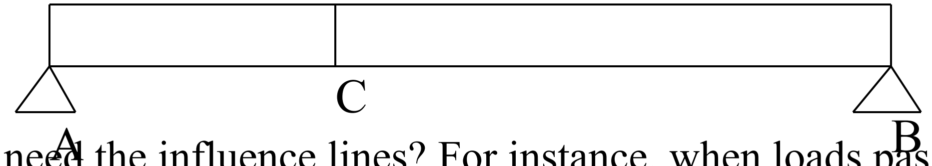
3. INFLUENCE LINES FOR STATICALLY DETERMINATE STRUCTURES

3. INFLUENCE LINES FOR STATICALLY DETERMINATE STRUCTURES - AN OVERVIEW

- Introduction - What is an influence line?
- Influence lines for beams
- Qualitative influence lines - Muller-Breslau Principle
- Influence lines for floor girders
- Influence lines for trusses
- Live loads for bridges
- Maximum influence at a point due to a series of concentrated loads
- Absolute maximum shear and moment

3.1 INTRODUCTION TO INFLUENCE LINES

- **Influence lines describe the variation of an analysis variable** (reaction, shear force, bending moment, twisting moment, deflection, etc.) **at a point** (say at C in Figure 6.1) ..
  …
  …



- Why do we need the influence lines? For instance, when loads pass over a structure, say a bridge, one needs to know when the maximum values of shear/reaction/bending-moment will occur at a point so that the section may be designed
- **Notations:**
  - **Normal Forces -** +ve forces cause +ve displacements in +ve directions
  - **Shear Forces -** +ve shear forces cause clockwise rotation & - ve shear force causes anti-clockwise rotation
  - **Bending Moments:** +ve bending moments cause "cup holding water" deformed shape

3.2 INFLUENCE LINES FOR BEAMS

- **Procedure:**

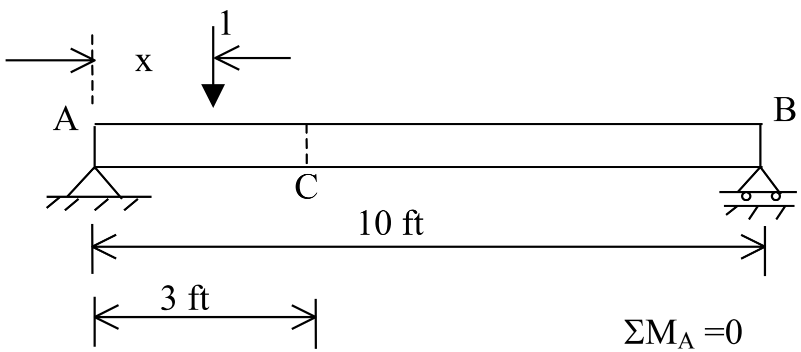
  (1) **Allow a unit load** (either 1b, 1N, 1kip, or 1 tonne) **to move over beam from left to right**

  (2) **Find the values** of shear force or bending moment, **at the point under consideration**, as the unit load moves over the beam from left to right

  (3) **Plot the values** of the shear force or bending moment, **over the length of the beam, computed for the point under consideration**

3.3 MOVING CONCENTRATED LOAD

3.3.1 Variation of Reactions R_A and R_B as functions of load position



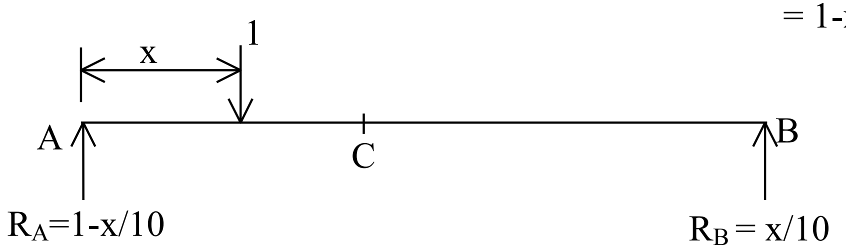
$$\Sigma M_A = 0$$

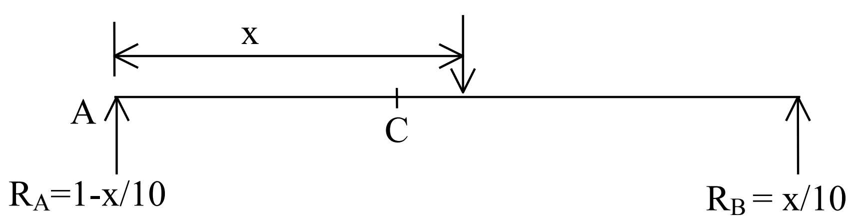
$$(R_B)(10) - (1)(x) = 0$$

$$R_B = x/10$$

$$R_A = 1 - R_B$$

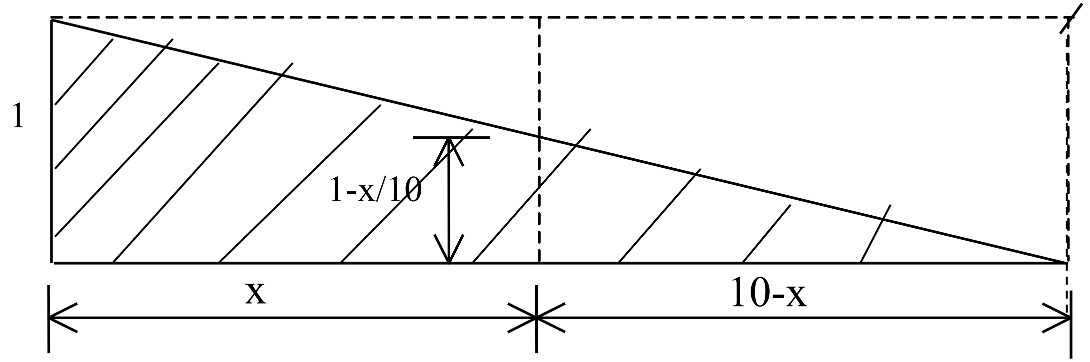
$$= 1 - x/10$$



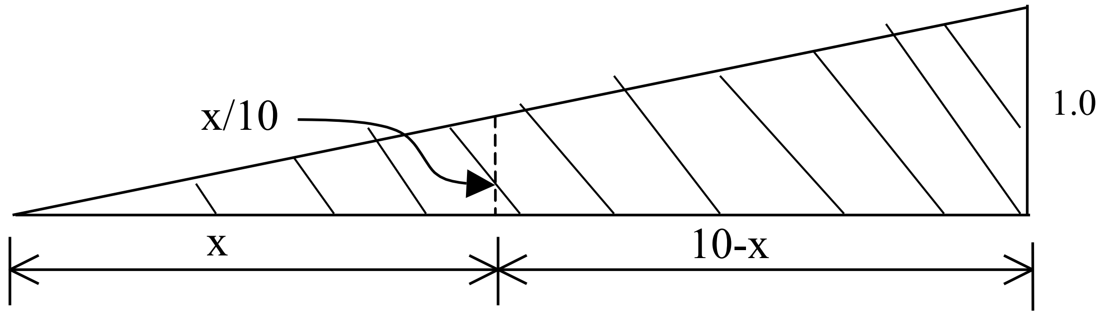


R_A occurs only at A; R_B occurs only at B

Influence line for R_A

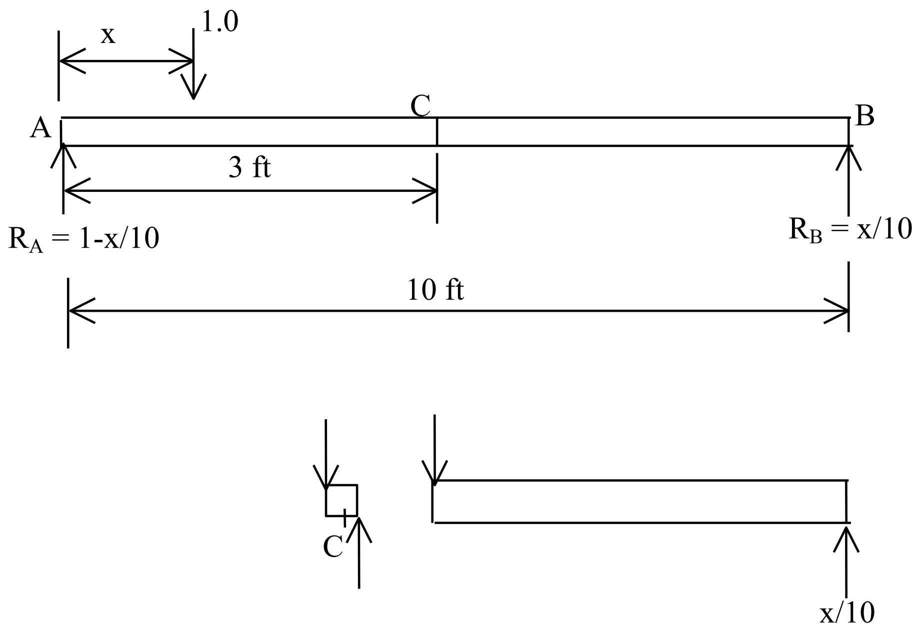


Influence line for R_B



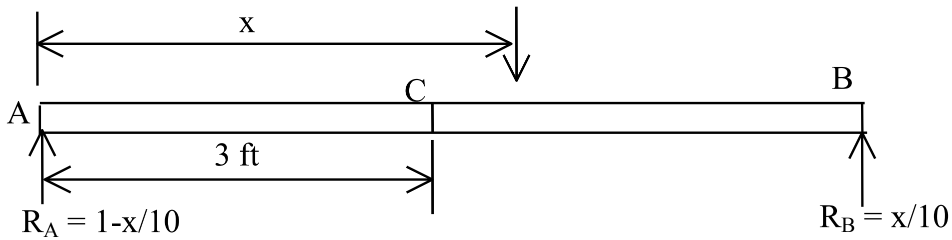
3.3.2 Variation of Shear Force at C as a function of load position

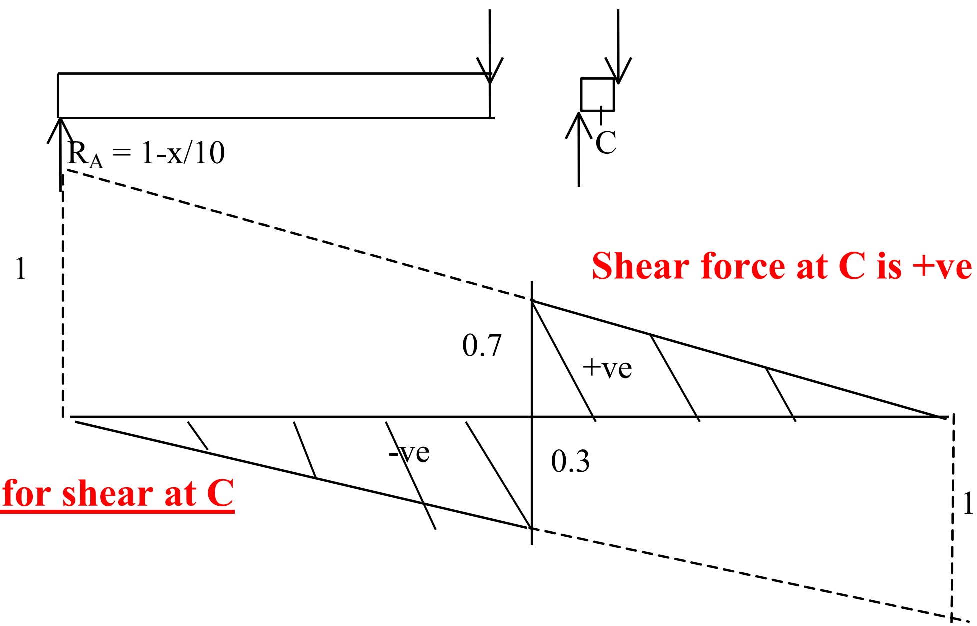
0 < x < 3 ft (unit load to the left of C)



Shear force at C is –ve, V_C =-x/10

3 < x < 10 ft (unit load to the right of C)



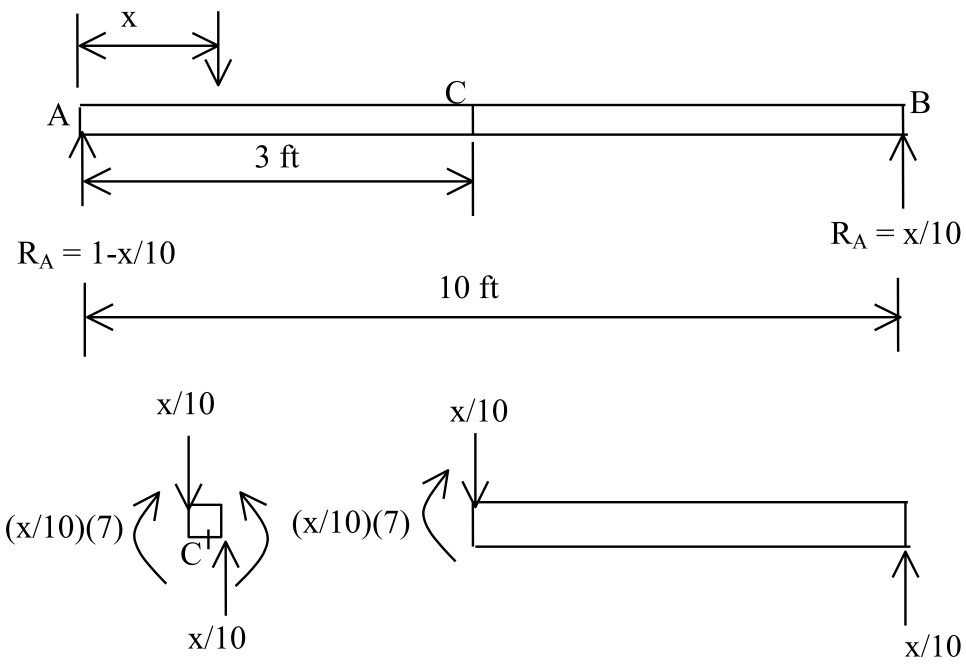


Shear force at C is +ve = 1-x/10

Influence line for shear at C

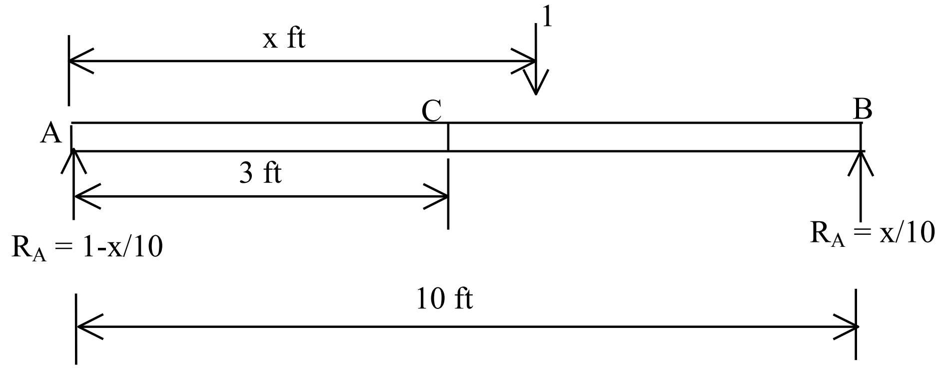
3.3.3 Variation of Bending Moment at C as a function of load position

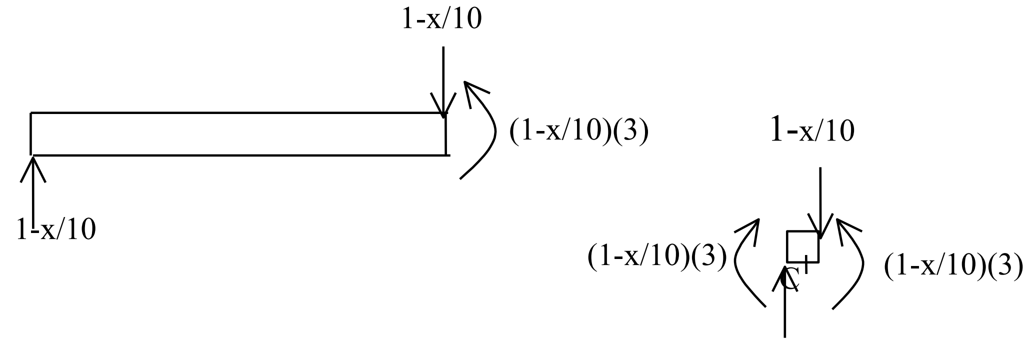
0 < x < 3.0 ft (Unit load to the left of C)



Bending moment is +ve at C

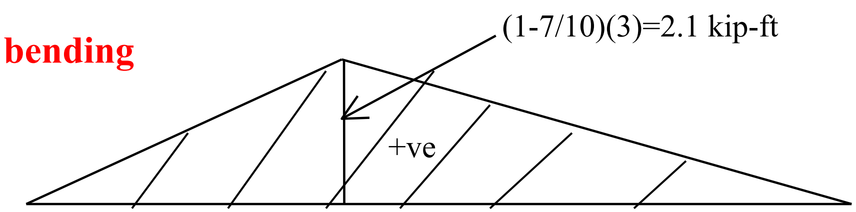
3 < x < 10 ft (Unit load to the right of C)





Moment at C is +ve

Influence line for bending Moment at C



3.4 QUALITATIVE INFLUENCED LINES - MULLER-BRESLAU'S PRINCIPLE

- The principle **gives only a procedure** to determine of the influence line of a parameter for a determinate or an indeterminate structure
- But **using the basic understanding of the influence lines**, the **magnitudes** of the influence lines also **can be computed**
- In order **to draw the shape of the influence lines properly**, the **capacity of the beam to resist the parameter investigated** (reaction, bending moment, shear force, etc.), **at that point, must be removed**
- The principle states that:**The influence line for a parameter** (say, reaction, shear or bending moment), at a point, **is to the same scale as the deflected shape of the beam**, when the beam is acted upon by that parameter.
  - The **capacity of the beam to resist that parameter**, at that point, **must be removed.**
  - **Then allow the beam to deflect under that parameter**
  - Positive **directions of the forces are the same** as before

3.5 PROBLEMS - 3.5.1 Influence Line for a Determinate Beam by Muller-Breslau's Method

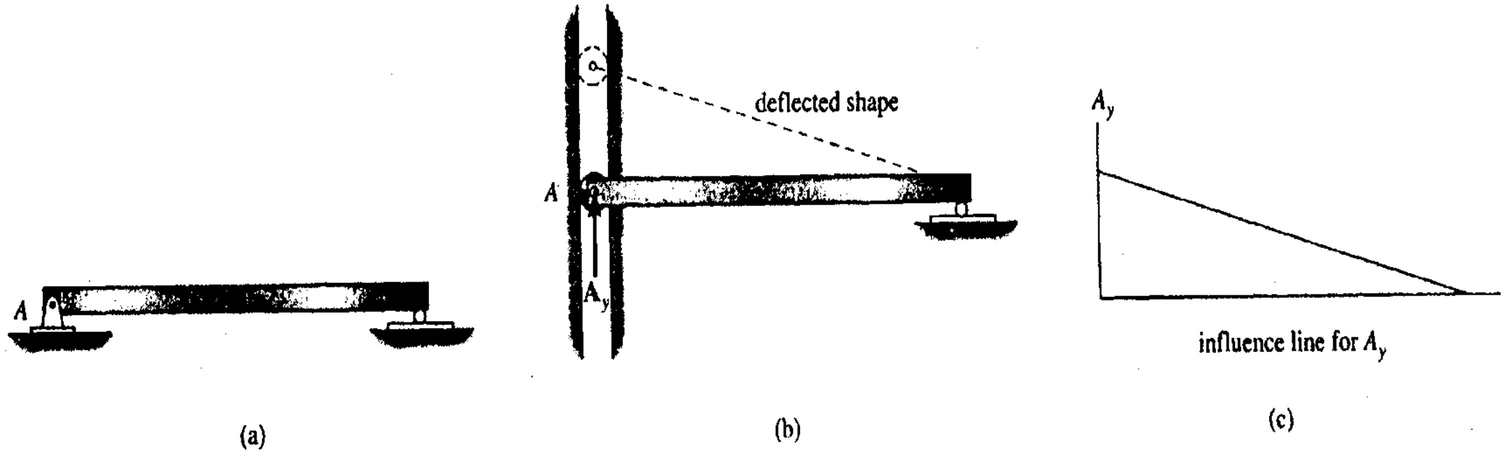


Fig. 6–12

Influence line for Reaction at A

3.5.2 Influence Lines for a Determinate Beam by Muller-Breslau's Method

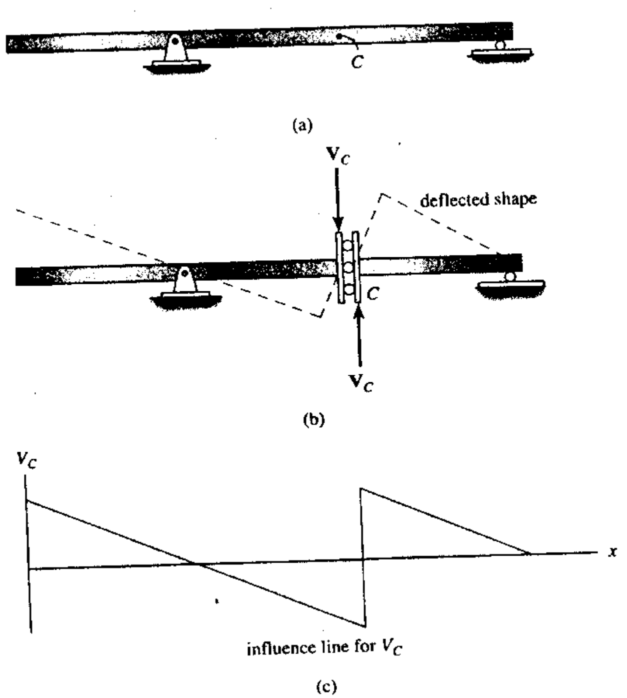


Fig. 6–13

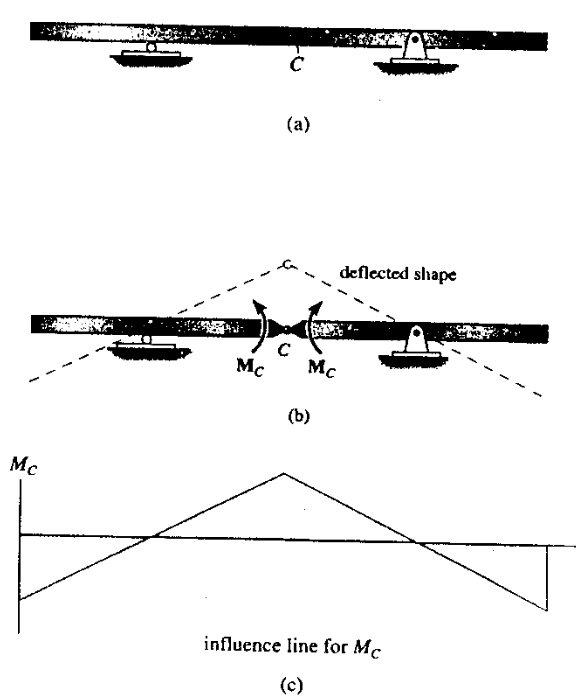


Fig. 6–14

<u>Influence Line for Shear at C</u>

<u>Influence Line for Bending Moment at C</u>

3.5.3 Influence Lines for an Indeterminate Beam by Muller-Breslau's Method

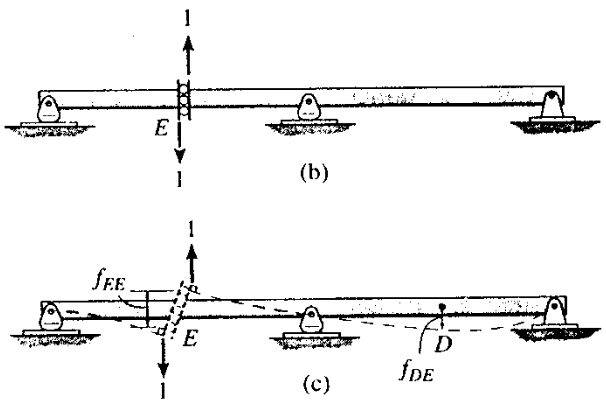


Fig. 9–24

Influence Line for Shear at E

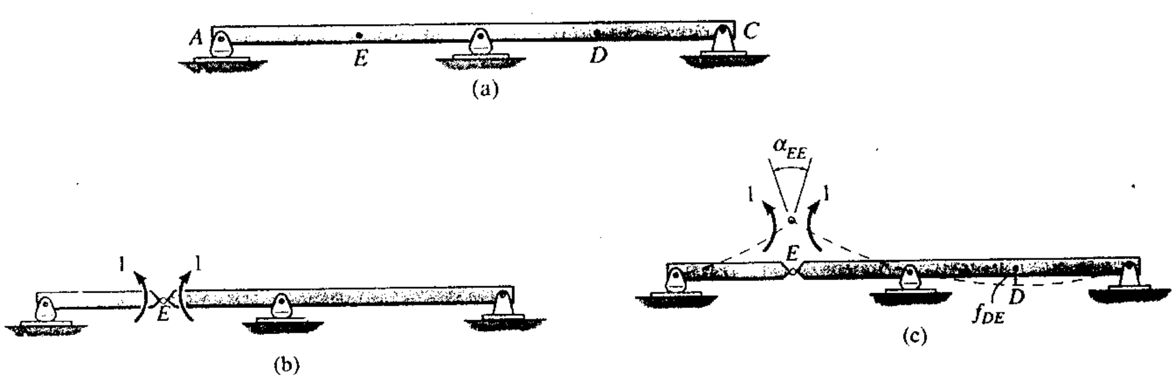
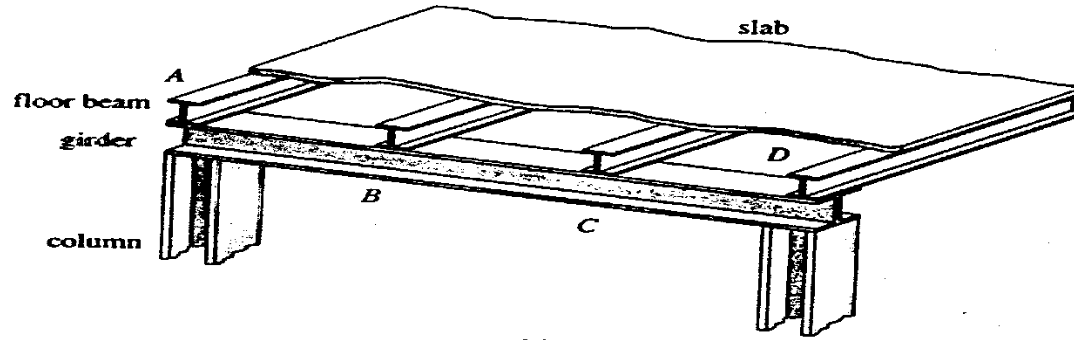


Fig. 9–25

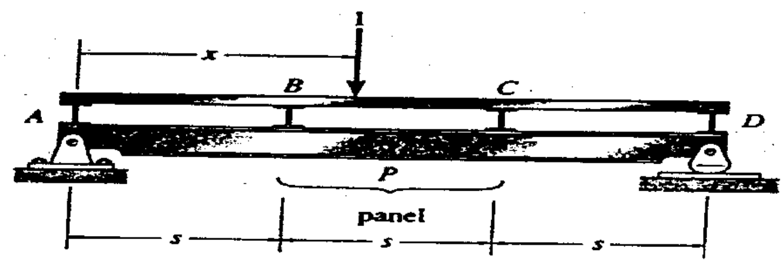
Influence Line for Bending Moment at E

3.6 INFLUENCE LINE FOR FLOOR GIRDERS

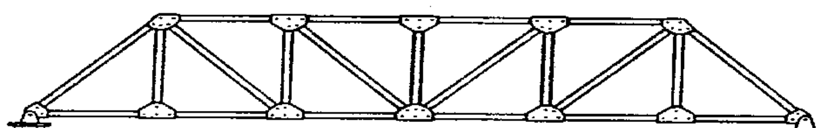
Floor systems are constructed as shown in figure below,



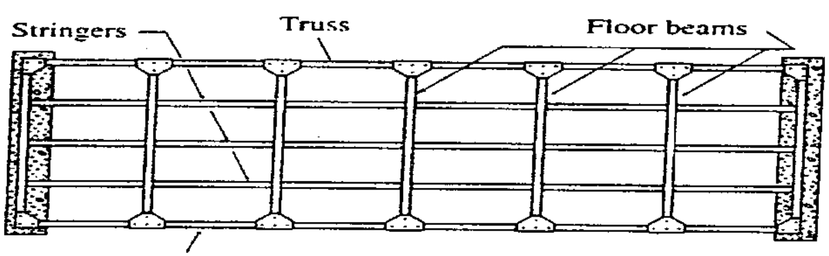
(a)



(b)



Elevation



Plan (deck not shown)

Fig. 8.14

3.6 INFLUENCE LINES FOR FLOOR GIRDERS (Cont'd)

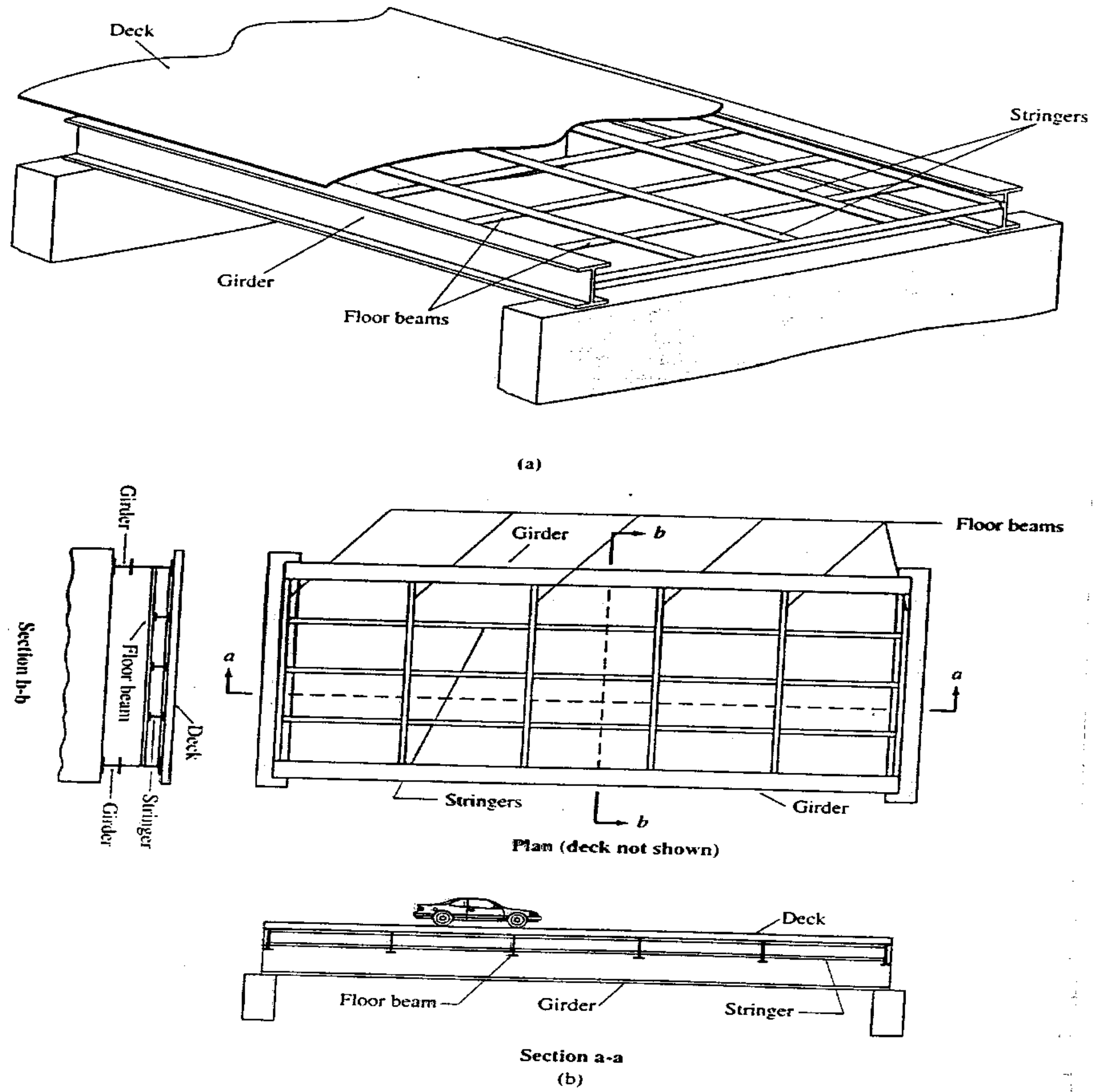
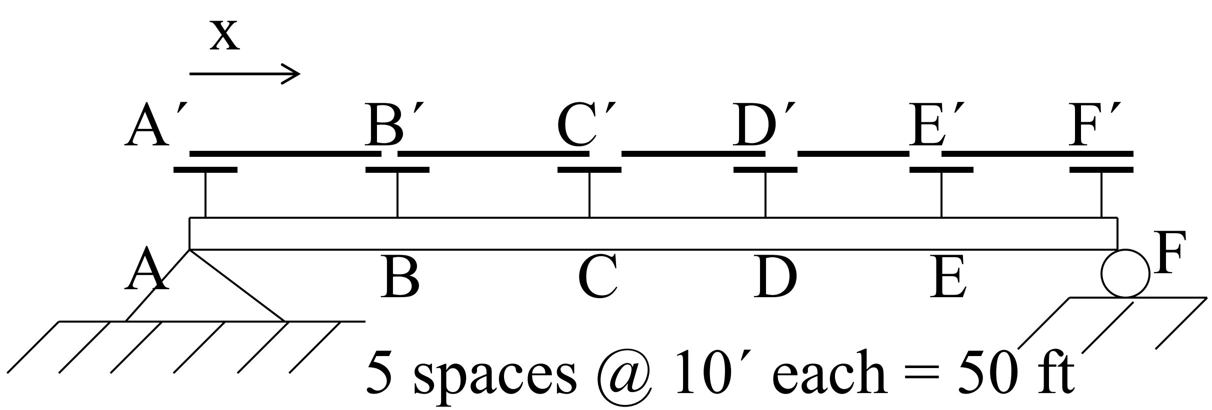


Fig. 8.10

3.6 INFLUENCE LINES FOR FLOOR GIRDERS (Cont'd)

3.6.1 Force Equilibrium Method:

Draw the Influence Lines for: (a) Shear in panel CD of the girder; and (b) the moment at E.



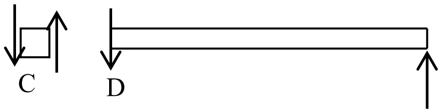
3.6.2 Place load over region A´B´ (0 < x < 10 ft)

Find the shear over panel CD

V_{CD}= - x/50

At x=0, V_{CD} = 0

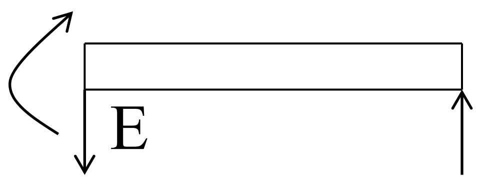
At x=10, V_{CD} = -0.2



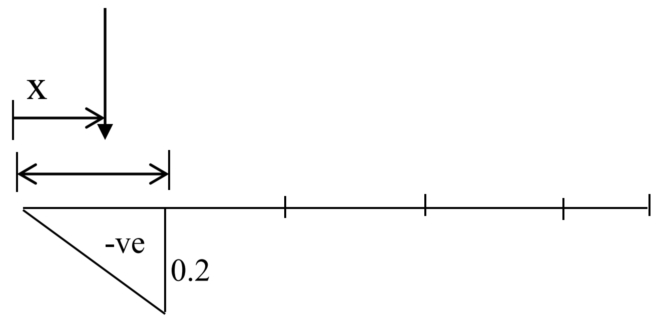
Find moment at E = +(x/50)(10)=+x/5

At x=0, M_E=0

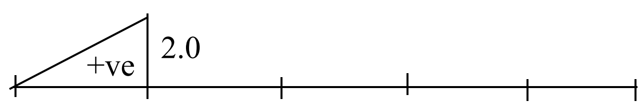
At x=10, M_E=+2.0



Continuation of the Problem



I. L. for V_{CD}



I. L. for M_E

Problem Continued -

3.6.3 Place load over region B′C′ (10 ft < x < 20ft)

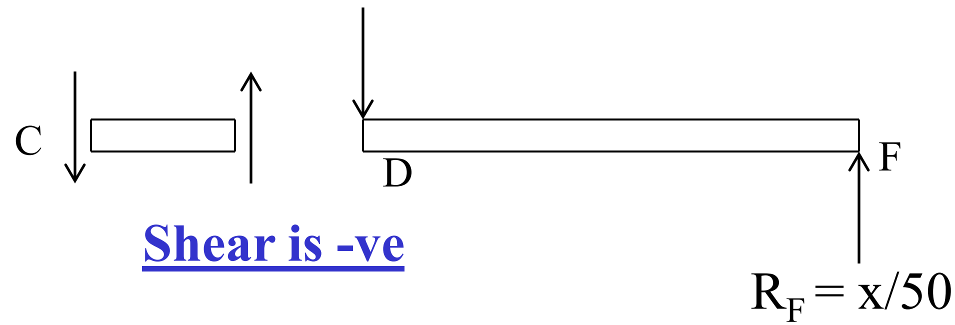
V_{CD} = -x/50 kip

At x = 10 ft

V_{CD} = -0.2

At x = 20 ft

V_{CD} = -0.4

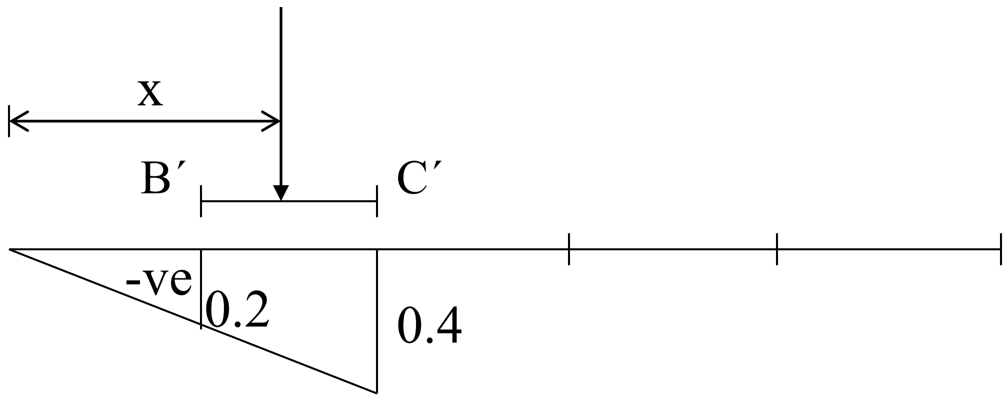


M_E = +(x/50)(10)

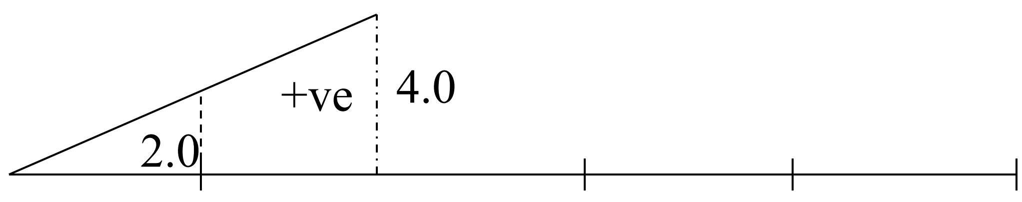
= +x/5 kip.ft

At x = 10 ft, M_E = +2.0 kip.ft

At x = 20 ft, M_E = +4.0 kip.ft



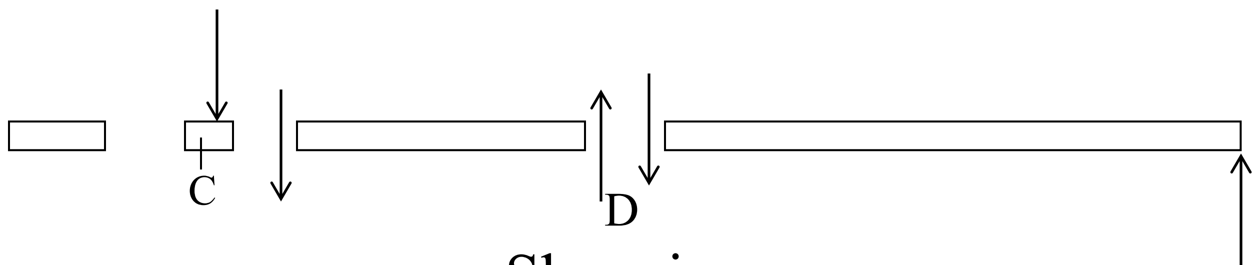
I. L. for V_{CD}



I. L. for M_E

3.6.4 Place load over region C′D′ (20 ft < x < 30 ft)

When the load is at C’ (x = 20 ft)

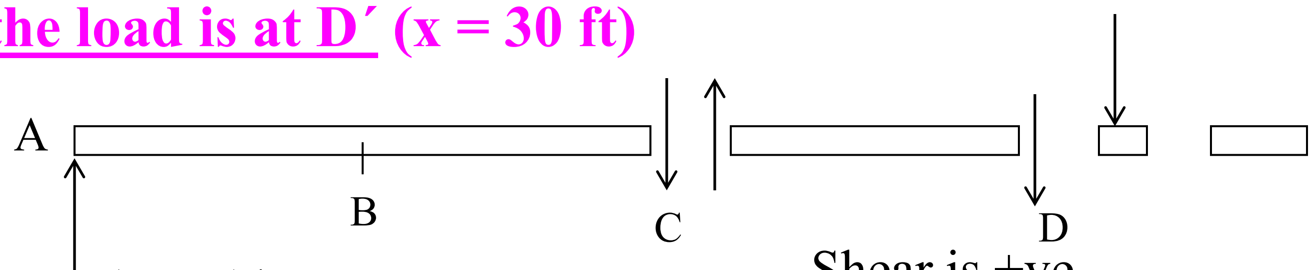


Shear is -ve

$R_F = 20/50$
$= 0.4$

$V_{CD} = -0.4$ kip

When the load is at D′ (x = 30 ft)

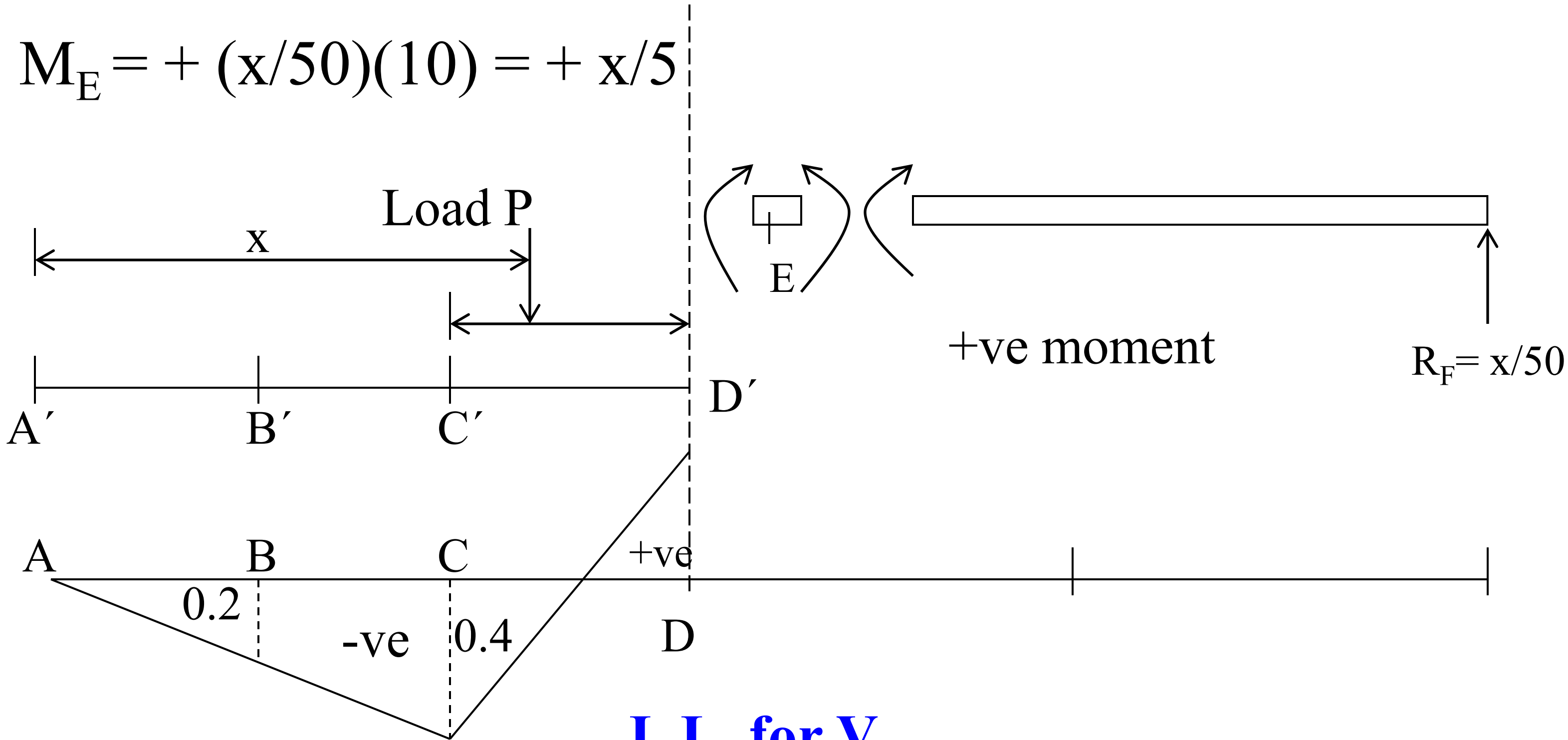


Shear is +ve

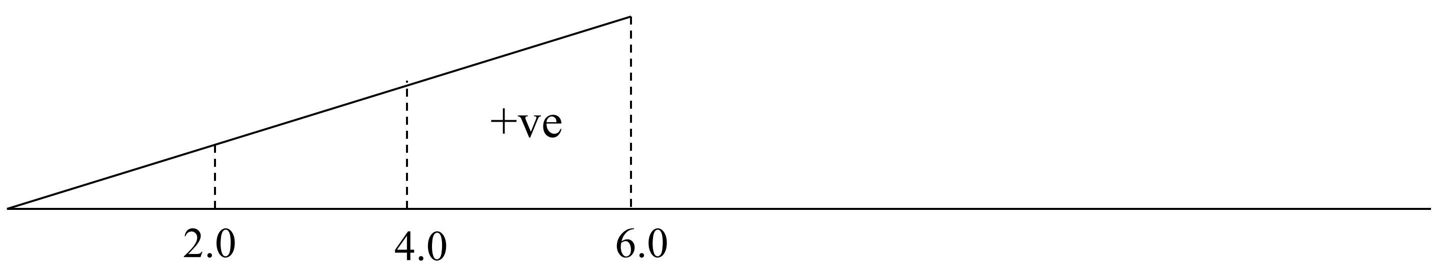
$R_A = (50 - x)/50$

$V_{CD} = + 20/50$
$= + 0.4$ kip

$$M_E = +\,(x/50)(10) = +\,x/5$$



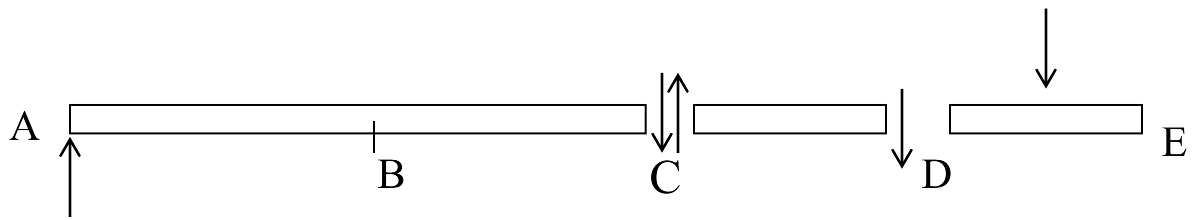
I. L. for V_{CD}



I. L. for M_E

3.6.5 Place load over region D′E′ (30 ft < x < 40 ft)

$V_{CD} = + (1\text{-}x/50)$ kip



$R_A = (1\text{-}x/50)$

Shear is +ve

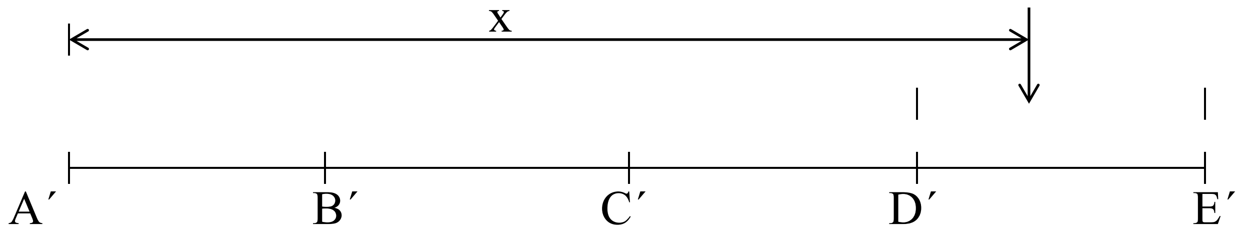
$M_E = +(x/50)(10)$
$= + x/5$ kip.ft

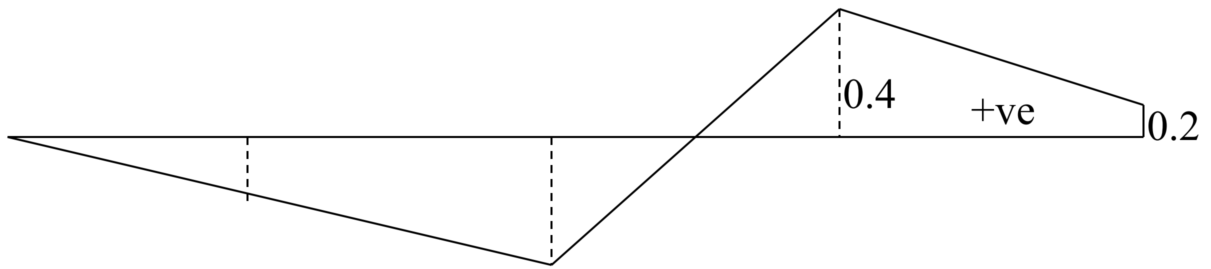
Moment is +ve

$R_F = x/50$

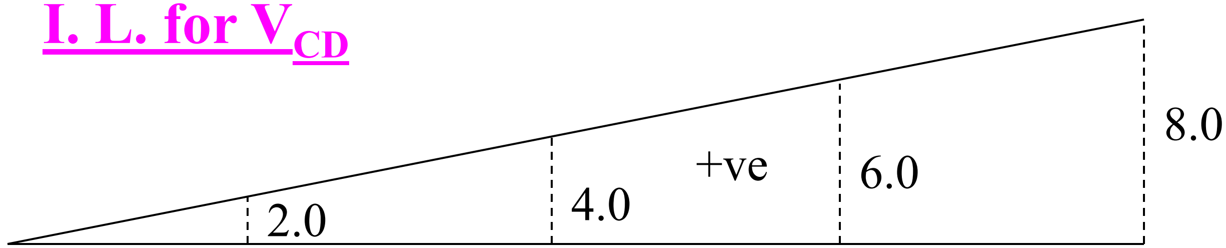
At x = 30 ft, $M_E = +6.0$
At x = 40 ft, $M_E = +8.0$

Problem continued





I. L. for V_{CD}

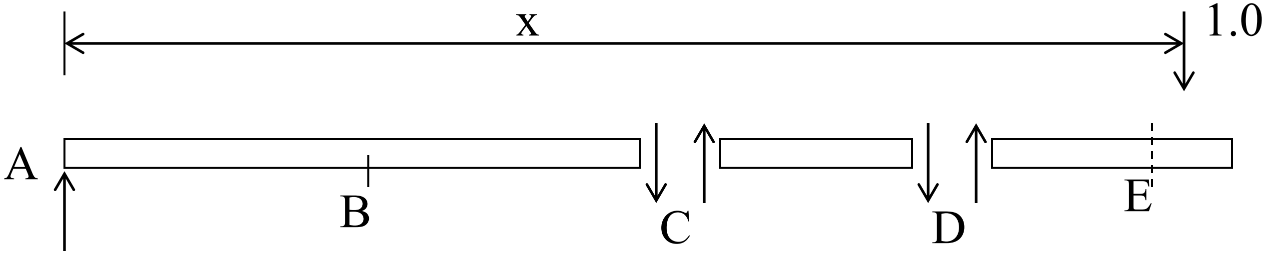


I. L. for M_E

3.6.6 Place load over region E′F′ (40 ft < x < 50 ft)

$V_{CD} = +\ 1\text{-}x/50$ At $x = 40$ ft, $V_{CD} = +\ 0.2$

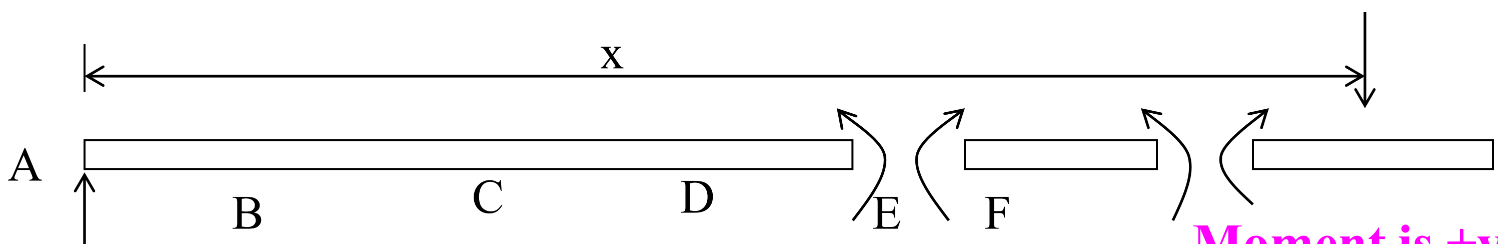
At $x = 50$ ft, $V_{CD} = 0.0$



$R_A = 1\text{-}x/50$

Shear is +ve

$M_E = +\ (1\text{-}x/50)(40) = (50\text{-}x)*40/50 = +(4/5)(50\text{-}x)$

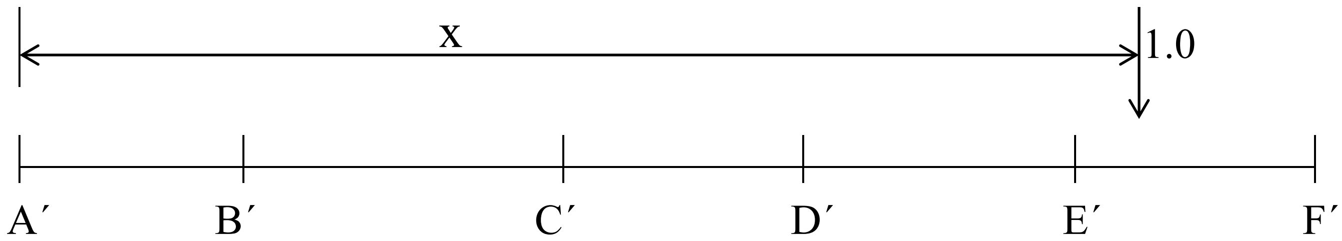


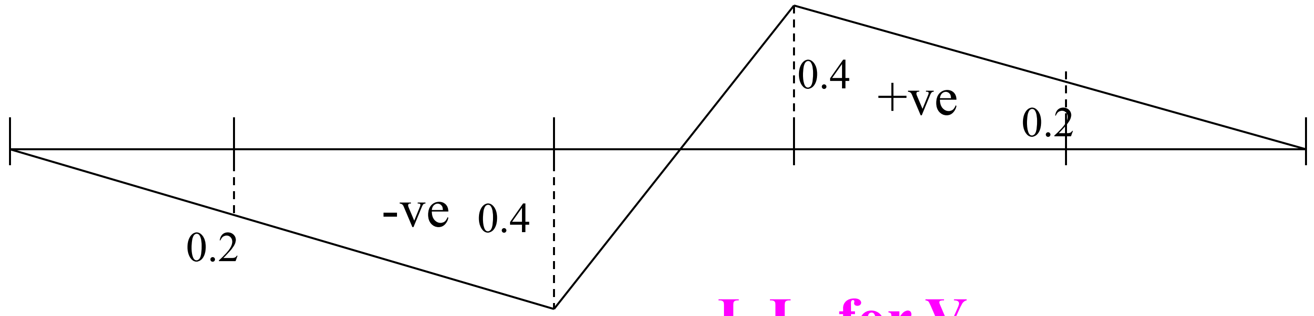
Moment is +ve

$R_A = 1\text{-}x/50$

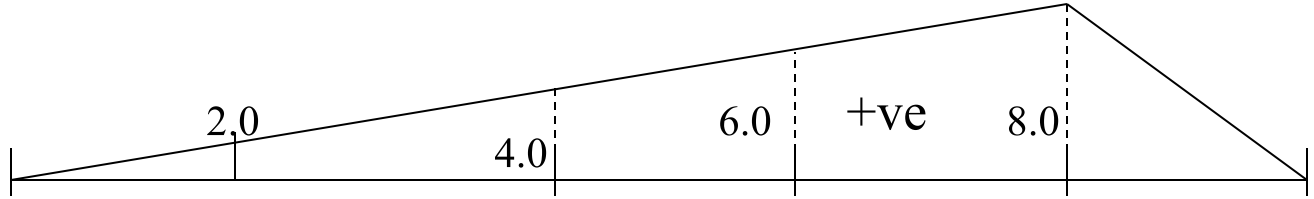
At $x = 40$ ft, $M_E = +\ 8.0$ kip.ft

At $x = 50$ ft, $M_E = 0.0$





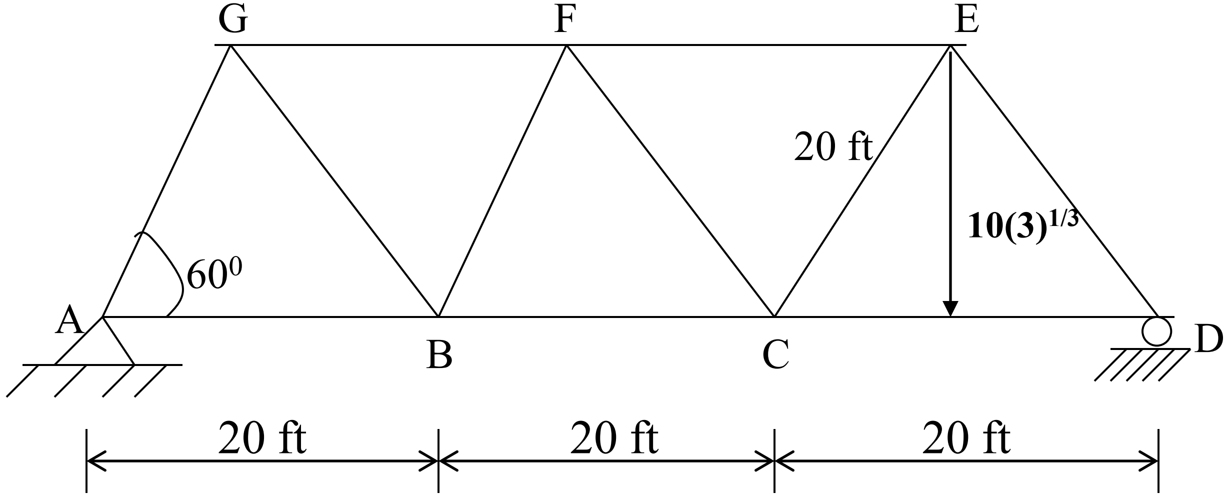
I. L. for V_{CD}



I. L. for M_E

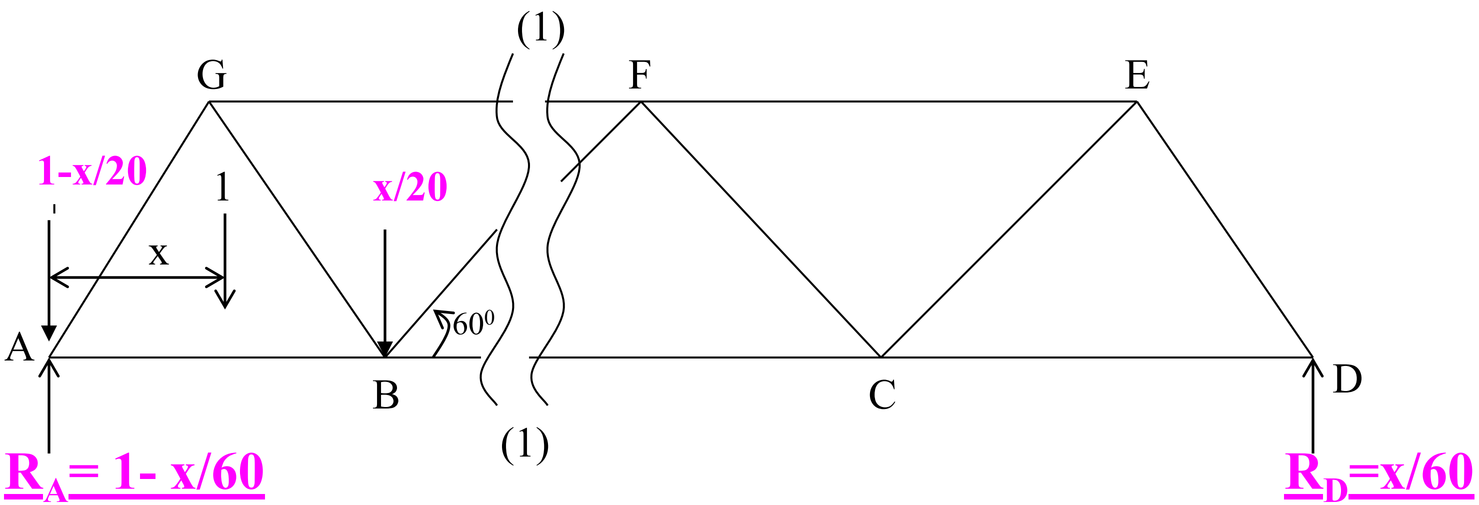
3.7 INFLUENCE LINES FOR TRUSSES

Draw the influence lines for: (a) Force in Member GF; and (b) Force in member FC of the truss shown below in Figure below



Problem 3.7 continued -
3.7.1 Place unit load over AB

(i) To compute GF, cut section (1) - (1)



At x = 0,
$F_{GF} = 0$
At x = 20 ft
$F_{GF} = - 0.77$

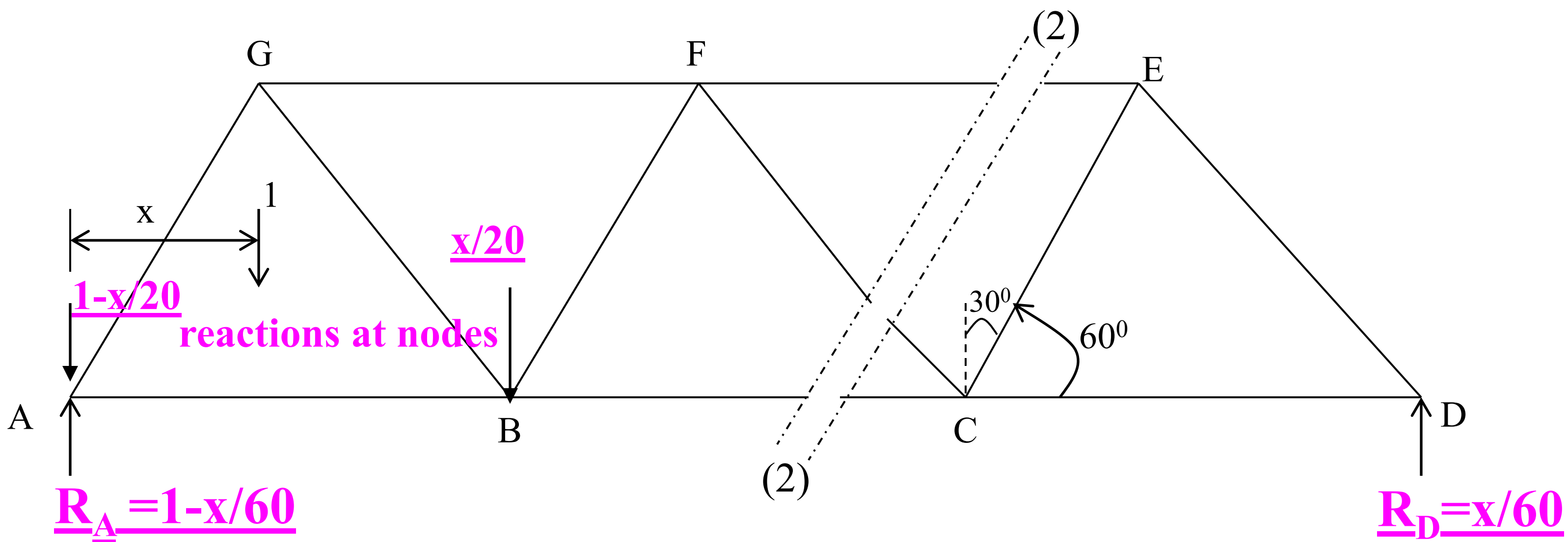
Taking moment about B to its right,

$(R_D)(40) - (F_{GF})(10\sqrt{3}) = 0$

$F_{GF} = (x/60)(40)(1/ 10\sqrt{3}) = x/(15 \sqrt{3})$ (-ve)

PROBLEM 3.7 CONTINUED -

(ii) To compute F_{FC}, cut section (2) - (2)



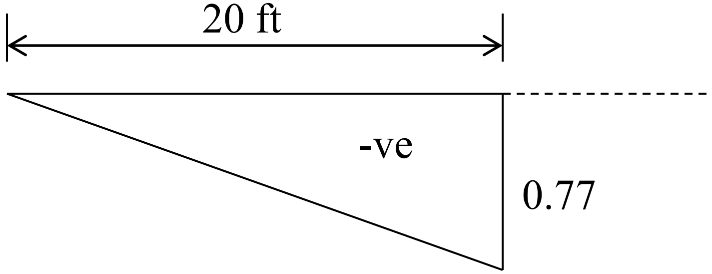
Resolving vertically over the right hand section

$$F_{FC} \cos 30^0 - R_D = 0$$

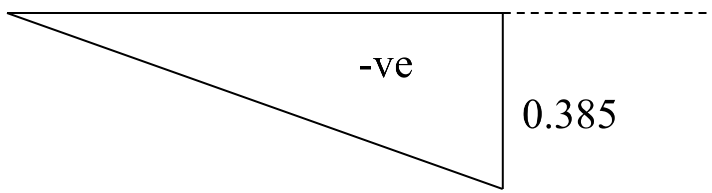
$$F_{FC} = R_D/\cos 30 = (x/60)(2/\sqrt{3}) = x/(30\sqrt{3}) \text{ (-ve)}$$

At x = 0, $F_{FC} = 0.0$

At x = 20 ft, $F_{FC} = -0.385$



I. L. for F_{GF}



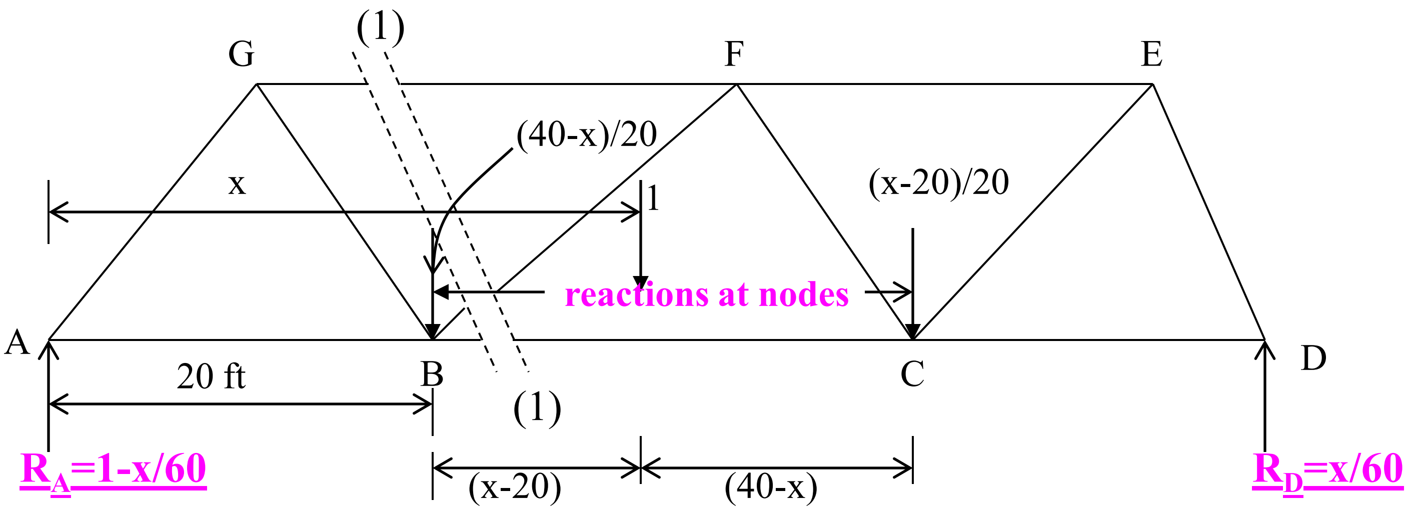
I. L. for F_{FC}

PROBLEM 3.7 Continued -

3.7.2 Place unit load over BC (20 ft < x <40 ft)

[Section (1) - (1) is valid for 20 < x < 40 ft]

(i) To compute F_{GF} use section (1) -(1)



Taking moment about B, to its left,

$$(R_A)(20) - (F_{GF})(10\sqrt{3}) = 0$$

$$F_{GF} = (20R_A)/(10\sqrt{3}) = (1-x/60)(2/\sqrt{3})$$

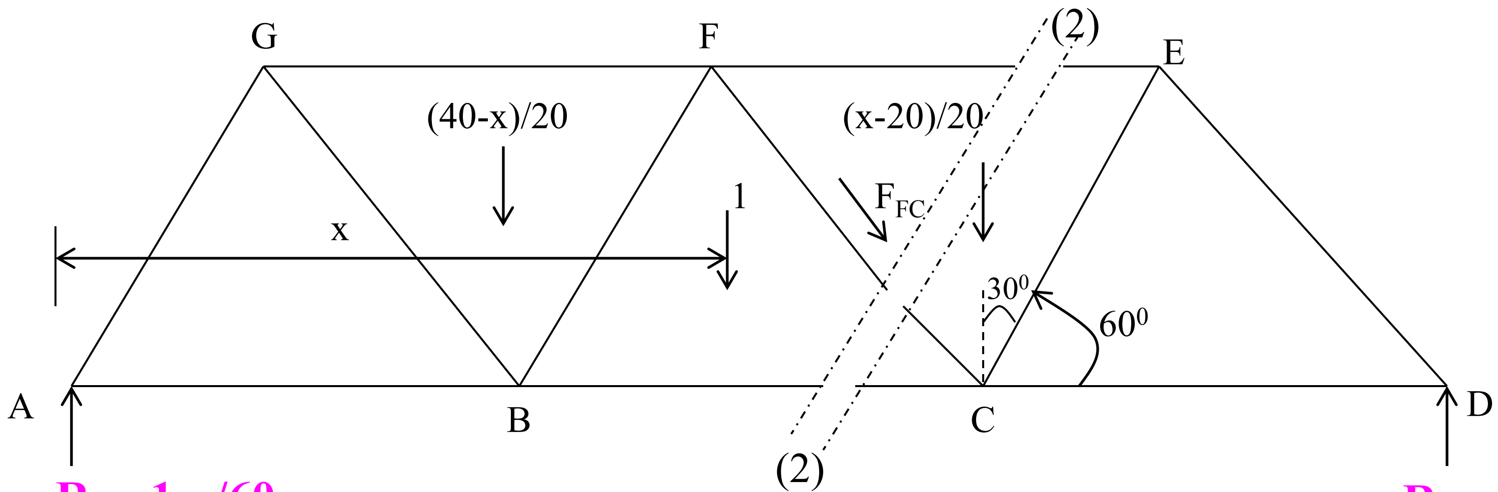
At x = 20 ft, F_{FG} = 0.77 (-ve)

At x = 40 ft, F_{FG} = 0.385 (-ve)

PROBLEM 6.7 Continued -

(ii) To compute F_{FC}, use section (2) - (2)

Section (2) - (2) is valid for 20 < x < 40 ft



Resolving force vertically, over the right hand section,

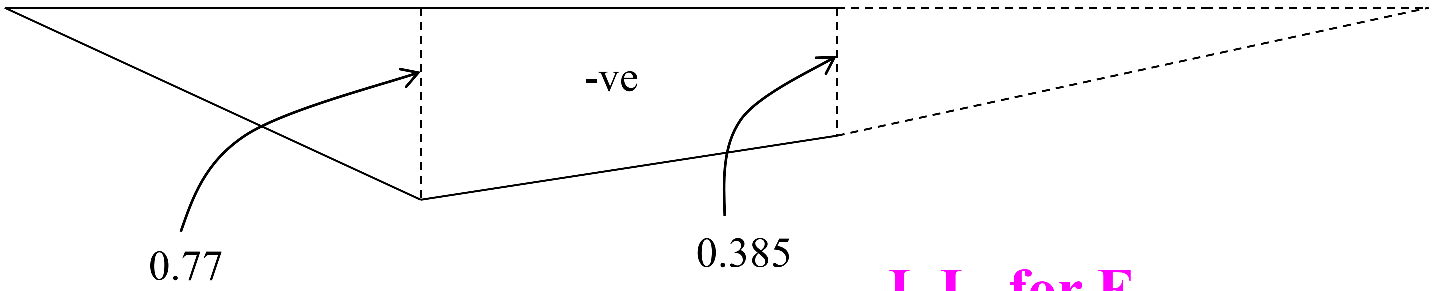
$$F_{FC}\cos 30 - (x/60) + (x-20)/20 = 0$$

$$F_{FC}\cos 30 = x/60 - x/20 + 1 = (1-2x)/60 \text{ (-ve)}$$

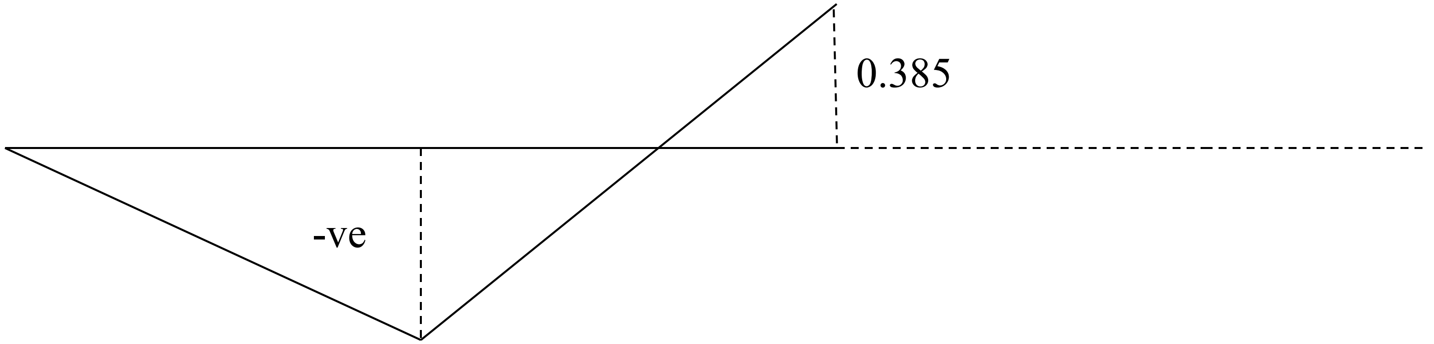
$$F_{FC} = ((60 - 2x)/60)(2/\sqrt{3}) \text{ -ve}$$

At x = 20 ft, $F_{FC} = (20/60)(2/\sqrt{3}) = 0.385$ (-ve)

At x = 40 ft, $F_{FC} = ((60\text{-}80)/60)(2/\sqrt{3}) = 0.385$ (+ve)



I. L. for F_{GF}

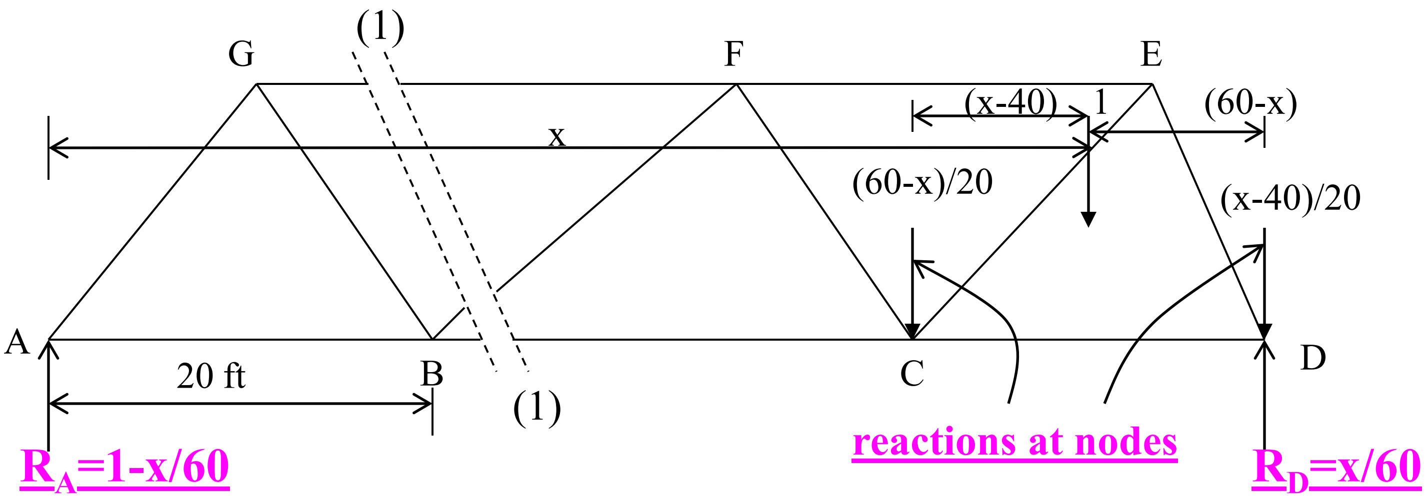


I. L. for F_{FC}

PROBLEM 3.7 Continued -

3.7.3 Place unit load over CD (40 ft < x <60 ft)

(i) To compute F_{GF}, use section (1) - (1)



Take moment about B, to its left,

$$(F_{FG})(10\sqrt{3}) - (R_A)(20) = 0$$

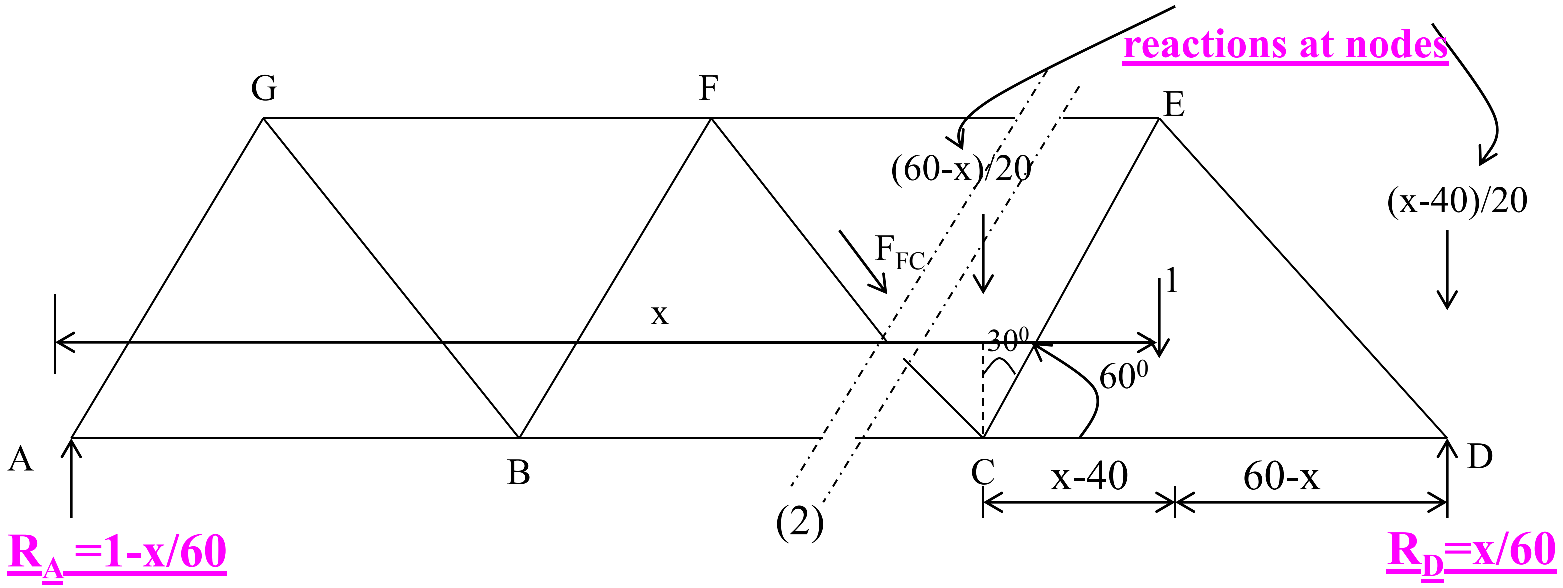
$$F_{FG} = (1-x/60)(20/10\sqrt{3}) = (1-x/60)(2/\sqrt{3}) \text{ -ve}$$

At x = 40 ft, F_{FG} = 0.385 kip (-ve)

At x = 60 ft, F_{FG} = 0.0

PROBLEM 3.7 Continued -

(ii) To compute F_{FG}, use section (2) - (2)



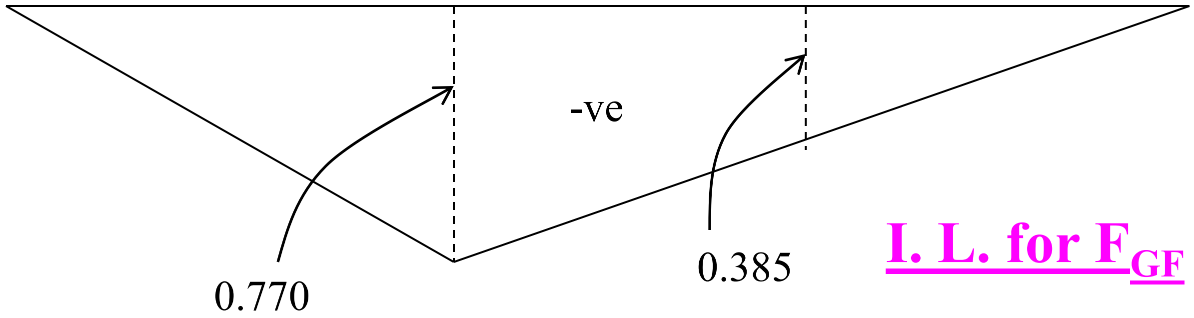
Resolving forces vertically, to the left of C,

$$(R_A) - F_{FC} \cos 30 = 0$$

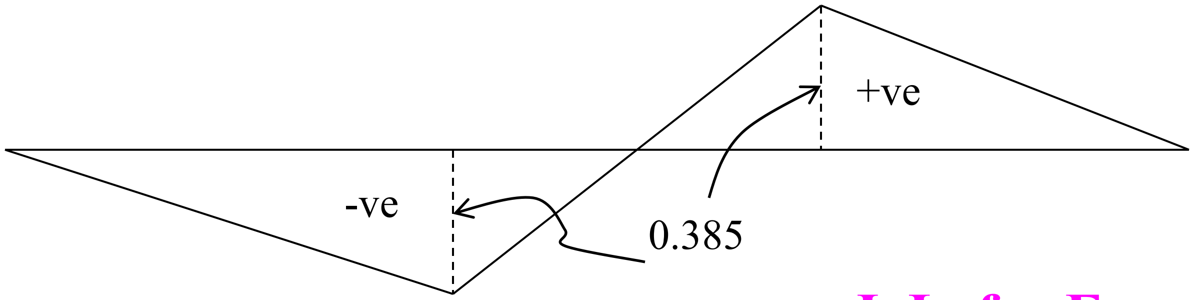
$$F_{FC} = R_A / \cos 30 = (1 - x/10)\,(2/\sqrt{3}) \text{ +ve}$$

At x = 40 ft, F_{FC} = 0.385 (+ve)

At x = 60 ft, F_{FC} = 0.0

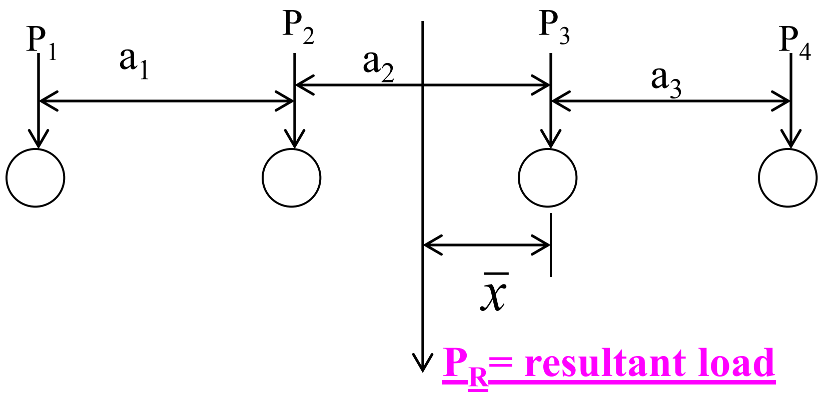


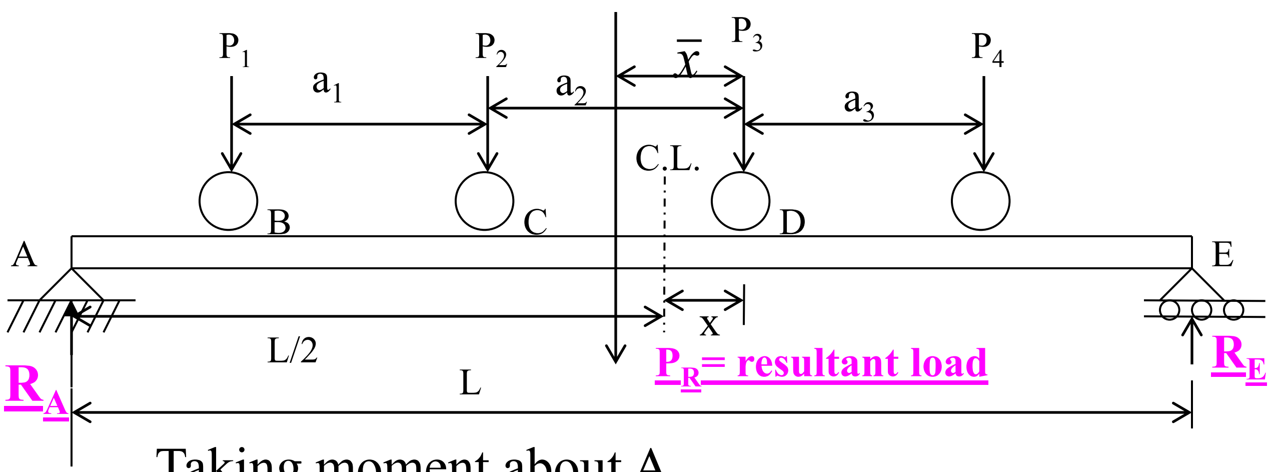
I. L. for F_{GF}



I. L. for F_{FC}

3.8 MAXIMUM SHEAR FORCE AND BENDING MOMENT UNDER A SERIES OF CONCENTRATED LOADS





Taking moment about A,

$$R_E \times L = P_R \times [L/2 - (\bar{x} - x)]$$

$$R_E = \frac{P_R}{L}(L/2 - \bar{x} + x)$$

Taking moment about E,

$$R_A \times L = P_R \times [L/2 + (\bar{x} - x)]$$

$$R_A = \frac{P_R}{L}(L/2 + \bar{x} - x)$$

$$M_D = R_A \times (L/2 + x) - P_1(a_1 + a_2) - P_2 \times a_2$$

$$= \frac{P_R}{L}(L/2 + \bar{x} - x)(L/2 + x) - P_1(a_1 + a_2) - P_2 \times (a_2)$$

$$\frac{dM_D}{dx} = 0$$

$$0 = \frac{P_R}{L}(L/2 + \bar{x} - x) + \frac{P_R}{L}(L/2 + x)(-1)$$

$$= \frac{P_R}{L}[(L/2) + \bar{x} - x - (L/2) - x]$$

$$i.e., \quad \bar{x} - 2x = 0$$

$$\bar{x} = 2x$$

$$x = \frac{\bar{x}}{2}$$

The centerline must divide the distance between the resultant of all the loads in the moving series of loads and the load considered under which maximum bending moment occurs.