Presentation on :

Solving Statically Indeterminate Structure by Stiffness Method

Introduction :

The structures which can not be analyzed by the equations of static equilibriums alone are called indeterminate structures.

This kind of structures consist of more members and/or more supports.

The excess members or reactions of an indeterminate structure are called redundant.

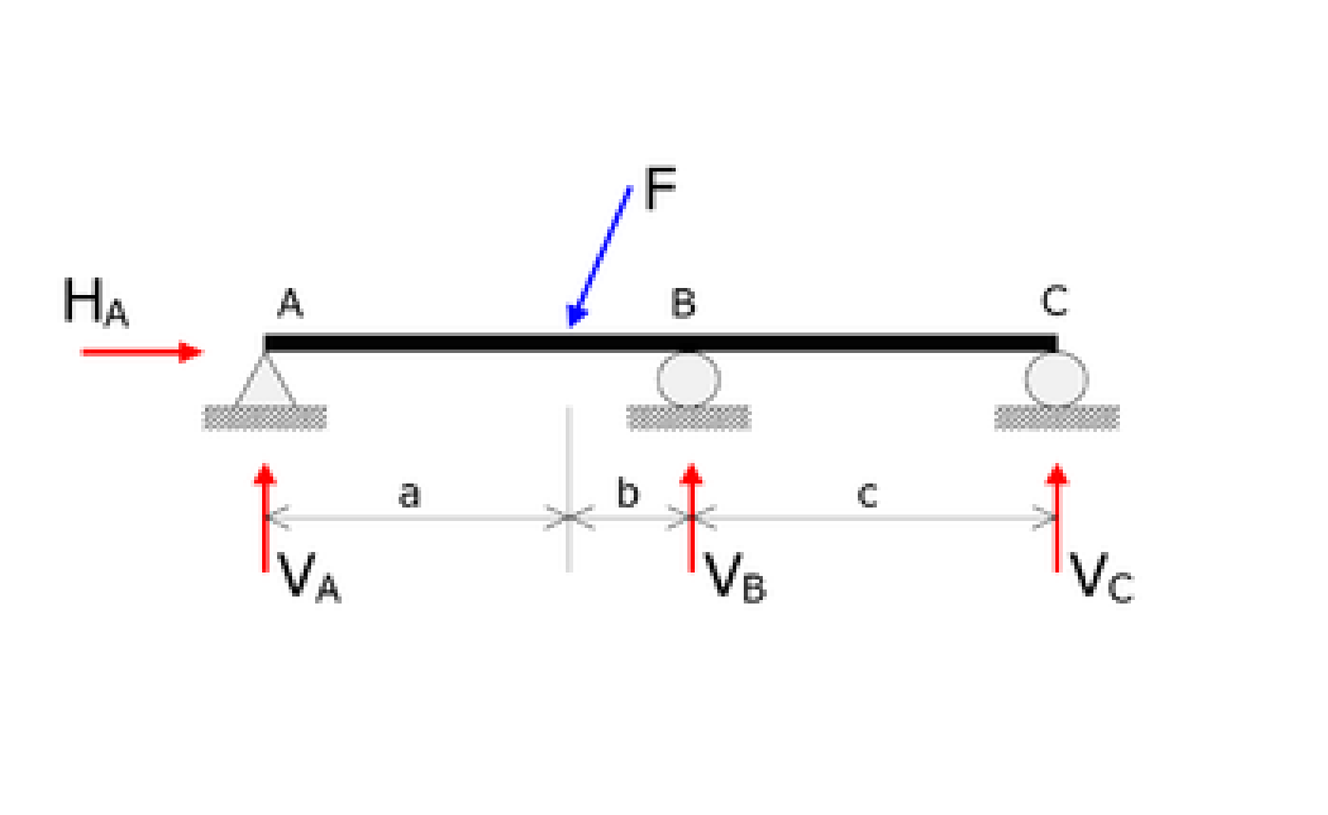


Figure : An indeterminate structure

Direct stiffness method :

It is a *matrix* method that makes use of the members' stiffness relations for computing member forces and displacements in structures.

In applying the method, additional restrains (supports) are added to fix all the degrees of freedom and the values of these restraints calculated. The restraints are then removed to allow deformations and restore equilibrium. The resulting equilibrium equations are solved for the displacements and subsequently the force actions are determined by compiling into a single matrix equation.

Analytical steps :

- At first , identify the degree of kinematic indeterminacy,
- Apply restrains and make it kinematically determinate (fully restraint structure) ,
- Apply loads on the fully restraint structure and calculate forces,
- Apply unknown displacements to the structure one at a time keeping all the other displacements zero and calculate forces corresponding to each degree of freedom,
- Write the equilibrium equations,
- Solve the equations in matrix form and obtain the value of unknown displacements,
- Calculate other reactions (reactions, SF,BM etc)

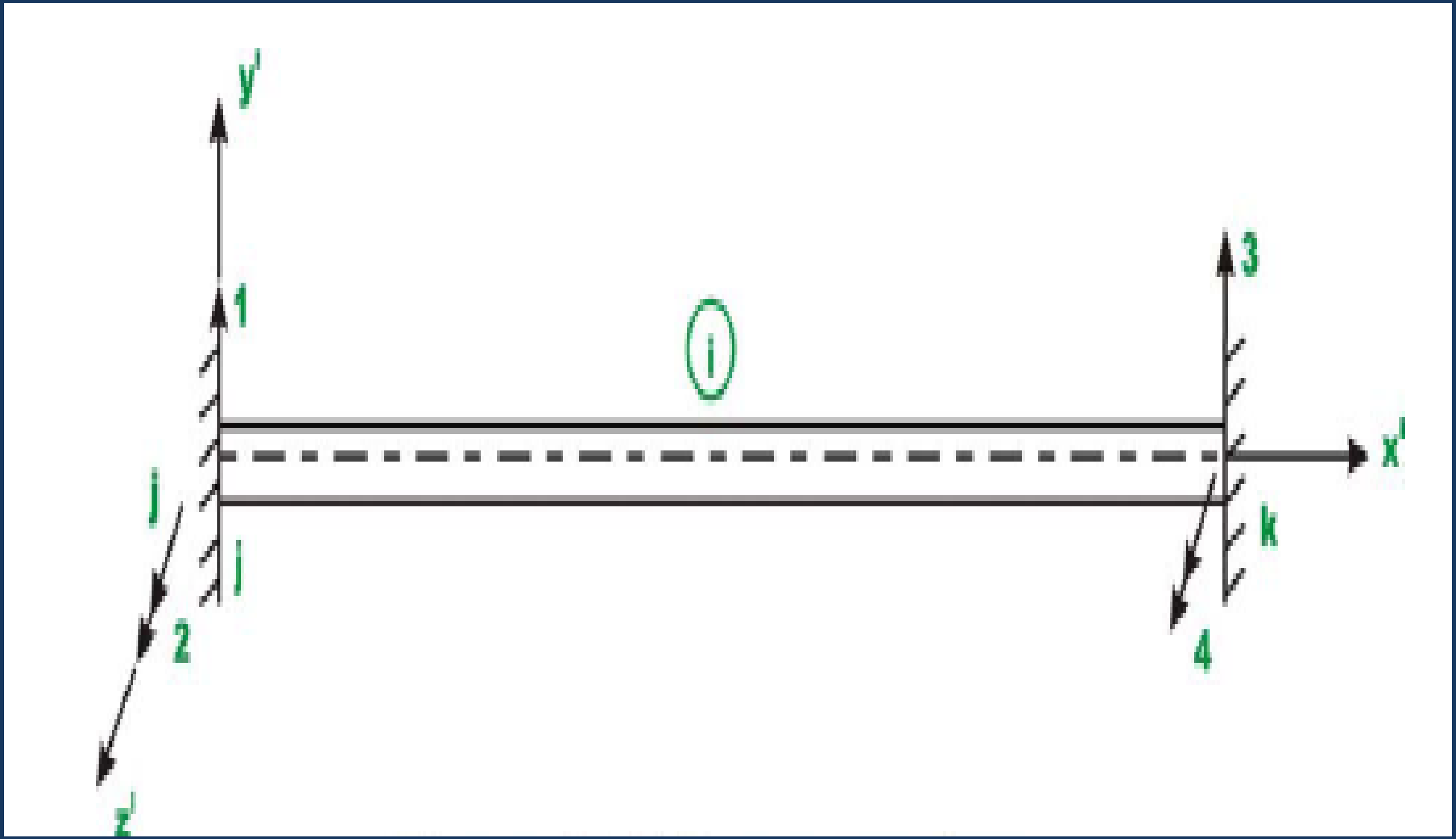
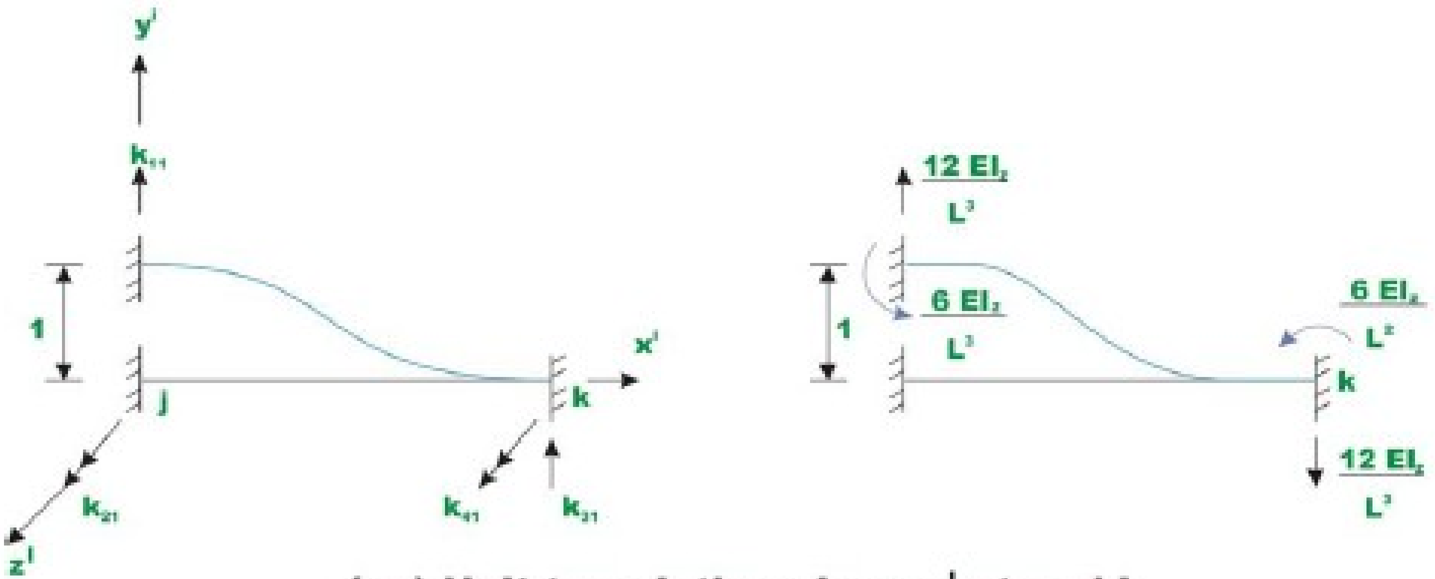
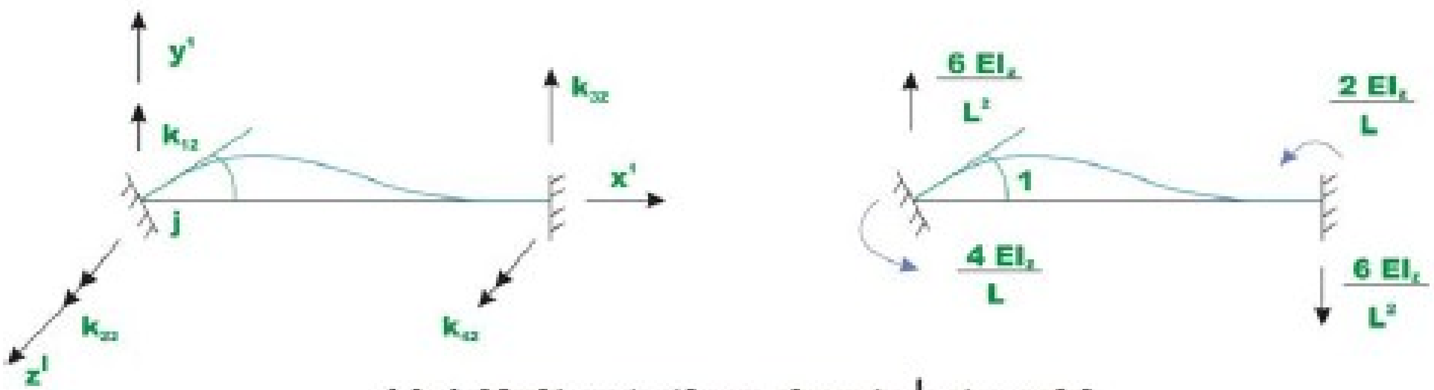


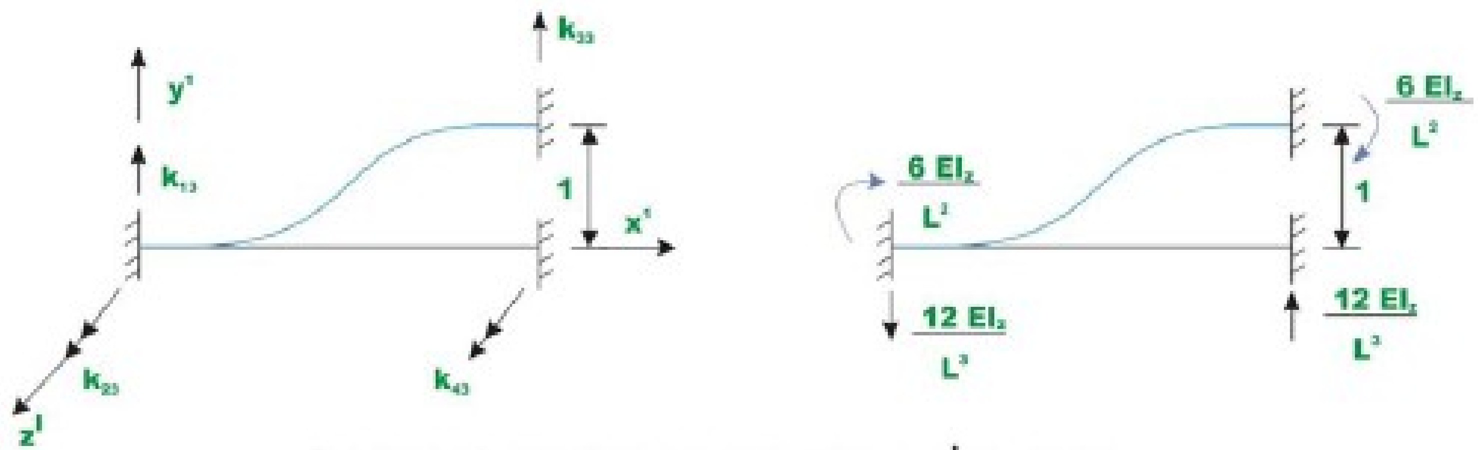
Figure: Beam member



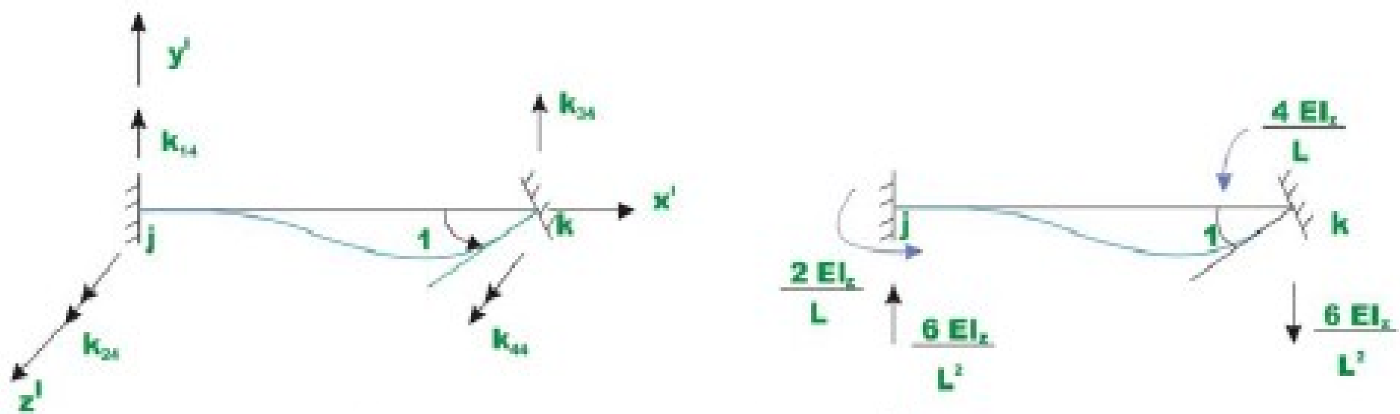
(a) Unit translation along y' at end j



(b) Unit rotation about z' at end j



(c) Unit displacement along y' at end k



(d) Unit rotation about z' at end k

Equilibrium Equation :

$P_m + ku = P_j$

Here,

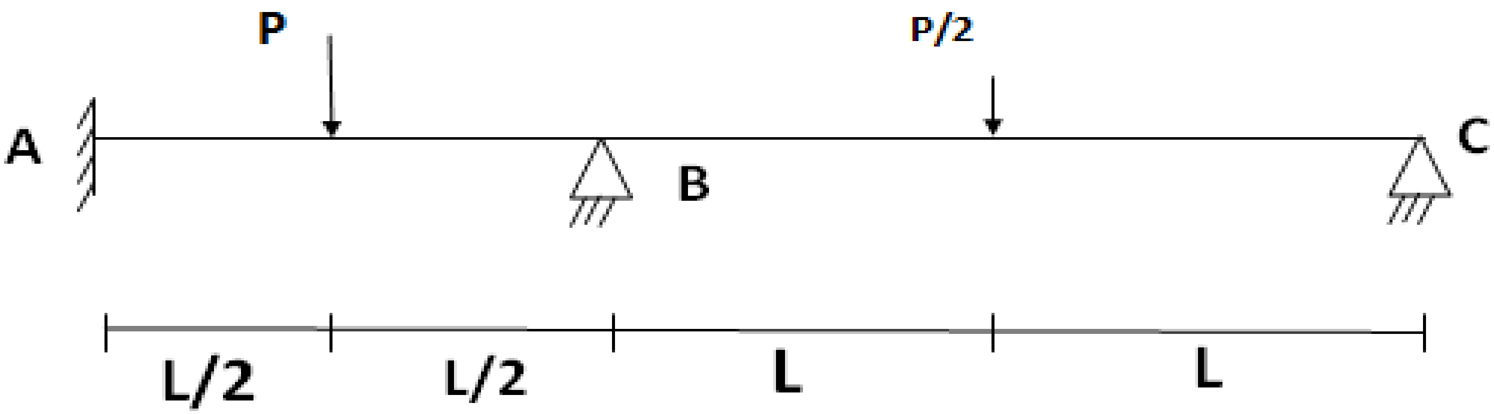
P_m = vector of member's characteristic forces.

k = member stiffness matrix which characterizes the member's resistance against deformations.

U = vector of member's characteristic displacements or deformations.

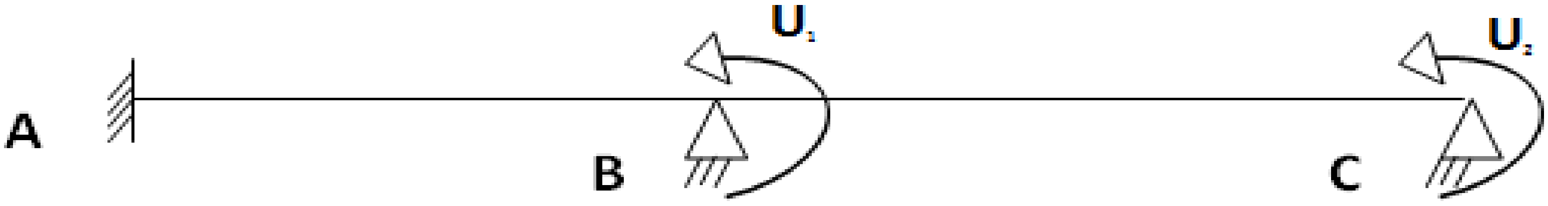
P_j = External moment.

Example:

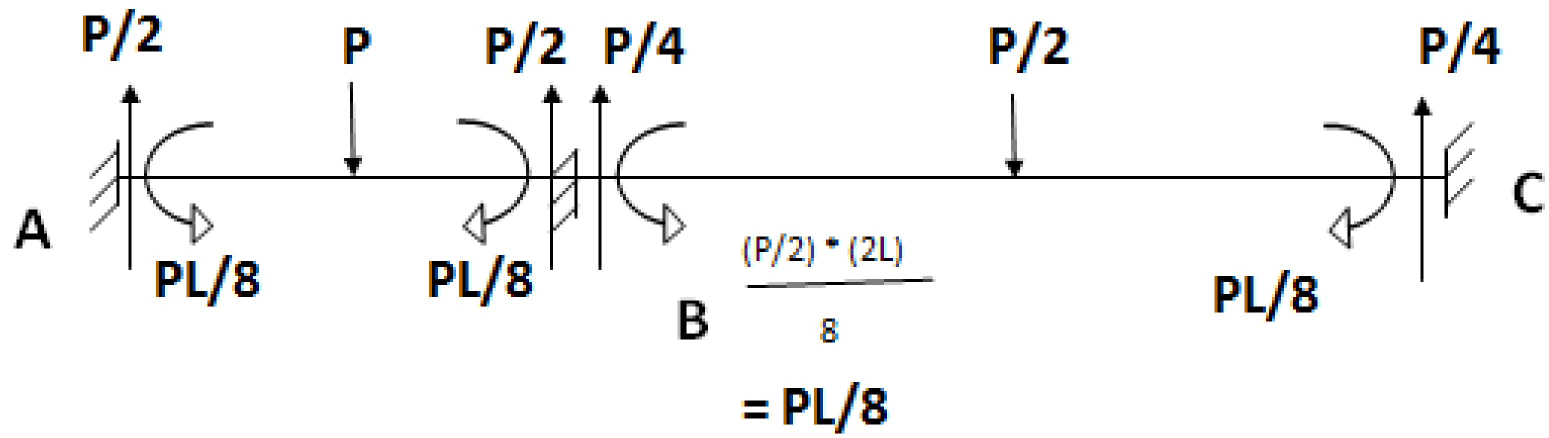


Solution :

Doki =(3*2)- (2+1+1)= 2



Fully restraint structure :



Member Force Matrix , P_m:

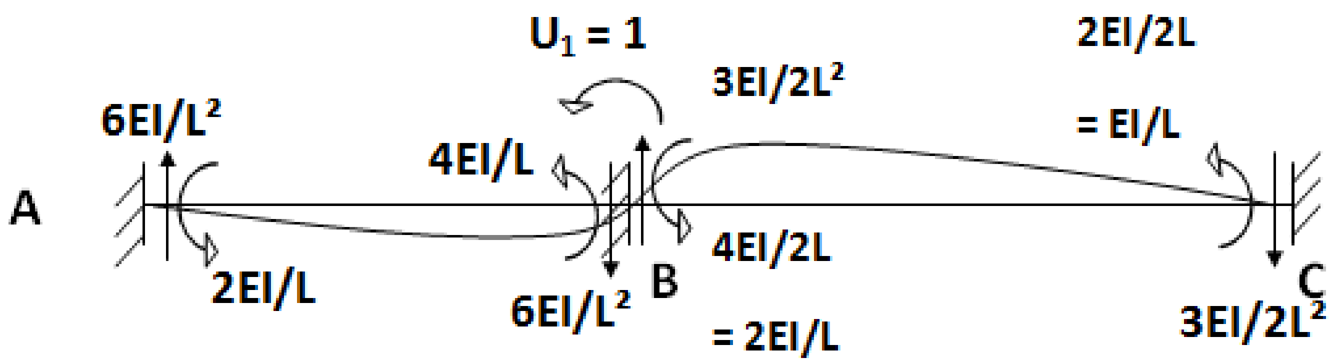
$$P_{m1} = -PL/8 + PL/8$$

$$P_{m2} = -PL/8$$

$$= \begin{pmatrix} 0 \\ -PL/8 \end{pmatrix}$$

Stiffness Matrix , k :

$u_1 = 1$ and $u_2 = 0$



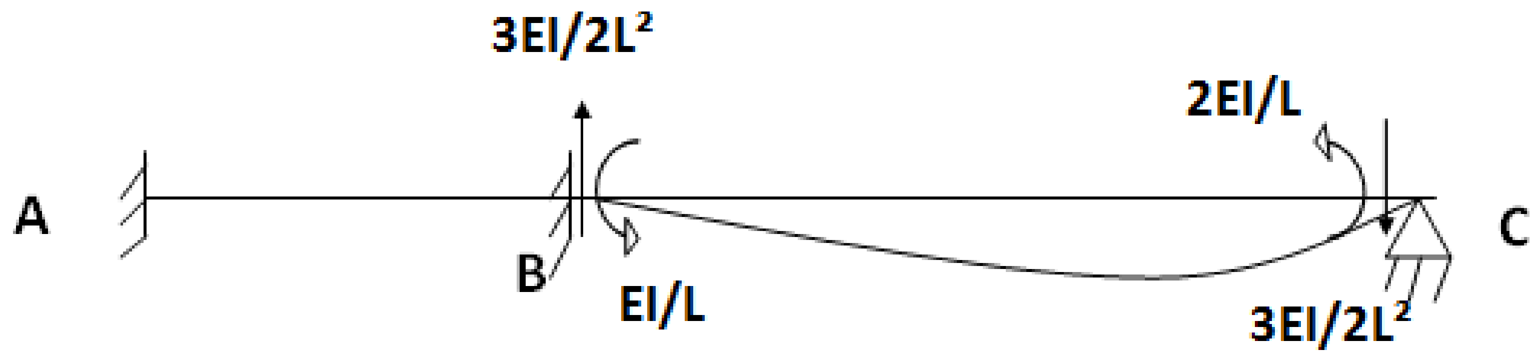
Here,

$K_{11} = 4EI/L + 2EI/L = 6EI/L$

$K_{21} = EI/L$

Now,

$u_2 = 1$, $u_1 = 0$



And,

$K_{22} = 2EI/L$

$K_{12} = EI/L$

$$EI/L \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} P_{m1} \\ P_{m2} \end{pmatrix}$$

$$U_1 = \frac{\begin{pmatrix} P_{m1} & k_{12} \\ P_{m2} & k_{22} \end{pmatrix}}{\begin{pmatrix} K_{11} & k_{12} \\ K_{21} & k_{22} \end{pmatrix}} \quad \text{and} \quad U_2 = \frac{\begin{pmatrix} k_{11} & P_{m1} \\ k_{21} & P_{m2} \end{pmatrix}}{\begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}}$$

$$EI/L \begin{pmatrix} 6 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ PL/8 \end{pmatrix}$$

$$U_2 = (PL/8) \,/\, (11EI/6L) = .0682\ PL^2/EI$$

$$EI/L\,(6u_1 + u_2) = 0$$

Or, $u_1 = -\ .0114\ PL^2/EI$

Benefits :

- The direct stiffness method was developed specifically to effectively and easily implement into computer software to evaluate complicated structures that contain a large number of elements.
- Nearly every finite element solver available is based on the direct stiffness method.
- Reduces computation time and reduce the required memory.
- One of the largest areas to utilize the direct stiffness method is the field of structural analysis where this method has been incorporated into modeling software. The software allows users to model a structure and, after the user defines the material properties of the elements, the program automatically generates element and global stiffness relationships.

Thank you