Cables and Arches

Cables: Assumptions

- Cable is perfectly flexible & inextensible
- No resistance to shear/bending: same as **truss bar**
- The force acting the cable is alw o the cable g

Only axial force!

Example 5.1 Under Concentrated Forces

Determine the tension in each segment of the cable. Also, what is the dimension *h*?

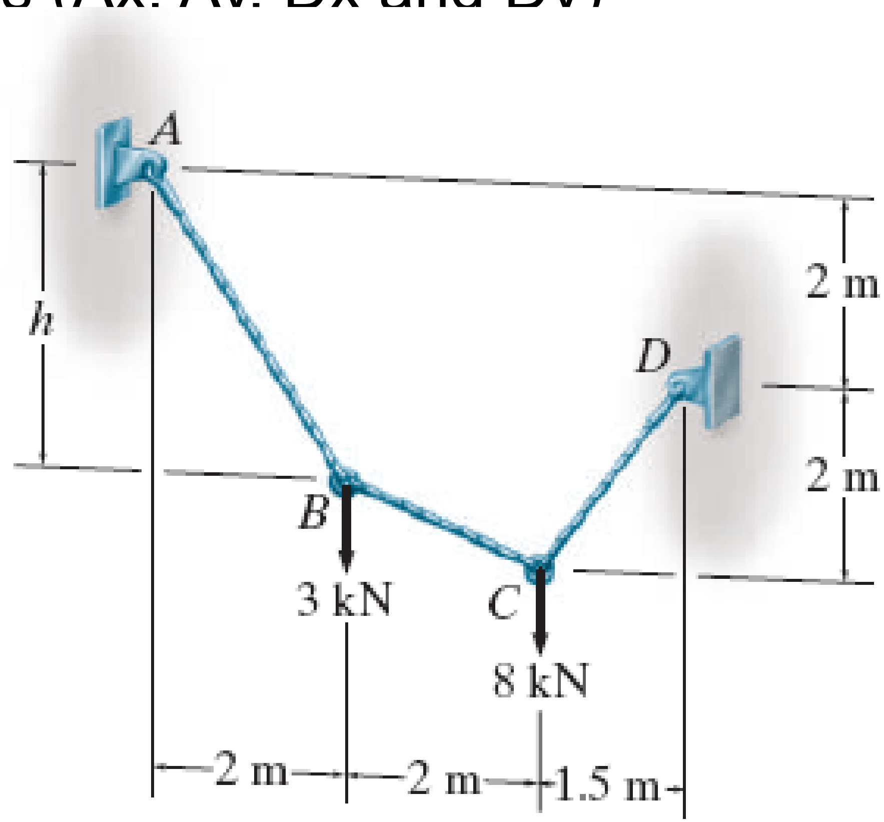
4 unknown external reactions (Ax, Ay, Dx and Dy)

3 unknown cable tensions

1 geometrical unknown h

8 unknowns

8 equilibrium conditions



Solution

$\Sigma M_A = 0$

$T_{CD}(3/5)(2m) + T_{CD}(4/5)(5.5m) - 3kN(2m) - 8kN(4m) = 0$

$T_{CD} = 6.79kN$

Joint equilibrium at C

$6.79kN(3/5) - T_{BC}\cos\theta_{BC} = 0$

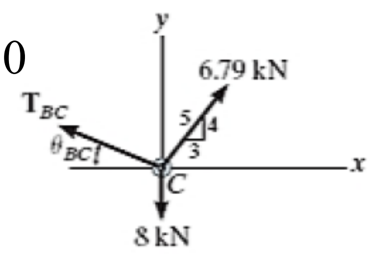
$6.79kN(4/5) - 8kN + T_{BC}\sin\theta_{BC} = 0$

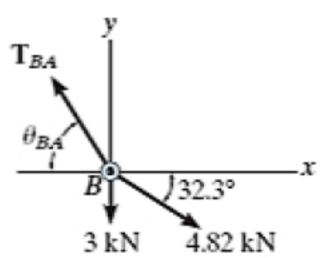
$\theta_{BC} = 32.3^o$ and $T_{BC} = 4.82kN$

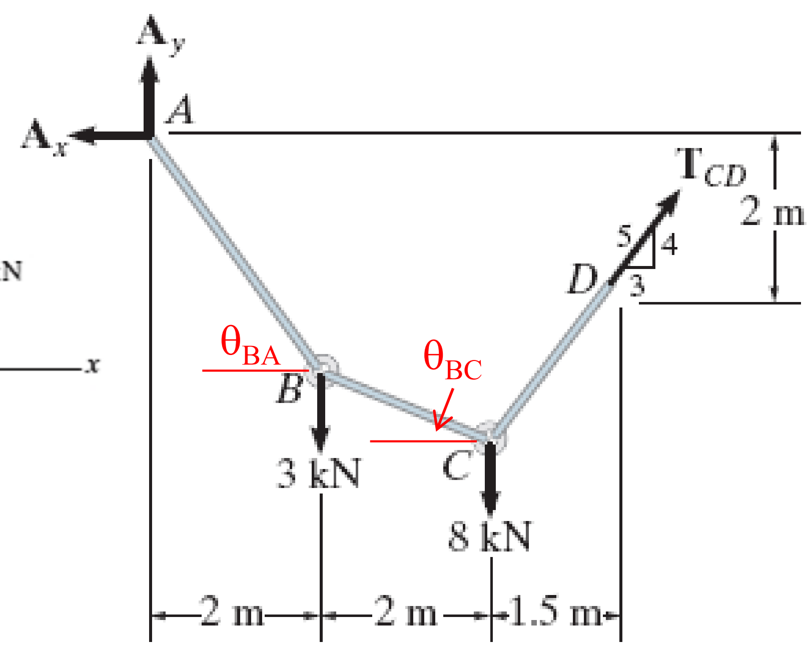
Similarly, Joint equilibrium at B

$\theta_{BA} = 53.8^o$ and $T_{BA} = 6.90kN$

$Finally, h = (2m)\tan 53.8^o = 2.74m$

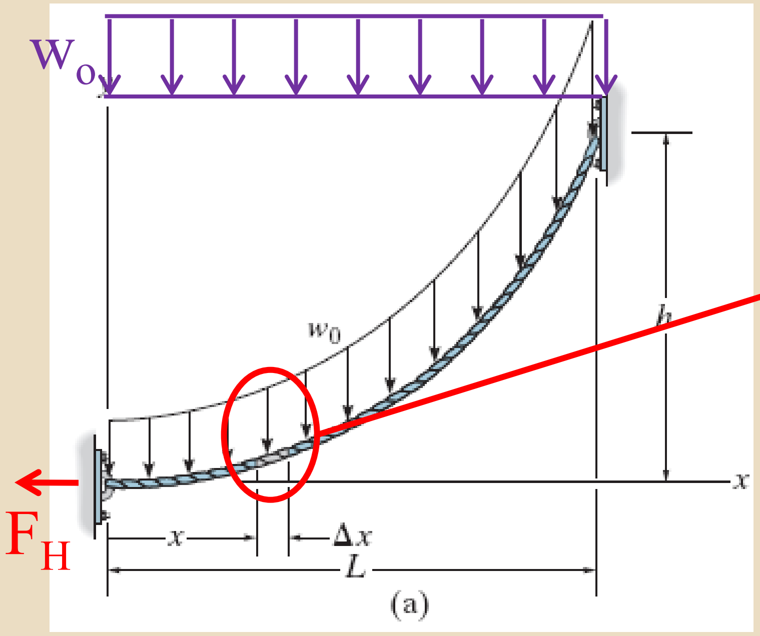




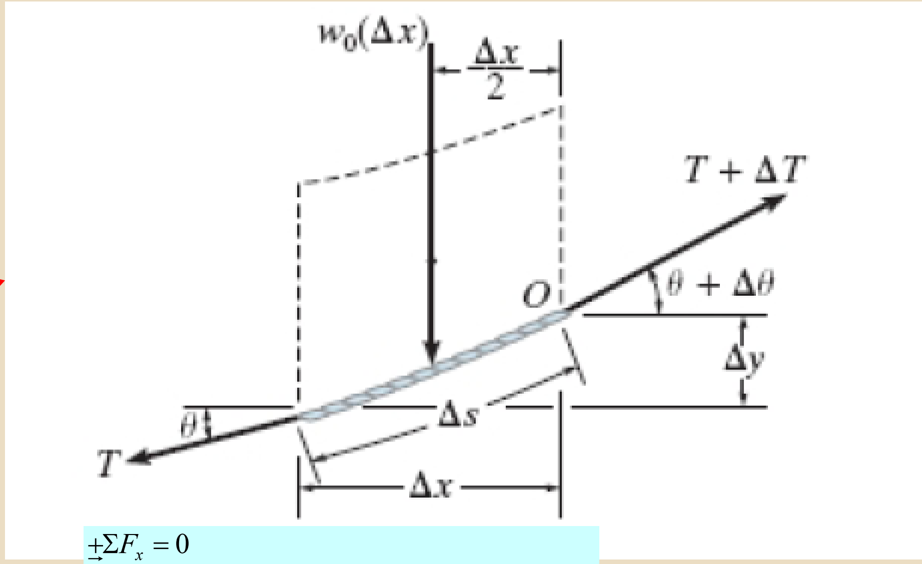


Cable subjected to a uniform distributed load

- Consider this cable under distributed vertical load w_o
- The cable force is not a constant.



$$\Delta(T\cos\theta) = \Delta T\cos\theta + T[\cos(\theta+\Delta\theta) - \cos\theta]$$



$$\overset{+}{\rightarrow}\Sigma F_x = 0$$
$$-T\cos\theta + (T+\Delta T)\cos(\theta+\Delta\theta) = 0$$

$$+\uparrow \Sigma F_y = 0$$
$$-T\sin\theta - w_o(\Delta x) + (T+\Delta T)\sin(\theta+\Delta\theta) = 0$$

With anti-clockwise as +ve

$$\Sigma M_0 = 0$$
$$w_o(\Delta x)(\Delta x/2) - T\cos\theta\Delta y + T\sin\theta\Delta x = 0$$

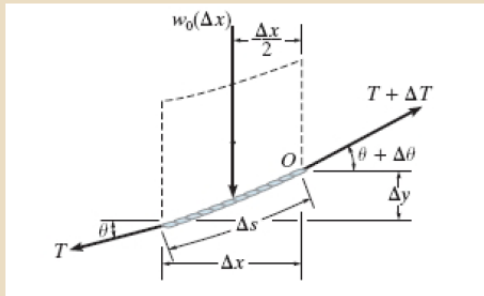
$$\frac{d(T\cos\theta)}{dx} = 0 \quad \text{eqn 1}$$
$$\frac{d(T\sin\theta)}{dx} = w_o \quad \text{eqn 2}$$
$$\frac{dy}{dx} = \tan\theta \quad \text{eqn 3}$$

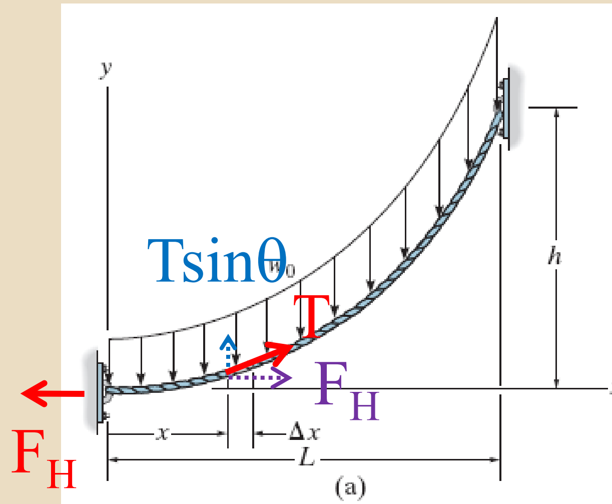
Cable subjected to a uniform distributed load

$$\frac{d(T\cos\theta)}{dx} = 0 \quad \text{eqn 1}$$

$$\frac{d(T\sin\theta)}{dx} = w_o \quad \text{eqn 2}$$

$$\frac{dy}{dx} = \tan\theta \quad \text{eqn 3}$$





- From Eqn 1 and let T = F_H at x = 0:

$$T\cos\theta = constant = F_H \quad \text{eqn 4}$$

- Integrating Eqn 2 realizing that Tsinθ = 0 at x = 0:

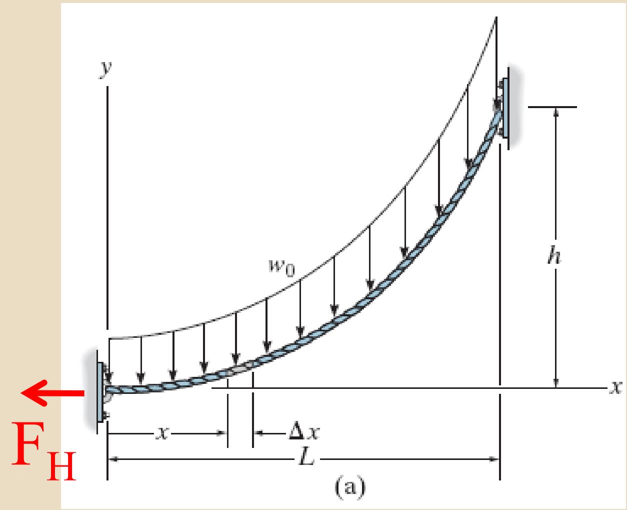
$$T\sin\theta = w_o x \quad \text{eqn 5}$$

- Eqn 5/Eqn 4:

$$\tan\theta = \frac{dy}{dx} = \frac{w_o x}{F_H} \quad \text{eqn 6}$$

Cable subjected to a uniform distributed load

$$\tan\theta = \frac{dy}{dx} = \frac{w_o x}{F_H} \quad \text{eqn 6}$$



- Performing an integration with y = 0 at x = 0 yields

$$y = \frac{w_o}{2F_H}x^2 \quad \text{eqn 7}$$

y = h at x = L

$$F_H = \frac{w_o L^2}{2h} \quad \text{eqn 8}$$

Cable profile: parabola

$$y = \frac{h}{L^2}x^2 \quad \text{eqn 9}$$

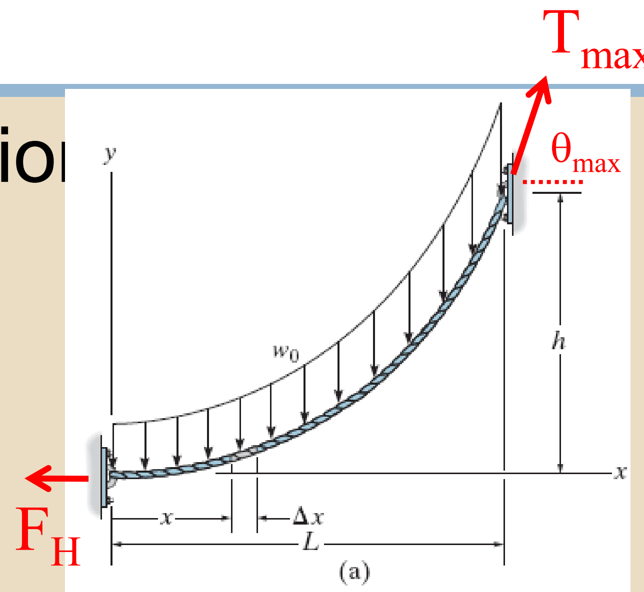
Cable subjected to a uniform distributed load

- Where and what is the max tension

$$T\cos\theta = F_H \quad \text{eqn 4}$$

$$T\sin\theta = w_o x \quad \text{eqn 5}$$

$$T = \sqrt{F_H{}^2 + (w_o x)^2}$$



- T is max when *x*=*L*

$$T_{max} = \sqrt{F_H{}^2 + (w_o L)^2} \quad \text{eqn 10}$$

$$F_H = \frac{w_o L^2}{2h} \quad \text{eqn 8}$$

$$T_{max} = w_o L\sqrt{1 + (L/2h)^2} \quad \text{eqn 11}$$

Cable subjected to a uniform distributed load

$$T\cos\theta = F_H$$

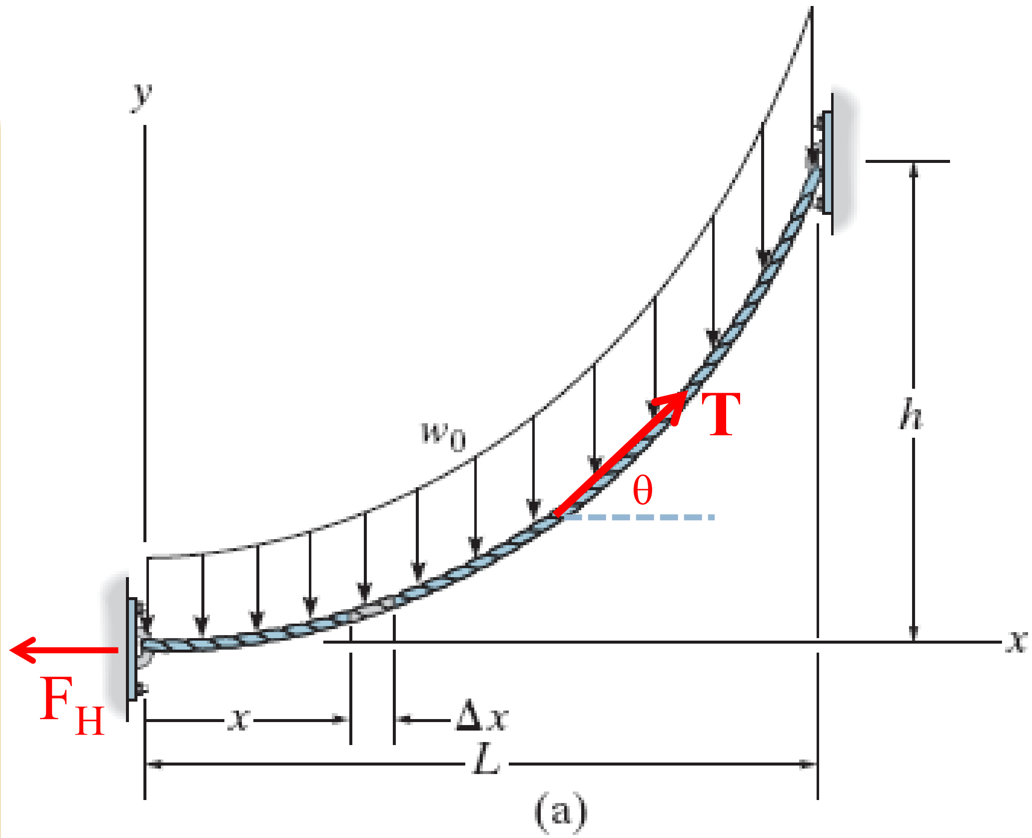
$$T\sin\theta = w_o x$$

$$\tan\theta = \frac{dy}{dx} = \frac{w_o x}{F_H}$$

$$F_H = \frac{w_o L^2}{2h}$$

$$y = \frac{h}{L^2}x^2$$

$$T_{\max} = w_o L\sqrt{1+(L/2h)^2}$$



Cable subjected to a uniform distributed load

- Neglect the cable weight which is uniform along the length
- A cable subjected to its own weight will take the form of a catenary curve $y = a\cosh\left(\frac{x}{a}\right)$
- This curve ~ parabolic for small sag-to-span ra

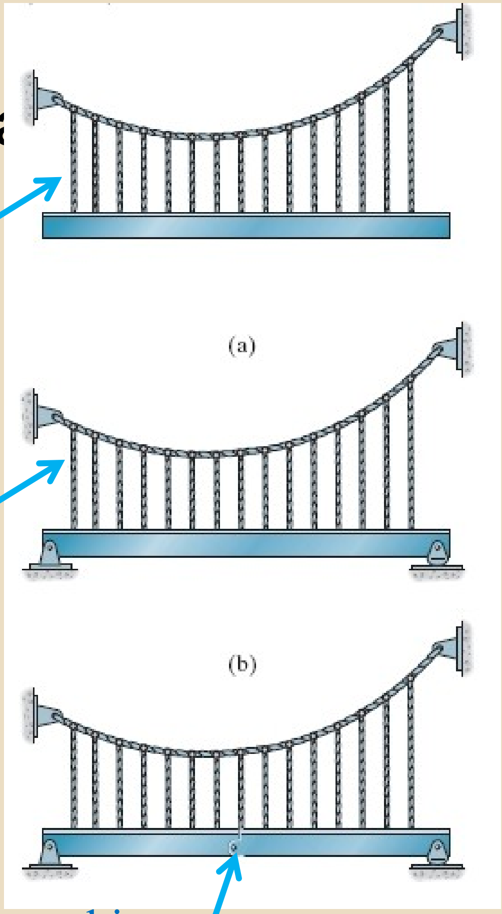
Wiki catenary

Hangers are close and uniformly spaced

If forces in the hangers are known then the structure can be analyzed

1 degree of indeterminacy

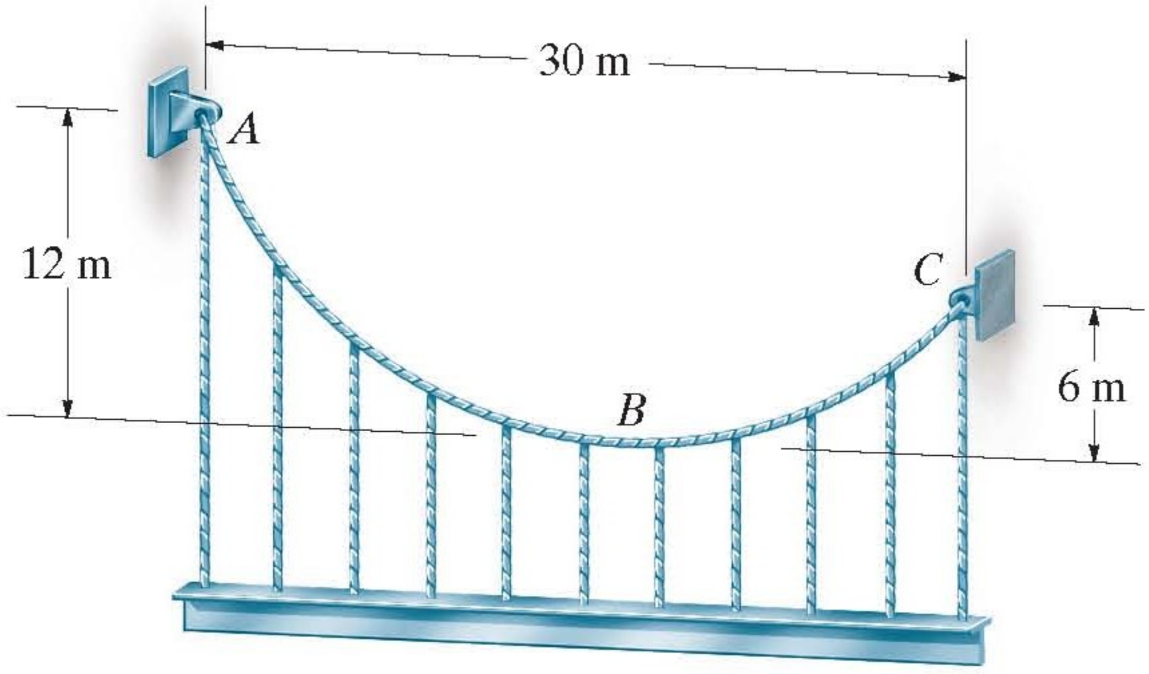
Determinate structure



hinge

Example 5.2

The cable supports a girder which weighs 12kN/m. Determine the tension in the cable at points A, B & C.

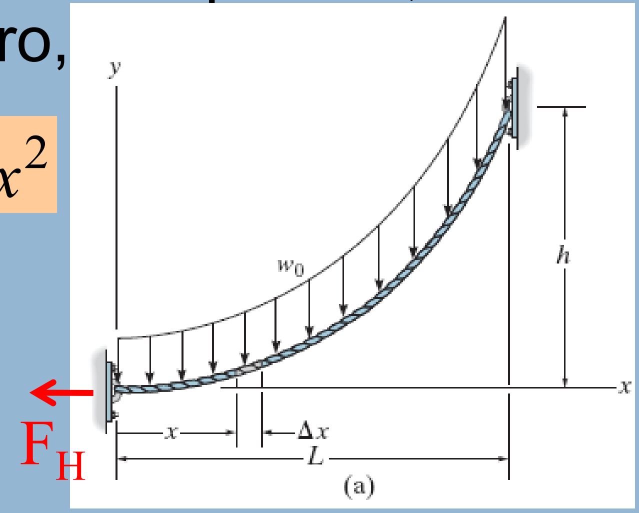


12kN/m

Solution

The origin of the coordinate axes is established at point B, the lowest point on the cable where slope is zero,

$$y = \frac{w_o}{2F_H}x^2 = \frac{12\text{kN/m}}{2F_H}x^2 = \frac{6}{F_H}x^2 \quad (1) = 0.0389x^2$$



Assuming point C is located x' from B:

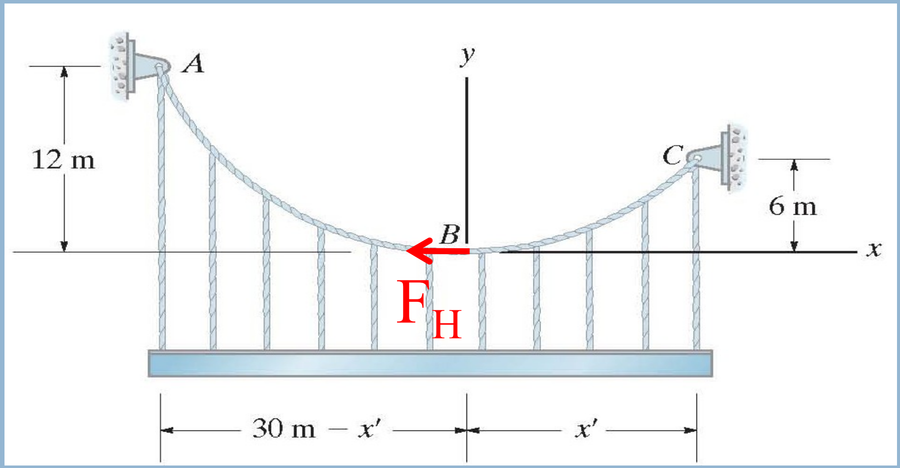
$$6 = \frac{6}{F_H}x'^2 \Rightarrow F_H = 1.0x'^2 \quad (2) = 154.4kN$$

From B to A:

$$12 = \frac{6}{F_H}[-(30 - x')]^2$$

$$12 = \frac{6}{1.0x'^2}[-(30 - x')]^2$$

$$x'^2 + 60x' - 900 = 0 \Rightarrow x' = 12.43m$$

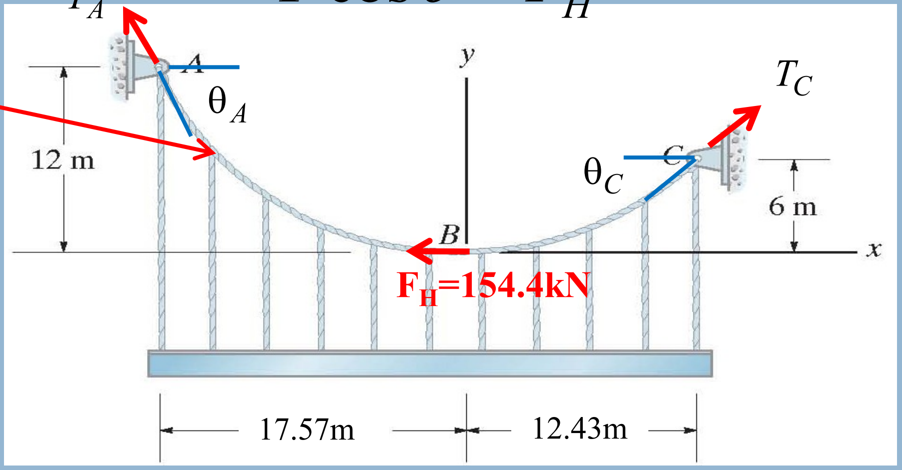


Solution

$$T\cos\theta = F_H$$

$$y = 0.0389x^2$$

$$\tan\theta = \frac{dy}{dx} = 0.0777x$$



$$\tan\theta_C = \left.\frac{dy}{dx}\right|_{x=12.43} = 0.966$$

$$\theta_C = 44.0^o$$

$$T_C = \frac{F_H}{\cos\theta_C} = \frac{154.4}{\cos 44.0^o} = 214.6kN$$

$$\tan\theta_A = \left.\frac{dy}{dx}\right|_{x=-17.57} = -1.366$$

$$\theta_A = -53.79^o$$

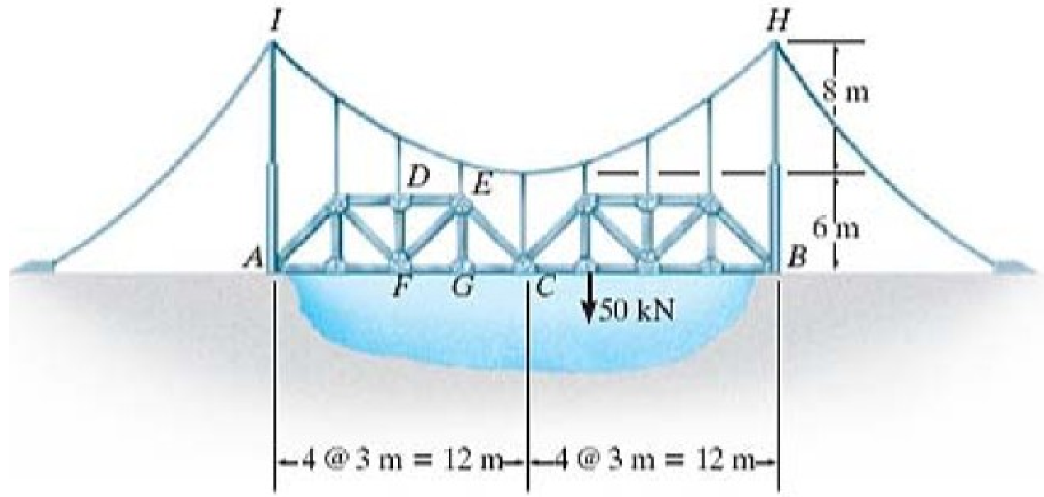
$$T_A = \frac{F_H}{\cos\theta_A} = 261.4kN$$

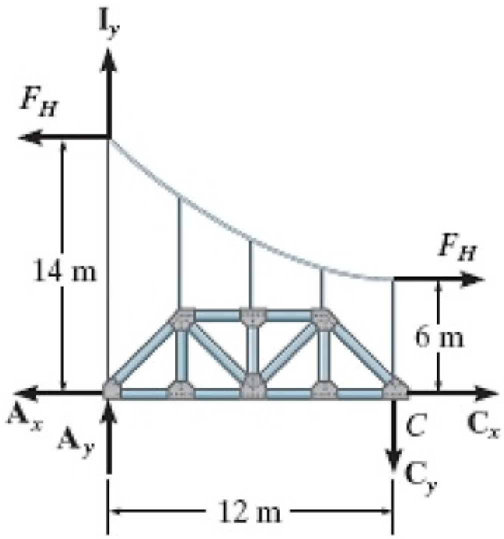
Example 5.3

- Determine the max tension in the cable IH

Assume the cable is parabolic

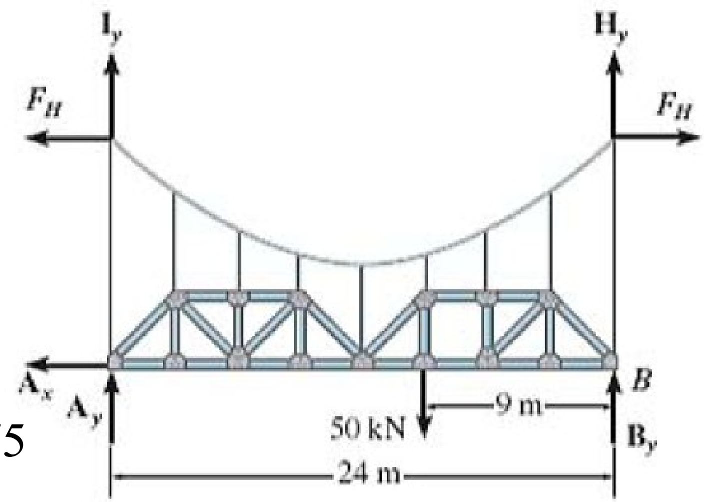
(under uniformly distributed load)





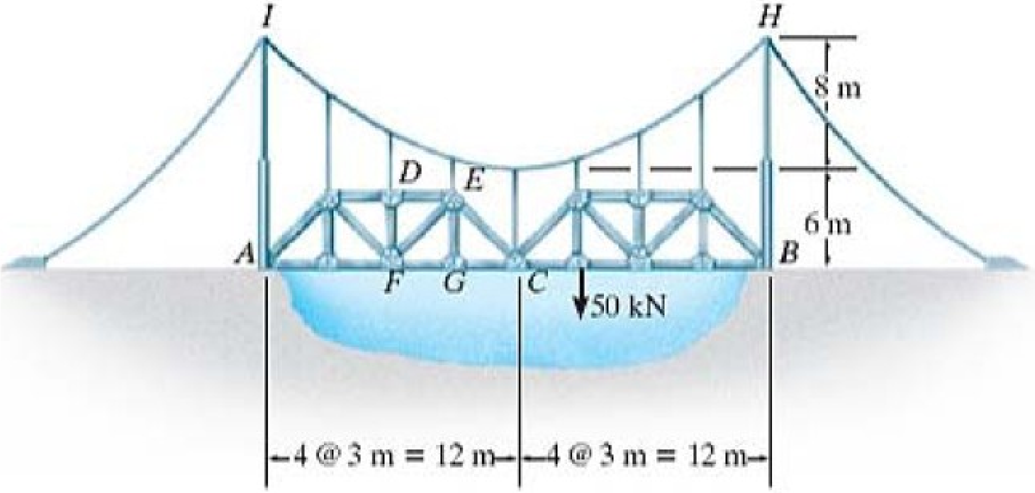
$$\sum M_C = 0 \quad \rightarrow \quad I_y + A_y = 0.667 F_H$$

$$F_H = 28.13\,kN$$

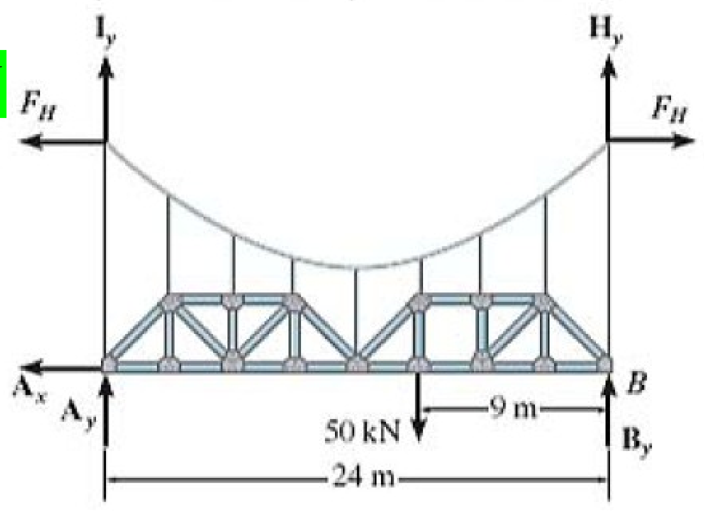


$$\sum M_B = 0 \quad \rightarrow \quad I_y + A_y = 18.75$$

Example 5.3



$$F_H = 28.13\ kN$$

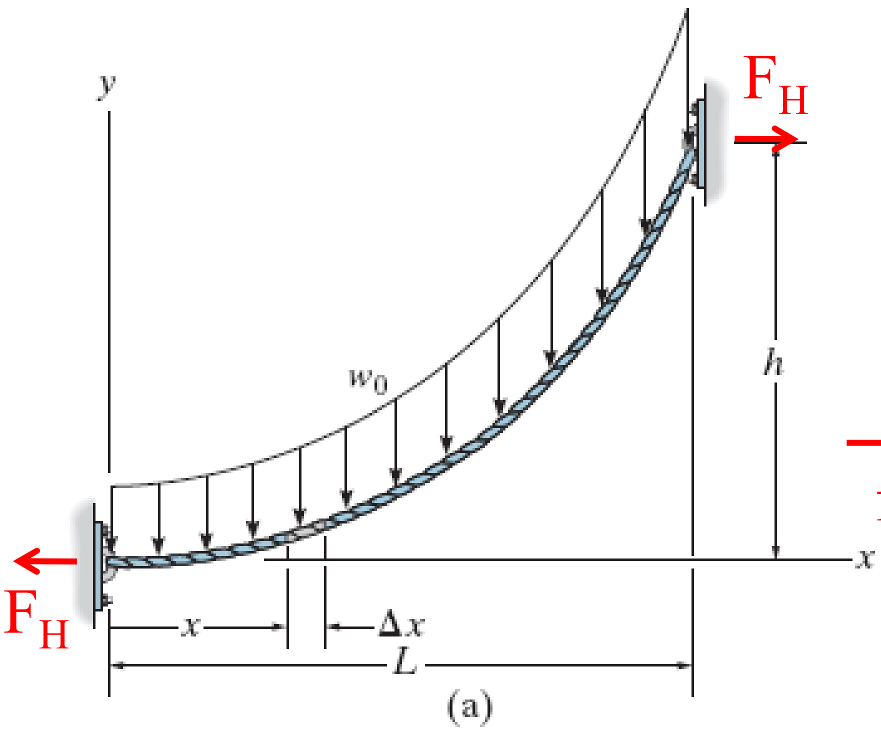


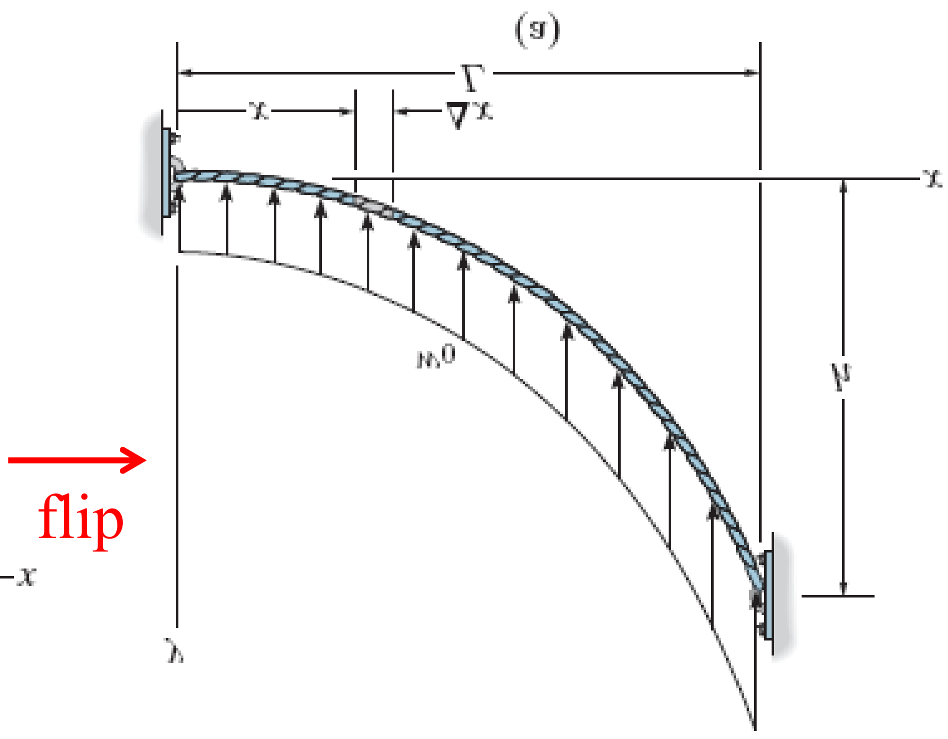
$$F_H = \frac{w_o L^2}{2h}$$

$$w_o = 3.13\ kN/m$$

$$T_{max} = w_o L \sqrt{1 + (L/2h)^2} = 46.9\ \text{kN}$$

Cable and Arch





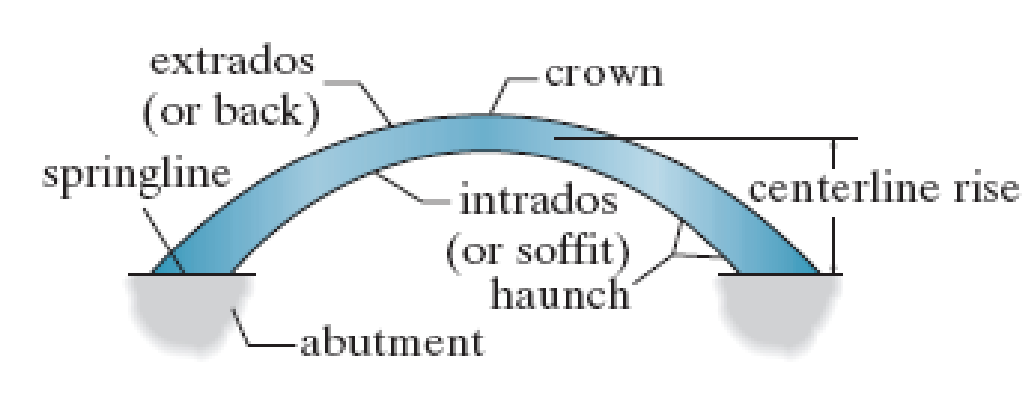
What if the load direction reverses?

Arches

- An arch acts as inverted cable so it receives compression
- An arch must also resist bendi d de

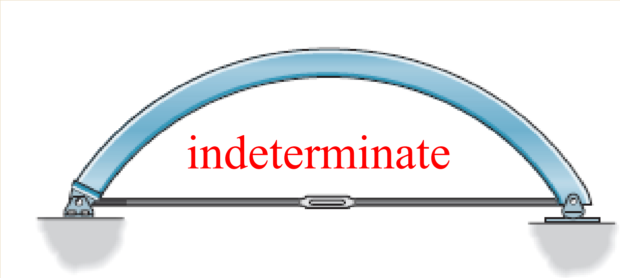
Arches

- Types of arches

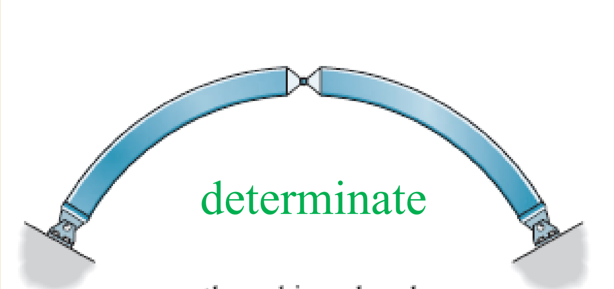


fixed arch
(a)

two-hinged arch
(b)

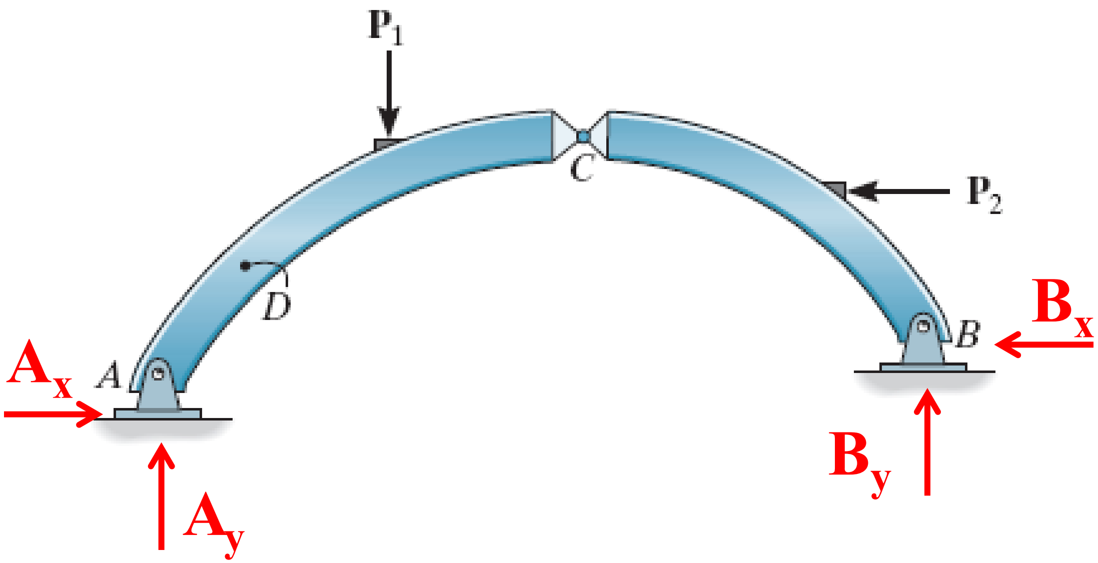


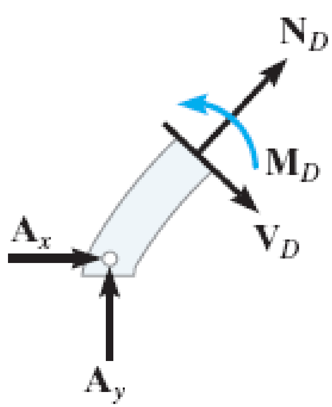
tied arch
(d)



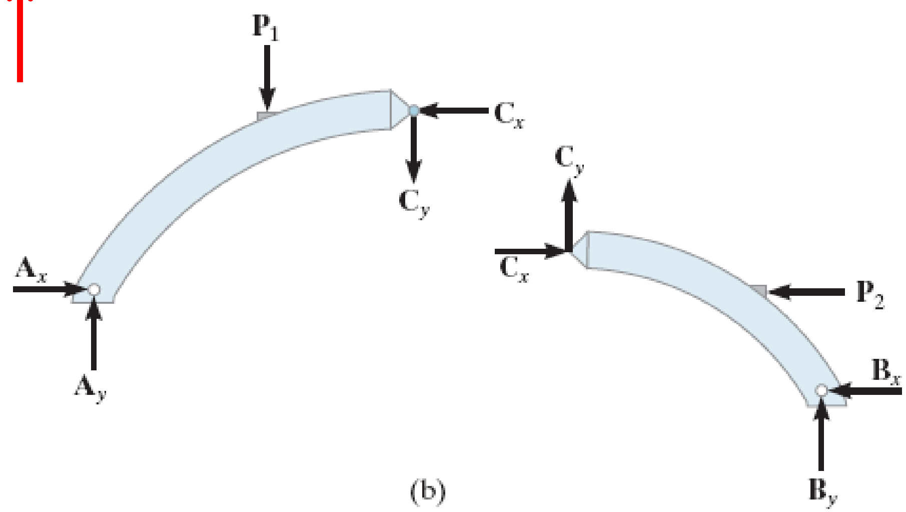
three-hinged arch
(c)

Three-Hinged Arch





(c)

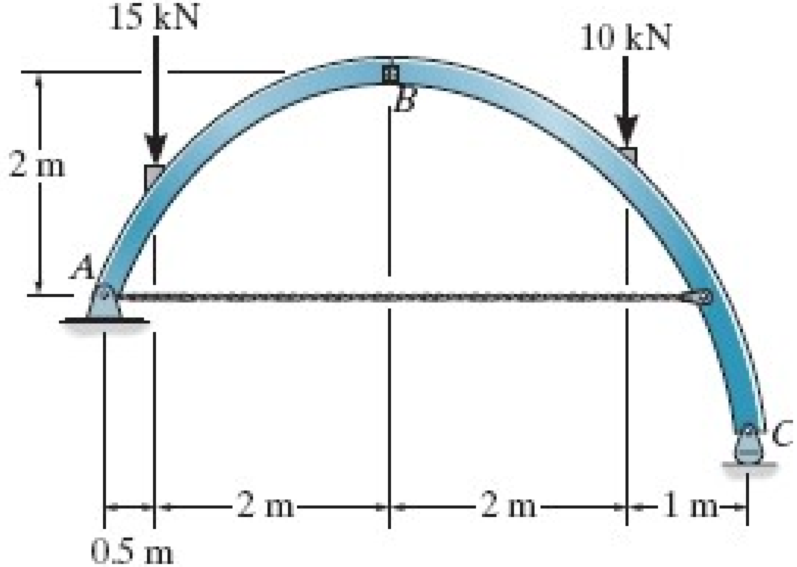


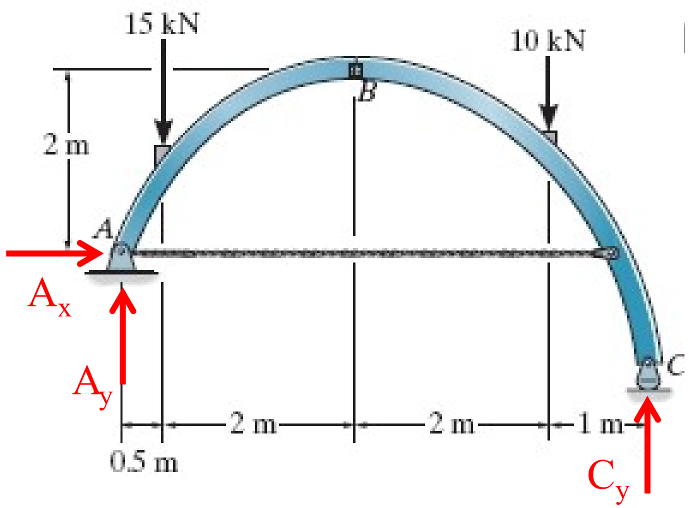
(b)

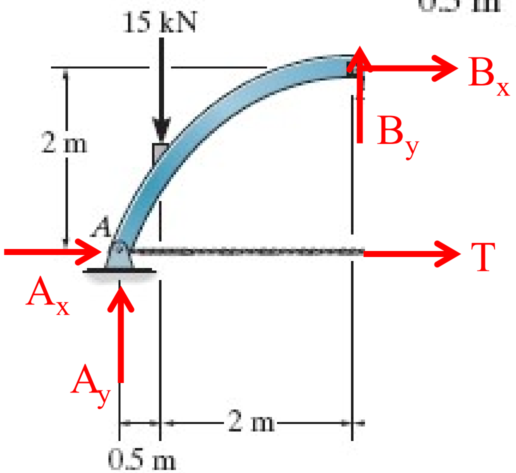
Problem 5-30

Determine reactions at A and C and the cable force

3 global Eqs



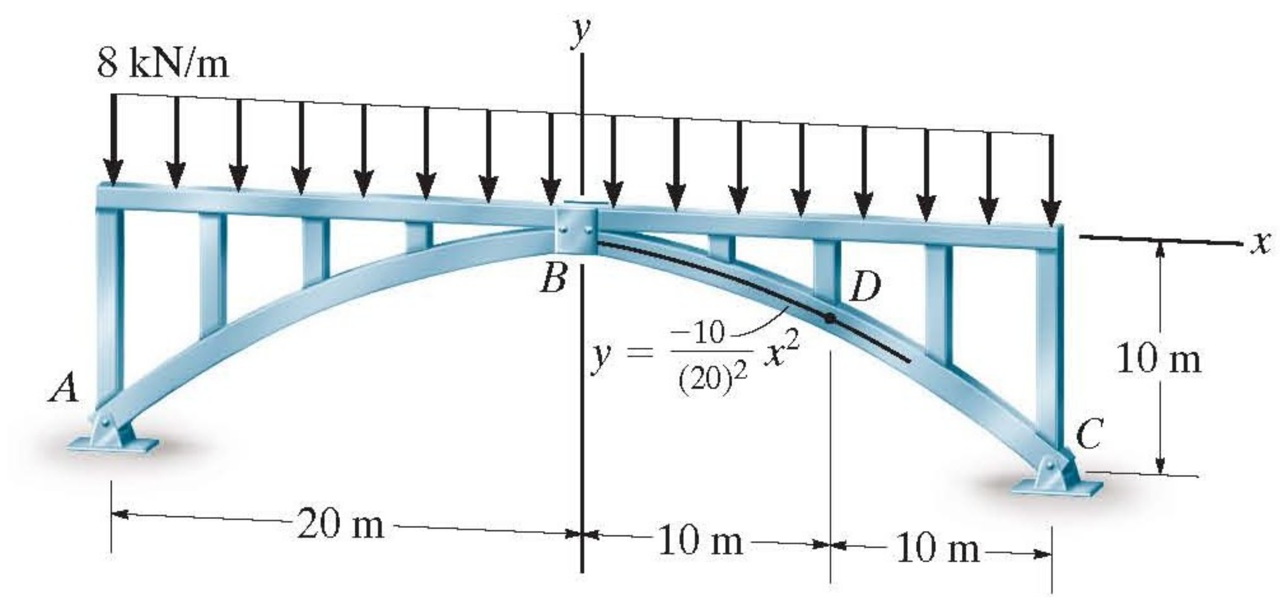




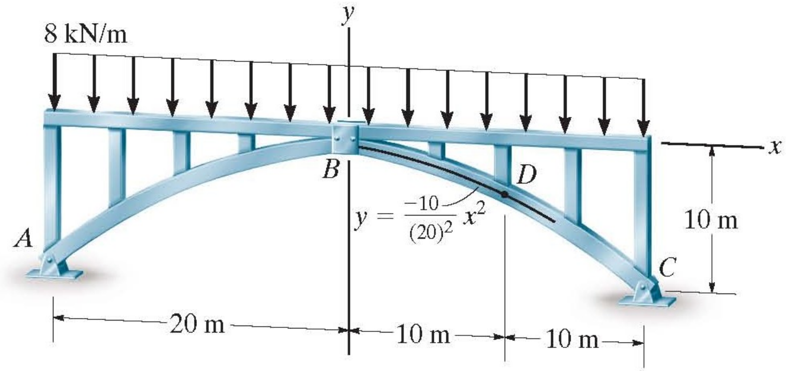
Example 5.4

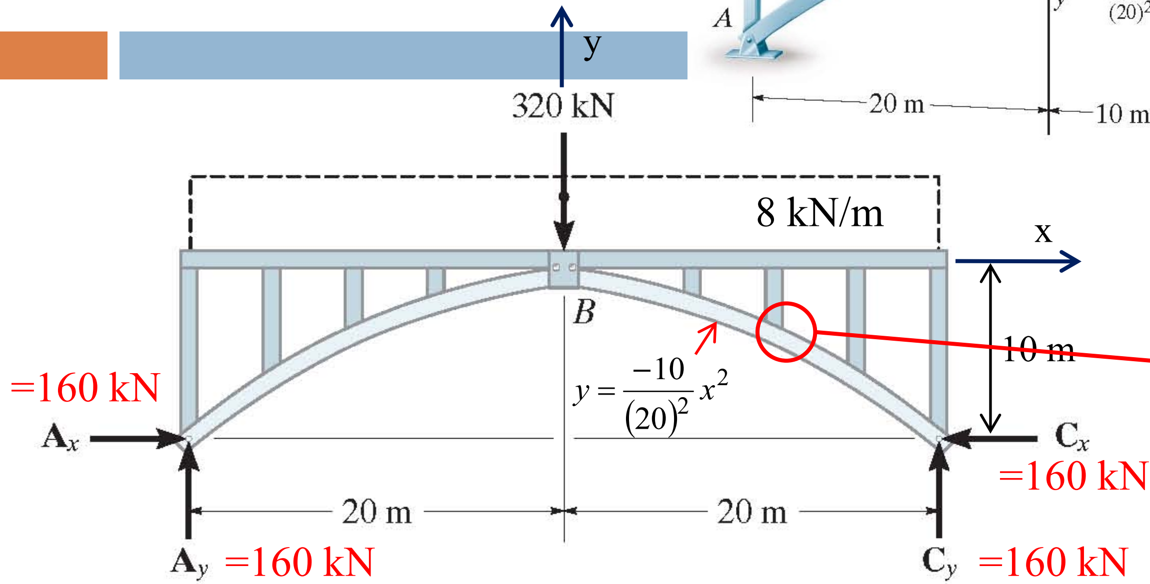
The three-hinged arch bridge has a parabolic shape and supports the uniform load. Assume the load is uniformly transmitted to the arch ribs.

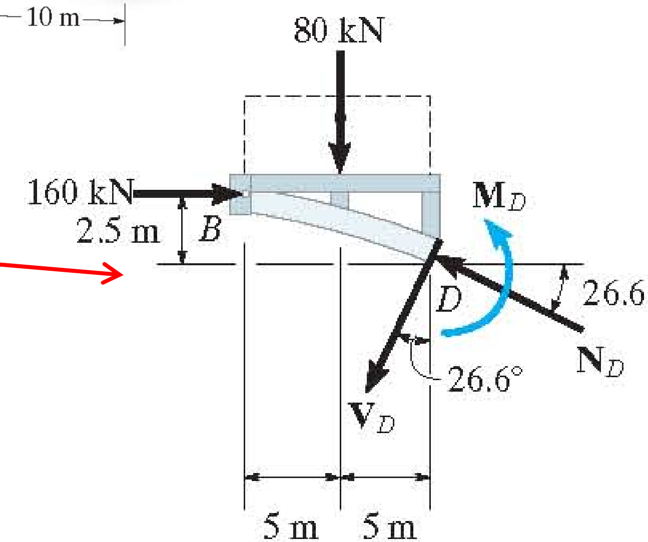
Show that the parabolic arch is subjected *only to axial compression* at an intermediate point such as point *D*.

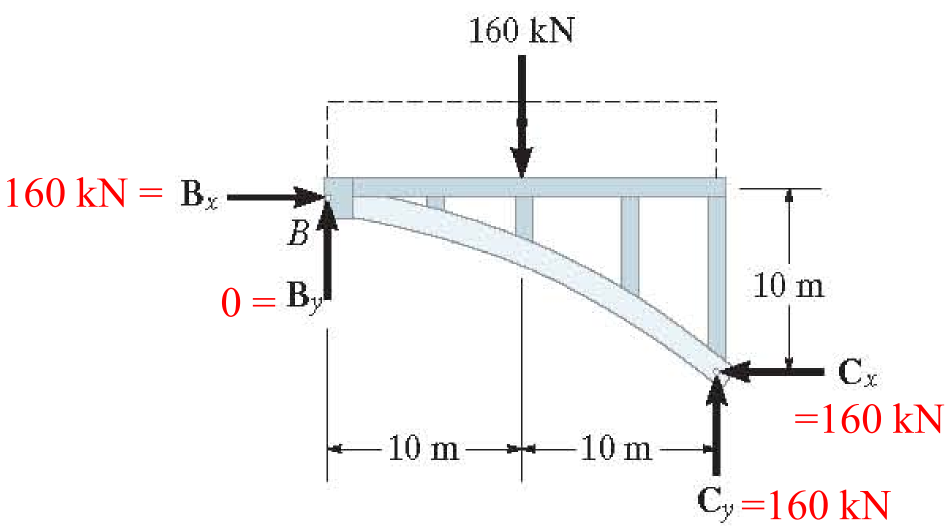


Solution









$$\tan\theta_D = \frac{dy}{dx} = \frac{-20}{(20)^2}x\bigg|_{x=10m} = -0.5$$

$$\theta_D = -26.6^{\circ}$$

$$N_D = 178.9kN$$

$$V_D = 0$$

$$M_D = 0$$