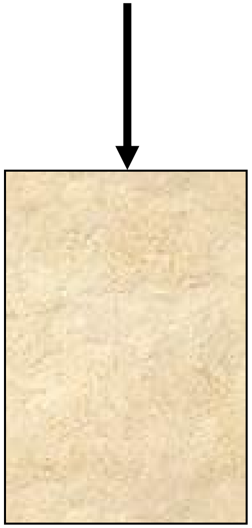
3. Footing placed rapidly on clay deposit



Note: UU test simulates the <u>short term condition</u> in the field. Thus, c_u can be used to analyze the short term behavior of soils

Unconfined Compression Test (UC Test)

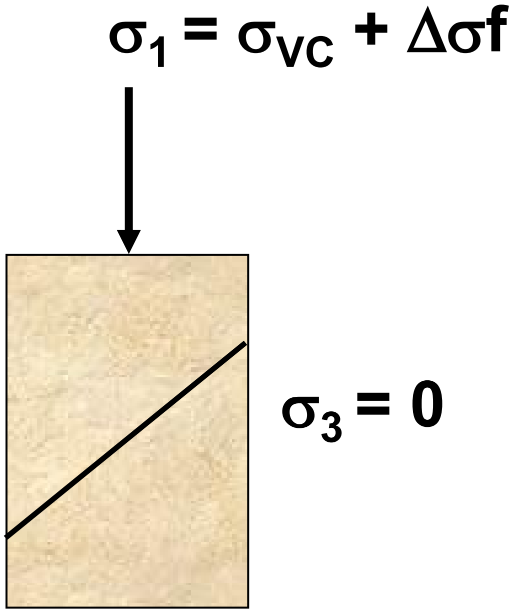
$$\sigma_1 = \sigma_{VC} + \Delta\sigma$$

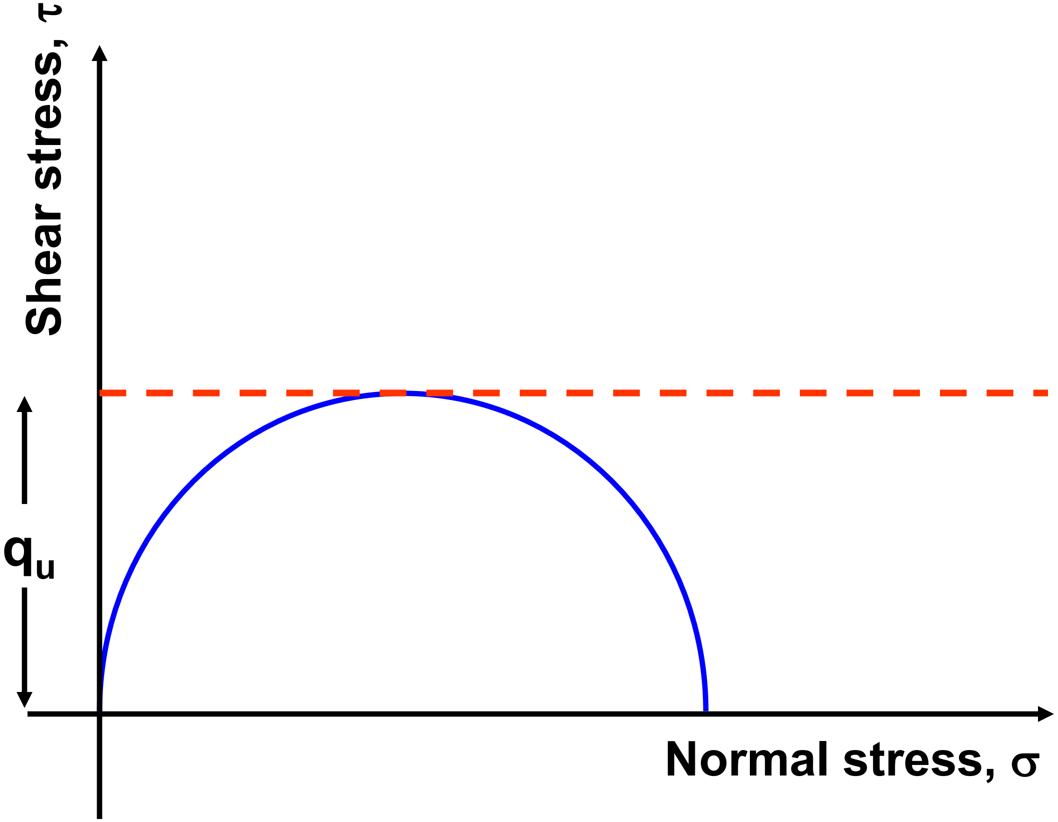


$$\sigma_3 = 0$$

Confining pressure is zero in the UC test

Unconfined Compression Test (UC Test)



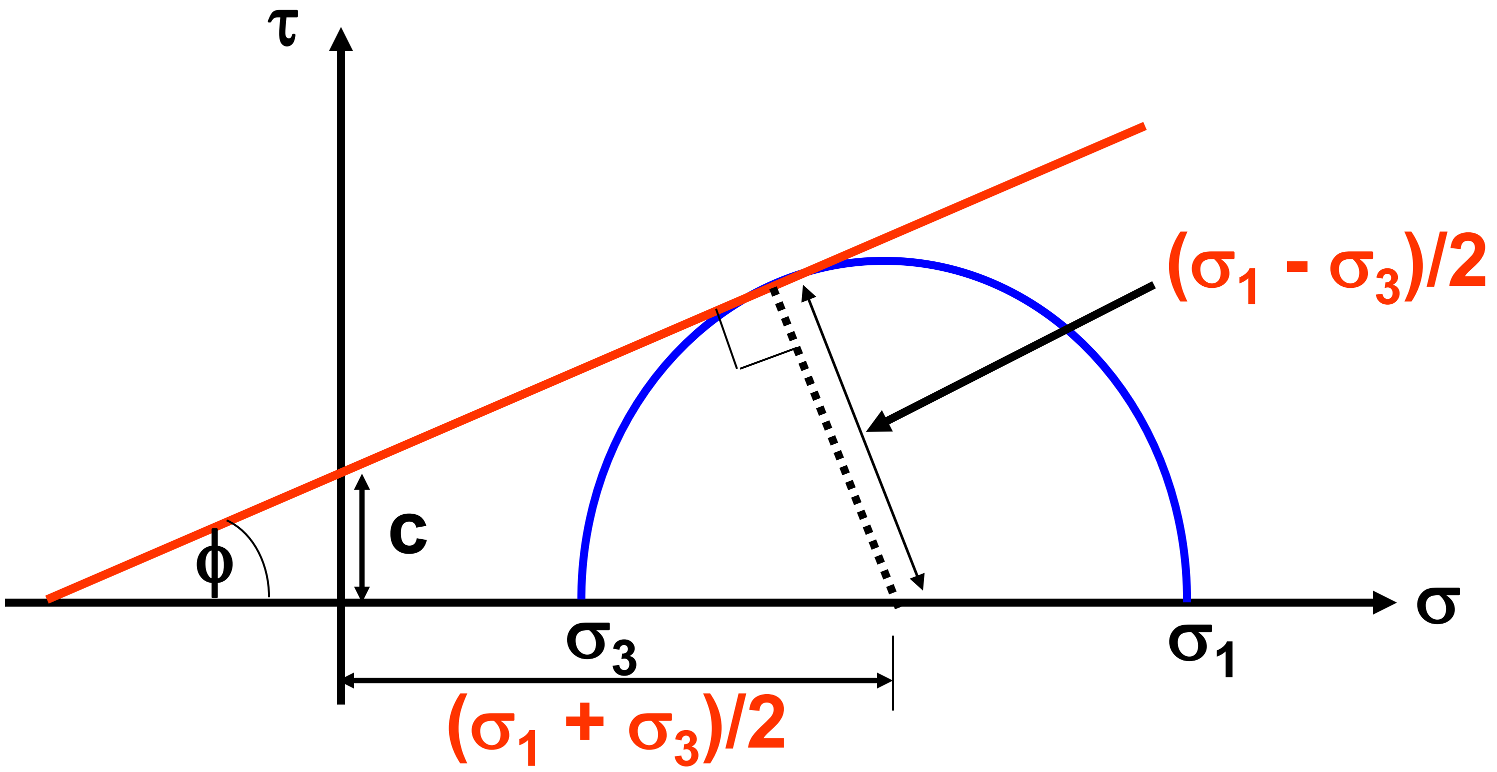


Note: Theoritically $q_u = c_u$, However in the actual case $q_u < c_u$ due to premature failure of the sample

Stress Invariants (*p* and *q*)

***p* (or *s*) = $(\sigma_1 + \sigma_3)/2$**

***q* (or *t*) = $(\sigma_1 - \sigma_3)/2$**

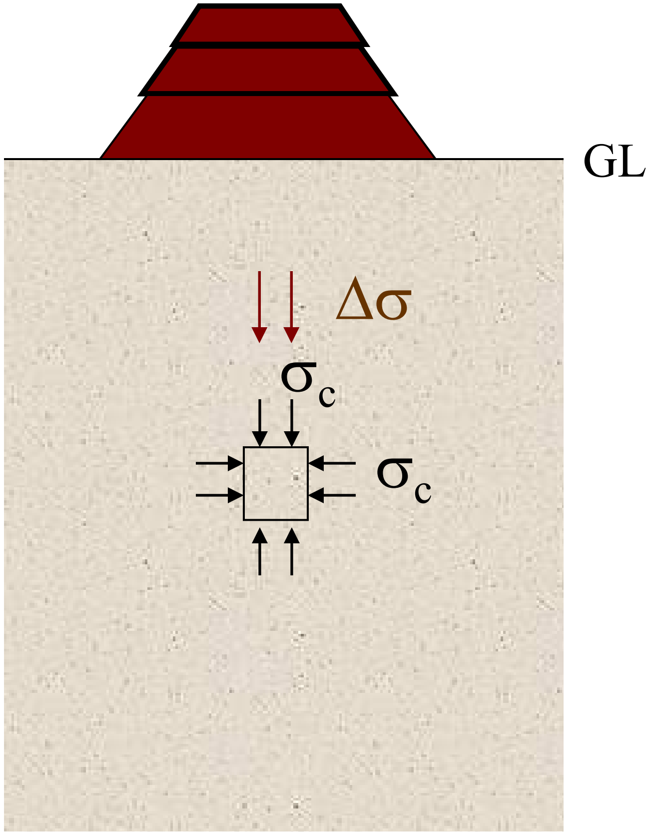


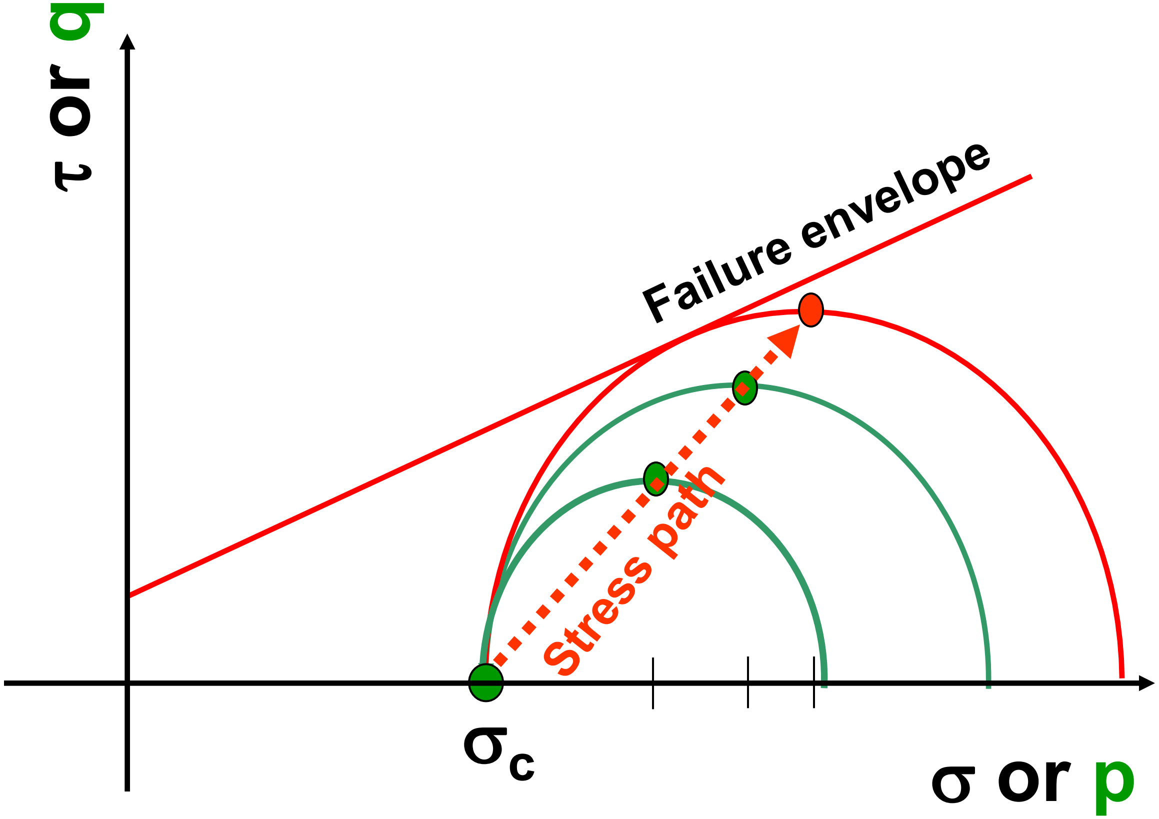
p and *q* can be used to illustrate the variation of the stress state of a soil specimen during a laboratory triaxial test

Stress Invariants (*p* and *q*)

p (or *s*) = $(\sigma_1 + \sigma_3)/2$

q (or *t*) = $(\sigma_1 - \sigma_3)/2$

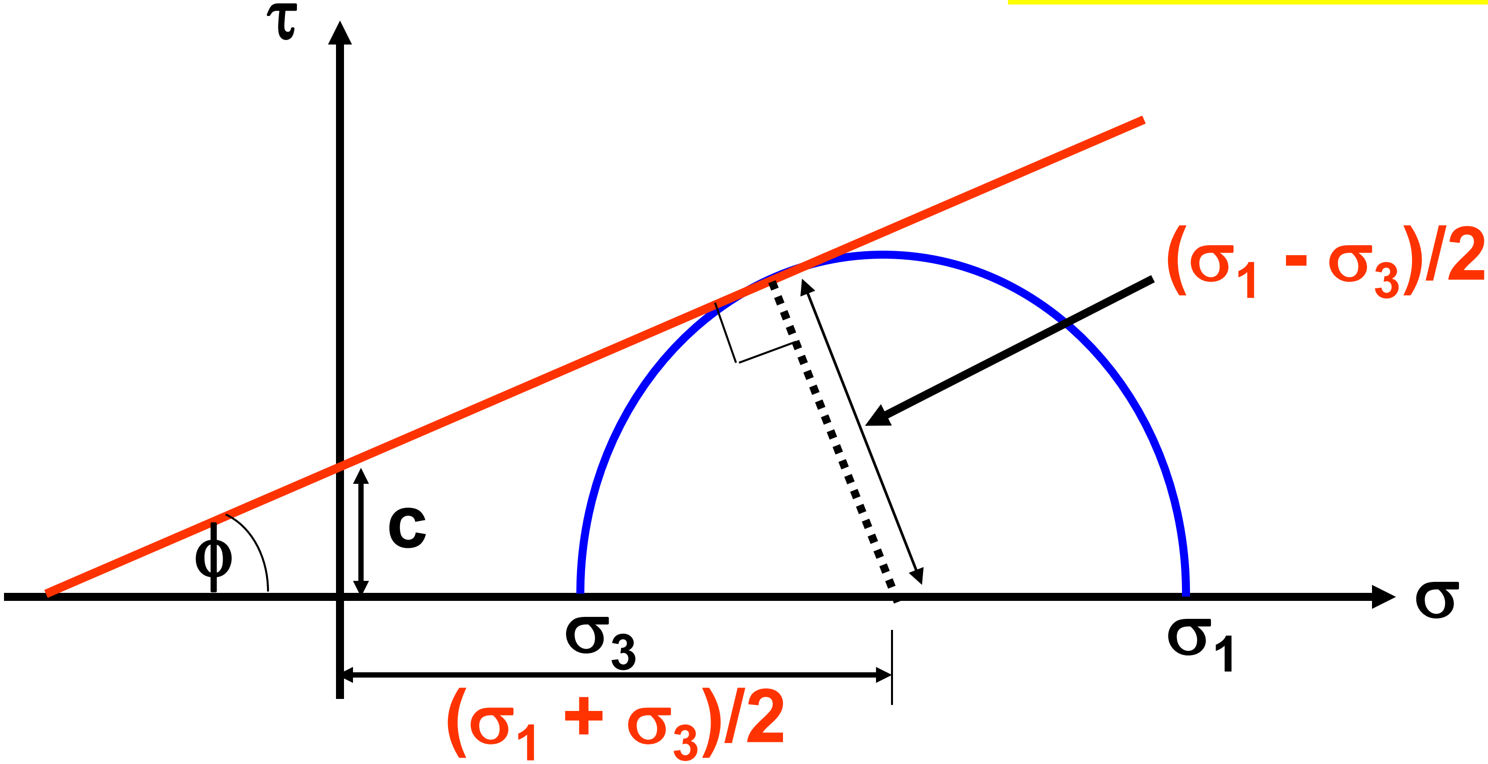




Mohr Coulomb failure envelope in terms of stress invariants

p (or *s*) = $(\sigma_1 + \sigma_3)/2$

q (or *t*) = $(\sigma_1 - \sigma_3)/2$



$$\left[c'Cot\phi' + \left(\frac{\sigma_1' + \sigma_3'}{2}\right)\right]Sin\phi' = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)$$

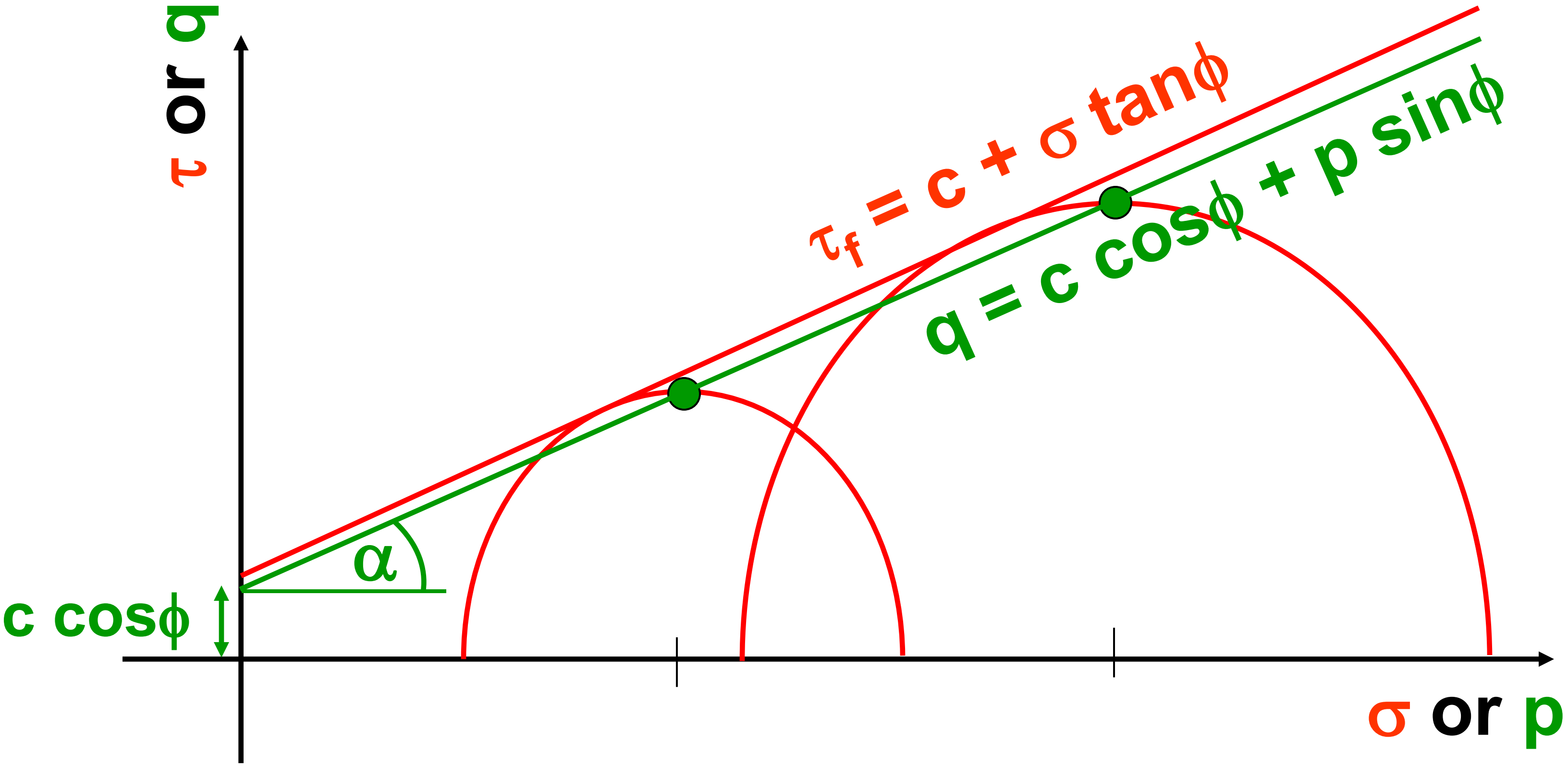
$$\frac{(\sigma_1' - \sigma_3')}{2} = \frac{(\sigma_1' + \sigma_3')}{2}Sin\phi' + c'Cos\phi'$$

$$q = pSin\phi' + c'Cos\phi'$$

Mohr Coulomb failure envelope in terms of stress invariants

p (or *s*) = $(\sigma_1 + \sigma_3)/2$

q (or *t*) = $(\sigma_1 - \sigma_3)/2$



Therefore, $\sin\phi = \tan\alpha$ → $\phi = \sin^{-1}(\tan\alpha)$

Stress path for CD triaxial test

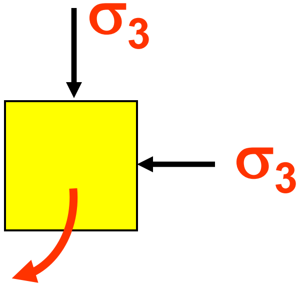
In CD tests pore water pressure is equal to zero. Therefore, total and effective stresses are equal

p, p' (or *s, s'*) = $(\sigma_1 + \sigma_3)/2$

= $(\sigma'_1 + \sigma'_3)/2$

q (or *t*) = $(\sigma_1 - \sigma_3)/2$

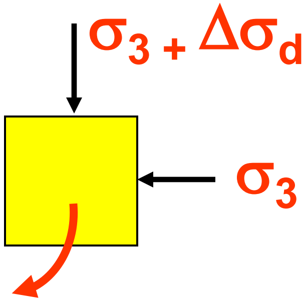
Step 1



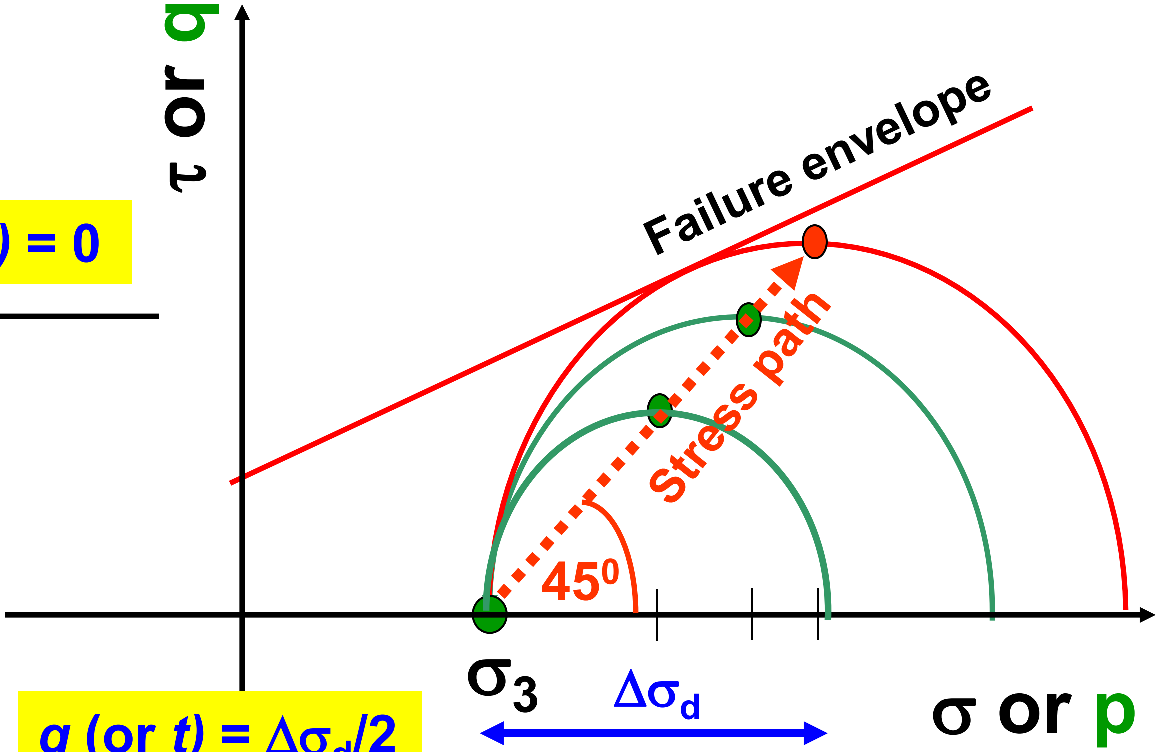
p, p' (or *s, s'*) = σ_3

q (or *t*) = 0

Step 2



p, p' (or *s, s'*) = $\sigma_3 + \Delta\sigma_d/2$



q (or *t*) = $\Delta\sigma_d/2$

Stress path for CU triaxial test

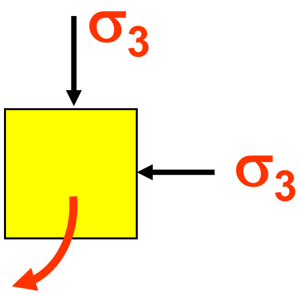
In CU tests pore water pressure develops during shearing

p (or s) = $(\sigma_1 + \sigma_3)/2$

p' (or s') = $(\sigma_1 + \sigma_3)/2 - u$

q (or t) = $(\sigma_1 - \sigma_3)/2$

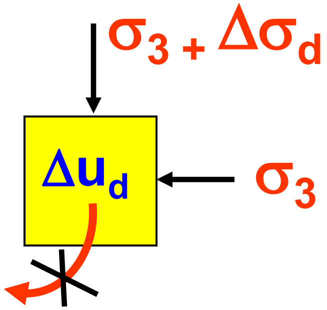
Step 1



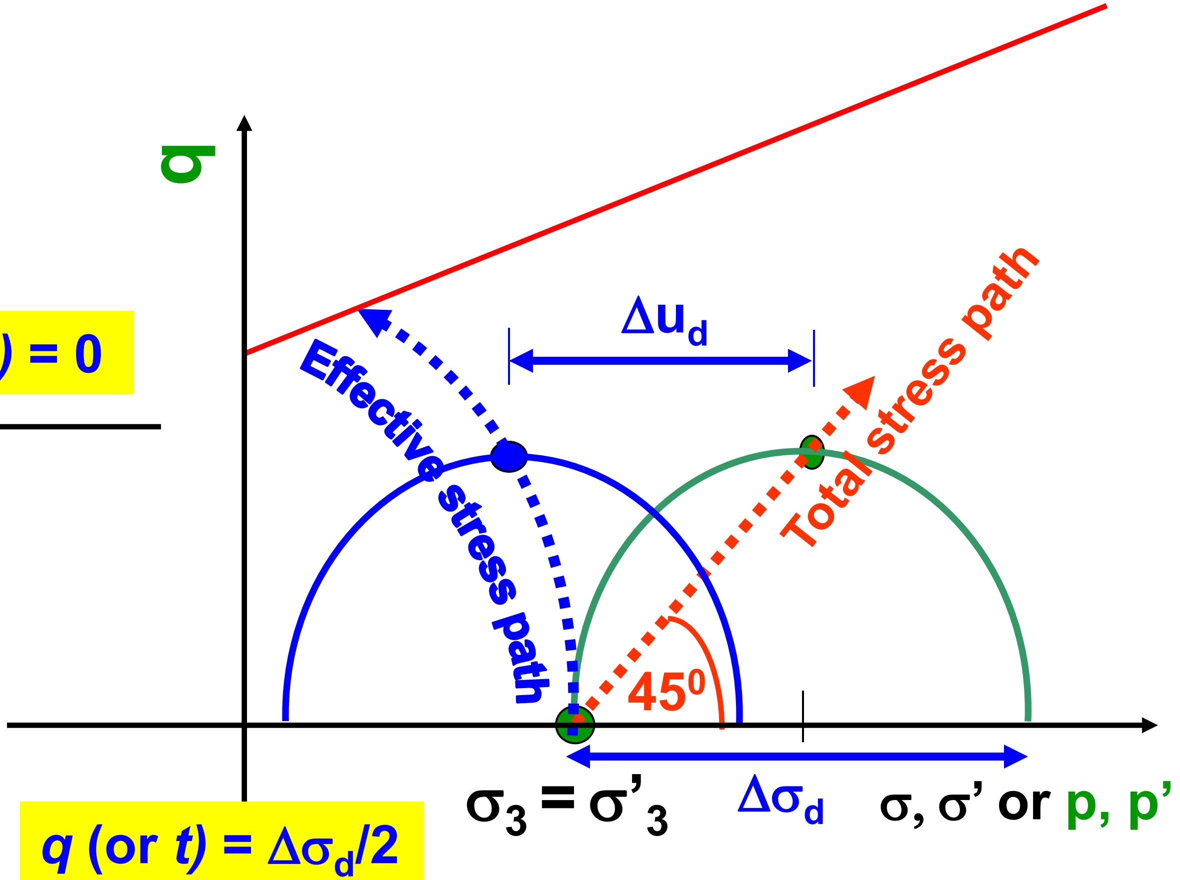
p, p' (or s, s') = σ_3

q (or t) = 0

Step 2



p (or s) = $\sigma_3 + \Delta\sigma_d/2$



q (or t) = $\Delta\sigma_d/2$

Other laboratory shear tests

❑ Direct simple shear test

❑ Torsional ring shear test

❑ Plane strain triaxial test

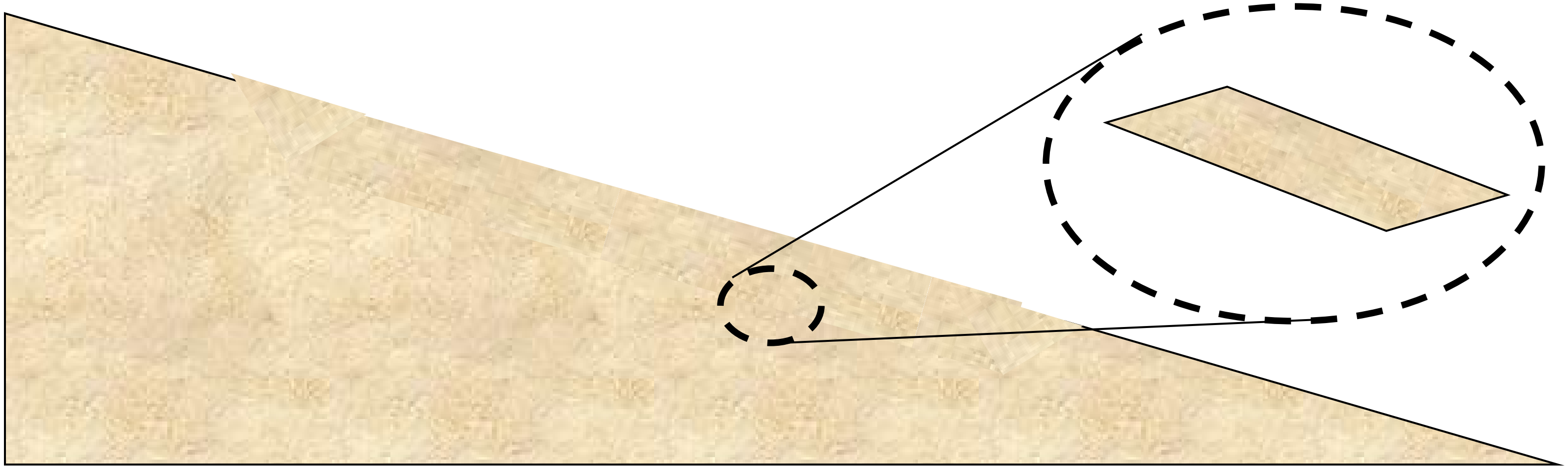
Other laboratory shear tests

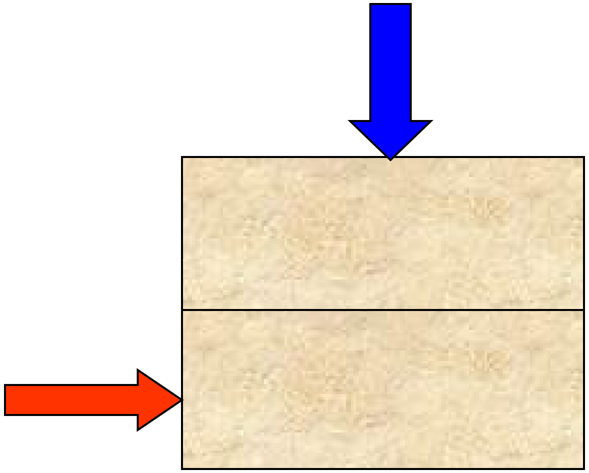
☐ Direct simple shear test

☐ Torsional ring shear test

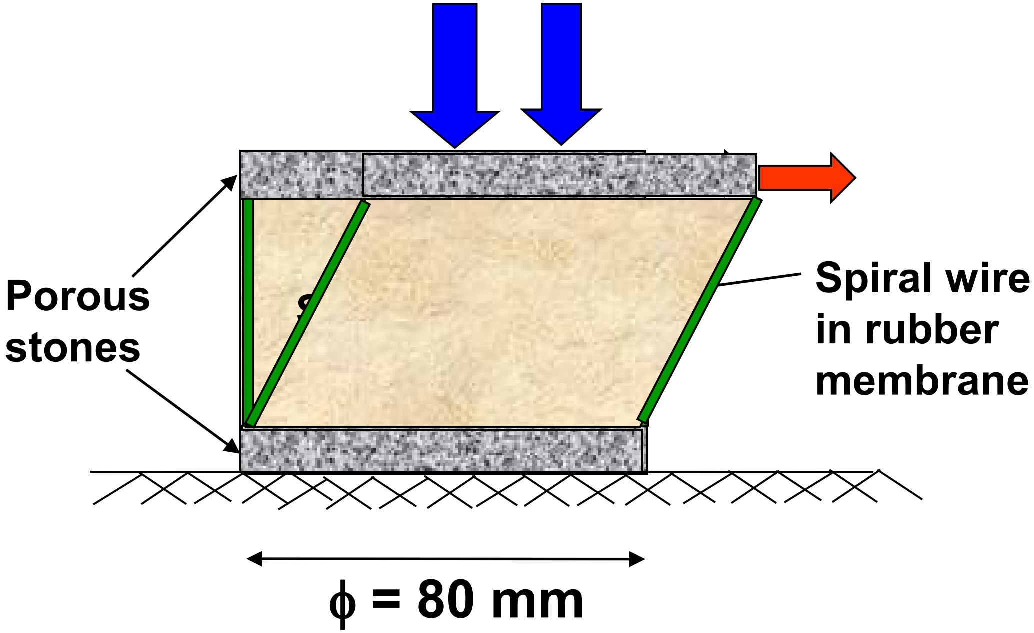
☐ Plane strain triaxial test

Direct simple shear test





Direct shear test

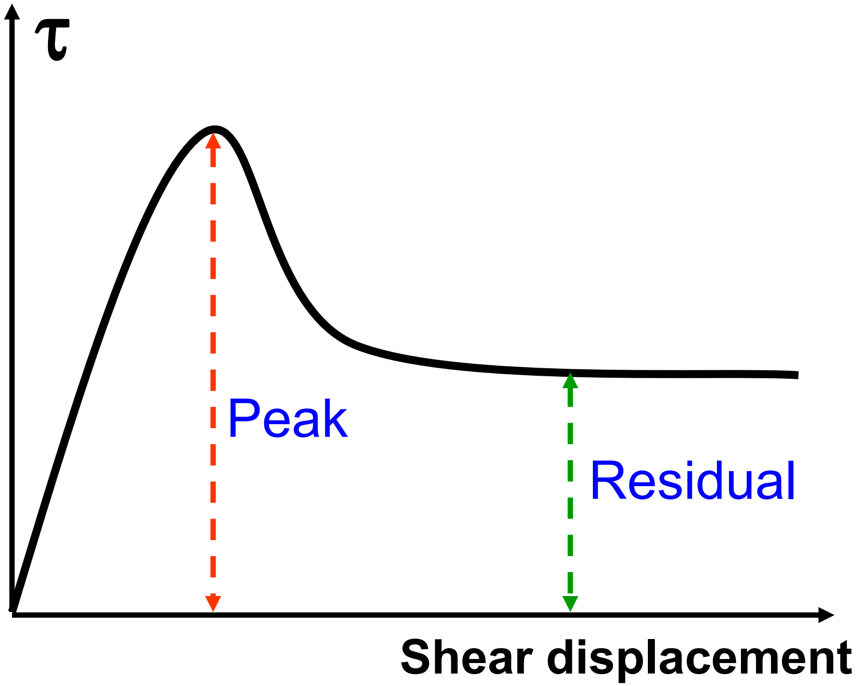


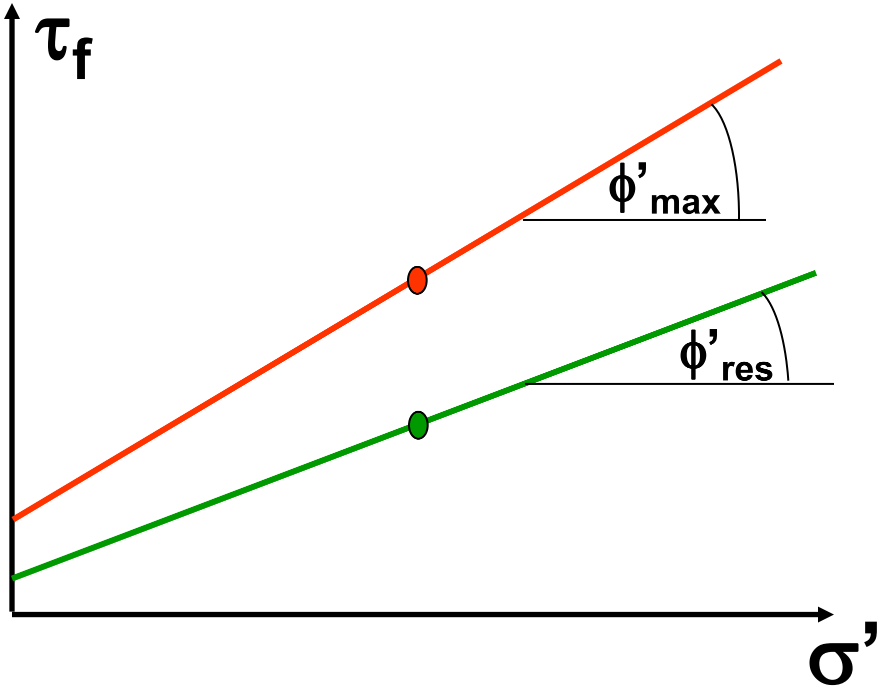
Direct simple shear test

Other laboratory shear tests

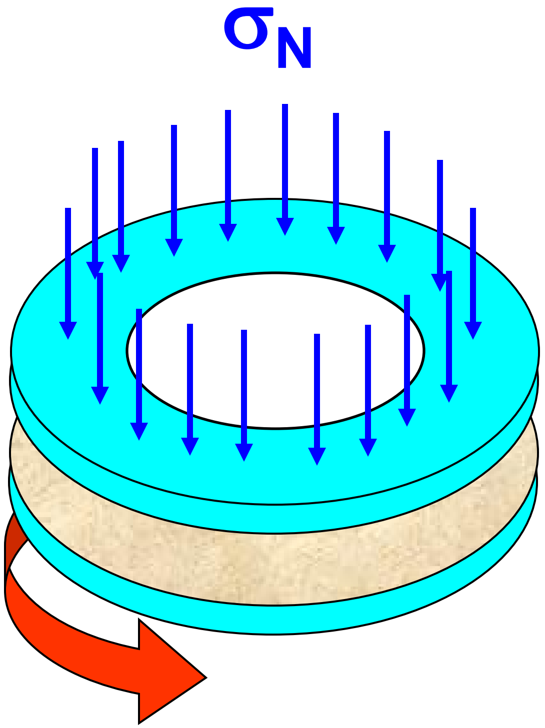
- ☐ Direct simple shear test
- ☐ Torsional ring shear test
- ☐ Plane strain triaxial test

Torsional ring shear test

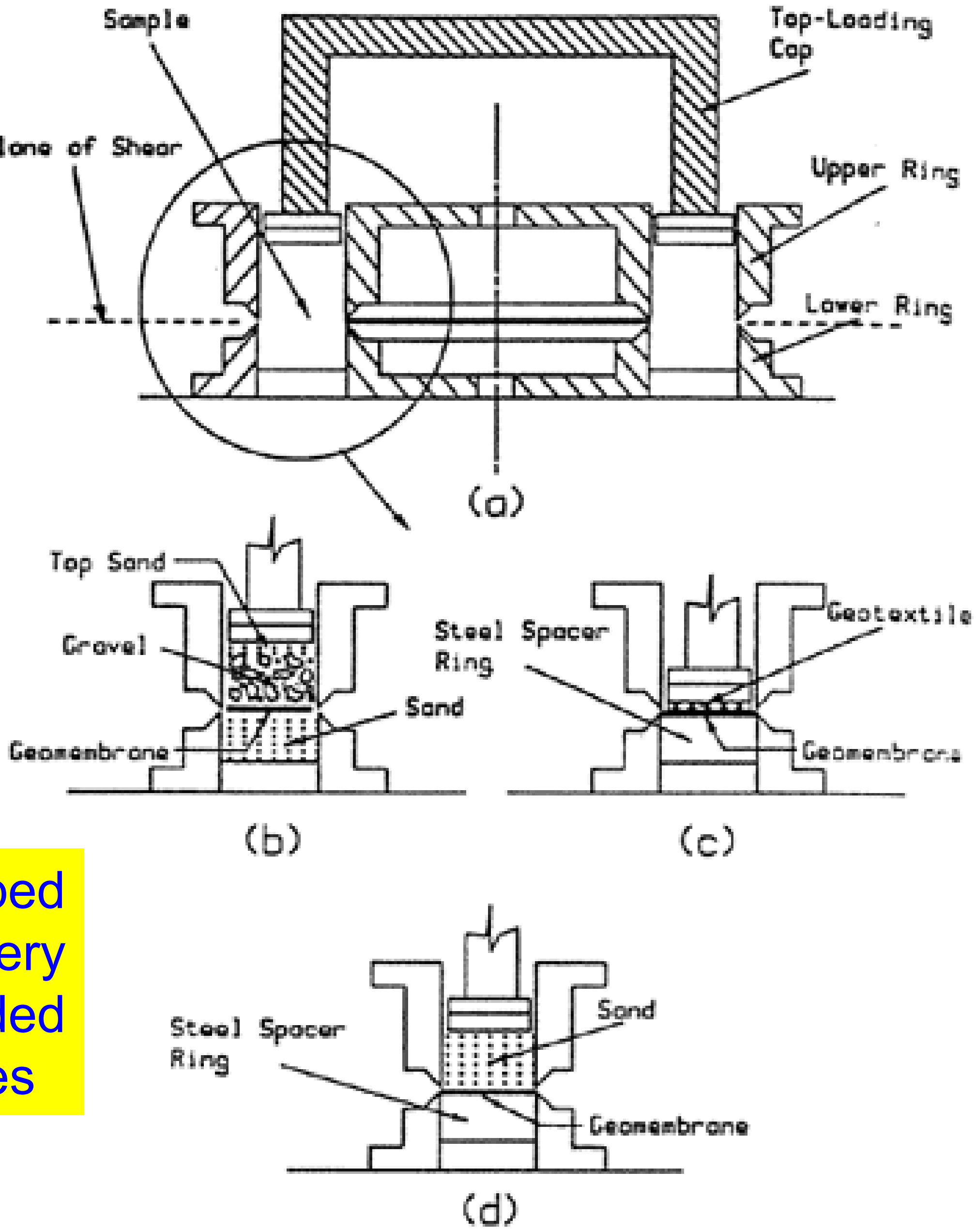




Torsional ring shear test



Preparation of ring shaped undisturbed samples is very difficult. Therefore, remoulded samples are used in most cases



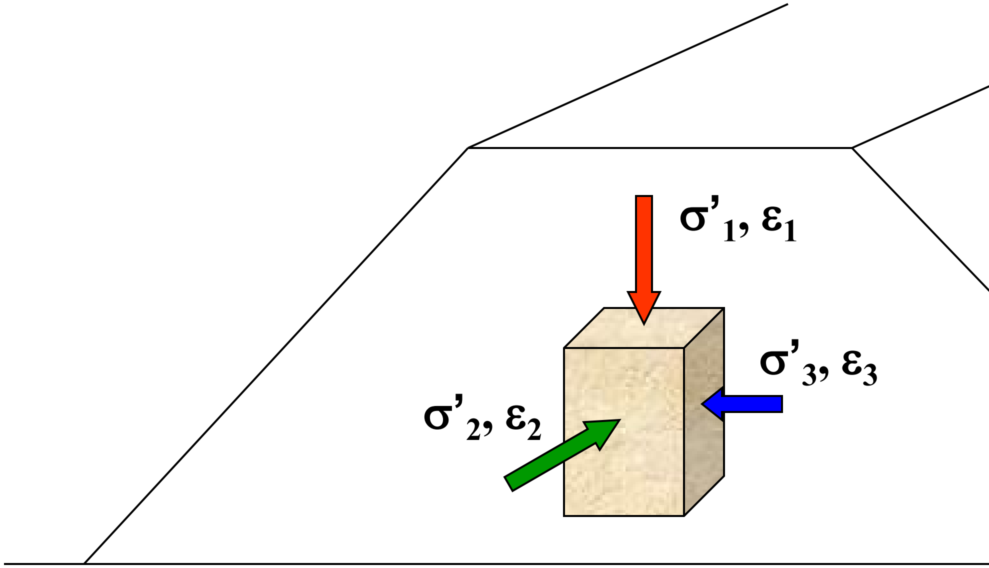
Other laboratory shear tests

❑ Direct simple shear test

❑ Torsional ring shear test

❑ Plane strain triaxial test

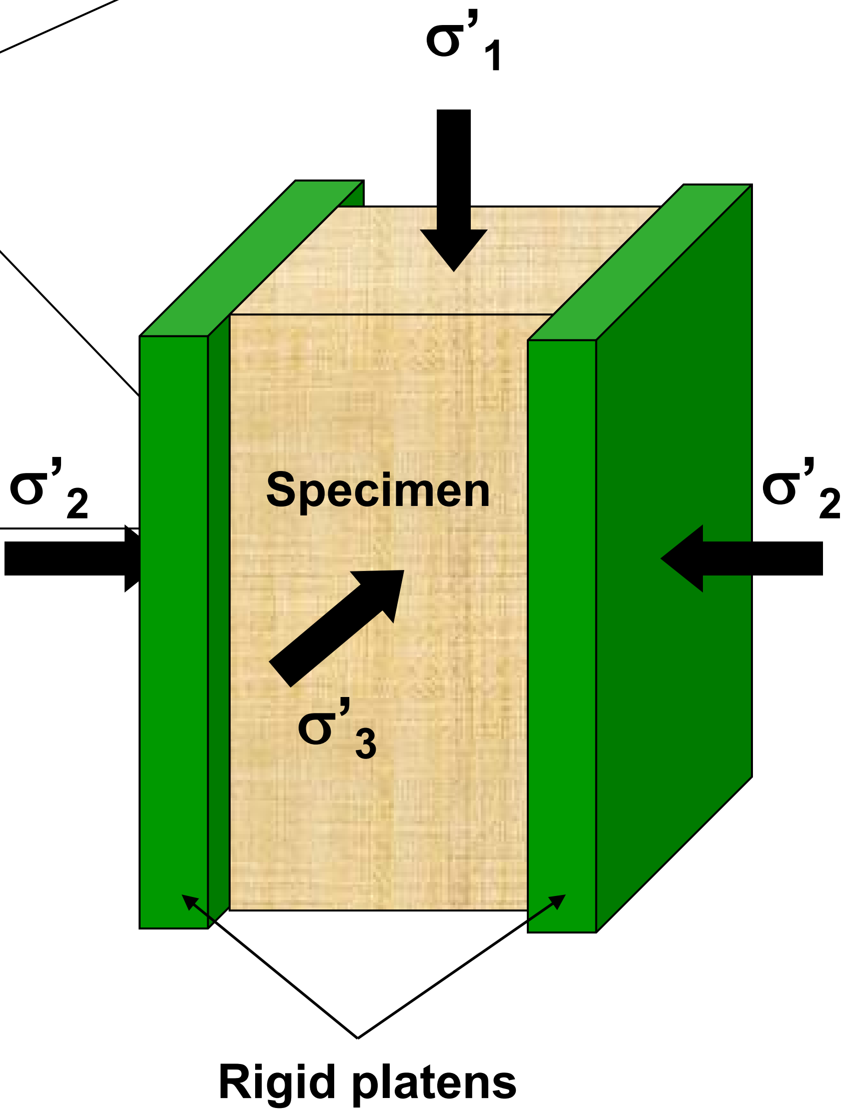
Plane strain triaxial test



Plane strain test

$\sigma'_2 \neq \sigma'_3$

$\varepsilon_2 = 0$



In-situ shear tests

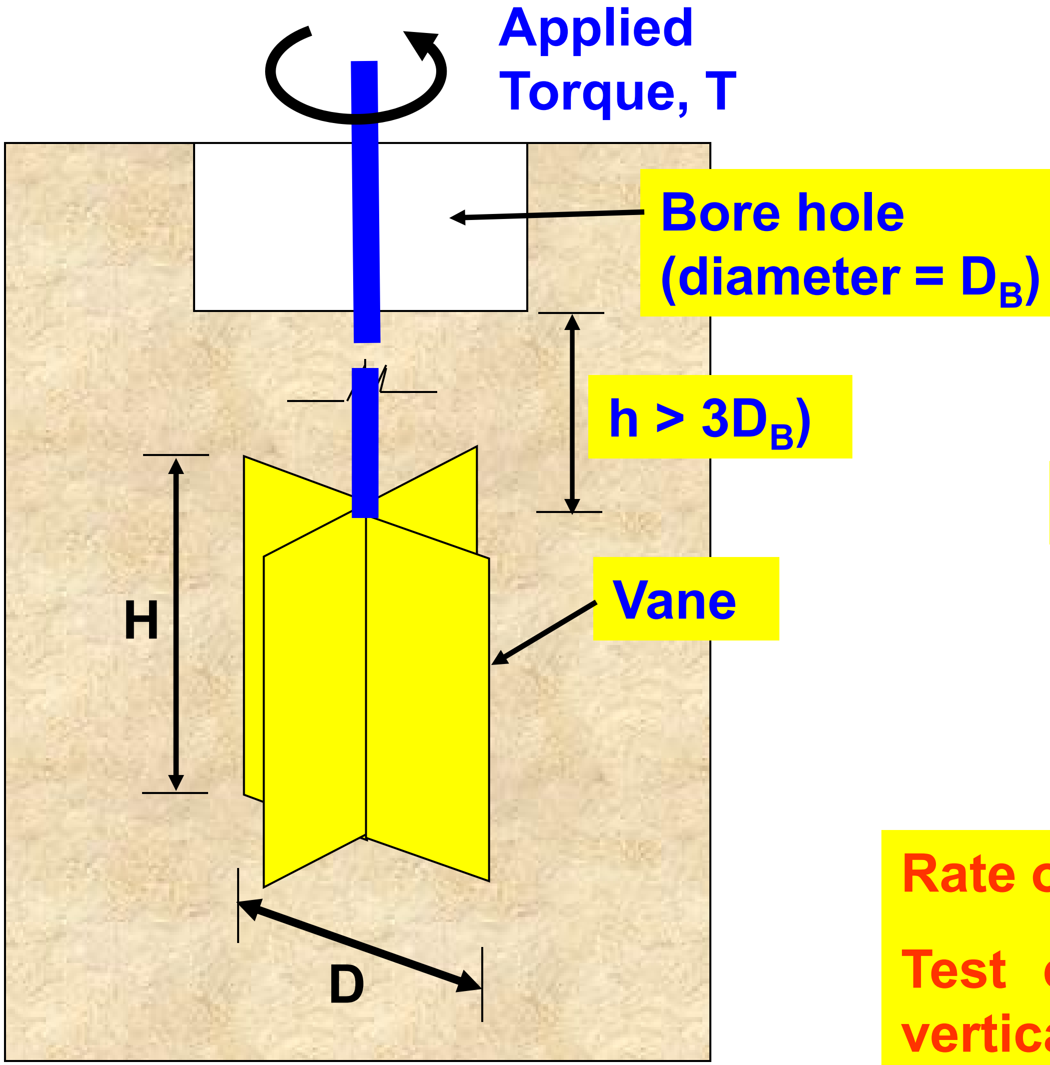
- **Vane shear test**
- **Torvane**
- **Pocket Penetrometer**
- **Pressuremeter**
- **Static Cone Penetrometer test (Push Cone Penetrometer Test, PCPT)**
- **Standard Penetration Test, SPT**

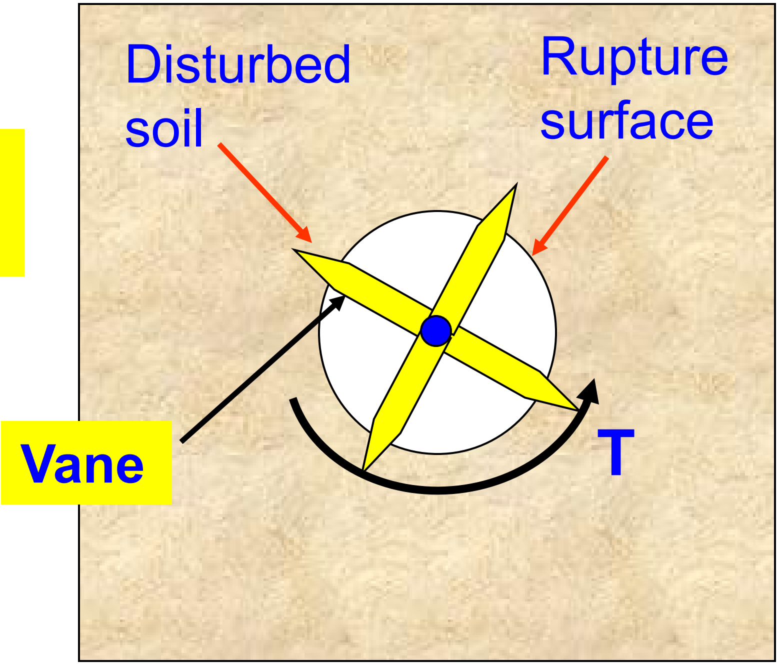
In-situ shear tests

- **Vane shear test** (suitable for soft to stiff clays)
- **Torvane**
- **Pocket Penetrometer**
- **Pressuremeter**
- **Static Cone Penetrometer test (Push Cone Penetrometer Test, PCPT)**
- **Standard Penetration Test, SPT**

Vane shear test

This is one of the most versatile and widely used devices used for investigating undrained shear strength (C_u) and sensitivity of soft clays



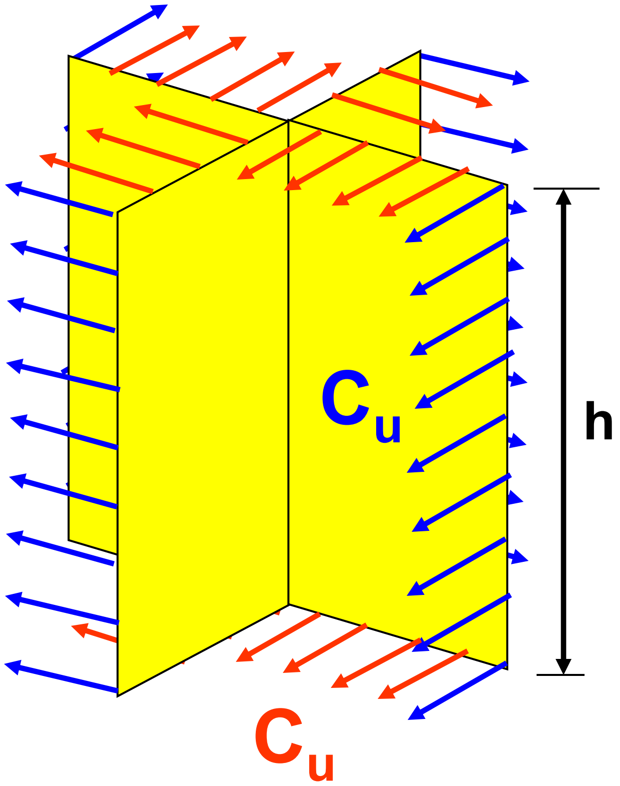


PLAN VIEW

Rate of rotation : $6^0 – 12^0$ per minute

Test can be conducted at 0.5 m vertical intervals

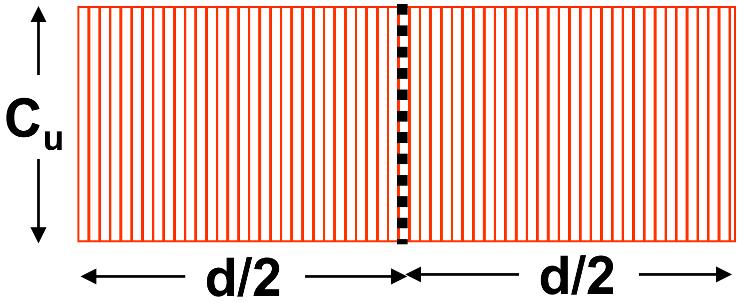
Vane shear test



Since the test is very fast, Unconsolidated Undrained (UU) can be expected

$$T = M_s + M_e + M_e = M_s + 2M_e$$

M_e – *Assuming a uniform distribution of shear strength*



$$M_e = \int_0^{\frac{d}{2}} (2\pi r dr).C_u r$$

$$M_e = 2\pi C_u \int_0^{\frac{d}{2}} r^2 dr = 2\pi C_u \left[\frac{r^3}{3}\right]_0^{\frac{d}{2}}$$

$$M_e = \frac{2\pi C_u}{3}\left[\frac{d^3}{8}\right] = \frac{\pi C_u d^3}{12}$$

Vane shear test

$$T = M_s + M_e + M_e = M_s + 2M_e$$

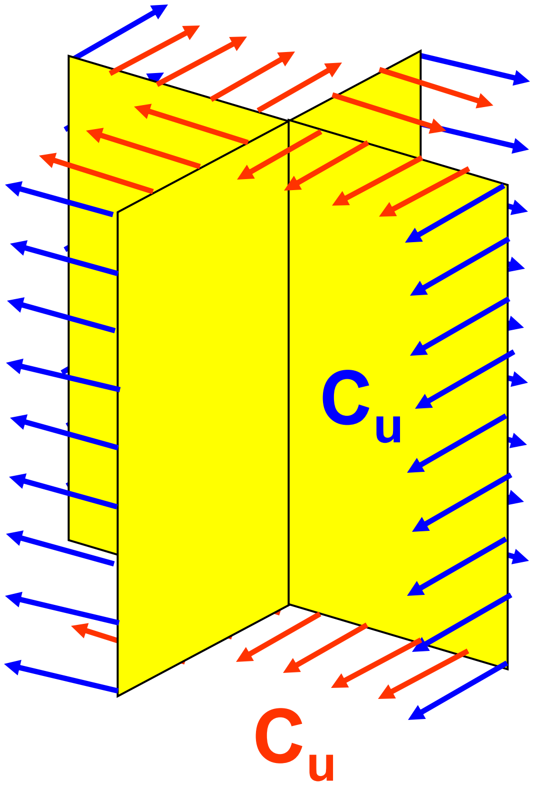
M_s – Shaft shear resistance along the circumference

$$M_s = \pi dhC_u \frac{d}{2} = \pi C_u \frac{d^2h}{2}$$

$$T = \pi C_u \frac{d^2h}{2} + \frac{\pi C_u d^3}{12} \times 2$$

$$T = \pi C_u \left(\frac{d^2h}{2} + \frac{d^3}{6} \right)$$

$$C_u = \frac{T}{\pi \left(\frac{d^2h}{2} + \frac{d^3}{6} \right)}$$

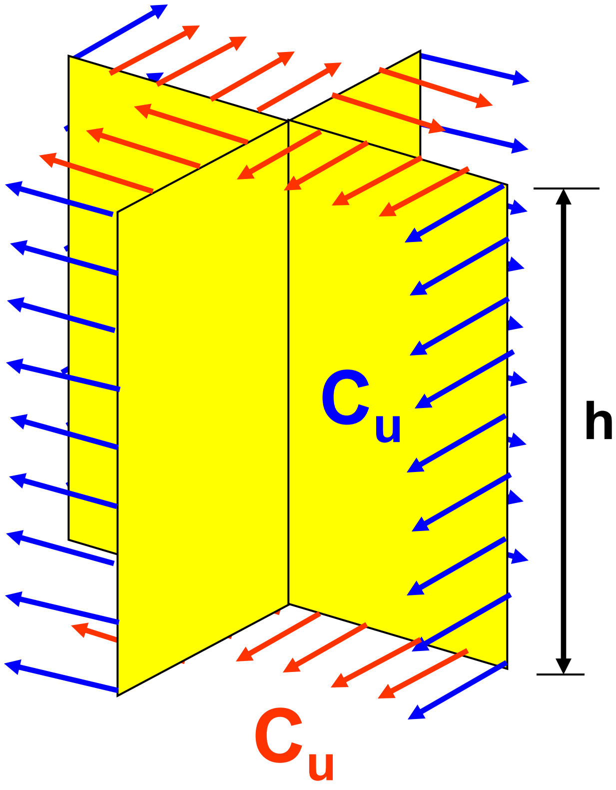


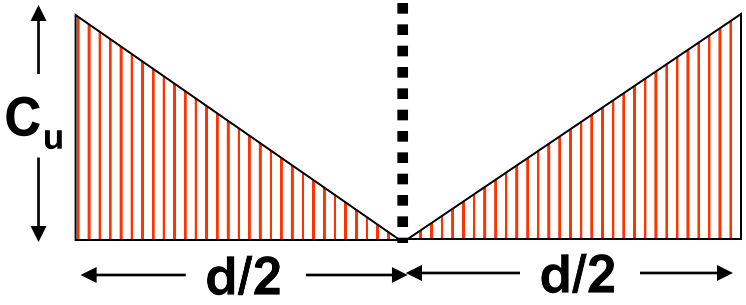
Since the test is very fast, Unconsolidated Undrained (UU) can be expected

Vane shear test

$T = M_s + M_e + M_e = M_s + 2M_e$

M_e – Assuming a triangular distribution of shear strength





$$C_u = \frac{T}{\pi\left(\frac{d^2h}{2} + \frac{d^3}{8}\right)}$$

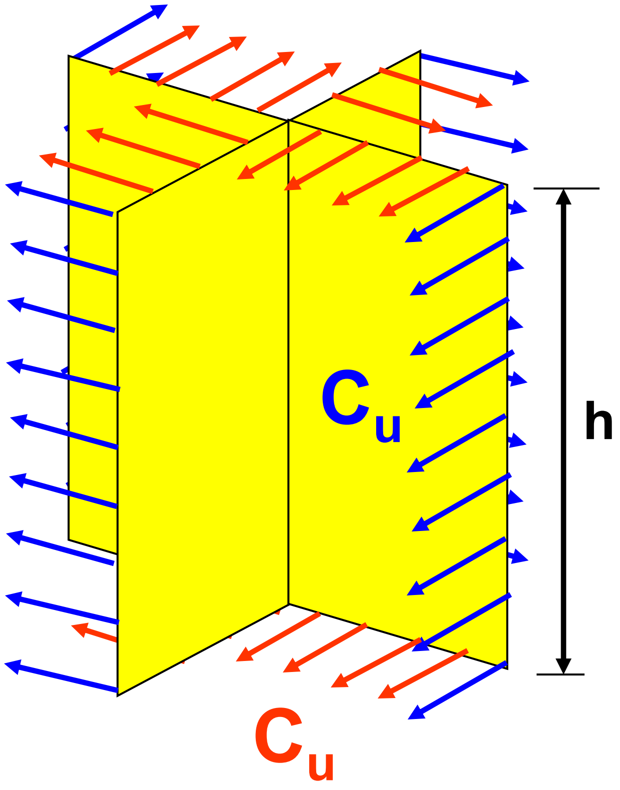
Since the test is very fast, Unconsolidated Undrained (UU) can be expected

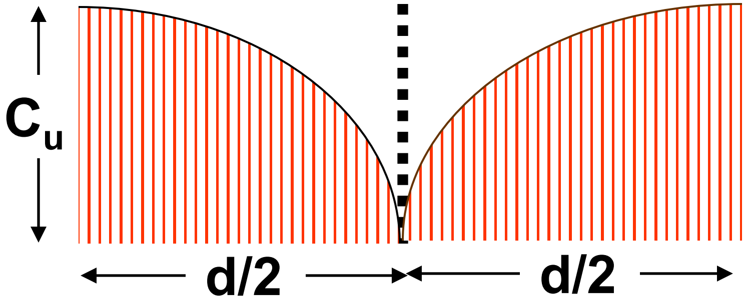
Can you derive this ???

Vane shear test

$$T = M_s + M_e + M_e = M_s + 2M_e$$

M_e – Assuming a parabolic distribution of shear strength



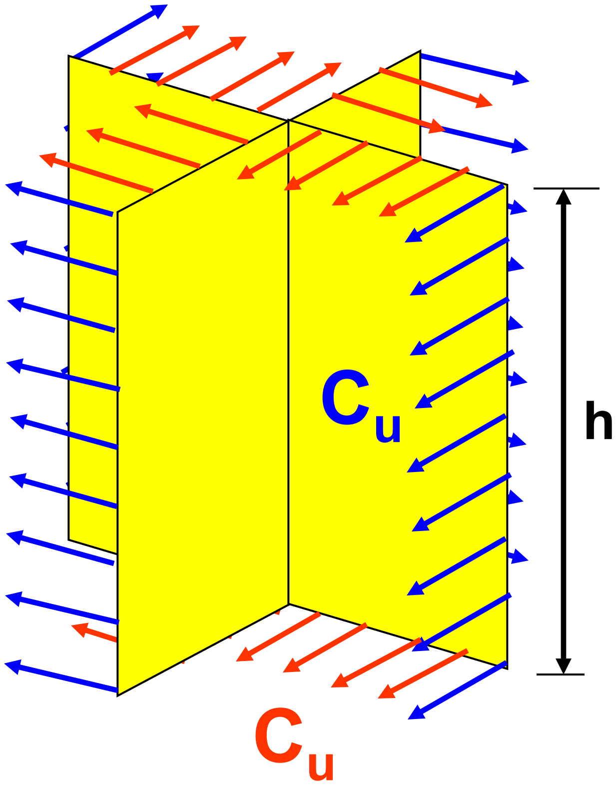


$$C_u = \frac{T}{\pi\left(\frac{d^2 h}{2} + \frac{3d^3}{20}\right)}$$

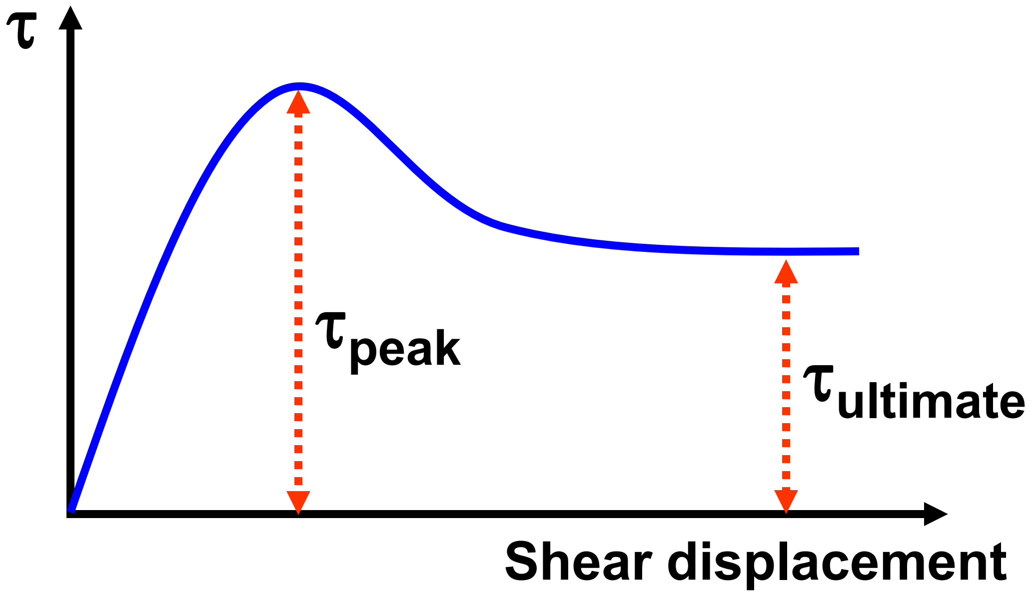
Since the test is very fast, Unconsolidated Undrained (UU) can be expected

Can you derive this ???

Vane shear test



After the initial test, vane can be rapidly rotated through several revolutions until the clay become remoulded



Since the test is very fast, Unconsolidated Undrained (UU) can be expected

$$Sensitivity = \frac{Peak\ Stength}{Ultimate\ Stength}$$

Some important facts on vane shear test

Insertion of vane into soft clays and silts disrupts the natural soil structure around the vane causing reduction of shear strength

The above reduction is partially regained after some time

C_u as determined by vane shear test may be a function of the rate of angular rotation of the vane

Correction for the strength parameters obtained from **vane shear test**

Bjerrum (1974) has shown that as the plasticity of soils increases, C_u obtained by vane shear tests may give unsafe results for foundation design. Therefore, he proposed the following correction.

$$C_{u(design)} = \lambda C_{u(vane\ shear)}$$

Where, λ = correction factor = 1.7 – 0.54 log (PI)

PI = Plasticity Index

In-situ shear tests

- ☐ Vane shear test
- ☐ Torvane (suitable for very soft to stiff clays)
- ☐ Pocket Penetrometer
- ☐ Pressuremeter
- ☐ Static Cone Penetrometer test (Push Cone Penetrometer Test, PCPT)
- ☐ Standard Penetration Test, SPT

Torvane

Torvane is a modification to the vane

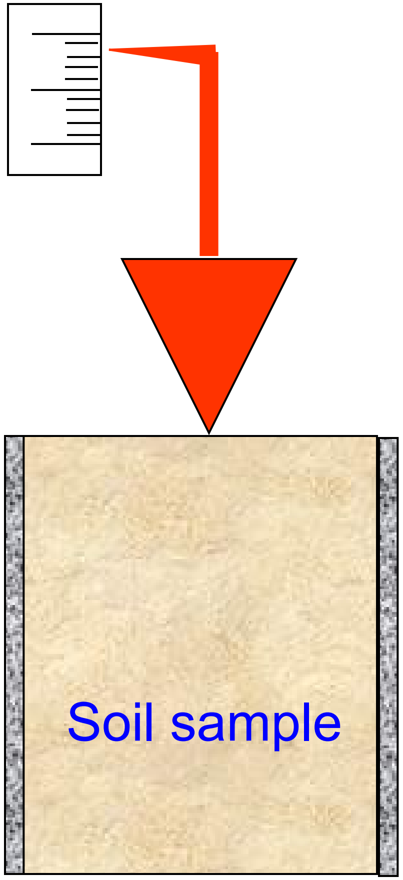
In-situ shear tests

- ☐ **Vane shear test**
- ☐ **Torvane**
- ☐ **Pocket Penetrometer** (suitable for very soft to stiff clays)
- ☐ **Pressuremeter**
- ☐ **Static Cone Penetrometer test (Push Cone Penetrometer Test, PCPT)**
- ☐ **Standard Penetration Test, SPT**

Pocket Penetrometer

Pushed directly into the soil. The unconfined compression strength (q_u) is measured by a calibrated spring.

Swedish Fall Cone (suitable for very soft to soft clays)



C_u ∞ Mass of the cone

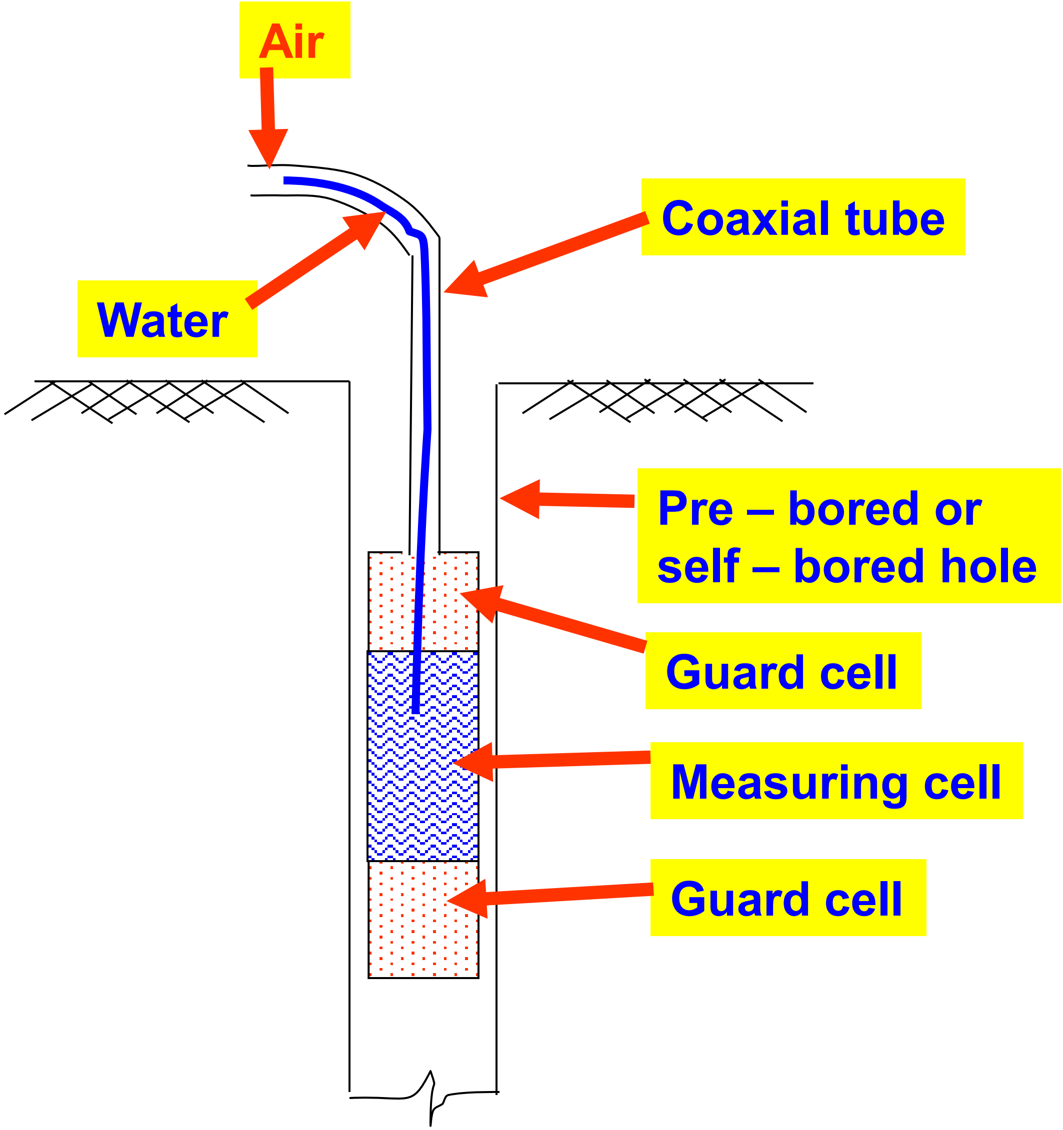
∞ $1/(\text{penetration})^2$

The test must be calibrated

In-situ shear tests

- ☐ **Vane shear test**
- ☐ **Torvane**
- ☐ **Pocket Penetrometer**
- ☐ **Pressuremeter** (suitable for all soil types)
- ☐ **Static Cone Penetrometer test (Push Cone Penetrometer Test, PCPT)**
- ☐ **Standard Penetration Test, SPT**

Pressuremeter



Air

Water

Coaxial tube

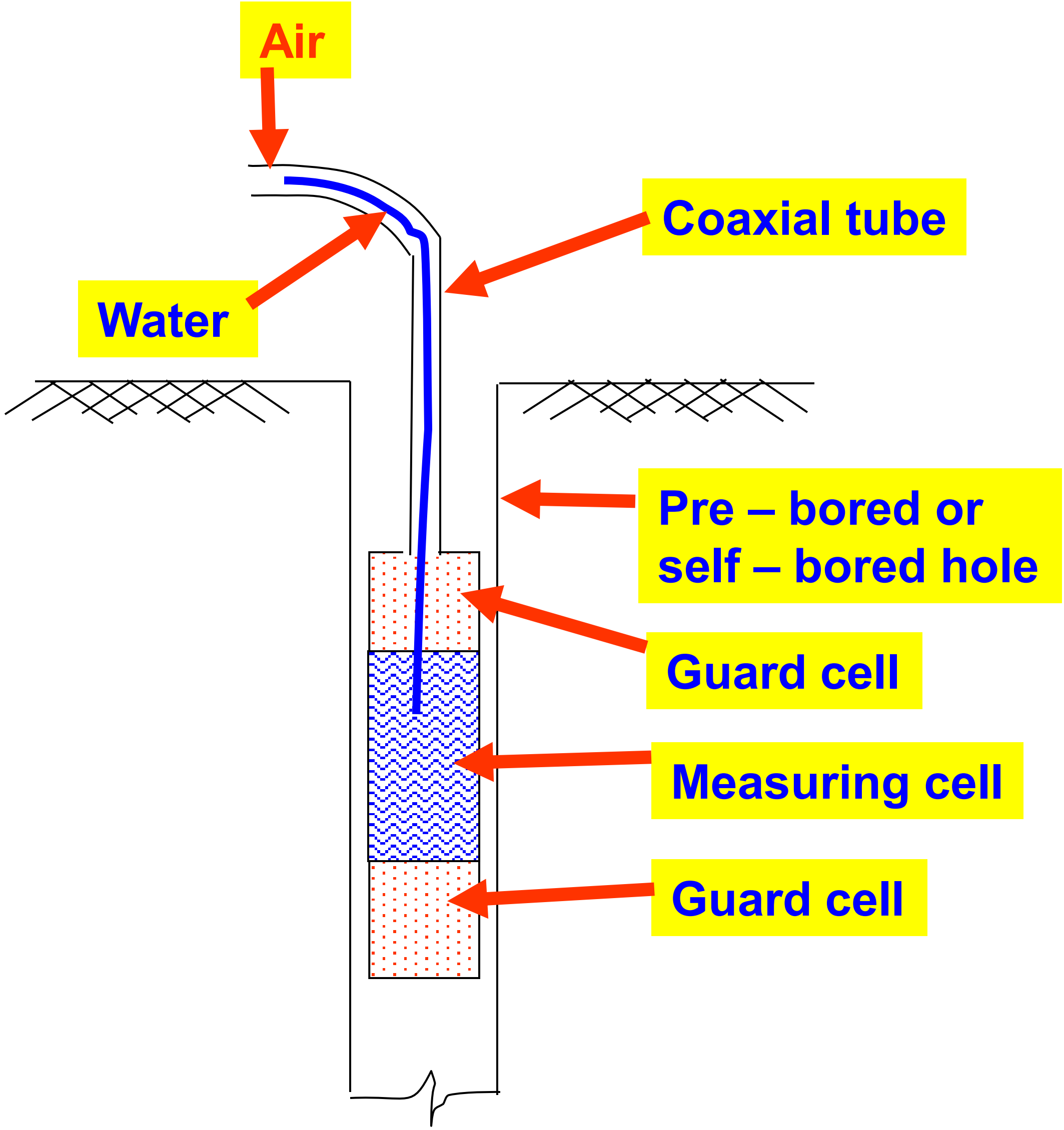
Pre – bored or self – bored hole

Guard cell

Measuring cell

Guard cell

Pressuremeter



Air

Water

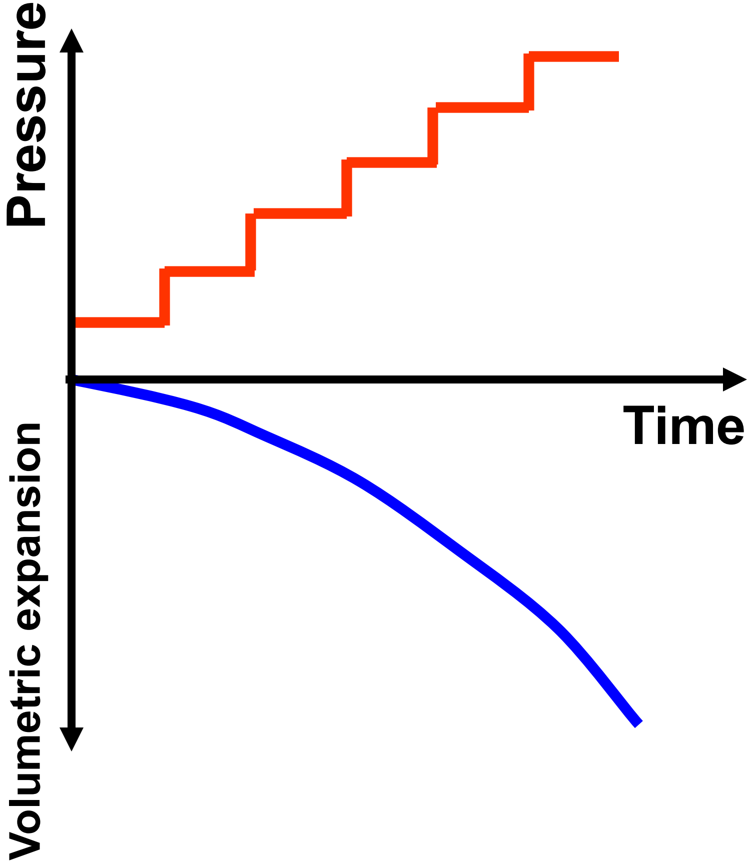
Coaxial tube

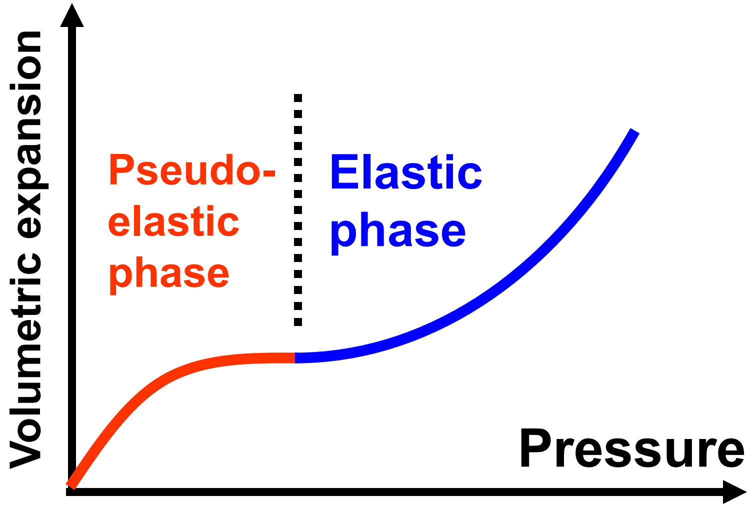
Pre – bored or self – bored hole

Guard cell

Measuring cell

Guard cell

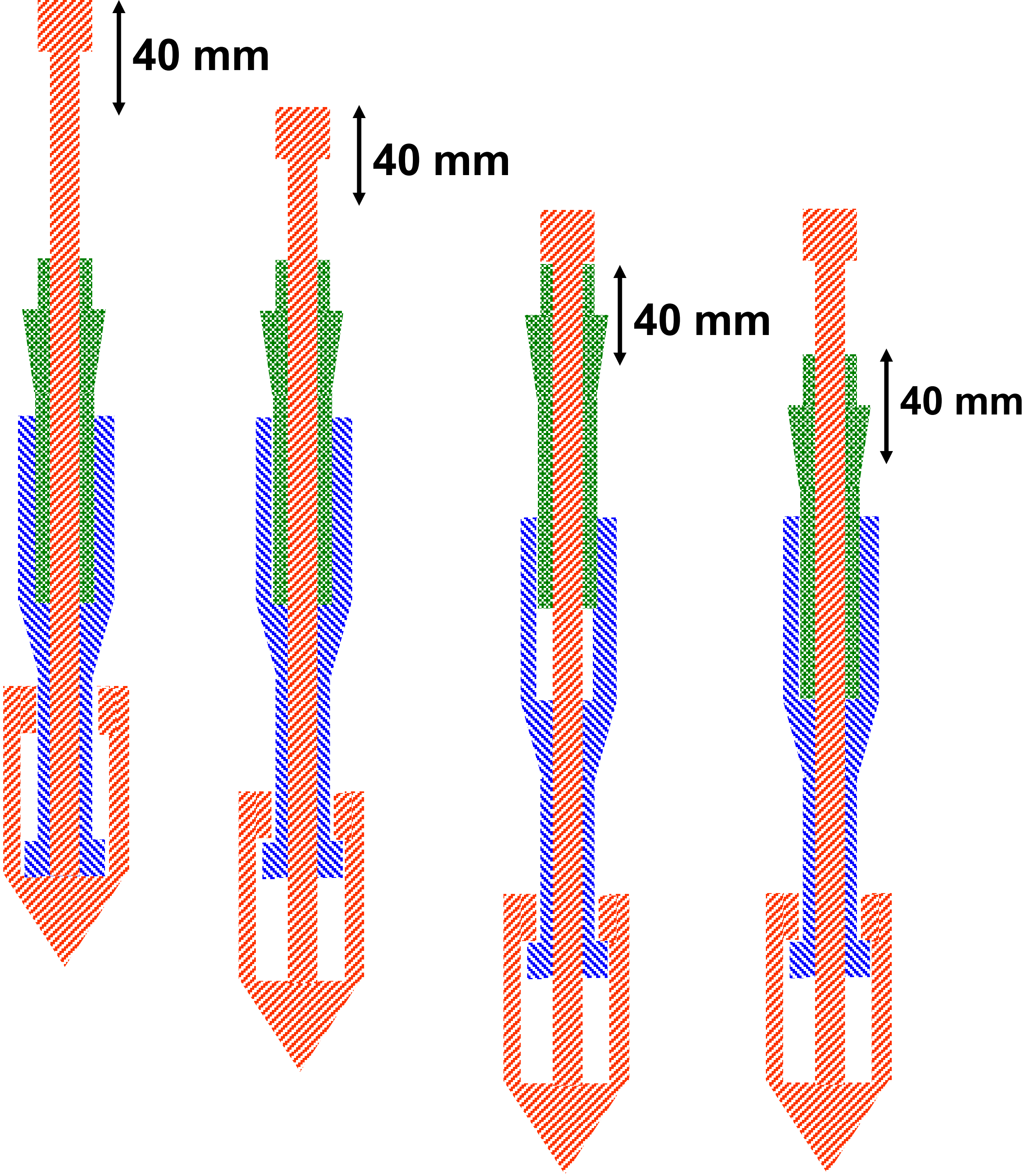




In-situ shear tests

- ☐ **Vane shear test**
- ☐ **Torvane**
- ☐ **Pocket Penetrometer**
- ☐ **Pressuremeter**
- ☐ **Static Cone Penetrometer test (Push Cone Penetrometer Test, PCPT)**
  (suitable for all soil types except very course granular materials)
- ☐ **Standard Penetration Test, SPT**

Static Cone Penetrometer test



Cone penetrometers with pore water pressure measurement capability are known as piezocones

Static Cone Penetrometer test

Force required for the inner rod to push the tip (F_c) and the total force required to push both the tip and the sleeve (F_c + F_s) will be measured

Point resistance (q_c) = F_c/ area of the tip

Sleeve resistance (q_s) = F_s/ area of the sleeve in contact with soil

Friction Ratio (f_r) = $q_s / q_c \times 100$ (%)

Various correlations have been developed to determine soil strength parameters (c, ϕ, ect) from f_r

In-situ shear tests

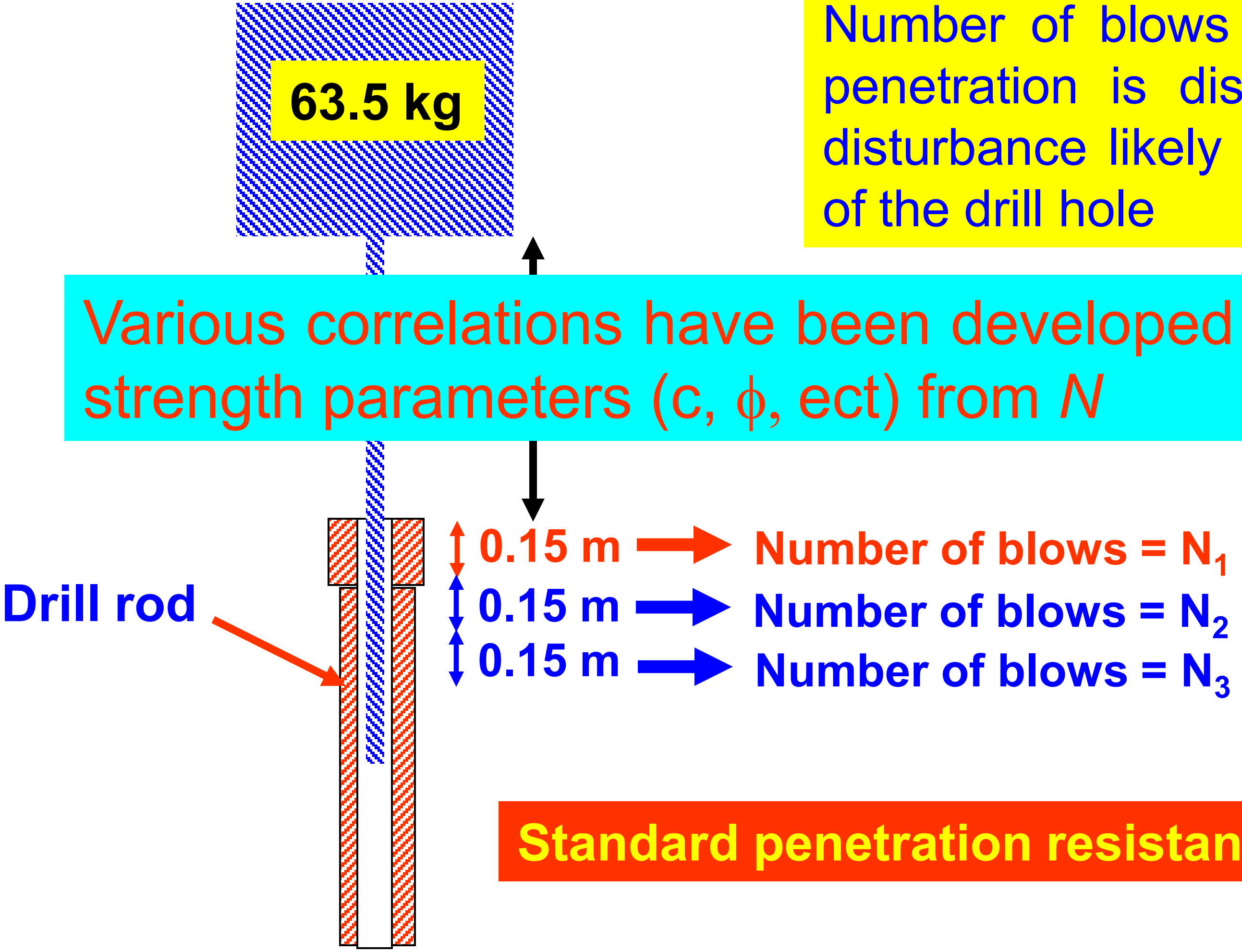
- ☐ Vane shear test
- ☐ Torvane
- ☐ Pocket Penetrometer
- ☐ Pressuremeter
- ☐ Static Cone Penetrometer test (Push Cone Penetrometer Test, PCPT)
- ☐ Standard Penetration Test, SPT (suitable for granular materials)

Standard Penetration Test, SPT

SPT is the most widely used test procedure to determine the properties of in-situ soils

Number of blows for the first 150 mm penetration is disregarded due to the disturbance likely to exist at the bottom of the drill hole

Various correlations have been developed to determine soil strength parameters (c, ϕ, ect) from *N*



Number of blows = N_1

Number of blows = N_2

Number of blows = N_3

Standard penetration resistance (SPT N) = $N_2 + N_3$

Various correlations for shear strength

For NC clays, the undrained shear strength (c_u) increases with the effective overburden pressure, σ'_0

$$\frac{c_u}{\sigma'_0} = 0.11 + 0.0037(PI)$$

Skempton (1957)

Plasticity Index as a %

For OC clays, the following relationship is approximately true

$$\left(\frac{c_u}{\sigma'_0}\right)_{Overconsoldated} \Bigg/ \left(\frac{c_u}{\sigma'_0}\right)_{NormallyConsolidated} = (OCR)^{0.8}$$

Ladd (1977)

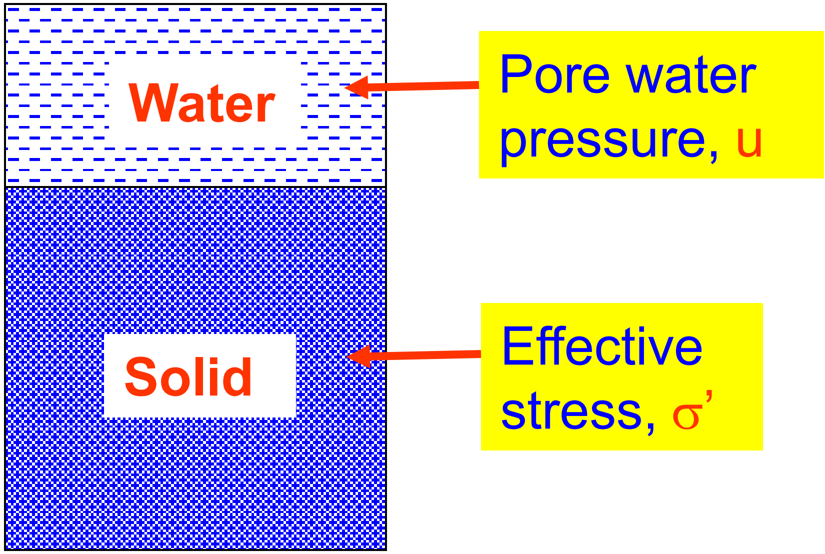
For NC clays, the effective friction angle (ϕ') is related to *PI* as follows

$$Sin\,\phi' = 0.814 - 0.234\log(IP)$$

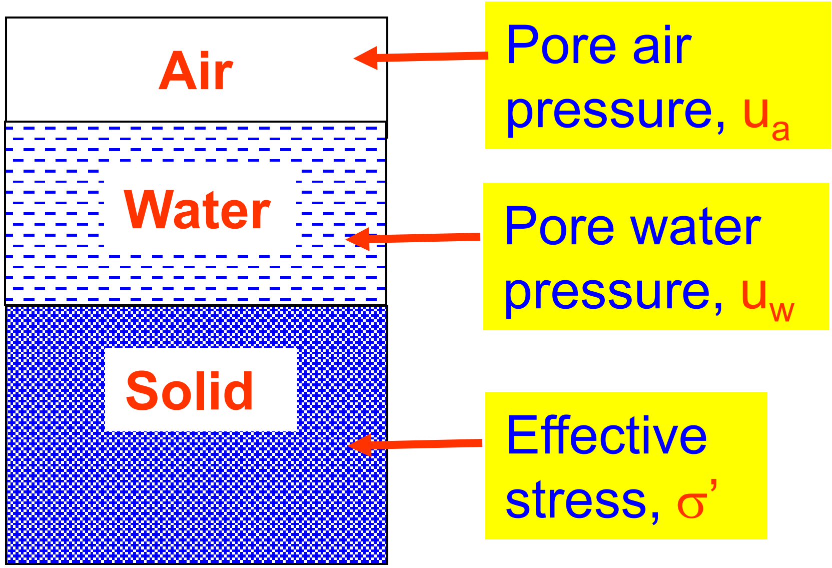
Kenny (1959)

Shear strength of partially saturated soils

In the previous sections, we were discussing the shear strength of saturated soils. However, in most of the cases, we will encounter unsaturated soils in tropical countries like Sri Lanka



Saturated soils



Unsaturated soils

Pore water pressure can be negative in unsaturated soils

Shear strength of partially saturated soils

Bishop (1959) proposed shear strength equation for unsaturated soils as follows

$$\tau_f = c' + \left[(\sigma_n - u_a) + \chi (u_a - u_w) \right] \tan \phi'$$

Where,

$\sigma_n - u_a$ = Net normal stress

$u_a - u_w$ = Matric suction

χ= a parameter depending on the degree of saturation

(χ = 1 for fully saturated soils and 0 for dry soils)

Fredlund et al (1978) modified the above relationship as follows

$$\tau_f = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b$$

Where,

$\tan\phi^b$ = Rate of increase of shear strength with matric suction

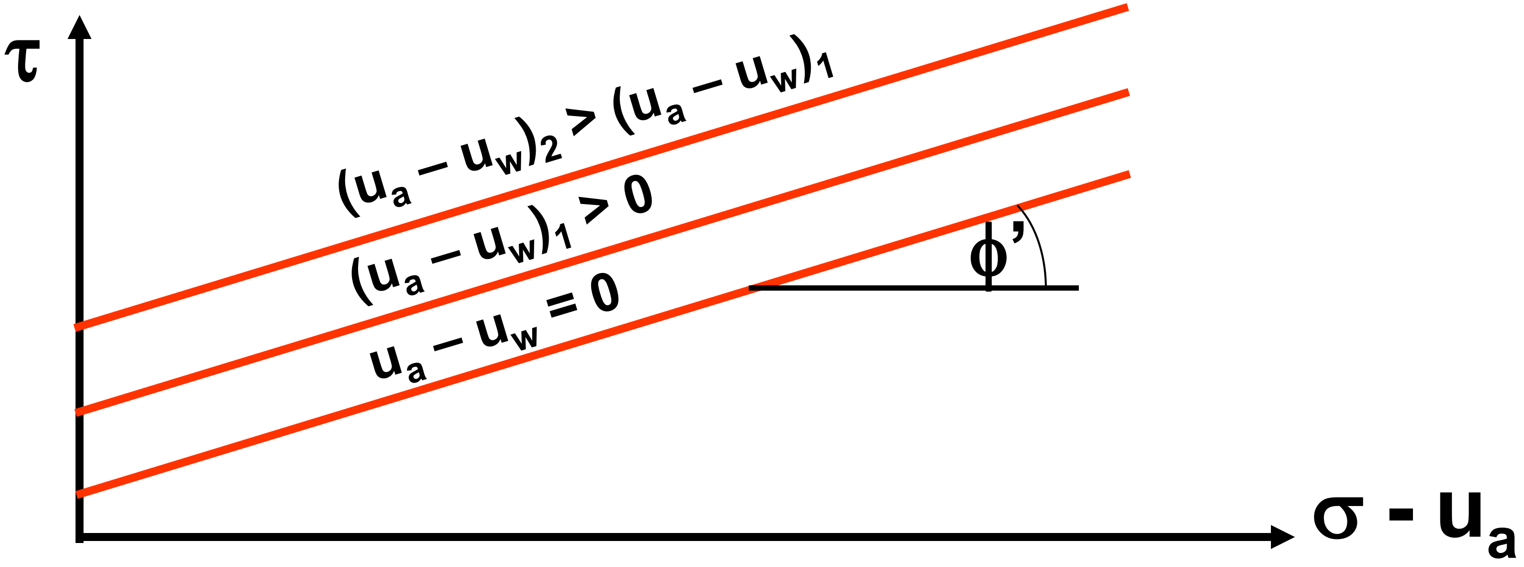
Shear strength of partially saturated soils

$$\tau_f = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b$$

Same as saturated soils

Apparent cohesion due to matric suction

Therefore, strength of unsaturated soils is much higher than the strength of saturated soils due to matric suction



How it become possible build a sand castle

$$\tau_f = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b$$

Same as saturated soils

Apparent cohesion due to matric suction