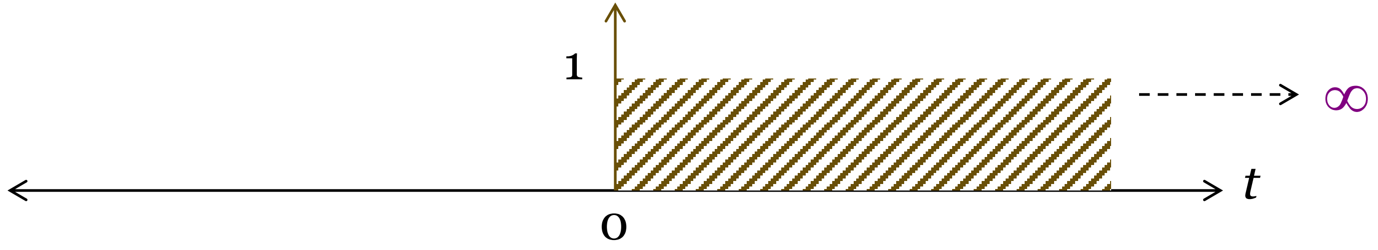
SOME INTERESTING SIGNALS

2. Step function $u(t)$



$$u(t) = 1 \quad \text{for} \quad t \geq 0$$
$$= 0 \quad \textit{otherwise}$$

A more vigorous mathematical treatment on signals

Deterministic Signals

A continuous time signal *x*(*t*) with finite energy

$$E_N = \int_{-\infty}^{\infty} \left|x(t)\right|^2 dt$$

Can be represented in the frequency domain

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \qquad \omega = 2\pi f$$

Satisfied Parseval's theorem

$$E_N = \int_{-\infty}^{\infty} \left|x(t)\right|^2 dt = \int_{-\infty}^{\infty} \left|X(f)\right|^2 df$$

Deterministic Signals

A discrete time signal *x*(*n*) with finite energy

$$E_N = \sum_{n=-\infty}^{\infty} \left| x(n) \right|^2$$

Can be represented in the frequency domain

Note: $X(\omega)$ *is periodic with period* $2\pi rad/\sec$

=

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \qquad x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

Satisfied Parseval's theorem

$$E_N = \sum_{n=-\infty}^{\infty} \left| x(n) \right|^2 = \int_{1/2}^{1/2} \left| X(f) \right|^2 df$$

Deterministic Signals

Energy Density Spectrum (*EDS*)

$$S_{xx}(f) = \left|X(f)\right|^2$$

Equivalent expression for the (*EDS*)

$$S_{xx}(f) = \sum_{m=-\infty}^{\infty} r_{xx}(m) e^{-j\omega m}$$

where

$$r_{xx}(m) = \sum_{n=-\infty}^{\infty} x^*(n) x(n+m)$$

*** Denotes complex conjugate**

Two Elementary Deterministic Signals

Impulse function: zero width and infinite amplitude

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1 \qquad \int_{-\infty}^{\infty} \delta(t) g(t)\,dt = g(0)$$

Discrete Impulse function

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$

Given $x(t)$ and $x(n)$, we have

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau \quad \textbf{and} \quad x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$

Two Elementary Deterministic Signals

Step function: A step response

$$u(t)=\begin{cases}1 & t\geq 0\\ 0 & otherwise\end{cases}$$

Discrete Step function

$$u(n)=\begin{cases}1 & n\geq 0\\ 0 & otherwise\end{cases}$$

Random Signals

Infinite duration and infinite energy signals

e.g. temperature variations in different places, each have its own waveforms.

Ensemble of time functions (random process): The set of all possible waveforms

Ensemble of all possible sample waveforms of a random process: $X(t,S)$, or simply $X(t)$.
t denotes time index and S denotes the set of all possible sample functions

A single waveform in the ensemble: $x(t,s)$, or simply $x(t)$.

Random Signals