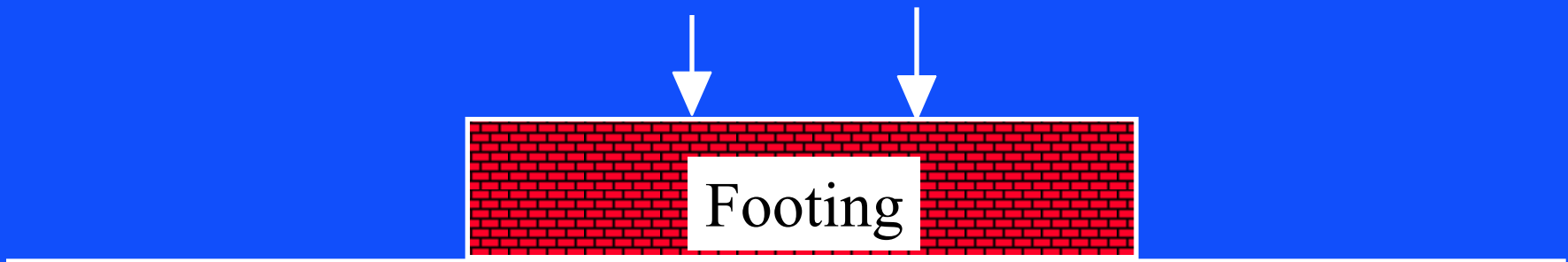
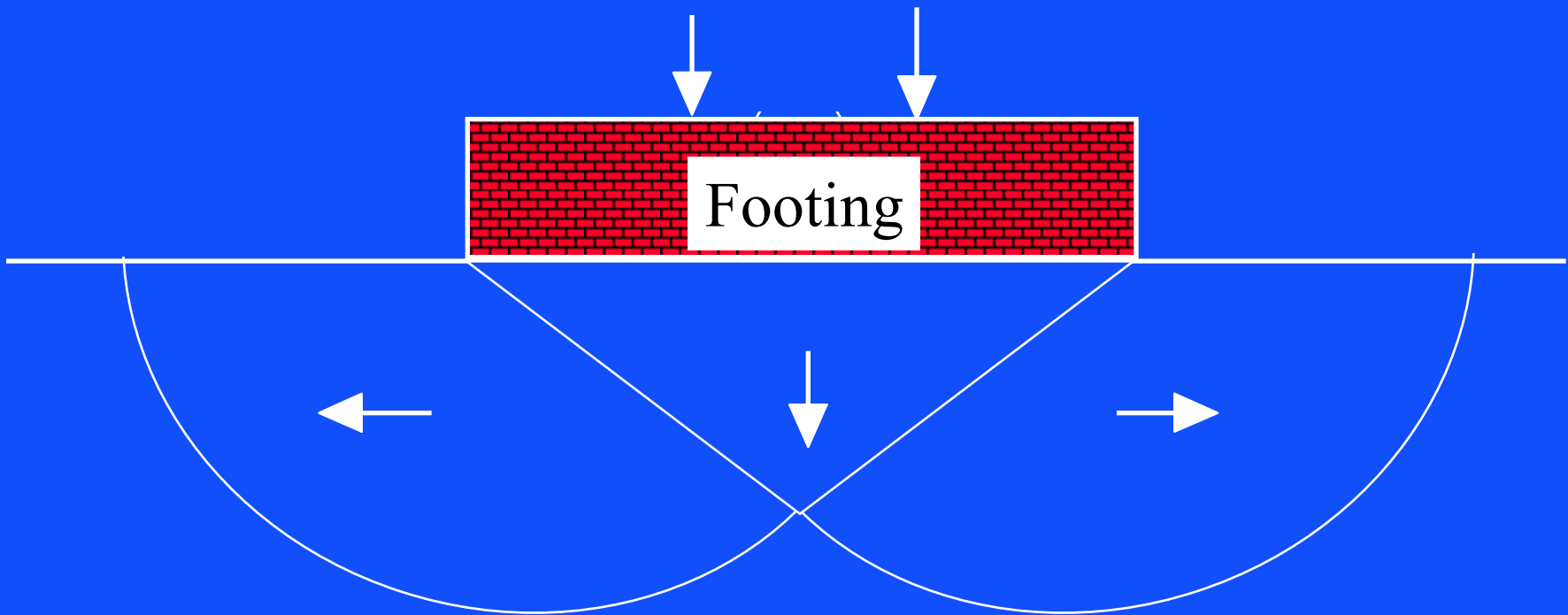


Bearing Capacity

Shallow Foundations

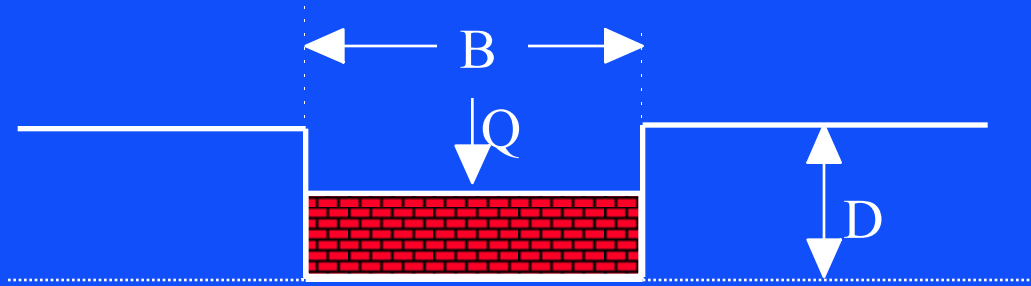


Shallow Foundations



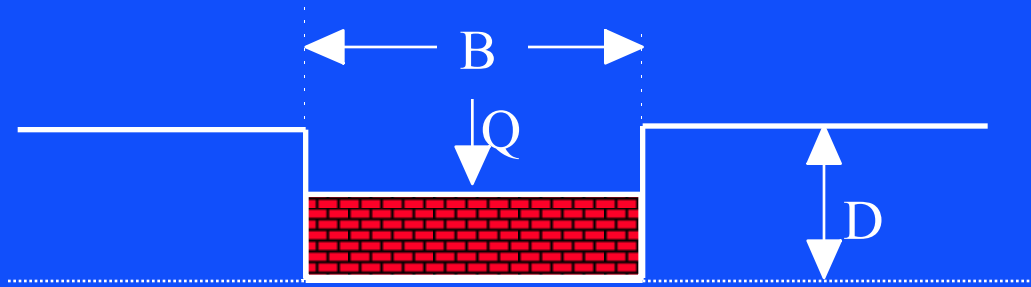
Shallow Foundations

Typical
Buried
Footing

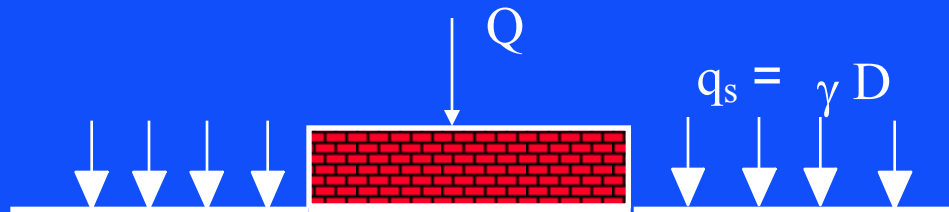


Shallow Foundations

Typical
Buried
Footing

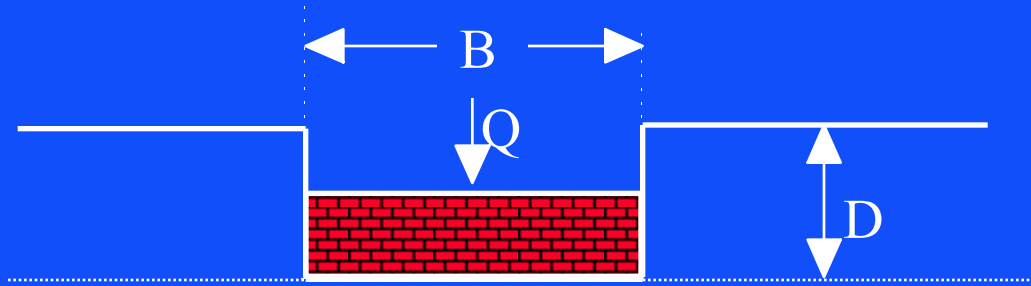


Equivalent
Surface
Footing

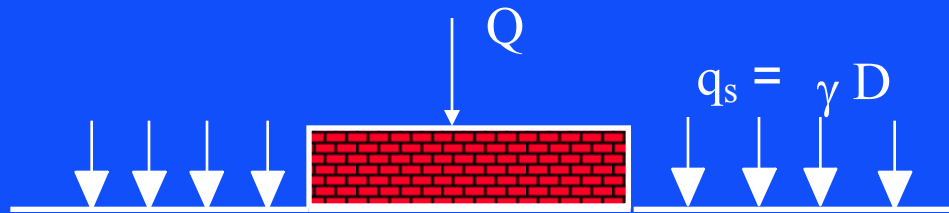


Shallow Foundations

Typical
Buried
Footing



Equivalent
Surface
Footing



Shallow Foundations have $D/B < 1$

Shallow Foundations

Methods of analysis

Lower bound approach •

failure stress state in equilibrium –

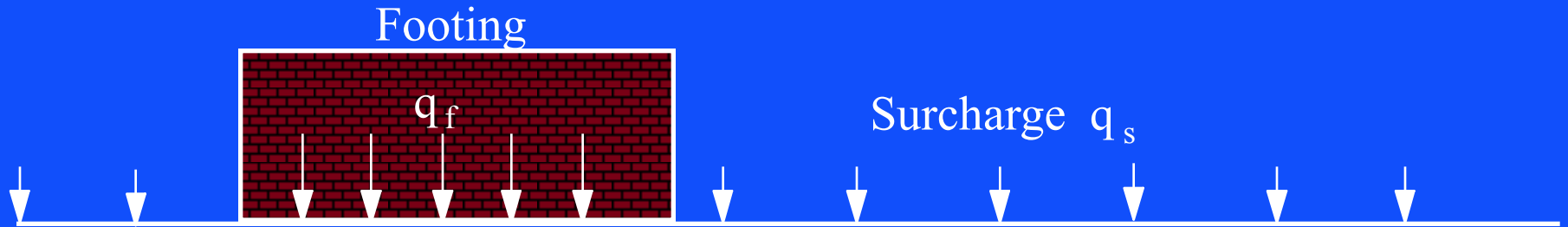
failure load less than or equal to true collapse –

Upper bound approach •

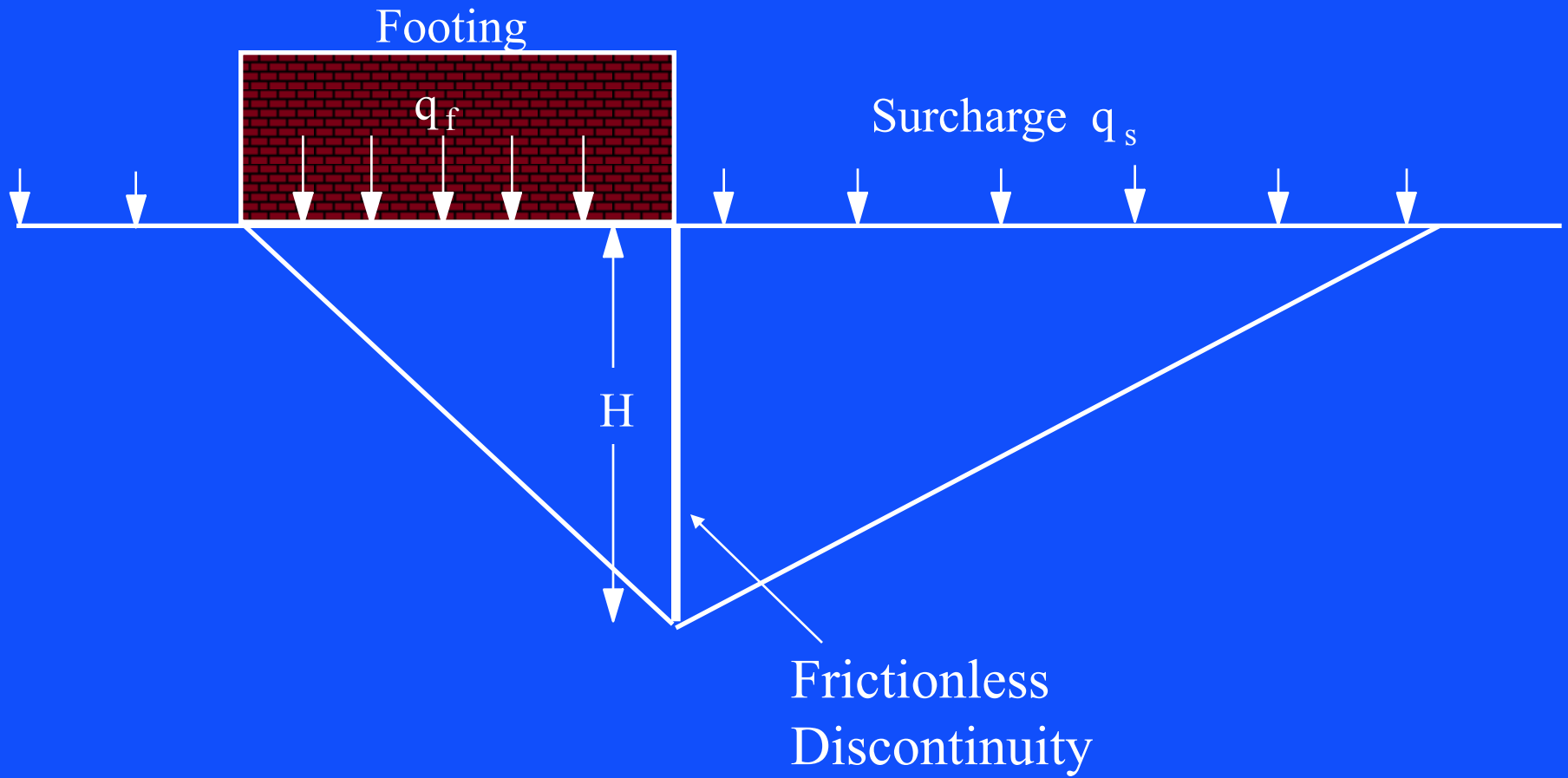
failure mechanism assumed –

failure load greater than or equal to true collapse –

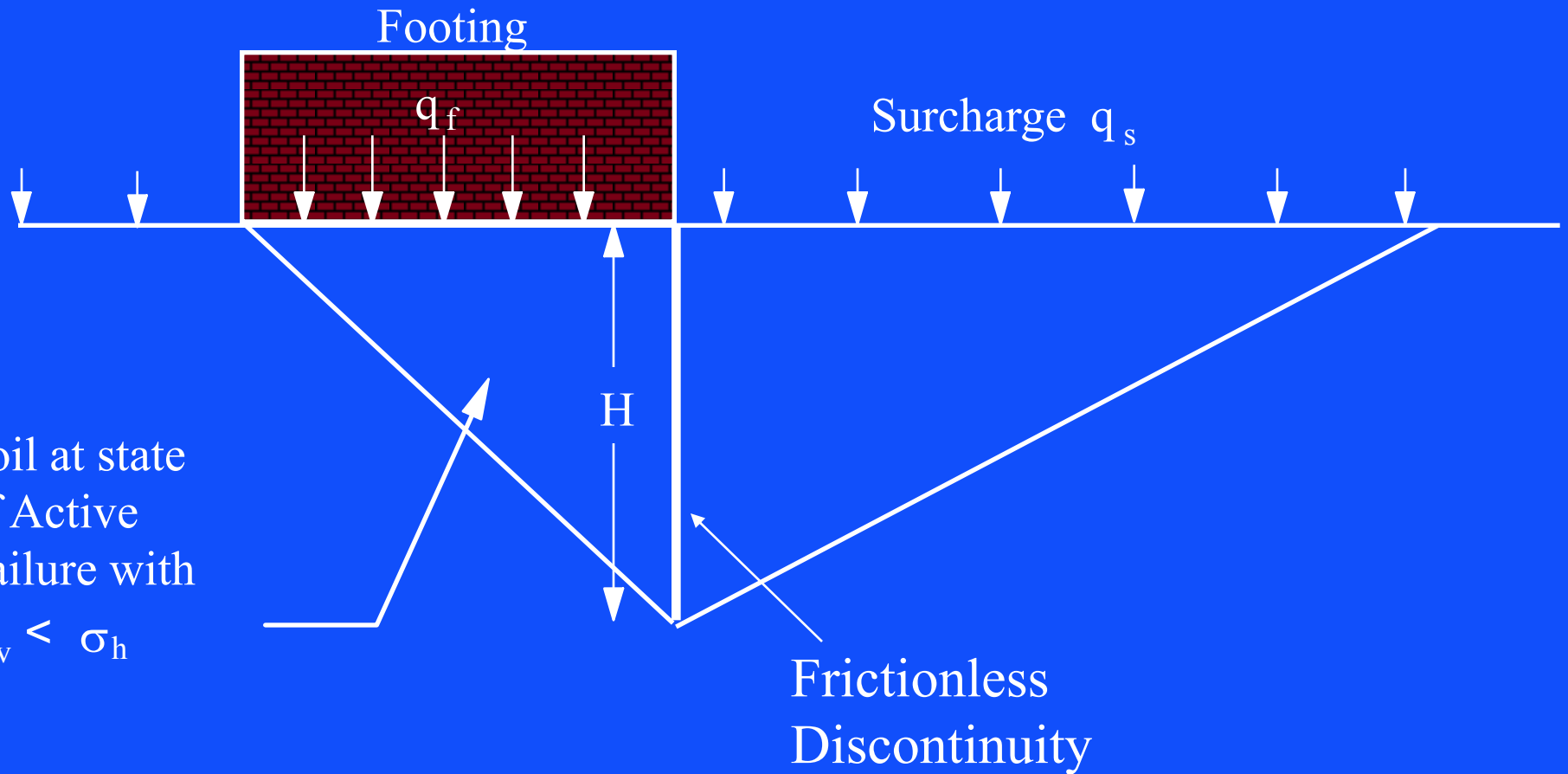
Shallow Foundations



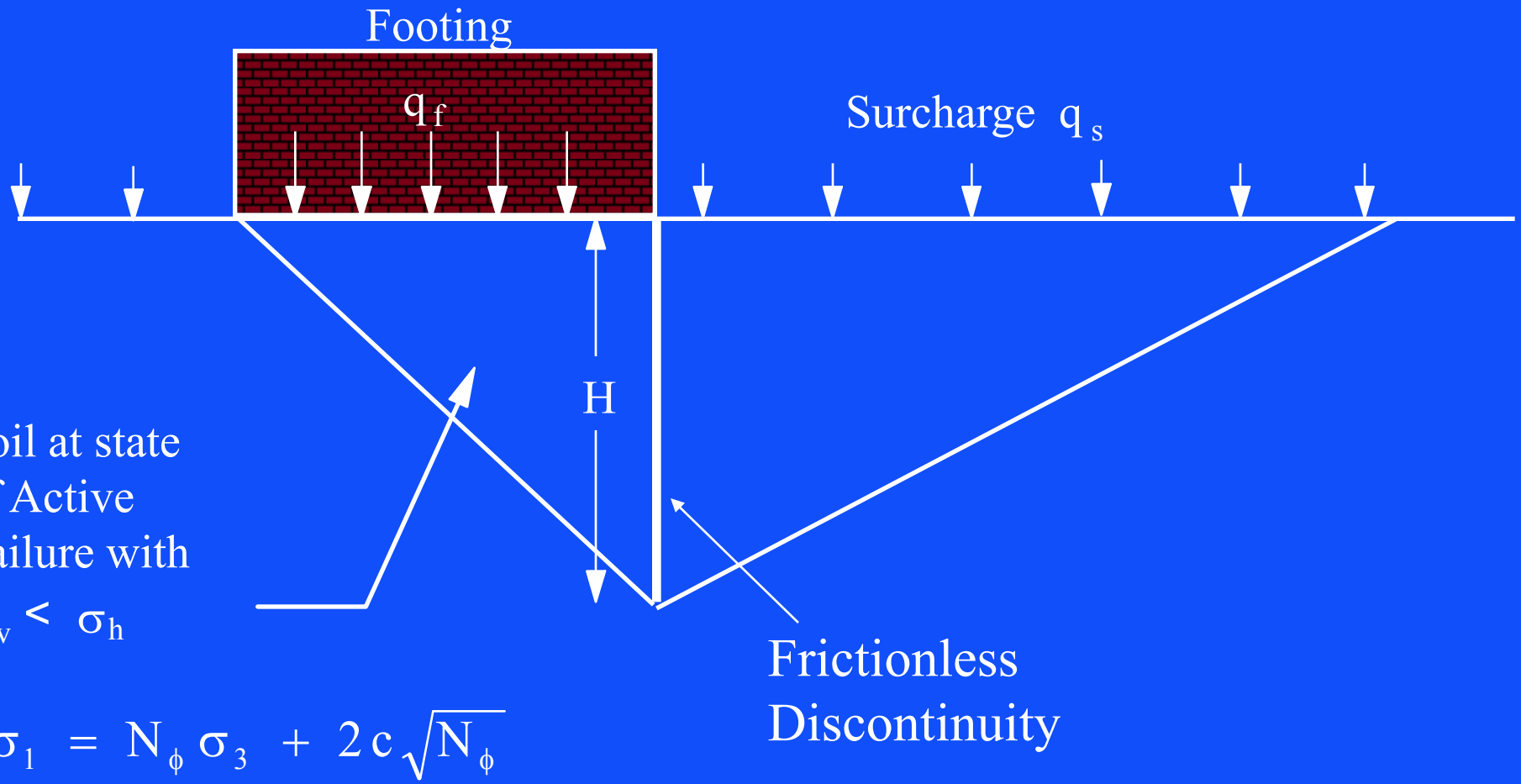
Shallow Foundations



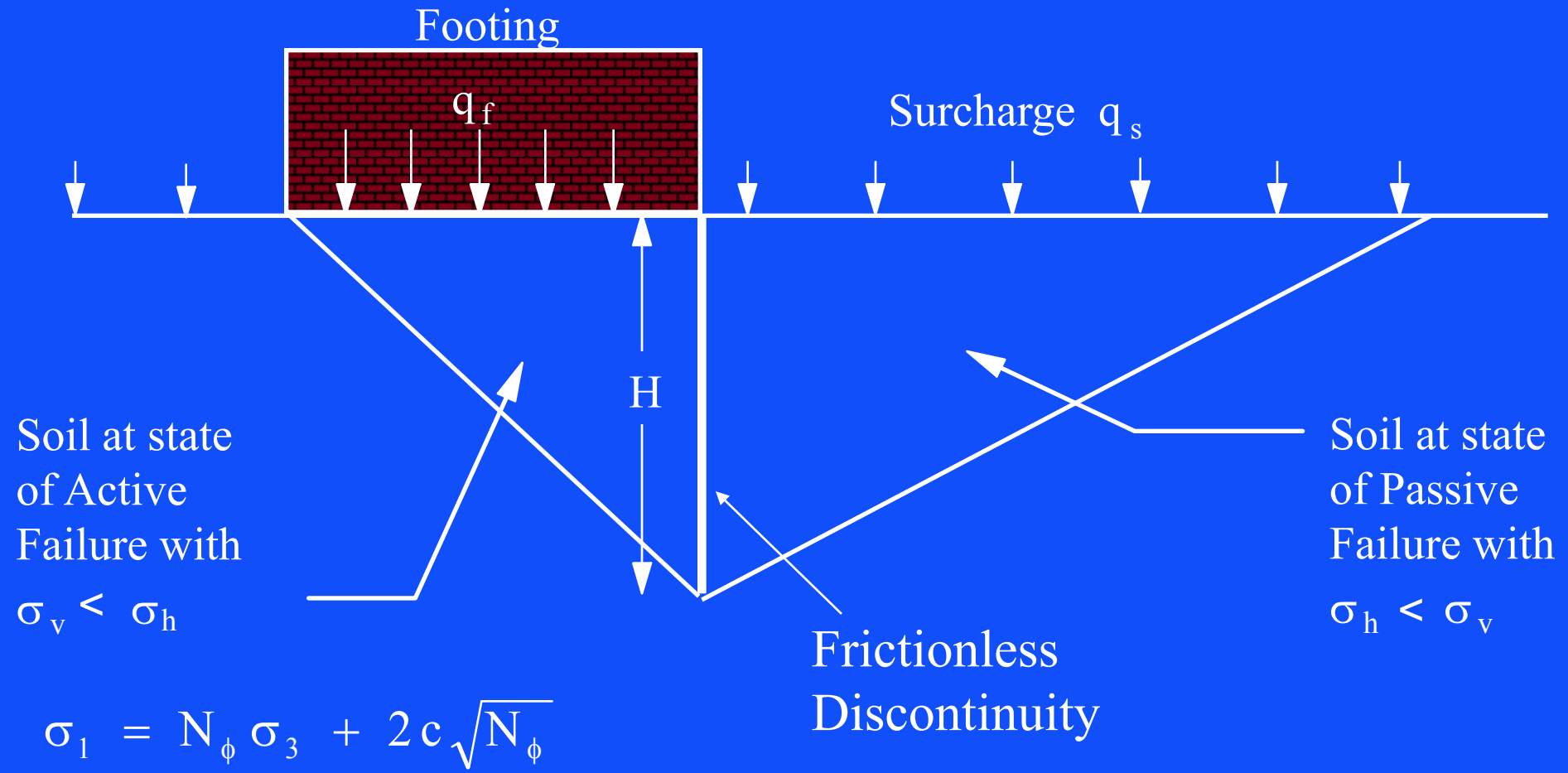
Shallow Foundations



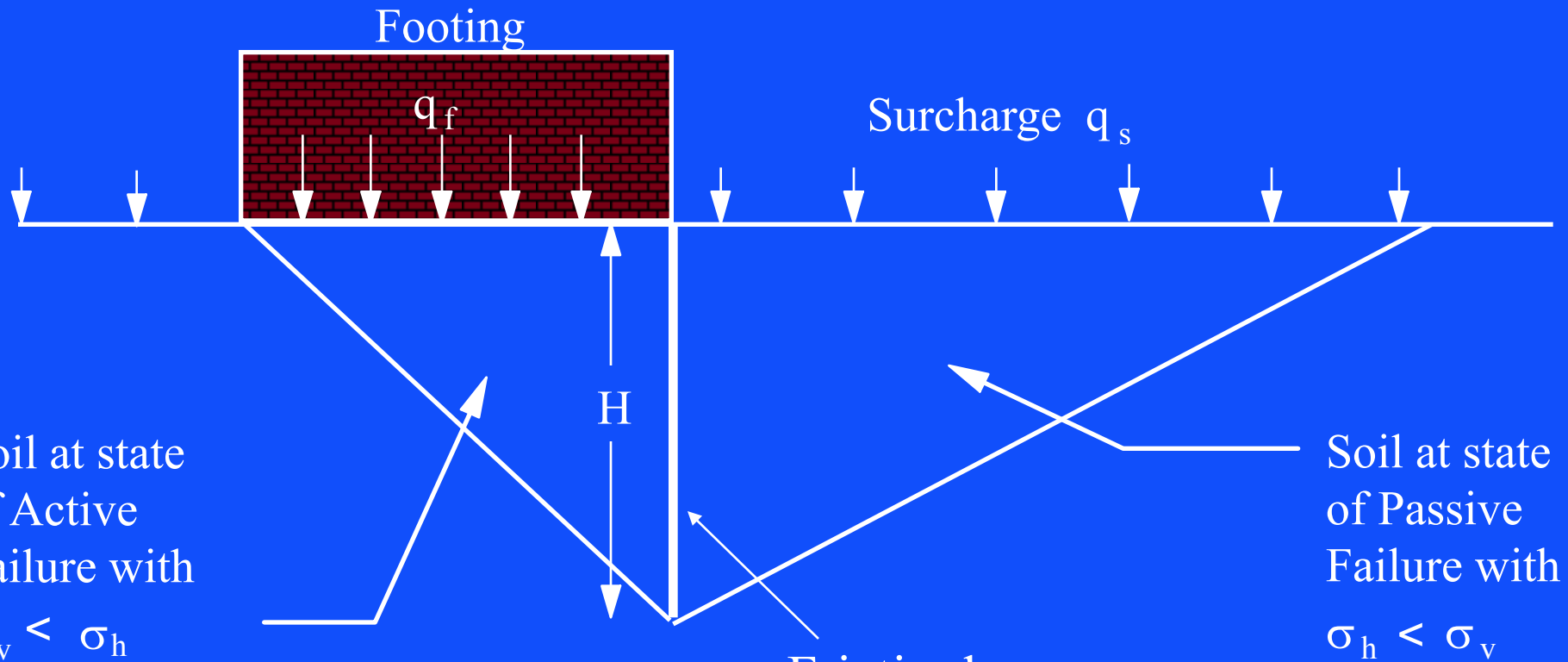
Shallow Foundations



Shallow Foundations



Shallow Foundations



Soil at state of Active Failure with $\sigma_v < \sigma_h$

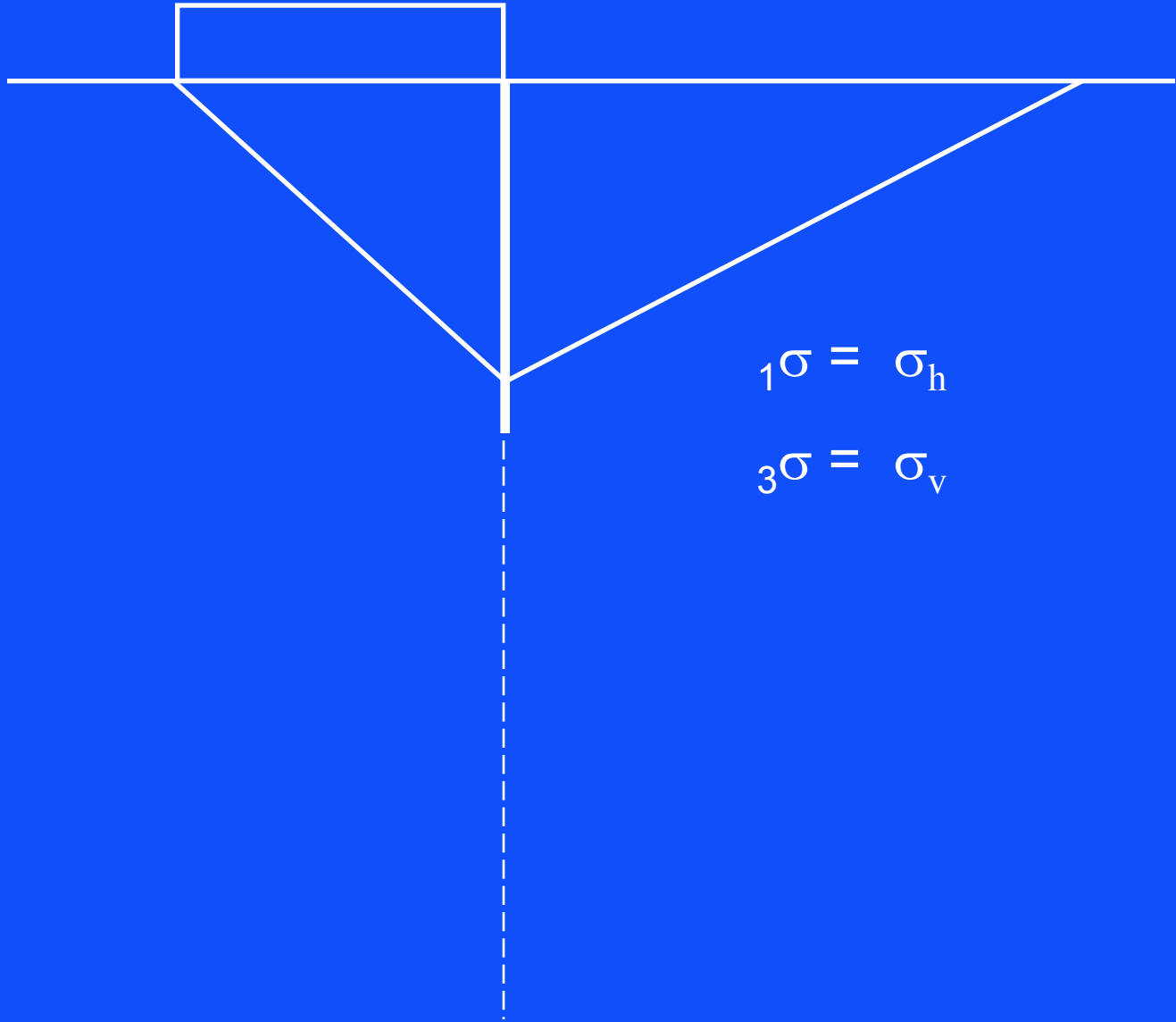
Soil at state of Passive Failure with $\sigma_h < \sigma_v$

Frictionless Discontinuity

$$\sigma_1 = N_\phi \sigma_3 + 2c \sqrt{N_\phi}$$

$$N_\phi = \frac{\sigma_1 + c \cot \phi}{\sigma_3 + c \cot \phi}$$

Shallow Foundations



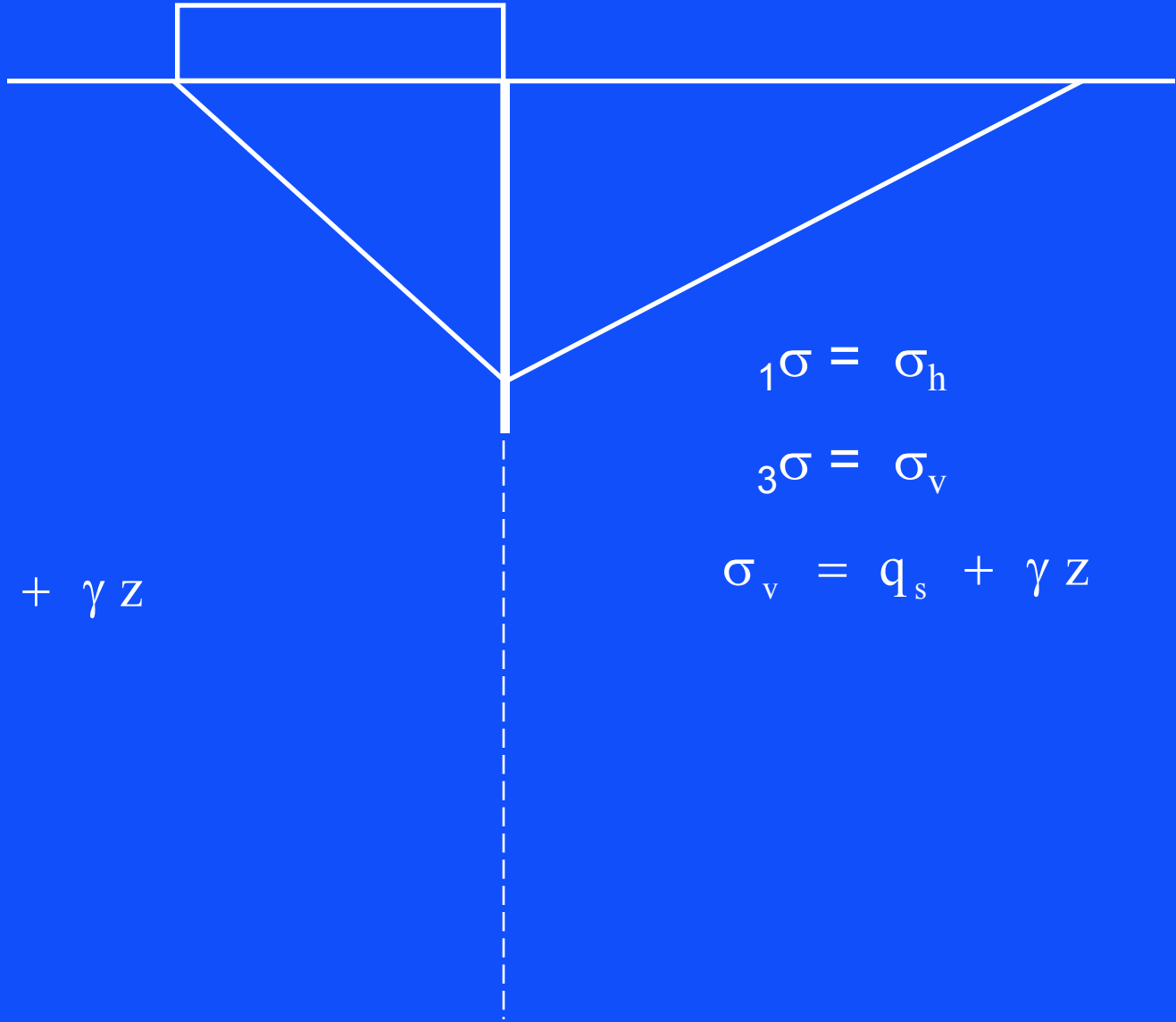
$$1\sigma = \sigma_v$$

$$3\sigma = \sigma_h$$

$$1\sigma = \sigma_h$$

$$3\sigma = \sigma_v$$

Shallow Foundations



$$1\sigma = \sigma_v$$

$$3\sigma = \sigma_h$$

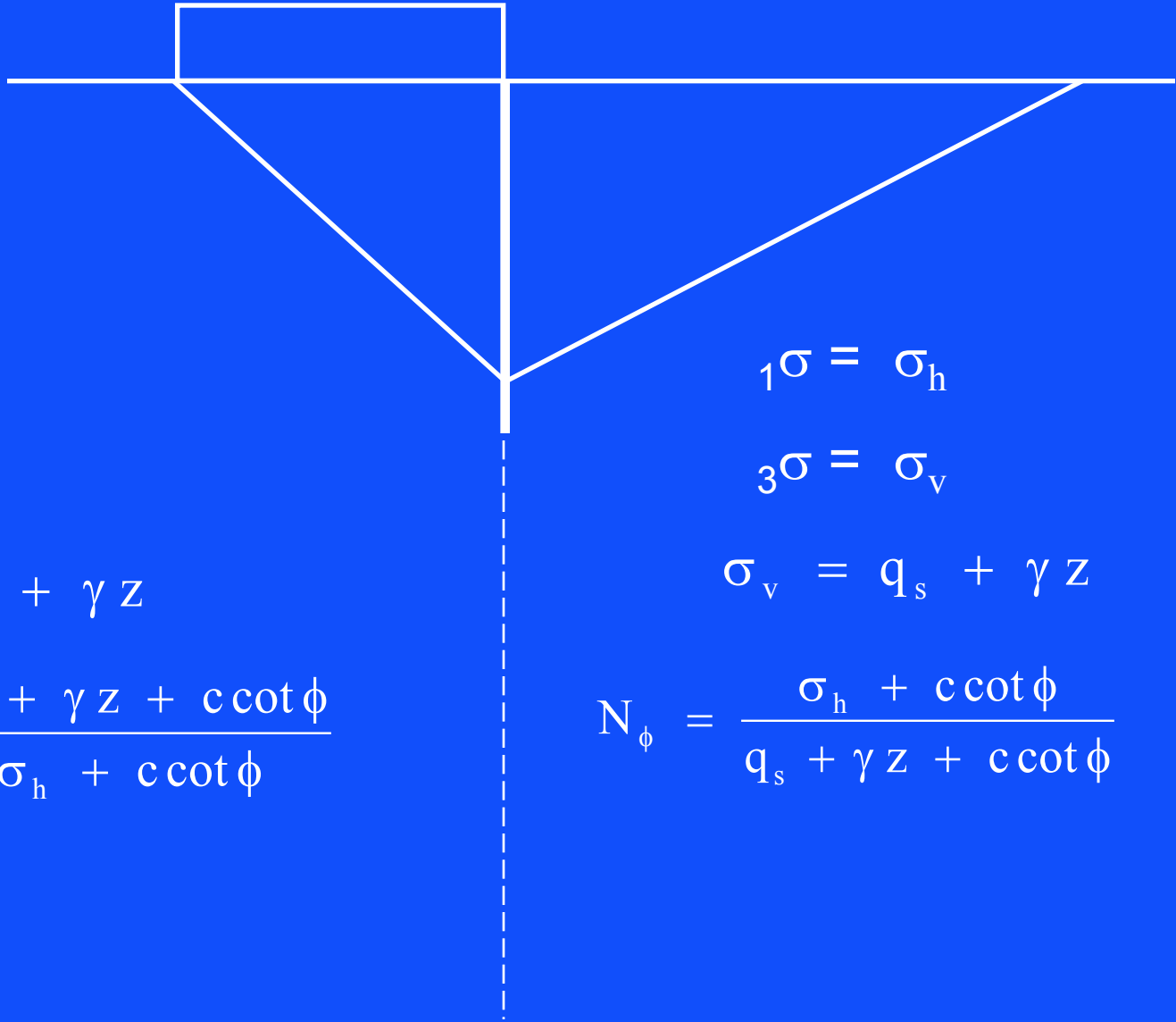
$$\sigma_v = q_f + \gamma z$$

$$1\sigma = \sigma_h$$

$$3\sigma = \sigma_v$$

$$\sigma_v = q_s + \gamma z$$

Shallow Foundations



$$1\sigma = \sigma_v$$

$$3\sigma = \sigma_h$$

$$\sigma_v = q_f + \gamma z$$

$$N_\phi = \frac{q_f + \gamma z + c \cot \phi}{\sigma_h + c \cot \phi}$$

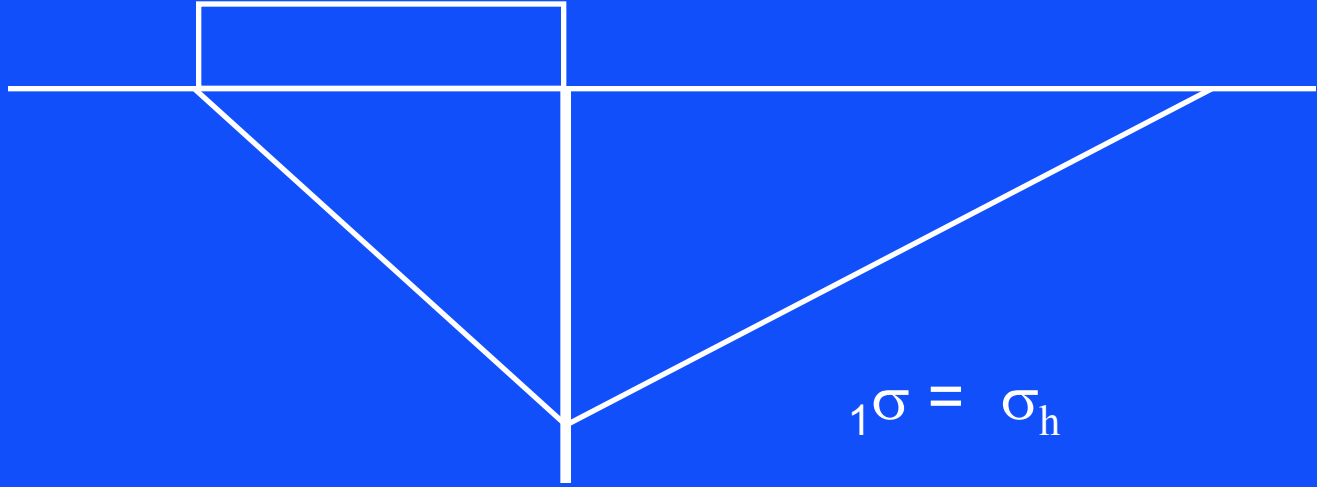
$$1\sigma = \sigma_h$$

$$3\sigma = \sigma_v$$

$$\sigma_v = q_s + \gamma z$$

$$N_\phi = \frac{\sigma_h + c \cot \phi}{q_s + \gamma z + c \cot \phi}$$

Shallow Foundations



$$1\sigma = \sigma_v$$

$$3\sigma = \sigma_h$$

$$\sigma_v = q_f + \gamma z$$

$$N_\phi = \frac{q_f + \gamma z + c \cot \phi}{\sigma_h + c \cot \phi}$$

$$\sigma_h = \frac{1}{N_\phi} (q_f + \gamma z + c \cot \phi) - c \cot \phi$$

$$1\sigma = \sigma_h$$

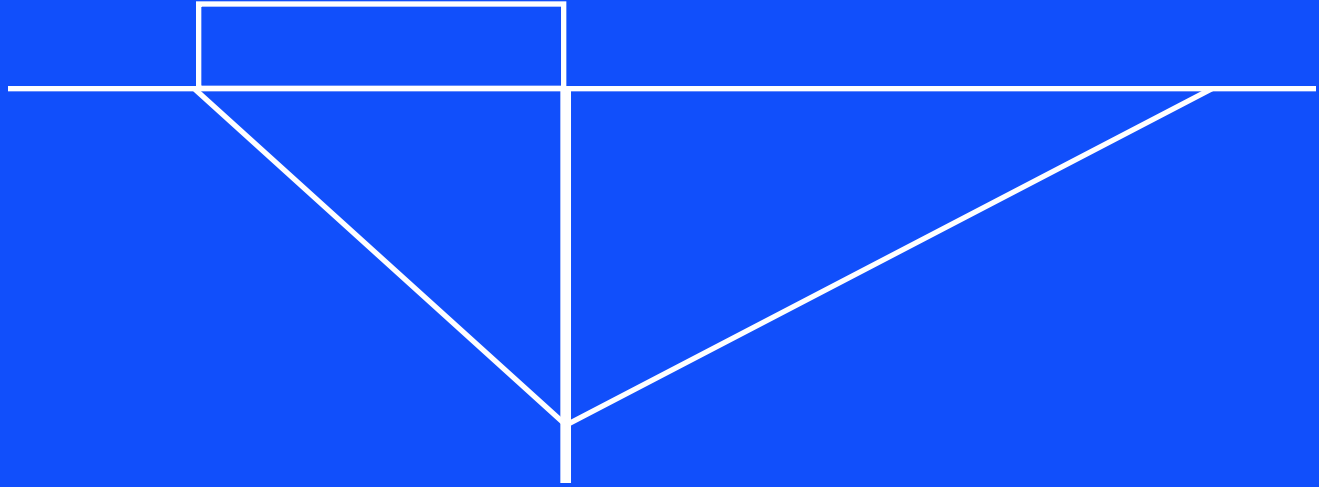
$$3\sigma = \sigma_v$$

$$\sigma_v = q_s + \gamma z$$

$$N_\phi = \frac{\sigma_h + c \cot \phi}{q_s + \gamma z + c \cot \phi}$$

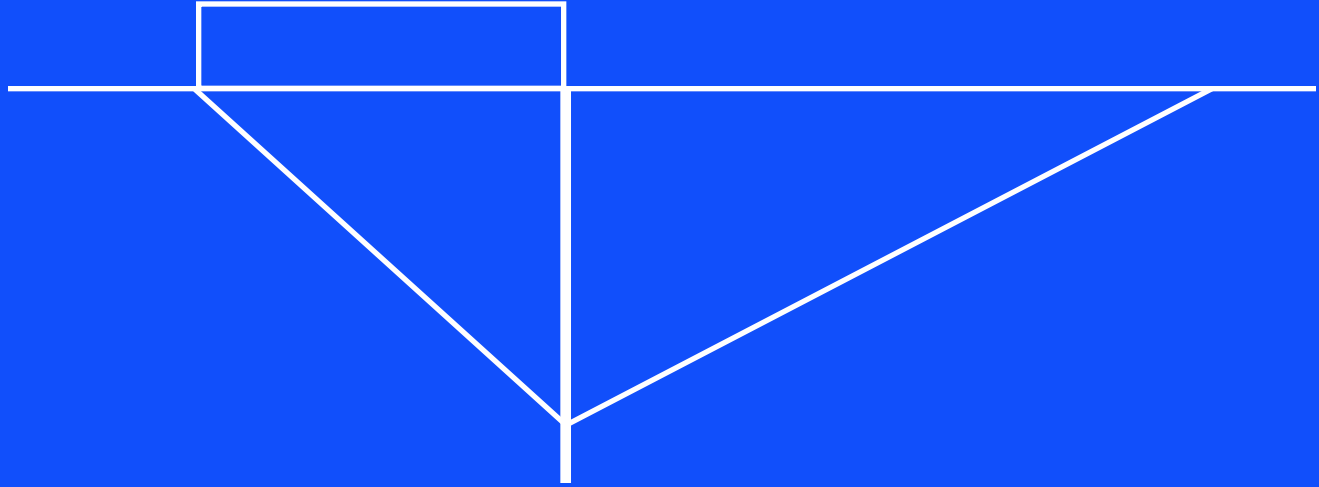
$$\sigma_h = N_\phi (q_s + \gamma z + c \cot \phi) - c \cot \phi$$

Shallow Foundations



$$\int_0^H (\sigma_h)_{\text{active}} dz = \int_0^H (\sigma_h)_{\text{passive}} dz$$

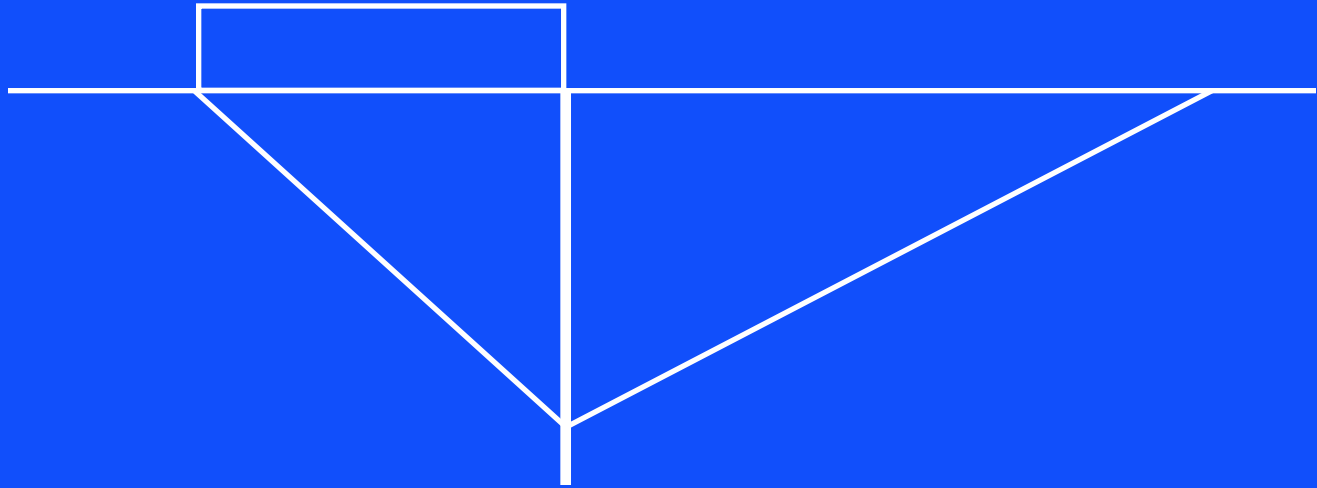
Shallow Foundations



$$\int_0^H (\sigma_h)_{\text{active}} dz = \int_0^H (\sigma_h)_{\text{passive}} dz$$

$$\frac{1}{N_\phi} \left[q_f H + \frac{\gamma H^2}{2} + c \cot \phi H \right] = N_\phi \left[q_s H + \frac{\gamma H^2}{2} + c \cot \phi H \right]$$

Shallow Foundations



$$\int_0^H (\sigma_h)_{\text{active}} dz = \int_0^H (\sigma_h)_{\text{passive}} dz$$

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$$q_f = q_s N_\phi^2 + \frac{\gamma H}{2} (N_\phi^2 - 1) + c \cot \phi (N_\phi^2 - 1)$$

Shallow Foundations

$$q_f = q_s N_\phi^2 + \frac{\gamma H}{2} (N_\phi^2 - 1) + c \cot \phi (N_\phi^2 - 1)$$

This solution will give a lower bound to the true solution •
because of the simplified stress distribution assumed in the
soil

Similar terms occur in all bearing capacity expressions. •
They are functions of the friction angle and

the surcharge applied to the soil surface •

the self weight of the soil •

cohesion •

Shallow Foundations

A general bearing capacity equation can be written •

$$q_f = q_s N_q + \frac{\gamma B}{2} N_\gamma + c N_c$$

Shallow Foundations

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The terms N_q , N_γ and N_c are known as the bearing capacity •
factors

Shallow Foundations

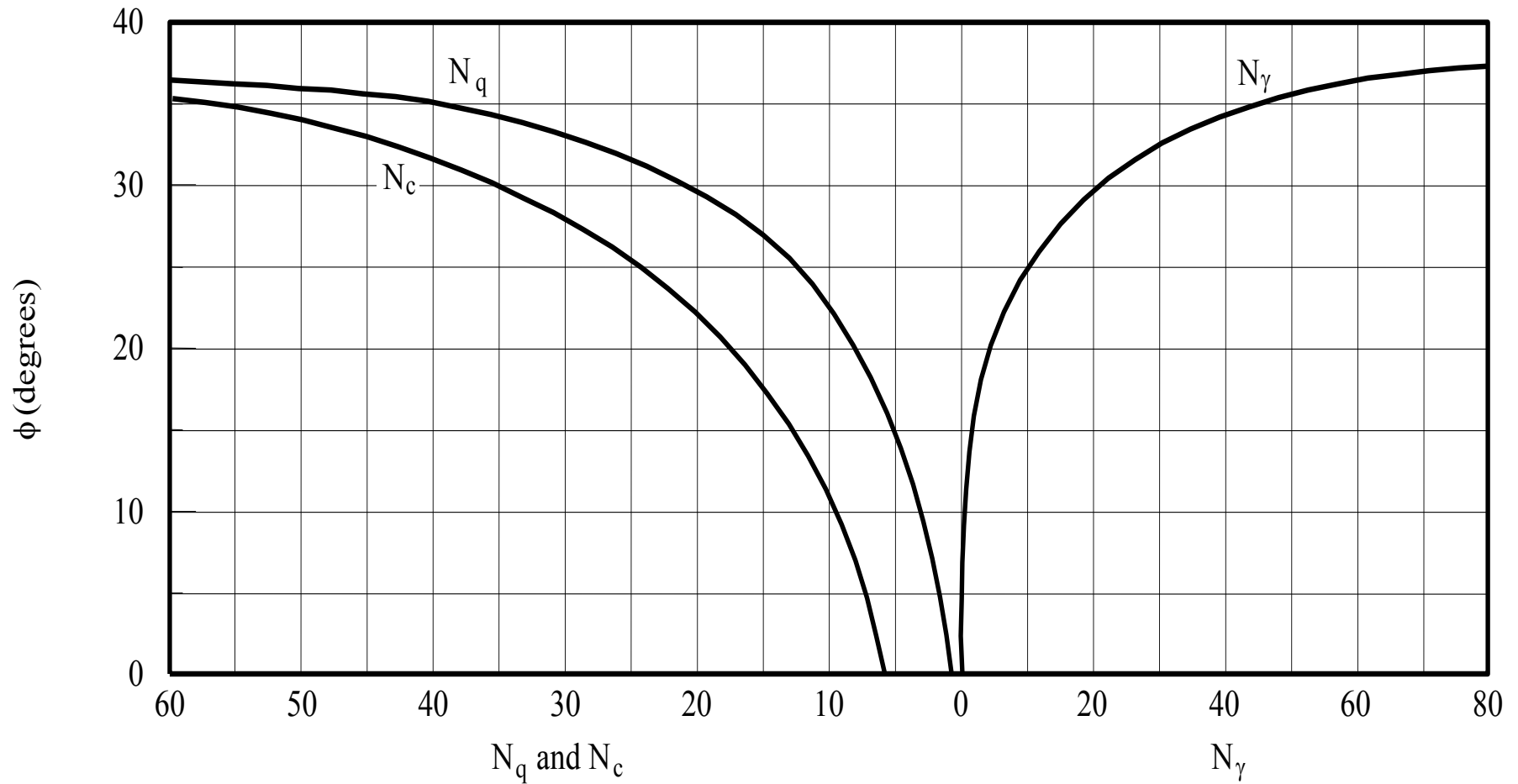
A general bearing capacity equation can be written •

$$q_f = q_s N_q + \frac{\gamma B}{2} N_\gamma + c N_c$$

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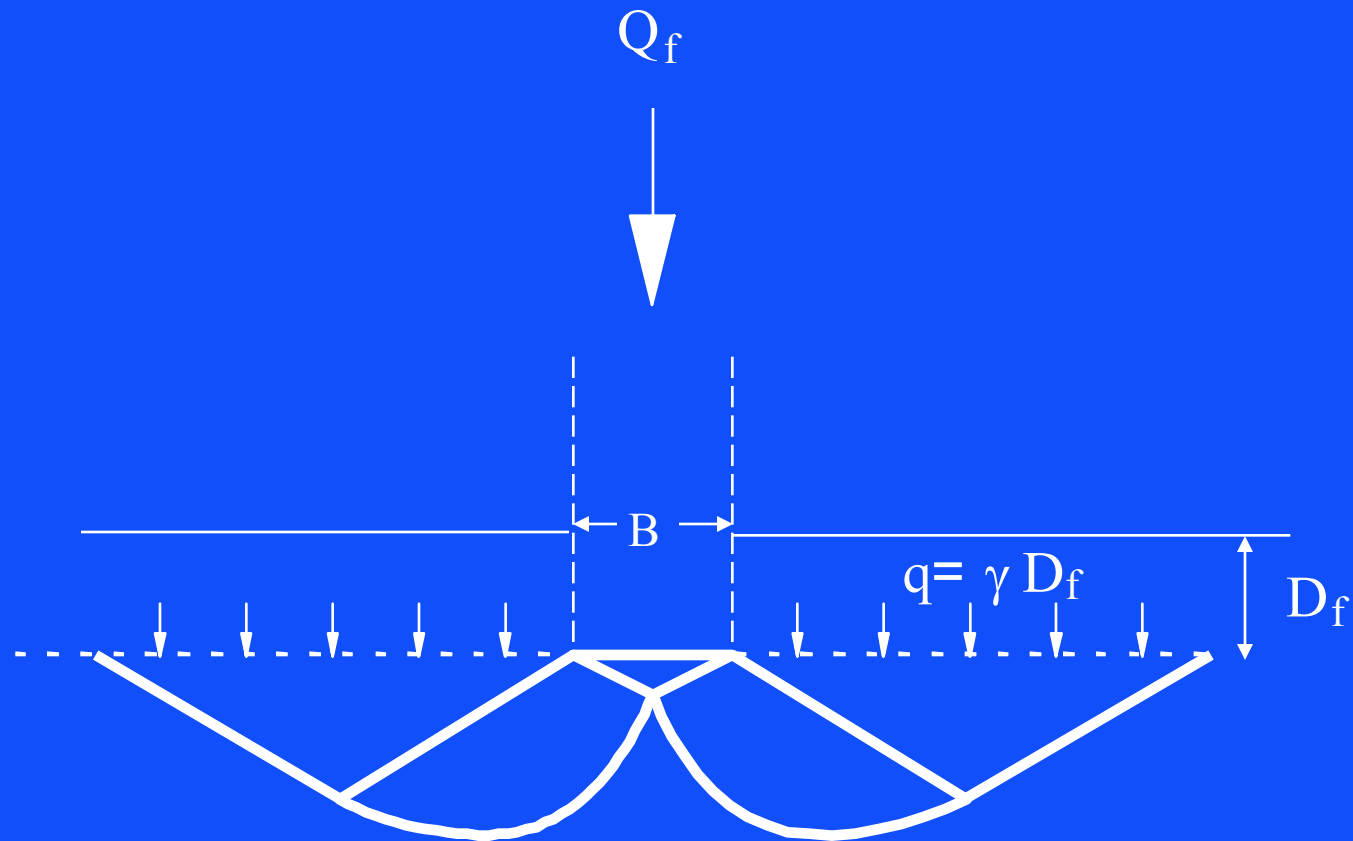
Values can be determined from charts •

Shallow Foundations



BEARING CAPACITY FACTORS [After Terzaghi and Peck (1948)]

Shallow Foundations



Mechanism analysed by Terzaghi

Effect of Foundation Shape

Continuous strip footing

$$q_f = q_s N_q + \frac{\gamma B}{2} N_\gamma + c N_c$$

Effect of Foundation Shape

Continuous strip footing

$$q_f = q_s N_q + \frac{\gamma B}{2} N_\gamma + c N_c$$

Square footing

$$q_f = q_s N_q + 0.4 \gamma B N_\gamma + 1.3 c N_c$$

Effect of Foundation Shape

Continuous strip footing

$$q_f = q_s N_q + \frac{\gamma B}{2} N_\gamma + c N_c$$

Square footing

$$q_f = q_s N_q + 0.4 \gamma B N_\gamma + 1.3 c N_c$$

Circular footing

$$q_f = q_s N_q + 0.6 \gamma B N_\gamma + 1.3 c N_c$$

Effective Stress Analysis

Effective stress analysis is needed to assess the long term foundation capacity. •

Total and effective stresses are identical if the soil is dry. •

The analysis is identical to that described above except that γ_{dry} rather than γ_{sat} , the parameters used in the equations are c' , ϕ rather than c_u , ϕ_u .

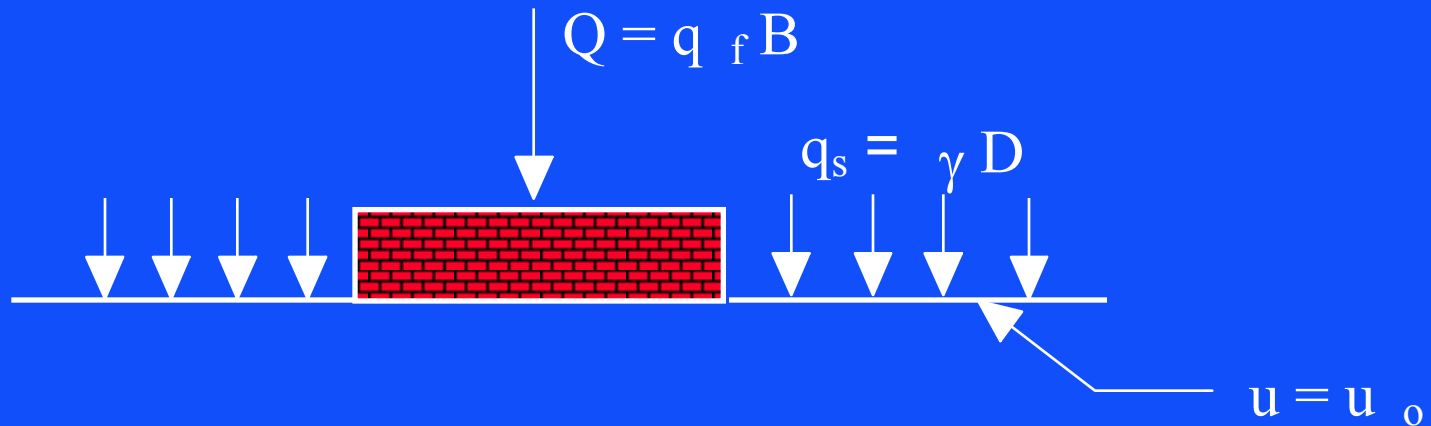
If the water table is more than a depth of $1.5 B$ (the footing width) below the base of the footing the water can be assumed to have no effect. •

Effective Stress Analysis

If the soil below the base of the footing is saturated, the analysis must account for the water pressures. •

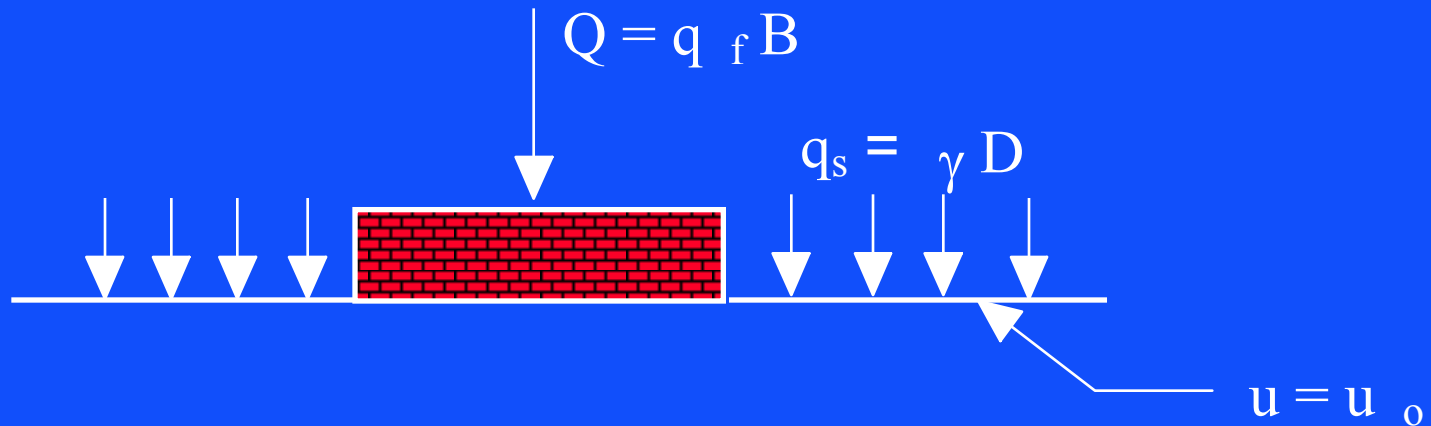
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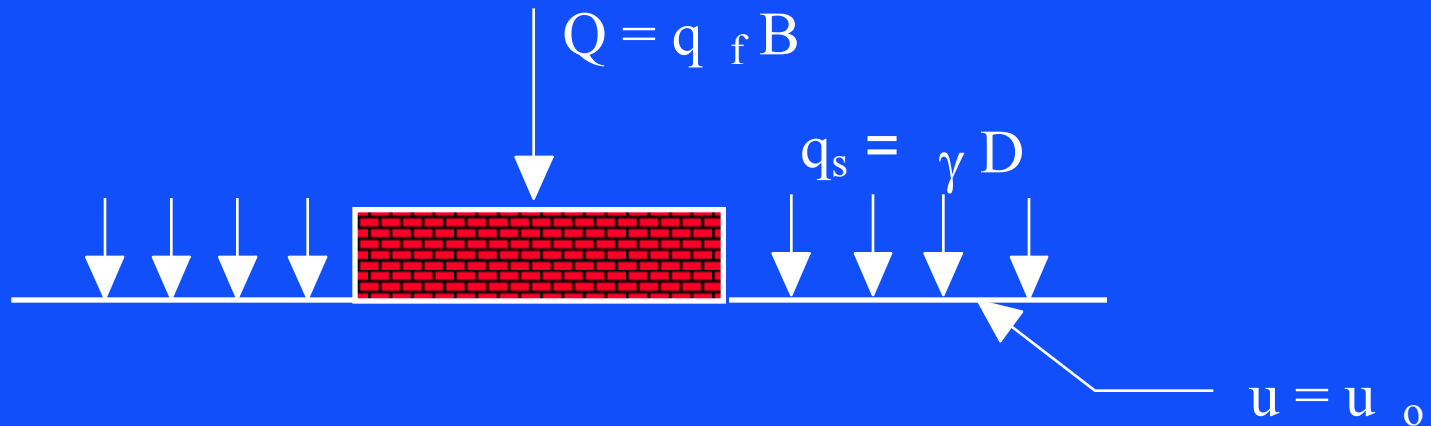


The effective bearing capacity

$$q'_f = q_f - u_o$$

Effective Stress Analysis

If the soil below the base of the footing is saturated, the analysis must account for the water pressures. •



The effective bearing capacity

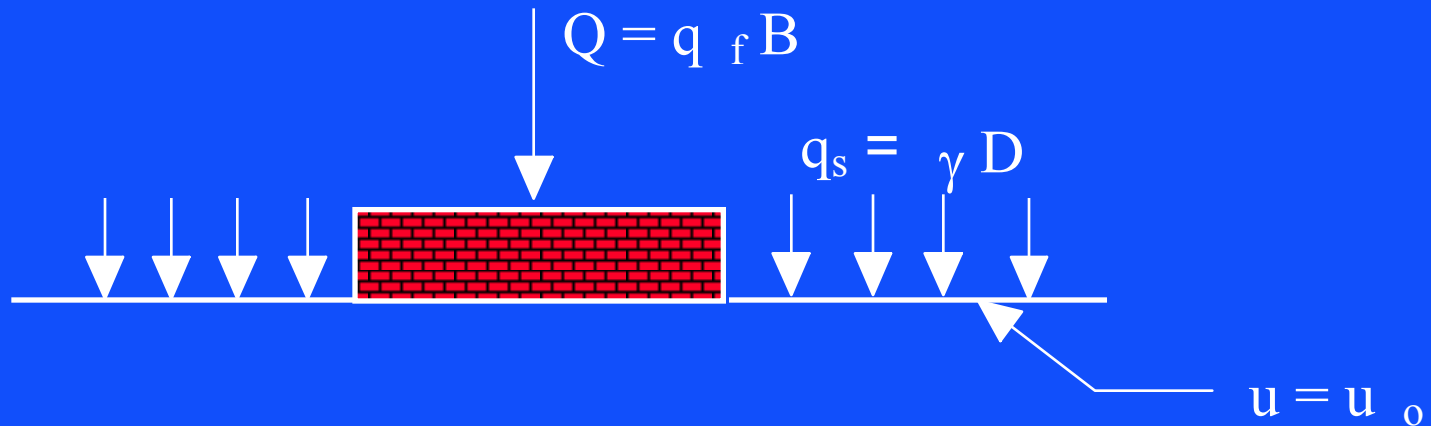
$$q'_f = q_f - u_o$$

The effective surcharge

$$q'_s = q_s - u_o$$

Effective Stress Analysis

If the soil below the base of the footing is saturated, the analysis must account for the water pressures.



The effective bearing capacity

$$q'_f = q_f - u_o$$

The effective surcharge

$$q'_s = q_s - u_o$$

The effective (submerged) unit weight

$$\gamma' = \gamma_{\text{sat}} - \gamma_w$$

Effective Stress Analysis

These effective quantities are required because Mohr Coulomb failure criterion must be expressed in terms of effective stress

$$N_{\phi} = \frac{\sigma'_1 + c' \cot \phi'}{\sigma'_3 + c' \cot \phi'}$$

Effective Stress Analysis

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$$N_{\phi} = \frac{\sigma'_1 + c' \cot \phi'}{\sigma'_3 + c' \cot \phi'}$$

The total vertical stress, pore pressure and effective vertical stress at any depth z beneath the footing are

$$\sigma_v = q_f + \gamma z$$

Effective Stress Analysis

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$$N_{\phi} = \frac{\sigma'_1 + c' \cot \phi'}{\sigma'_3 + c' \cot \phi'}$$

The total vertical stress, pore pressure and effective vertical stress at any depth z beneath the footing are

$$\sigma_v = q_f + \gamma z$$

$$u = u_o + \gamma_w z$$

Effective Stress Analysis

These effective quantities are required because Mohr Coulomb failure criterion must be expressed in terms of effective stress

$$N_{\phi} = \frac{\sigma'_1 + c' \cot \phi'}{\sigma'_3 + c' \cot \phi'}$$

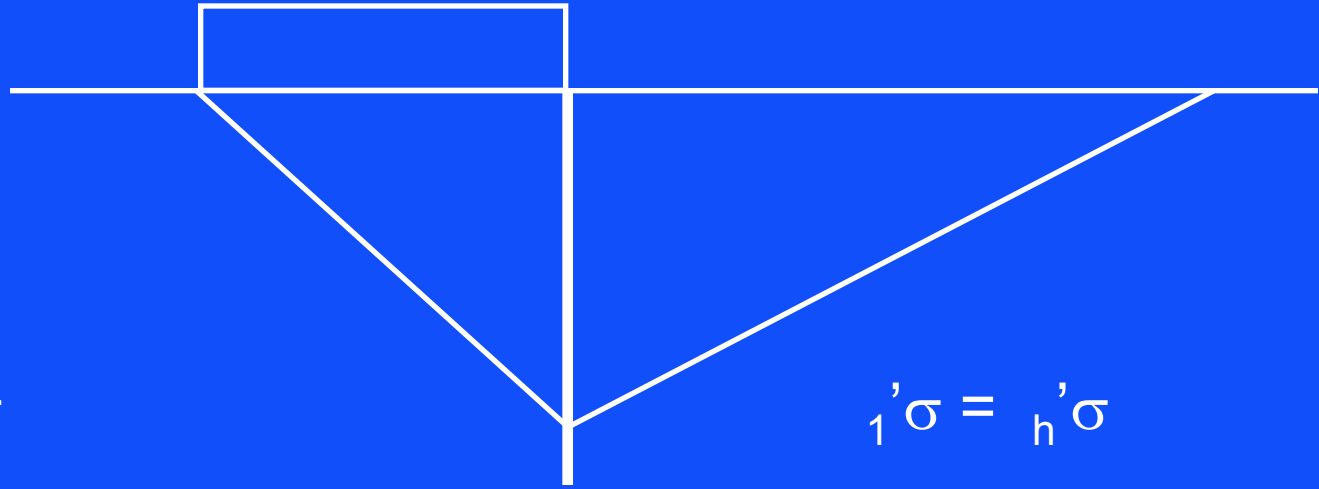
The total vertical stress, pore pressure and effective vertical stress at any depth z beneath the footing are

$$\sigma_v = q_f + \gamma z$$

$$u = u_o + \gamma_w z$$

$$\sigma'_v = \sigma_v - u = q'_f + \gamma' z$$

Effective Stress Analysis



$${}_1'\sigma = {}_v'\sigma$$

$${}_3'\sigma = {}_h'\sigma$$

$$\sigma'_v = q'_f + \gamma'z$$

$$N_\phi = \frac{q'_f + \gamma'z + c' \cot \phi'}{\sigma'_h + c' \cot \phi'}$$

$$\sigma'_h = \frac{1}{N_\phi} (q'_f + \gamma'z + c' \cot \phi') - c' \cot \phi'$$

$${}_1'\sigma = {}_h'\sigma$$

$${}_3'\sigma = {}_v'\sigma$$

$$\sigma'_v = q'_s + \gamma'z$$

$$N_\phi = \frac{\sigma'_h + c' \cot \phi'}{q'_s + \gamma'z + c' \cot \phi'}$$

$$\sigma'_h = N_\phi (q'_s + \gamma'z + c' \cot \phi') - c' \cot \phi'$$

Effective Stress Analysis

The simple analysis leads to

$$q'_f = q'_s N_\phi^2 + \frac{\gamma' H}{2} (N_\phi^2 - 1) + c' \cot \phi' (N_\phi^2 - 1)$$

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This is similar to the previous expression except that now all terms involve effective quantities.

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This is similar to the previous expression except that now all terms involve effective quantities.

As before a general expression can be written with the form

$$q'_f = q'_s N_q + \frac{\gamma' B}{2} N_\gamma + c' N_c$$

Effective Stress Analysis

The simple analysis leads to

$$q'_f = q'_s N_\phi^2 + \frac{\gamma' H}{2} (N_\phi^2 - 1) + c' \cot \phi' (N_\phi^2 - 1)$$

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$$q'_f = q'_s N_q + \frac{\gamma' B}{2} N_\gamma + c' N_c$$

The Bearing Capacity Factors are identical to those from Total Stress Analysis

Effective Stress Analysis

The simple analysis leads to

$$q'_f = q'_s N_\phi^2 + \frac{\gamma' H}{2} (N_\phi^2 - 1) + c' \cot \phi' (N_\phi^2 - 1)$$

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As before a general expression can be written with the form

$$q'_f = q'_s N_q + \frac{\gamma' B}{2} N_\gamma + c' N_c$$

The Bearing Capacity Factors are identical to those from Total Stress Analysis

Note that the Total Bearing Capacity $q_f = q'_f + u_o$

Effective Stress Analysis

Analysis has so far considered

- soil strength parameters
- rate of loading (drained or undrained)
- groundwater conditions (dry or saturated)
- foundation shape (strip footing, square or circle)

Other important factors include

- soil compressibility
- embedment ($D/B > 1$)
- inclined loading
- eccentric loading
- non-homogeneous soil

Effective Stress Analysis

More theoretically accurate bearing capacity factors are given on pages 69 to 71 of the Data Sheets •

In practice the Terzaghi factors are still widely used. •

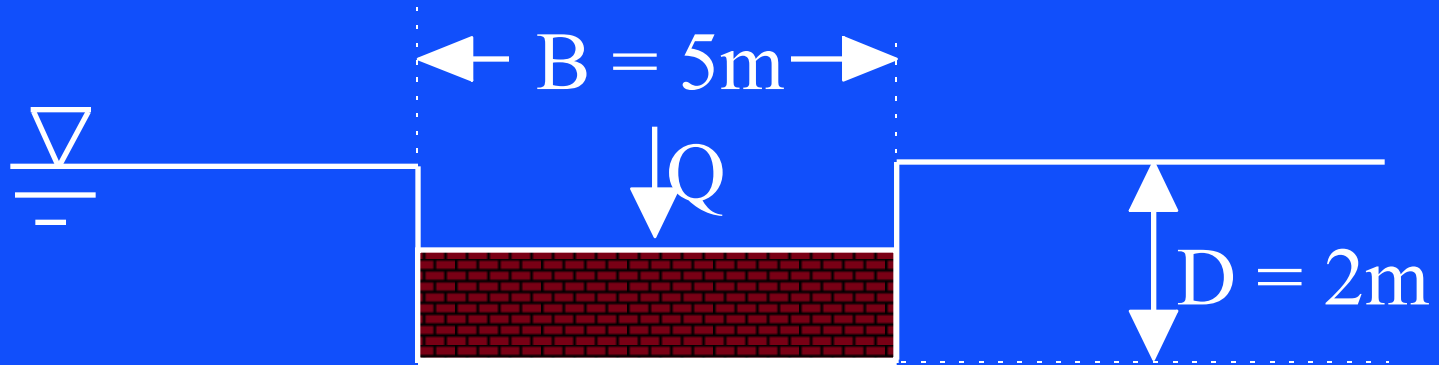
The bearing capacity equation assumes that the effects of c' , γ , and ϕ' can be superimposed. •

This is not correct as there is an interaction between the three effects because of the plastic nature of the soil response. •

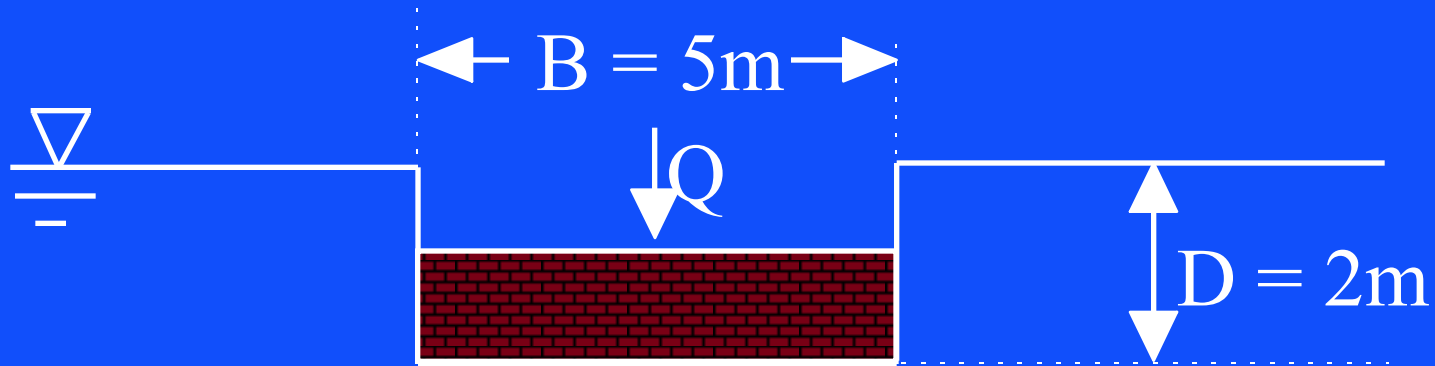
Effective Stress Analysis

- The formulae give the ultimate bearing capacity
- Significant deformations and large settlements may occur before general bearing failure occurs
- Local failure (yield) will occur at some depth beneath the footing at a load less than the ultimate collapse load
- The zone of plastic (yielding) soil will then spread as the load is increased. Only when the failure zone extends to the surface will a failure mechanism exist.
- A minimum load factor of 3 against ultimate failure is usually adopted to keep settlements within acceptable bounds, and to avoid problems with local failure.

Example



Example

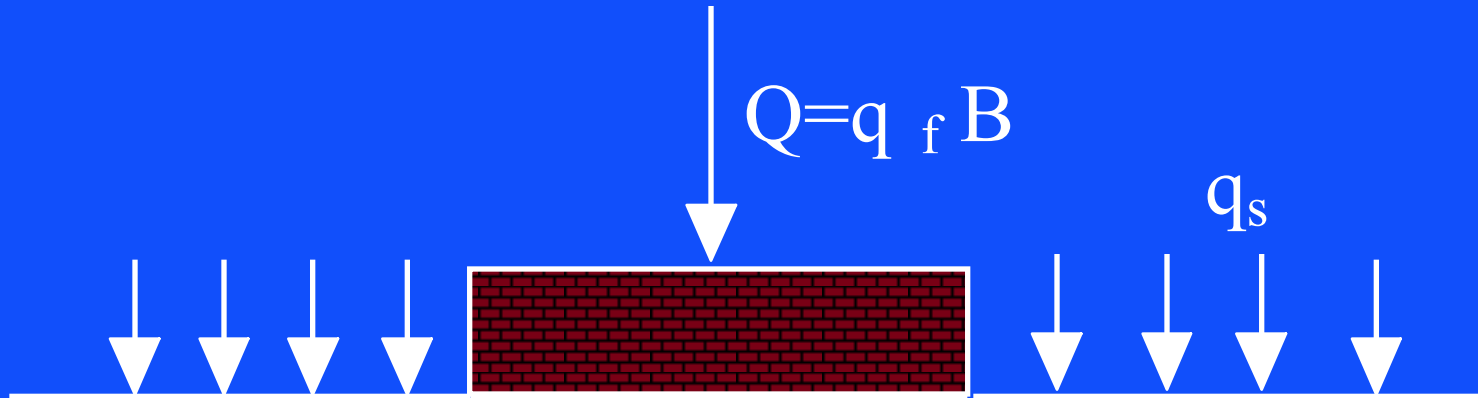


Determine short term and long term ultimate capacity given

$$c_u = 25 \text{ kN/m}^2, \phi_u = 0, c' = 2 \text{ kN/m}^2, \\ \phi' = 25^\circ, \text{ and } \gamma_{\text{sat}} = 15 \text{ kN/m}^2.$$

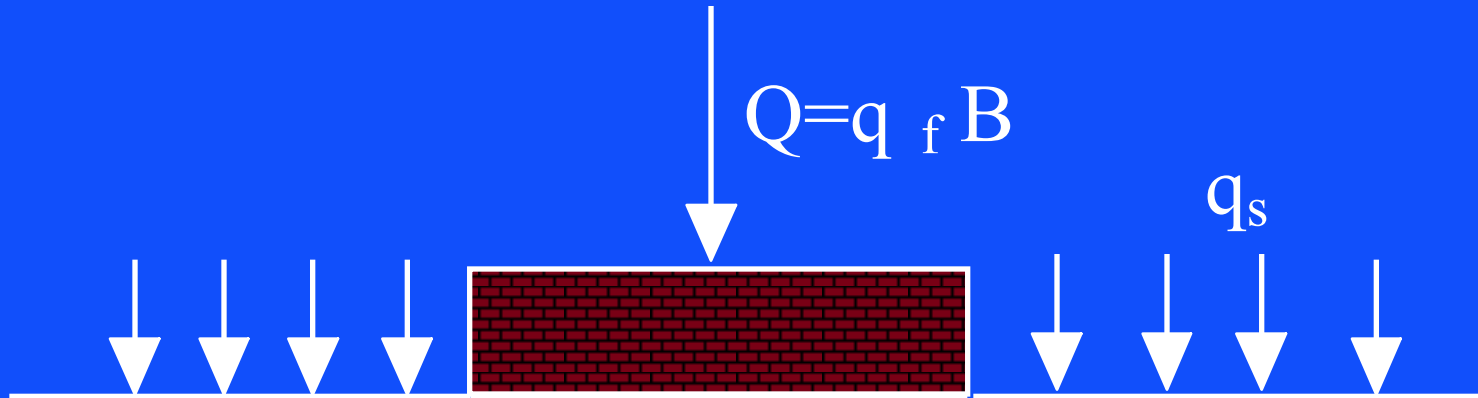
Example

Equivalent surface footing



Example

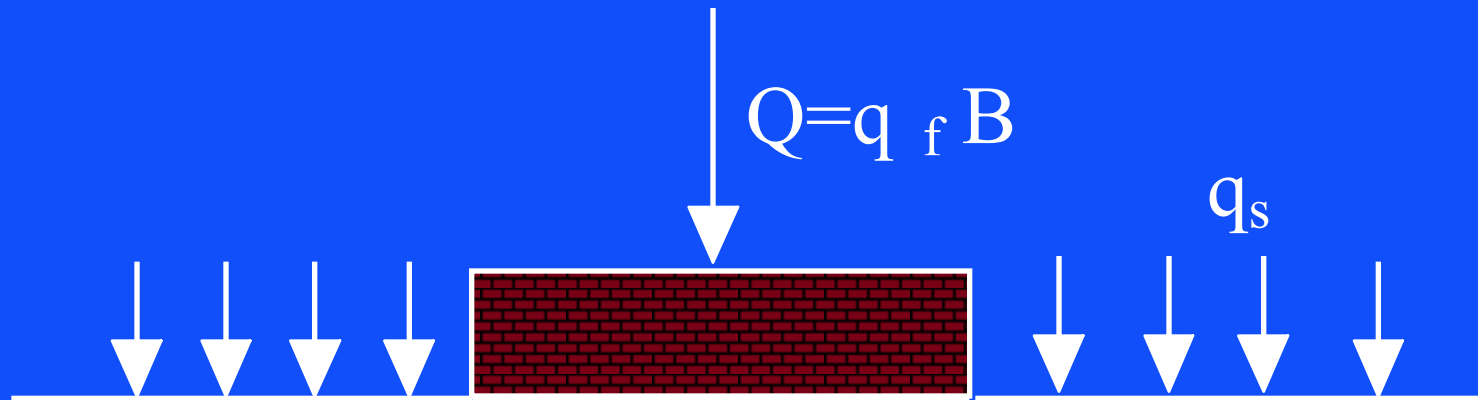
Equivalent surface footing



Short term - Undrained (total stress) analysis

Example

Equivalent surface footing

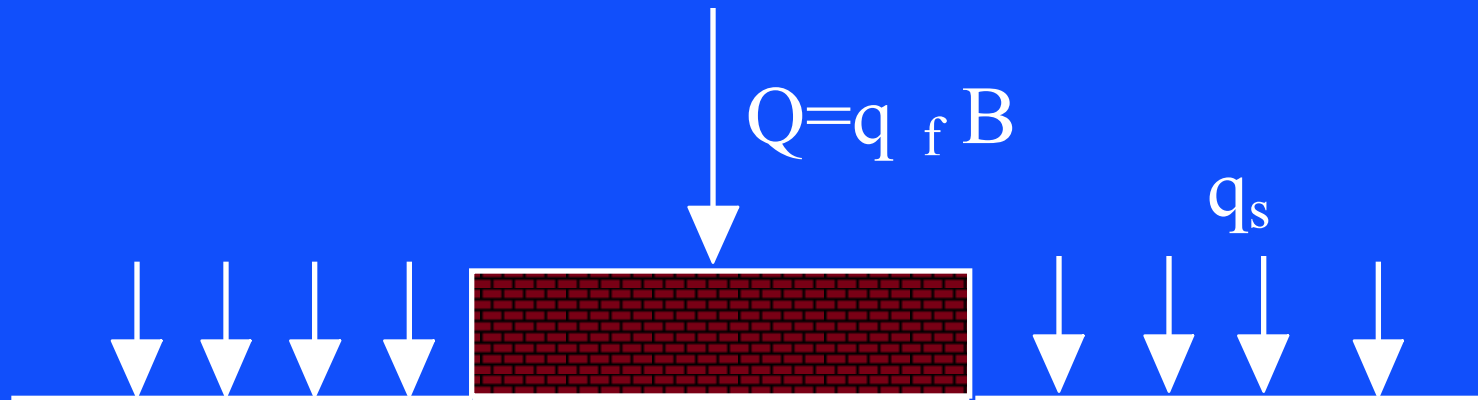


Short term - Undrained (total stress) analysis

Position of water table not important - soil must be saturated

Example

Equivalent surface footing

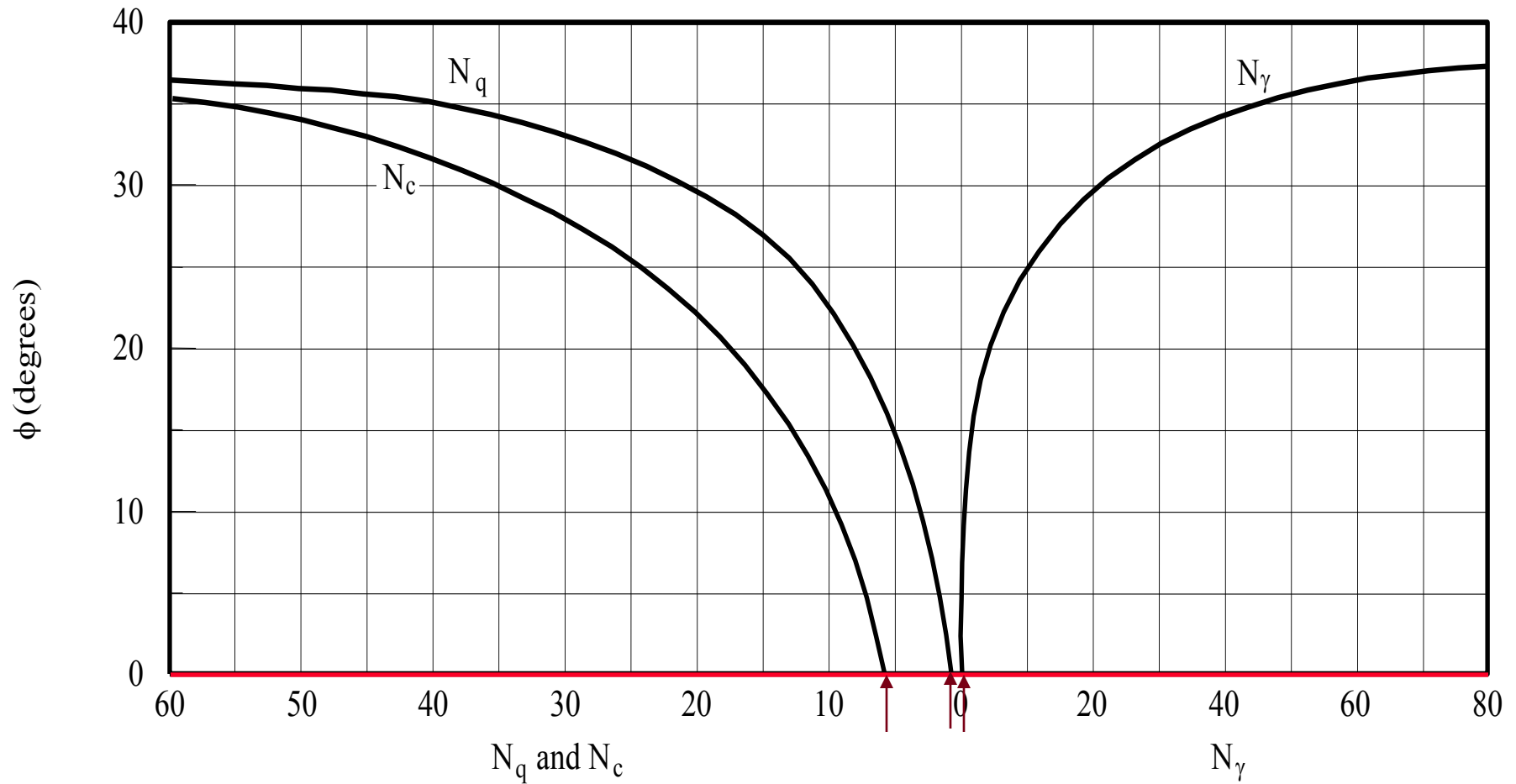


Short term - Undrained (total stress) analysis

Position of water table not important - soil must be saturated

$$q_s = \gamma_{\text{sat}} D = 15 \times 2 = 30 \text{ kPa}$$

Example



$$5.14 = \text{and } N_c 0 = , N_\gamma 1 = N_q 0 = \phi_u$$

Example

Short term capacity

$$q_f = q_s N_q + \frac{\gamma B}{2} N_\gamma + c N_c$$

Example

Short term capacity

$$q_f = q_s N_q + \frac{\gamma B}{2} N_\gamma + c N_c$$

$$q_f \cdot 30 = \left[25 + 0 + 1 \right] \text{ kPa (Bearing capacity)} \cdot 158.5 = 5.14$$

Example

Short term capacity

$$q_f = q_s N_q + \frac{\gamma B}{2} N_\gamma + c N_c$$

$$q_f = 30 = 25 + 0 + 1 \text{ kPa (Bearing capacity)} \quad 158.5 = 5.14$$

$$Q = q_f \cdot B = 158.5 \text{ kN/m (Bearing Force)} \quad 792.5 = 5$$

Example

Long term capacity

Effective stress (fully drained) analysis

Example

Long term capacity

Effective stress (fully drained) analysis

$$q_s = 30 \text{ kPa}$$

Example

Long term capacity

Effective stress (fully drained) analysis

$$q_s = 30 \text{ kPa}$$

$$u_o = 2 \times 9.8 = 19.6 \text{ kPa}$$

Example

Long term capacity

Effective stress (fully drained) analysis

$$q_s = 30 \text{ kPa}$$

$$u_o = 2 \times 9.8 = 19.6 \text{ kPa}$$

$$q'_s = 10.4 \text{ kPa}$$

Example

Long term capacity

Effective stress (fully drained) analysis

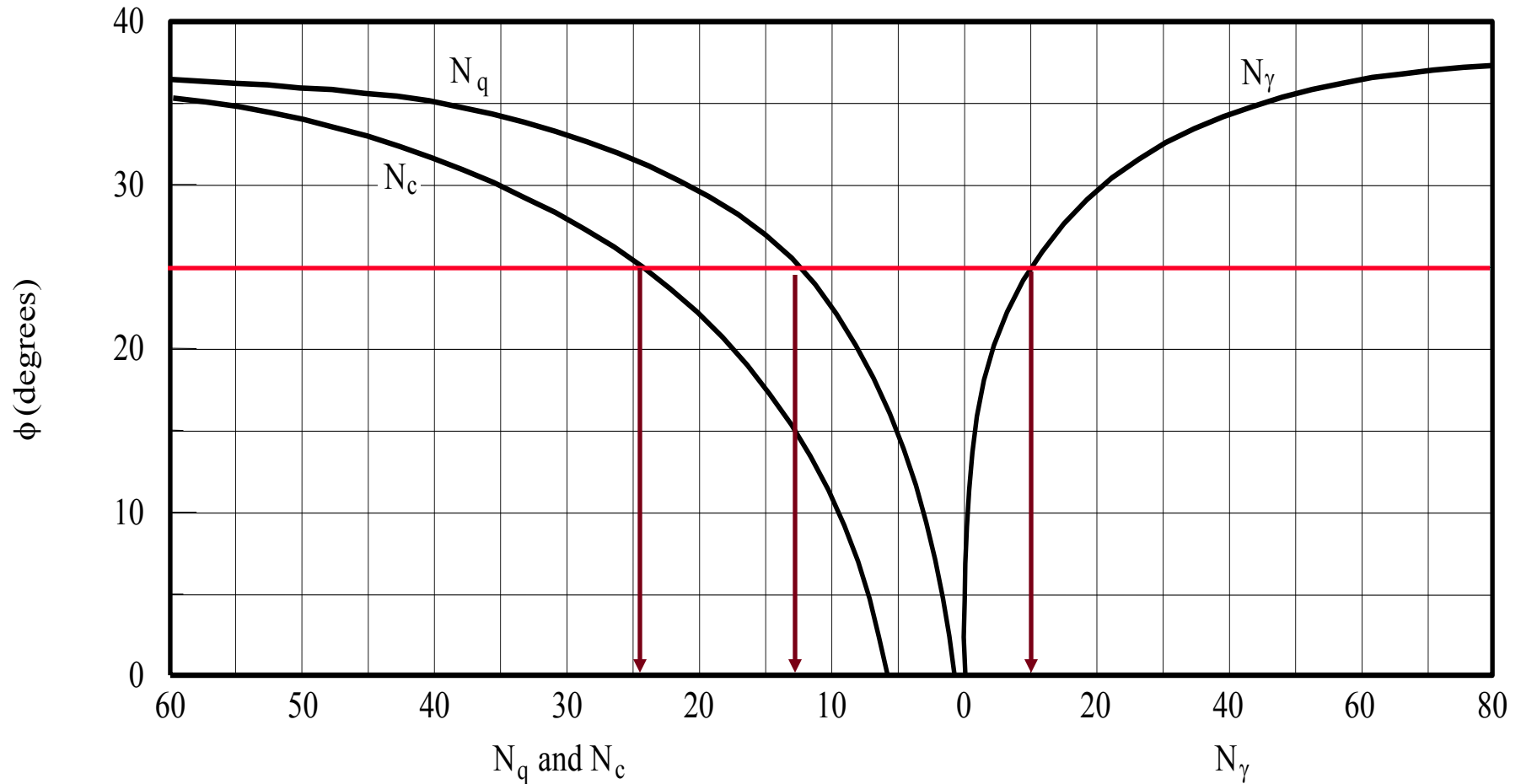
$$q_s = 30 \text{ kPa}$$

$$u_o = 2 \times 9.8 = 19.6 \text{ kPa}$$

$$q'_s = 10.4 \text{ kPa}$$

$$\gamma' = 15 - 9.8 = 5.2 \text{ kPa}$$

Example



$$24.5 = \phi \text{ and } N_c 10 = N_\gamma 13 = N_q 25$$

Example

Long term capacity

$$q'_f = q'_s N_q + \frac{\gamma' B}{2} N_\gamma + c' N_c$$

$$q'_f = 10.4 \times 13 + 0.5 \times 5.2 \times 5 \times 10 + 2 \times 24.5 = 314.2 \text{ kPa}$$

Example

Long term capacity

$$q'_f = q'_s N_q + \frac{\gamma' B}{2} N_\gamma + c' N_c$$

$$q'_f = 10.4 \times 13 + 0.5 \times 5.2 \times 5 \times 10 + 2 \times 24.5 = 314.2 \text{ kPa}$$

$$q_f = 314.2 + 19.6$$

$$= 333.8 \text{ kPa}$$

Example

Long term capacity

$$q'_f = q'_s N_q + \frac{\gamma' B}{2} N_\gamma + c' N_c$$

$$q'_f = 10.4 \times 13 + 0.5 \times 5.2 \times 5 \times 10 + 2 \times 24.5 = 314.2 \text{ kPa}$$

$$q_f = 314.2 + 19.6$$

$$= 333.8 \text{ kPa}$$

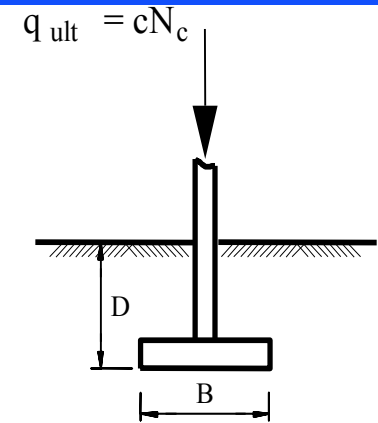
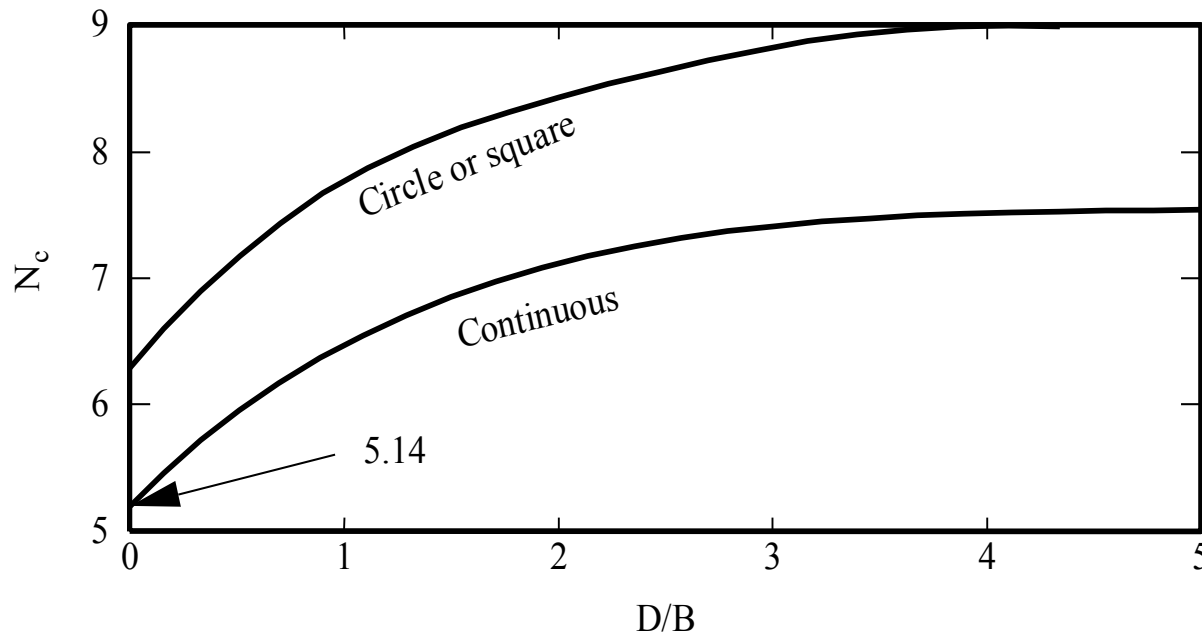
$$Q = 1669 \text{ kN/m}$$

0 = Total Stress Analysis ϕ_u

$$q_f = N_c c_u + q_s$$

0 = Total Stress Analysis ϕ_u

$$q_f = N_c c_u + q_s$$



N_c (for rectangle)

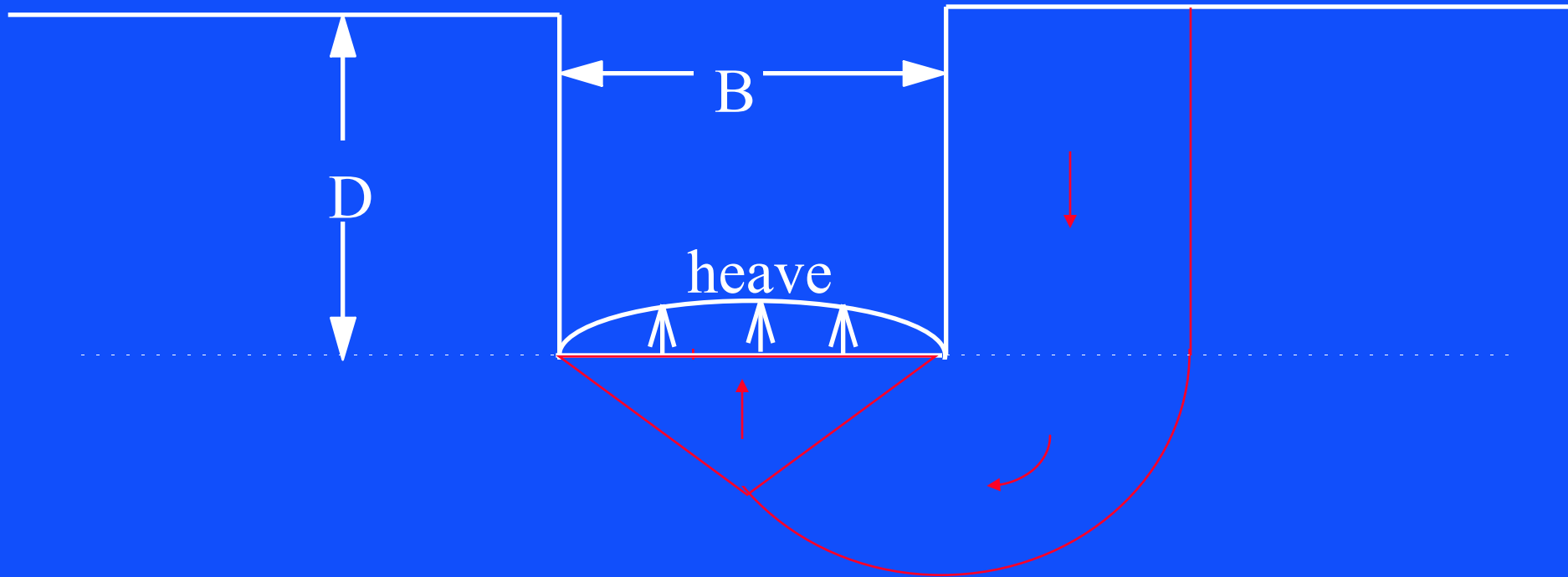
$$= (0.84 + 0.16 \frac{B}{L}) N_c \text{ (square)}$$

L = Length of footing

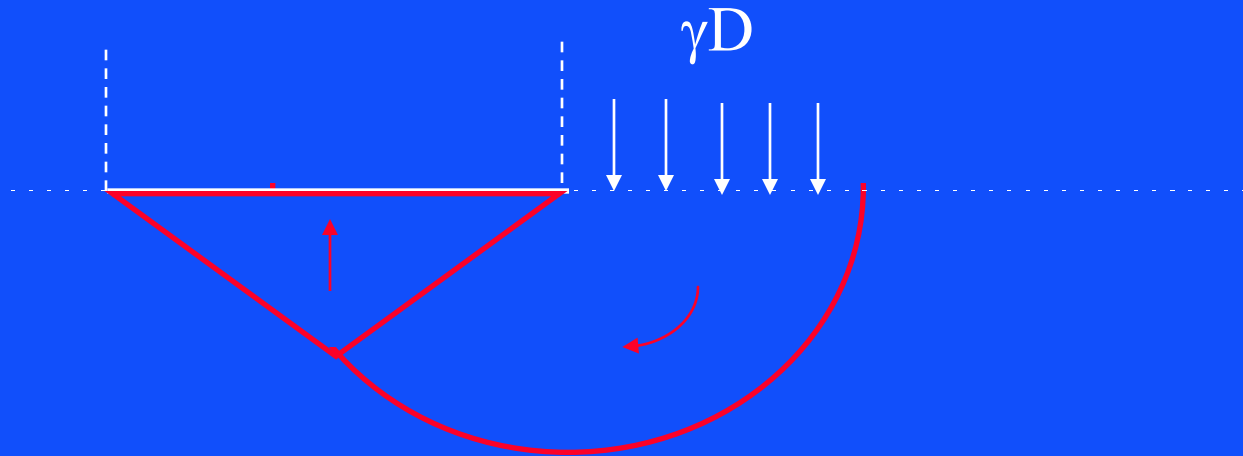
ULTIMATE BEARING CAPACITY OF CLAY ($\phi = 0$ only) (After A.W. Skempton)

$$q_f = cN_c + \gamma D$$

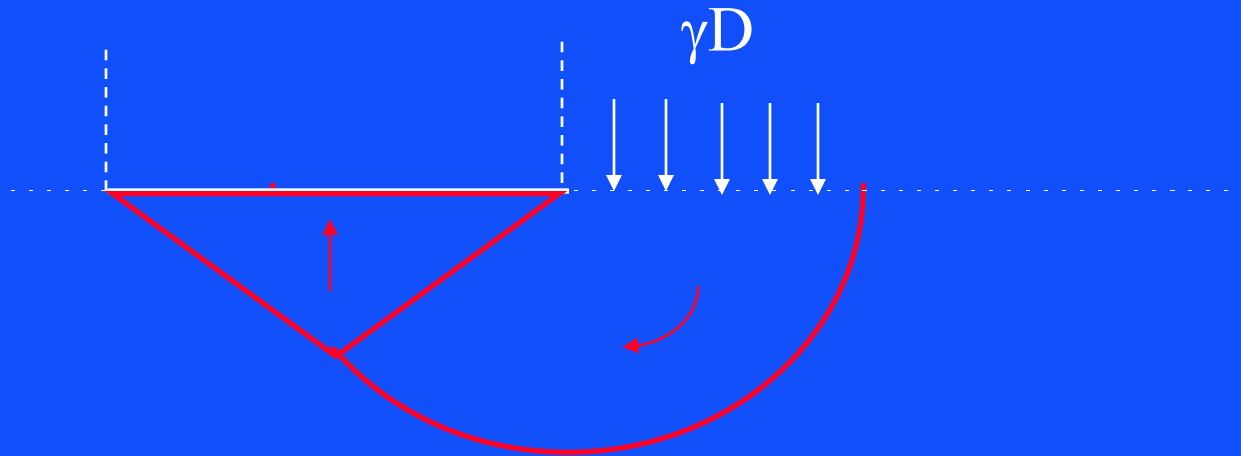
Bottom heave into excavations



Bottom heave into excavations



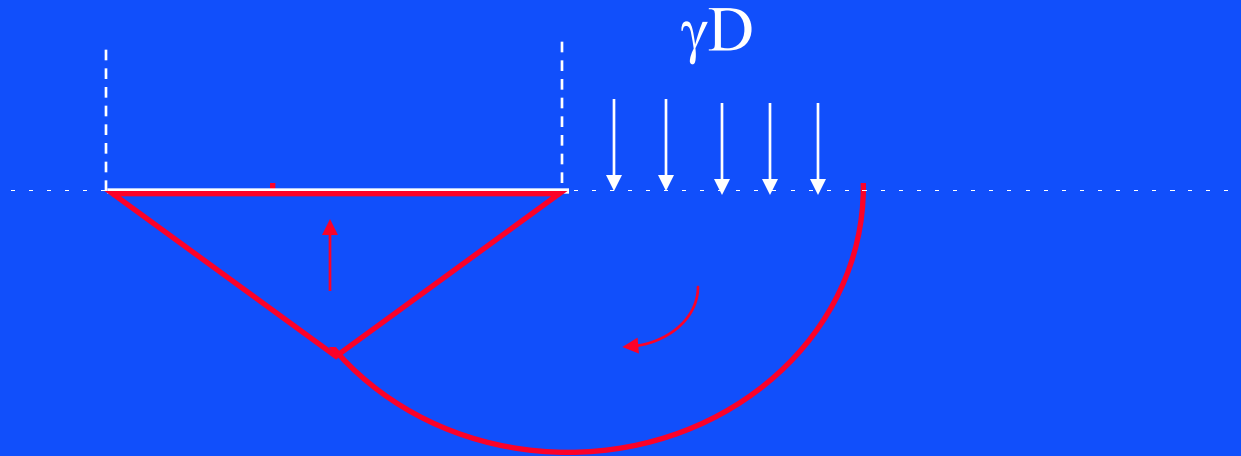
Bottom heave into excavations



For $\phi = 0$, and constant undrained strength c_u

The bearing capacity (pressure) = $c_u N_c$

Bottom heave into excavations

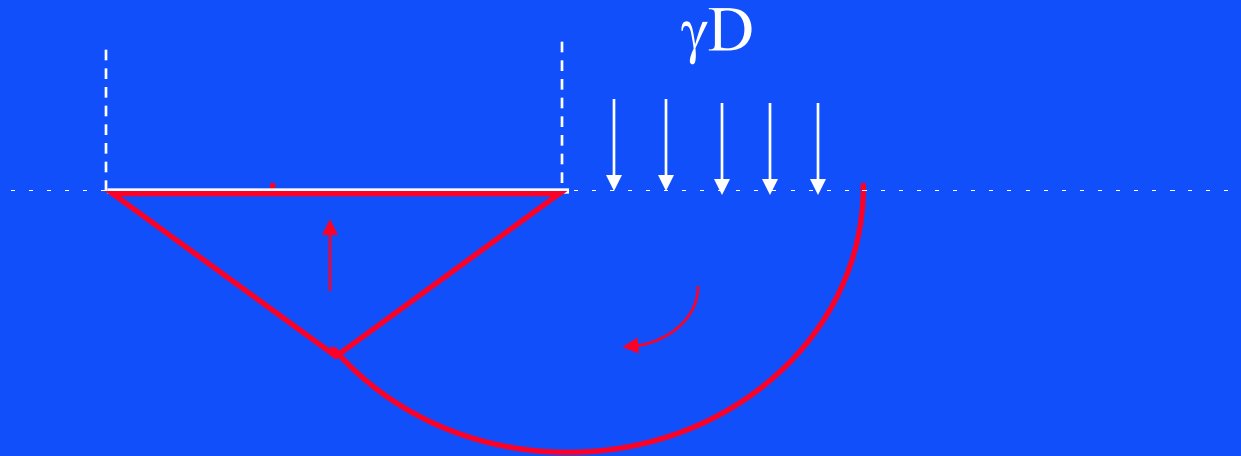


For $\phi = 0$, and constant undrained strength c_u

The bearing capacity (pressure) = $c_u N_c$

The driving pressure causing failure = γD

Bottom heave into excavations



For $\phi = 0$, and constant undrained strength c_u

The bearing capacity (pressure) = $c_u N_c$

The driving pressure causing failure = γD

and the Factor of Safety = $\frac{\text{Bearing capacity}}{\text{Stress causing failure}} = \frac{c_u N_c}{\gamma D}$