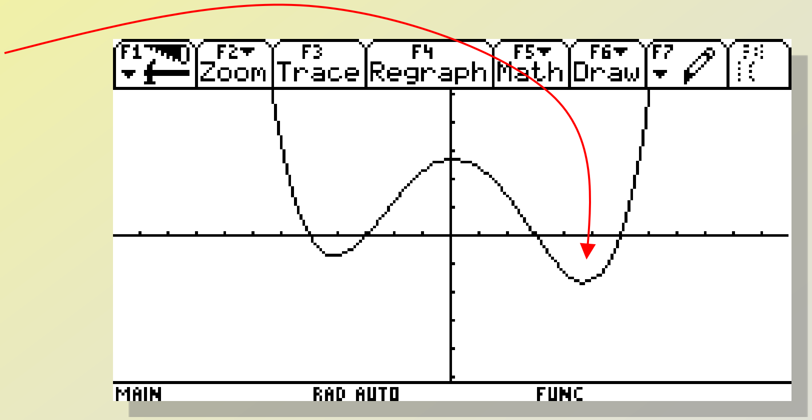
Extrema of

Function

Definition

- Absolute max or min is the largest/smallest possible value of the function
- Absolute extrema often coincide with relative extrema



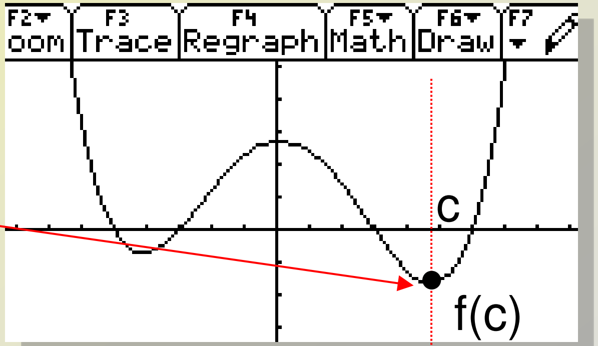
- A function may have several relative extrema
  - It never has more than one absolute max or min

- Given f(x) defined on interval
  - The number c belongs to the interval
- Then f(c) is the absolute minimum of *f* on the interval if

$$f(x) \geq f(c)$$

  - … f … val

Reminder – the absolute max or min is a y-value, not an x-value

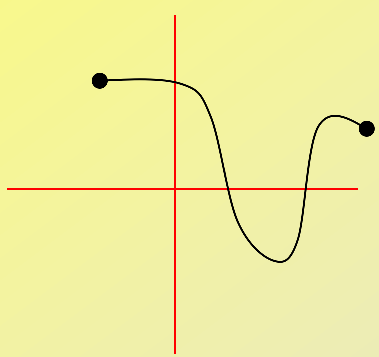


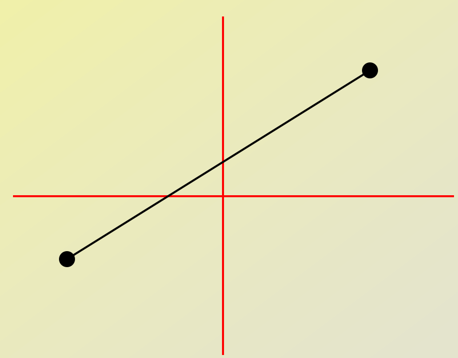
- Similarly f(c) is the absolute maximum if

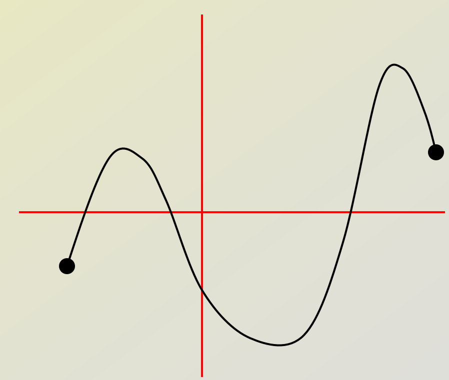
$f(x) \leq f(c)$ for all x in the interval

Functions on Closed Interval

- Extreme Value Theorem
  - A function *f* on continuous close interval [a, b] will have both an absolute max and min on the interval







- Find all absolute maximums, minimums

Example

- For the functions and intervals given, determine the absolute max and min

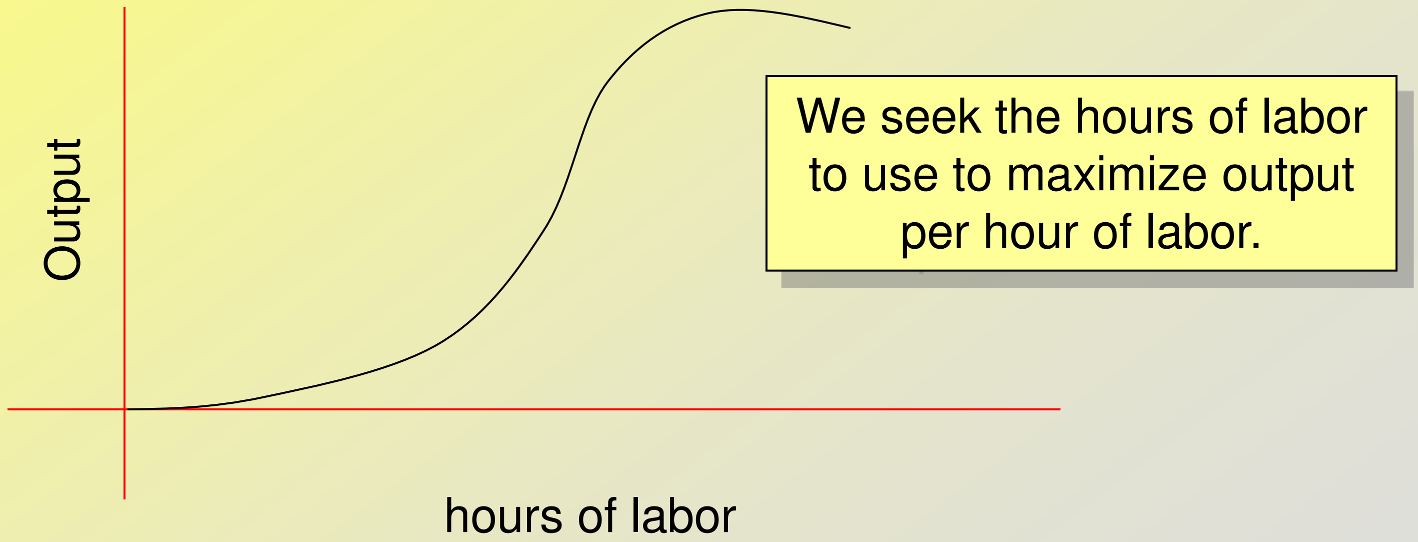
$$f(x) = x^4 - 32x^2 - 7 \quad \text{on } [-5, 6]$$

$$y = \frac{8+x}{8-x} \quad \text{on } [4, 6]$$

$$f(x) = \left(x^2 + 18\right)^{2/3} \quad \text{on } [-3, 3]$$

Graphical Interpretation

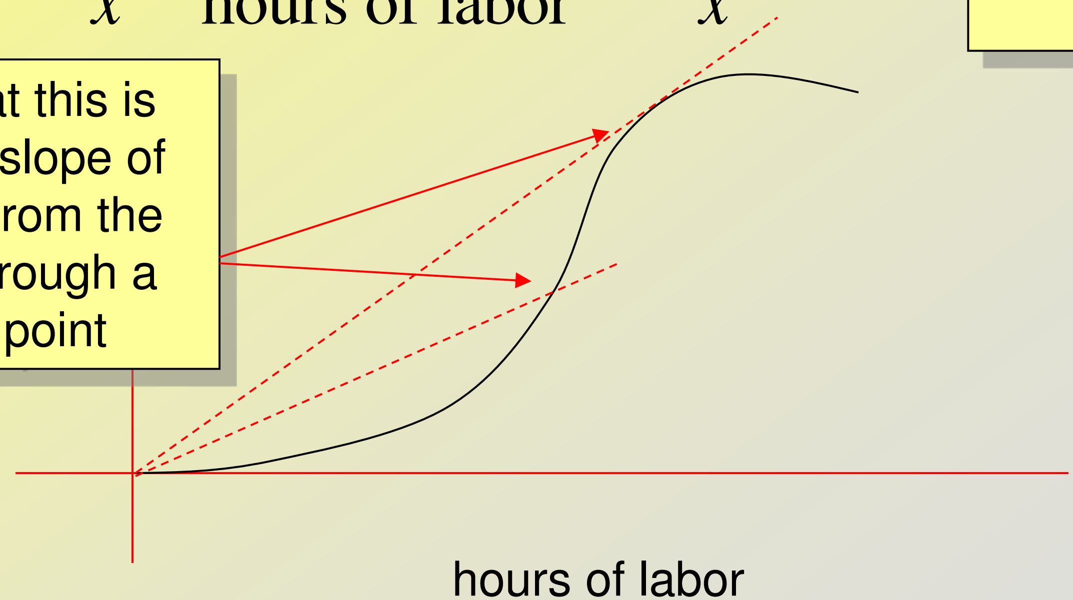
- Consider a graph that shows production output as a function of hours of labor used



- For any point on the curve
  - x-coordinate measures hours of labor
  - y-coordinate measures output
  - Thus $\dfrac{y}{x} = \dfrac{\text{output}}{\text{hours of labor}} = \dfrac{f(x)}{x}$

We seek to maximize this value

Note that this is also the slope of the line from the origin through a given point



- It can be shown that what we seek is the solution to the equation

$$f'(x) = \frac{f(x)}{x}$$

Now we have the (x, y) where the line through the origin and tangent to the curve is the steepest