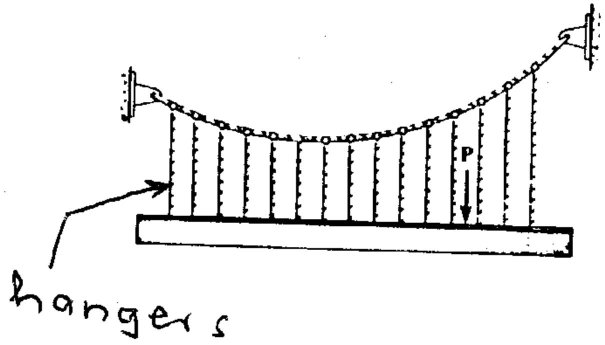
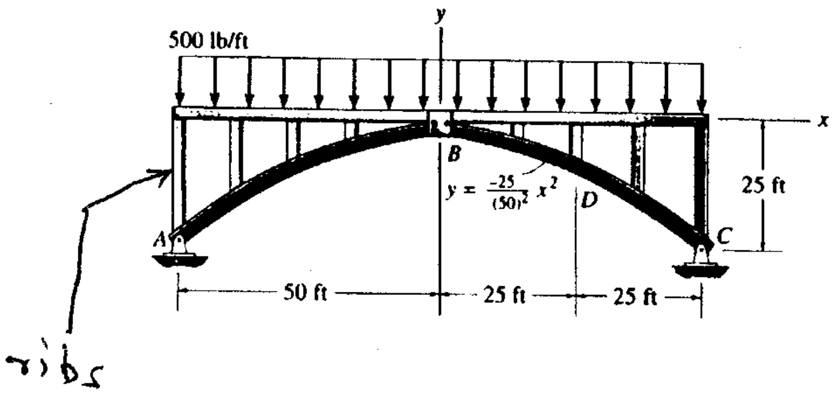
2. CABLES AND ARCHES

2.1 INTRODUCTION

2.1 Introduction

- **Cables** carry applied loads & **develop mostly tensile stresses** - Loads applied through hangers - Cables near the end supporting structures experience bending moments and shear forces
- **Arches** carry applied loads and develop **mainly in-plane compressive stresses**; three-hinged, two-hinged and fixed arches - Loads applied through ribs - Arch sections near the rib supports and and arches, other than three-hinged arches, experience bending moments and shear forces





2.1 INTRODUCTION (Cont'd)

Cables

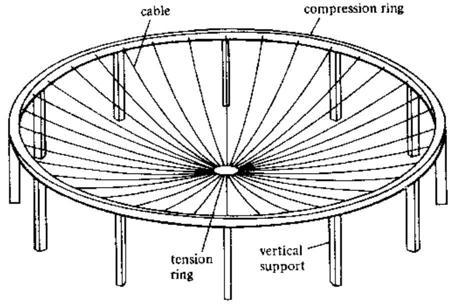
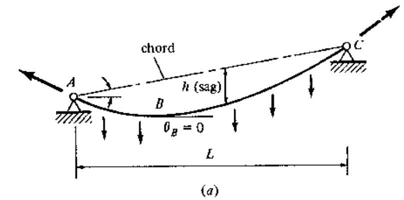


FIG. 5.1 (*left*) Cable supported roof composed of three element: cables, a center tension ring, and an outer compression ring.



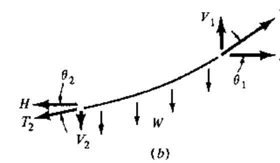


FIG. 5.2 (*right*) Vertically loaded cables: (*a*) cable with an inclined chord. h, the vertical distance between the chord and the cable is called the *sag*. (*b*) freebody of a cable segment carrying vertical loads; Although the resultant cable force T varies with the slope of the cable, $\Sigma F_x = 0$ requires that H, the horizontal component of T is constant from section to section.

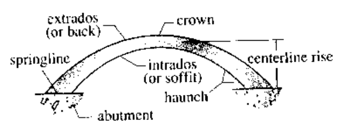


Fig. 5–7

5.4 Arches

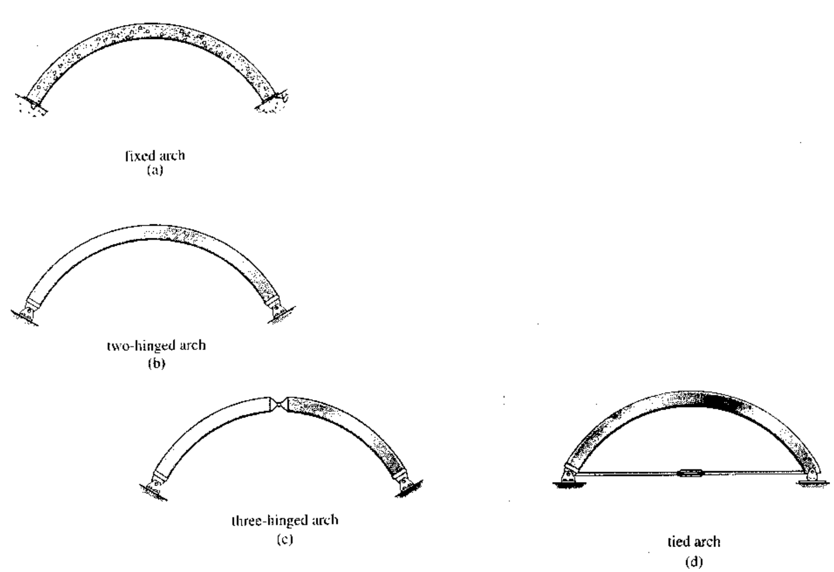


Fig. 5–8

2.1 INTRODUCTION (Cont'd)

- In cables, the **loads applied through hangers is considered to be a uniformly distributed load** - in the same manner, the **loads distributed to the arches through the ribs are considered to be uniformly distributed**
- **Cable type structures** - Suspension roof, suspension bridges, cable cars, guy-lines, transmission lines, etc.
- **Arch type structures** - Arches, domes, shells, vaults

2.2 ANALYSIS OF CABLE

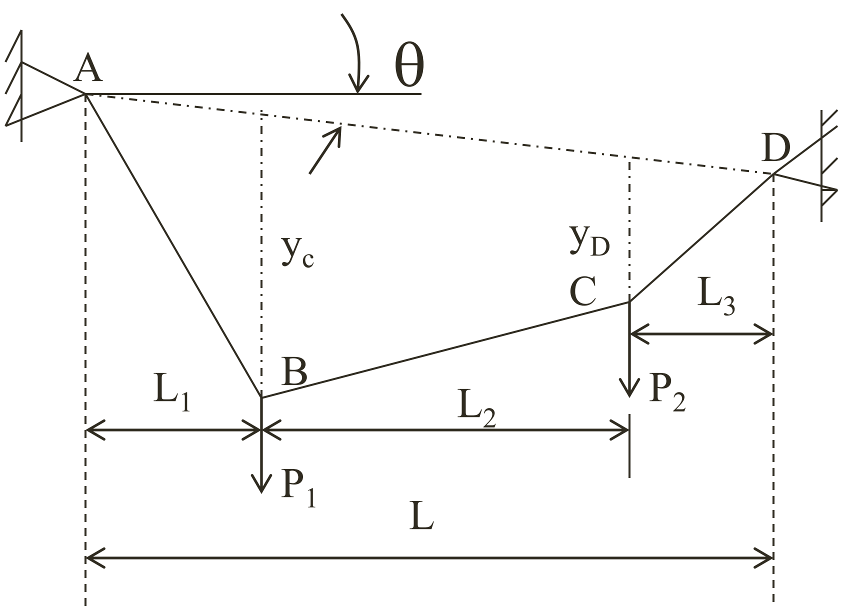
2.2.1 Assumptions

- Cable is **flexible and in-extensible**; hence does not resist any bending moment or shear force (this is not always true - e.g., fatigue of cables); **self weight of cable neglected** when external loads act on the cable
- Since only axial tensile forces are carried by the cable, the **force in the cable is tangential to the cable profile**
- **Since it is in-extensible, the length is always constant**; as a consequence of the cable profile not changing its length and form, it is **assumed to be a rigid body during analysis**
- **Even when a moving load is acting on the cable**, the **load is assumed to be uniformly distributed** over the cable (since the cable profile is not assumed to change)

2.2 ANALYSIS OF CABLE (Cont'd)

- **2.2.2 Cables subjected to concentrated loads**
- When the weight of the cable is neglected in analysis and is subjected to only concentrated loads, the **cable takes the form of several straight line segments; the shape is called as funicular polygon.** Consider for instance the cable shown in Figure 5.1

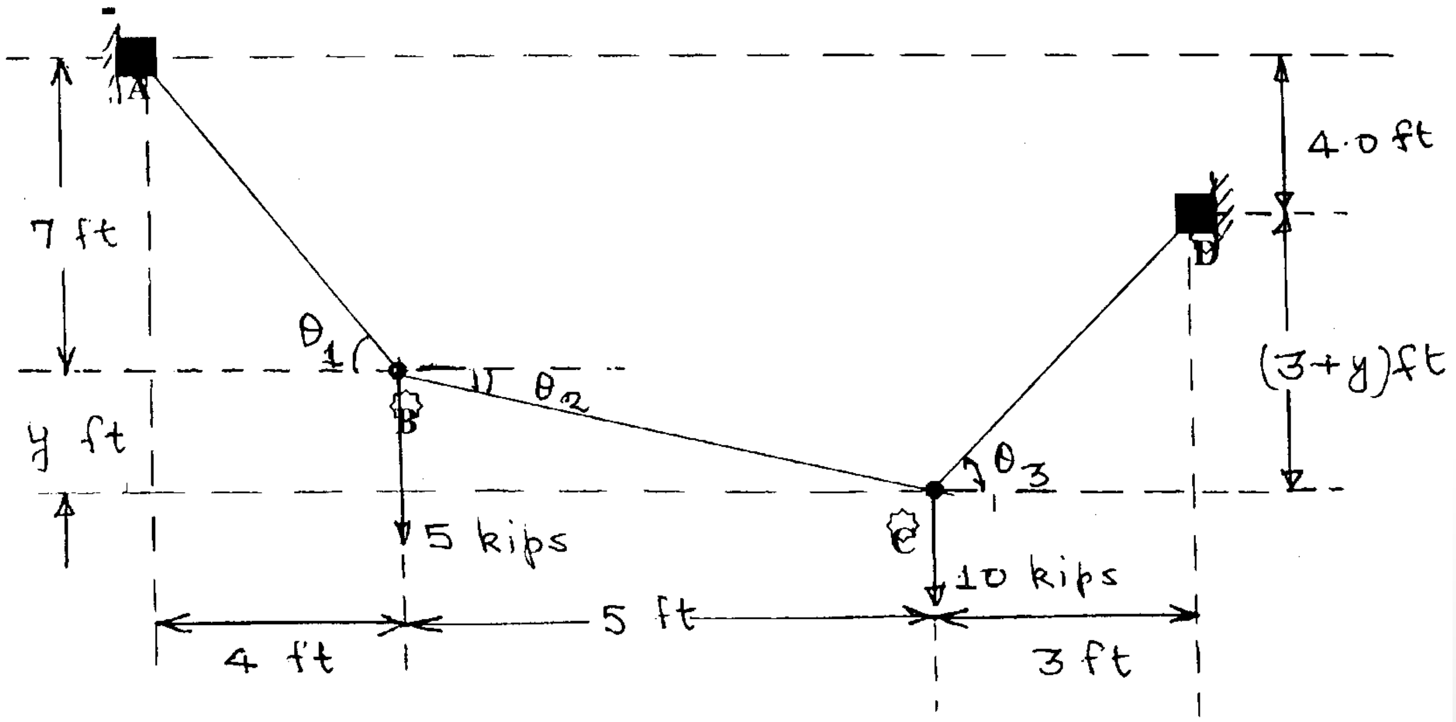
Figure 5.1



2.2 ANALYSIS OF CABLE (Cont'd)

- **2.2.2 Cable under concentrated loads (Cont'd)**
- In figure 5.1, the known parameters are L_1, L_2, L_3, P_1 & P_2 - the **unknowns are the four support reactions at A and B, the three cable tensions (in the three segments) and the two sags (y_C and y_D) - 9 unknowns**
- **Eight force equilibrium equations can be written at the four nodes** and **we need to have one more condition** to solve the problem - **This is met** by assuming something about the cable, either its total length, or one of its sags (say y_C or y_D)

- **2.2.2 Cable under concentrated loads (Cont'd)**
- Problem 5.1: Determine the tension in each segment of the cable, shown below, and the total length of the cable



2.2 ANALYSIS OF CABLE - FOR CONCENTRATED LOADS (Cont'd)

- $AB = \sqrt{(7^2 + 4^2)} = 8.062\,ft; \quad BC = \sqrt{(y^2 + 5^2)}$
- $\cos(\theta_1) = 4/(8.062) = 0.4962; \quad \sin(\theta_1) = 7/(8.062) = 0.8683$
- $\cos(\theta_2) = 5/\sqrt{(y^2 + 5^2)}; \quad \sin(\theta_2) = y \big/ \sqrt{(y^2 + 5^2)}$
- $CD = \sqrt{[(3+y)^2 + 3^2]}; \quad \cos(\theta_3) = 3 \big/ \sqrt{[(3+y)^2 + 3^2]}; \quad \sin(\theta_3)$

  $= (3+y)/\sqrt{[(3+y)^2 + 3^2]}; \quad \tan(\theta_3) = (3+y) \big/ 3$
- Considering horizontal and vertical equilibrium at B,

  $\sum F_H = 0; \quad \sum F_V = 0$
- $BA\cos(\theta_1) - BC\cos(\theta_2) = 0.0 \quad \therefore BC = BA \times (0.4962)/\cos(\theta_2);$

and $BA\sin(\theta_1) - 5 - BC\sin(\theta_2) = 0$;

$\therefore BA = 5/[0.8683 - 0.4962\tan(\theta_2)] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(I)$

2.2 ANALYSIS OF CABLE - FOR CONCENTRATED LOADS (Cont'd)

- Considering equilibrium at C, $\sum F_H = 0, \quad \& \quad \sum F_V = 0$;
- $BC\cos(\theta_2) - CD\cos(\theta_3) = 0; \quad CD = BA \times (0.4962)/(\cos(\theta_3));$
- $BC\sin(\theta_2) + CD\sin(\theta_3) - 10 = 0$;

$$\therefore BA = 10/(0.4962\tan(\theta_2) + 0.4962\tan(\theta_3))................(II)$$

- **Dividing equation (I) by (II),**
- $[0.8683 - 0.4962\tan(\theta_2)]/[0.4962 \times (\tan(\theta_2) + \tan(\theta_3)] = 1/2$
- Substituting for $\tan(\theta_2)$ and $\tan(\theta_3)$ in terms of y and solving,
- y = 2.6784 ft
- BA = 8.2988 kips; BC = 4.6714 kips and CD = 8.815 kips;
- Total length of cable = 8.062 + 5.672 + 6.422 = 20.516 ft

2.3 CABLES SUBJECTED TO UNIFORMLY DISTRIBUTED LOADS

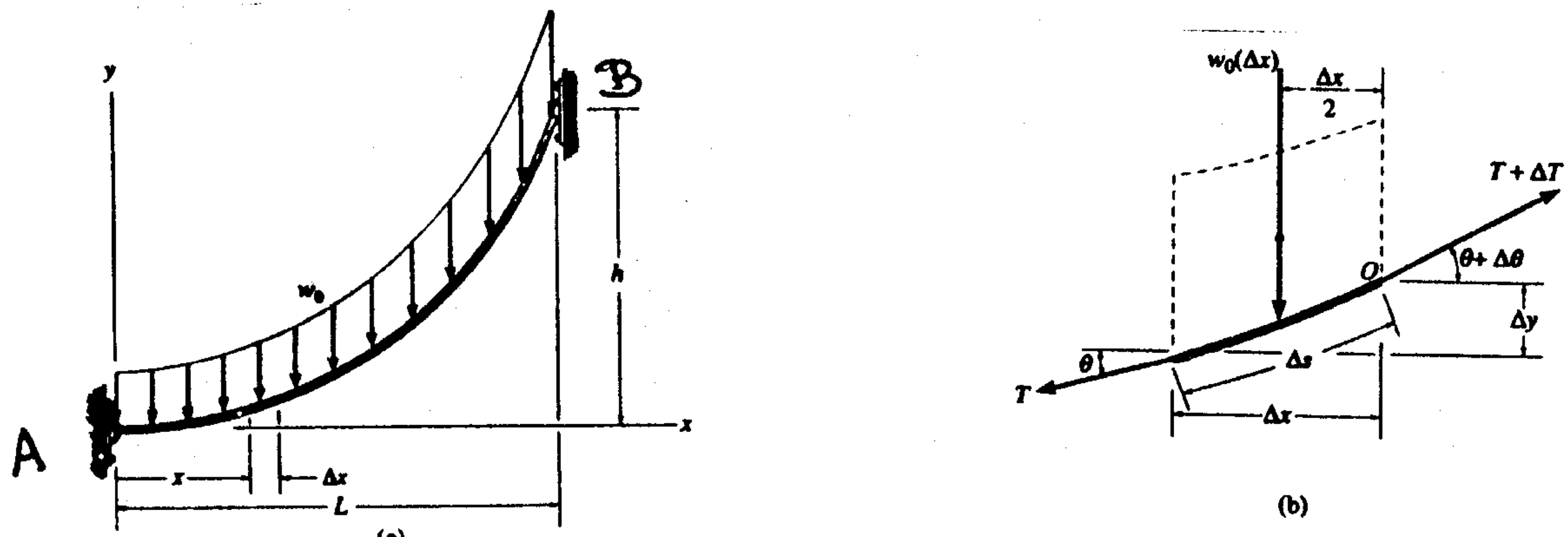


Fig. 5–3

- $\sum F_x = 0; \quad \therefore -T\cos(\theta) + (T + \Delta T)\cos(\theta + \Delta\theta) = 0.......(A)$

- $\sum F_y = 0; \quad \therefore -T\sin(\theta) - w_0\Delta X + (T + \Delta T)\sin(\theta + \Delta\theta) = 0.......(B)$

$\sum M_o = 0; \quad \therefore (w_0\Delta x)(\Delta x/2) - T\cos(\theta)\Delta y + T\sin(\theta)\Delta x = 0......(C)$

2.3 CABLES SUBJECTED TO UNIFORMLY DISTRIBUTED LOADS (Cont'd)

- Equation (A) reduces to : $\Delta T\cos(\theta) - T\sin(\theta)\Delta\theta = 0;$
- $\rightarrow \frac{d}{dx}[T\cos(\theta)] = 0;$ integrating $T\cos(\theta) = Cons\tan t = F_H ....(D)$
- Equation (B) reduces to: $\Delta T\sin(\theta) + T\cos(\theta)\Delta\theta - w_0\Delta x = 0;$

  this equation can be rewritten as $\frac{d}{dx}[T\sin(\theta)] = w_0 ....(E)$
- Equation (C) reduces to $-T\cos(\theta)\Delta y + T\sin(\theta)\Delta x = 0;$ this equation reduces to $\frac{dy}{dx} = \tan(\theta) ....(F)$
- From equation (E), one gets $T\sin(\theta) = w_0 x ....(G)$, using the condition that $\theta = 0$ at x = 0,
- From equation (D) and (G), dividing one by the other (G/D), one obtains $\tan(\theta) = w_0 x / F_H = \frac{dy}{dx}$ from Eqn. (F); and integrating further, $y = w_0 x^2 /(2F_H) + B.$ At x = 0, y = 0. This leads to the final form given by $y = w_0 x^2 /(2F_H)$

2.3 CABLES SUBJECTED TO UNIFORMLY DISTRIBUTED LOADS (Cont'd)

- $y = w_0 x^2 /(2F_H)$ …..This is the equation for a parabola.
- Using the condition, at x = L, y = h, one obtains that $F_H = w_0 L^2 /(2h)$; hence $y = h(x/L)^2$
- Considering the point B, $T_{mas} = \sqrt{[F_H^2 + (w_0 L)^2}$

$$= \sqrt{[(w_0 L/(2h))^2 + (w_0 L)^2]} = (w_0 L)\sqrt{[(L/(2h)^2 + 1]}$$

2.4 ADDITIONAL CONSIDERATIONS FOR CABLE SUPPORTED STRUCTURES

- **Forces on cable bridges**: Wind drag and lift forces - Aero-elastic effects should be considered (vortex-induced oscillations, flutter, torsional divergence or lateral buckling, galloping and buffeting).
- **Wind tunnel tests**: To examine the aerodynamic behavior
- **Precaution to be taken against**: Torsional divergence or lateral buckling due to twist in bridge; Aero-elastic stability caused by geometry of deck, frequencies of vibration and mechanical damping present; Galloping due to self-excited oscillations; Buffeting due to unsteady loading caused by velocity fluctuations in the wind flow