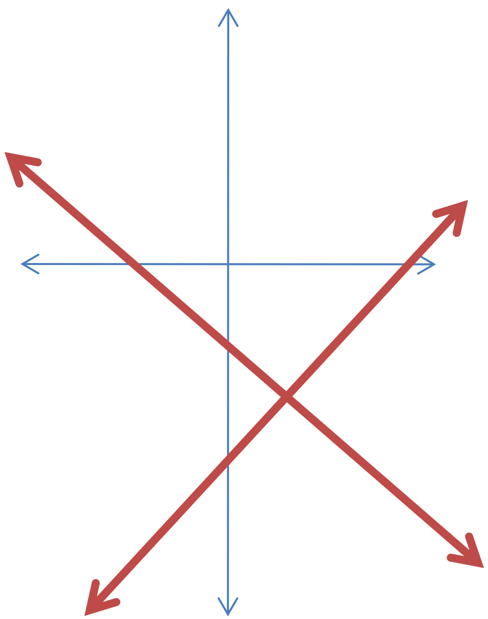
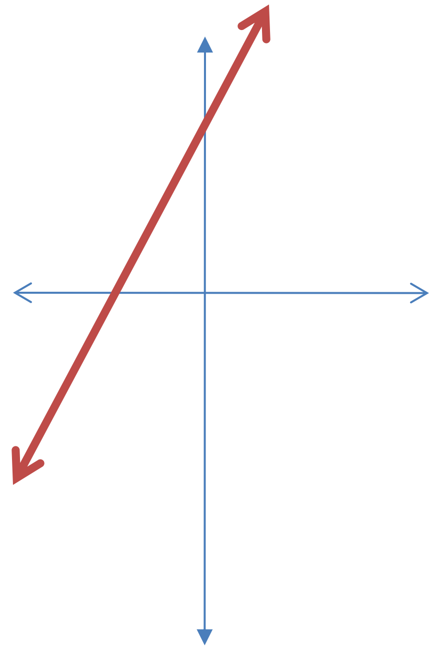
CONSISTENT AND INCONSISTENT SYSTEM

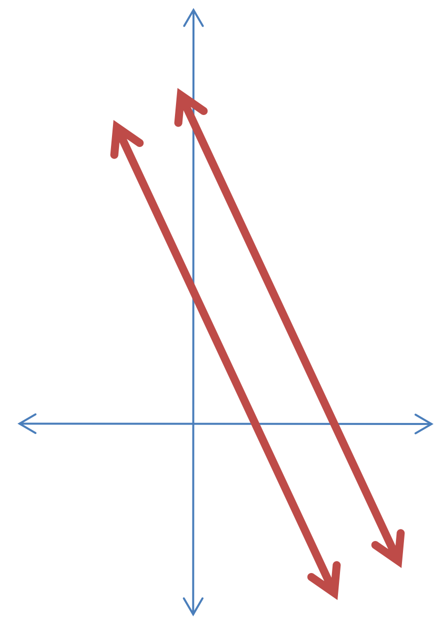
When the graph of two linear equations are drawn in the coordinate plane they may be related to each other as shown below.



- **FIGURE 1.**
- **$x + y = -2$**
- **$x - 2y = 7$**
- The graphs intersect in **exactly** one point.

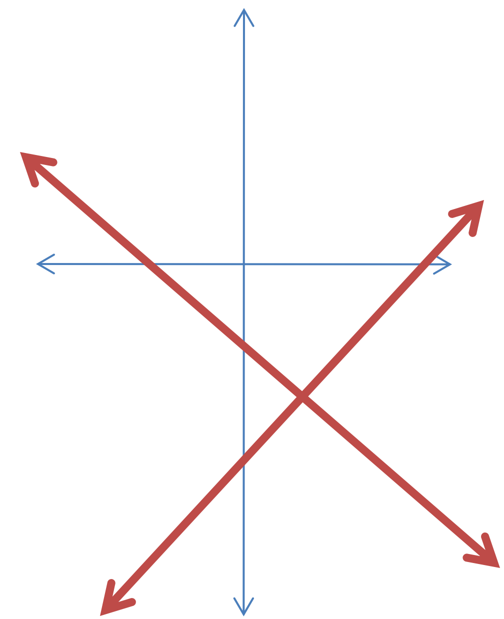


- **FIGURE 2.**
- **$y = 2x + 3$**
- **$2y = 4x + 6$**
- The graphs **coincide; that is, they have an *infinite number*** of points in common.



- **FIGURE 3.**
- **$3x + y = 2$**
- **$3x + y = 7$**
- The graphs are parallel; that is, they have **no points** in common.

The graphs in figures 1 and 2 have at least one point in common. The system are said to be CONSISTENT.



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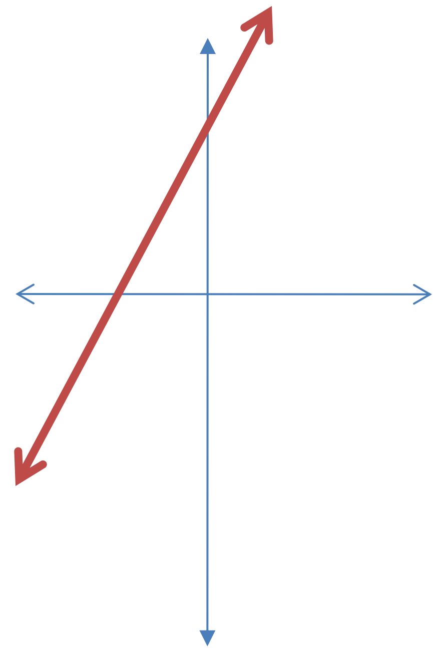

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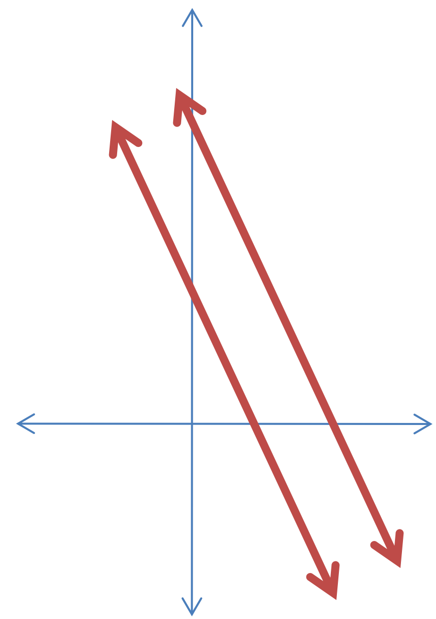

- **FIGURE 3.**
- **3x + y = 2**
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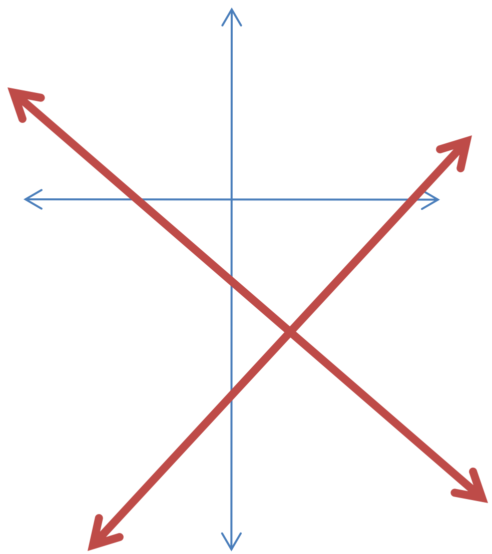
The graphs in figures 3 have NO point in common. This system is said to be <u>INCONSISTENT.</u>

CONSISTENT



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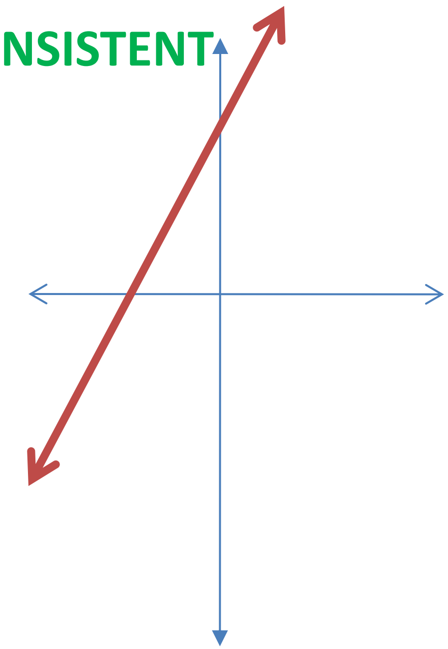
CONSISTENT



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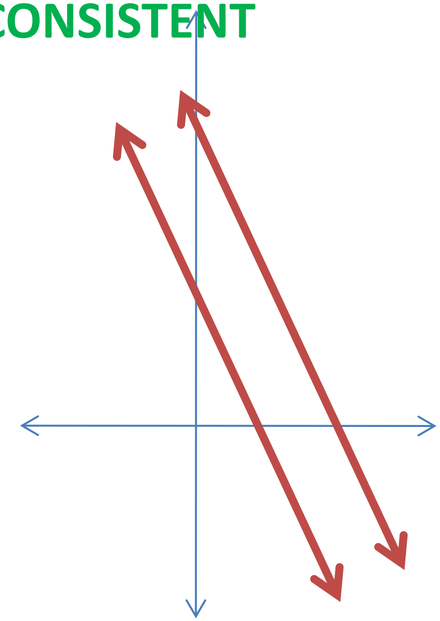
INCONSISTENT



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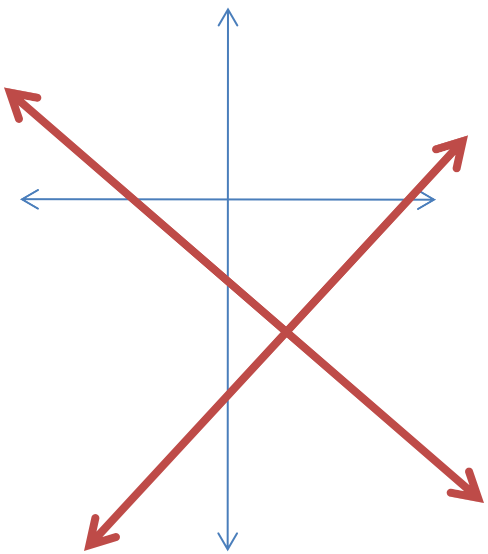
DEFINITIONS

- A **CONSISTENT SYSTEM** of equations or inequalities is one whose solution set contains *at least one ordered pair.*
- An I**NCONSISTENT SYSTEM** of equations or inequalities is one whose solution set is *the empty set.*

Write the equations of the system below in slope-intercept form.

CONSISTENT



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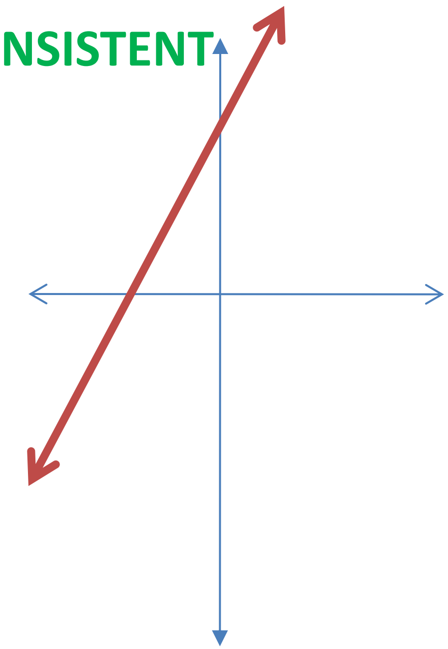

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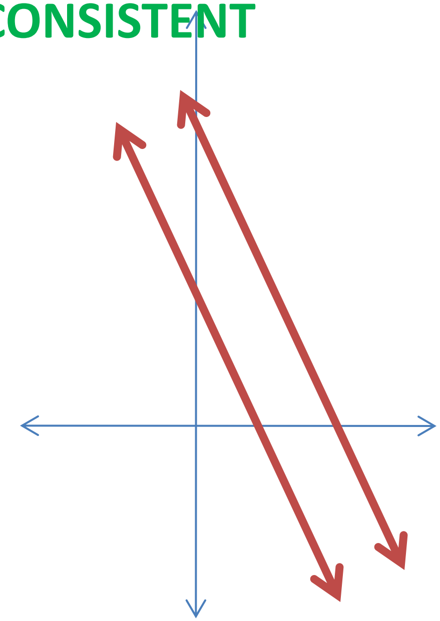

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- System 1
- $x + y = -2$ (1)
- $x - 2y = 7$ (2)
- Slope-intercept form
- $y = -x - 2$ (1)
- $y = \frac{1}{2}x - \frac{7}{2}$ (2)

- System 2
- $y = 2x + 3$ (1)
- $2y = 4x + 6$ (2)
- Slope-intercept form
- $y = 2x + 3$ (1)
- $y = \frac{4}{2}x - \frac{6}{2}$ (2)

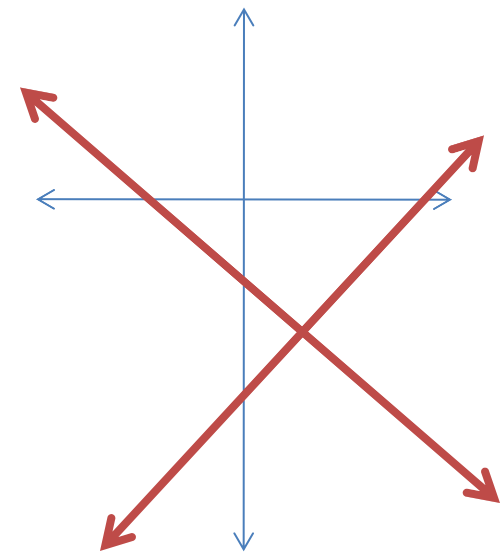
- System 3
- $3x + y = 2$ (1)
- $3x + y = 7$ (2)
- Slope-intercept form
- $y = -3x + 2$ (1)
- $y = -3x + 7$ (2)

- System 1
- $x + y = -2$ (1)
- $x - 2y = 7$ (2)
- Slope-intercept form
- $y = -x - 2$ (1)
- $y = \frac{1}{2}x - \frac{7}{2}$ (2)
- For system 1, exactly one ordered pair satisfies both equations. For this system,
- $m_1 = -1$ & $m_2 = \frac{1}{2}$
- Thus, $m_1 \neq m_2$

- System 2
- $y = 2x + 3$ (1)
- $2y = 4x + 6$ (2)
- Slope-intercept form
- $y = 2x + 3$ (1)
- $y = \frac{4}{2}x - \frac{6}{2}$ (2)
- For system 2, every ordered pair that satisfies equation 1 also satisfies Equation 2. The system is DEPENDENT. For this system
- $m_1 = 2$ & $m_2 = 2$
- Also , $b_1 = 3$ & $b_2 = 3$

- **System 3**
- **$3x + y = 2$ (1)**
- **$3x + y = 7$ (2)**
- **Slope-intercept form**
- **$y = -3x + 2$ (1)**
- **$y = -3x + 7$ (2)**
- For system 3, no ordered pair satisfies both equations. For this system,
- ***$m_1 = -3$ & $m_2 = -3$ Also, $b_1 = 2$ & $b_2 = 7$***
- **Thus, *$m_1 = m_2$ and* $b_1 \neq b_2$.**

CONSISTENT & INDEPENDENT



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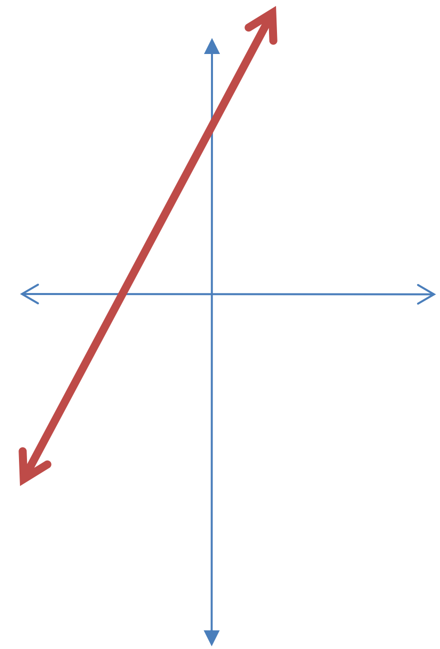

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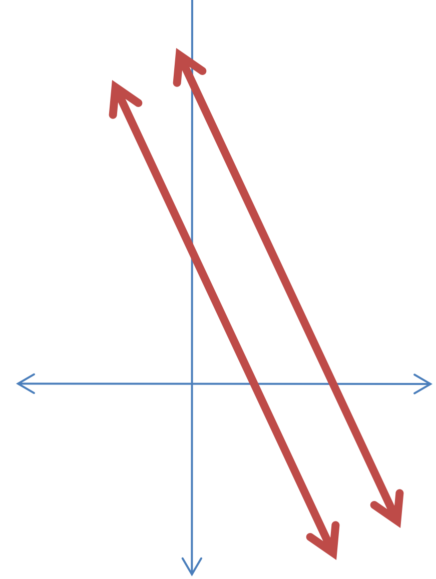

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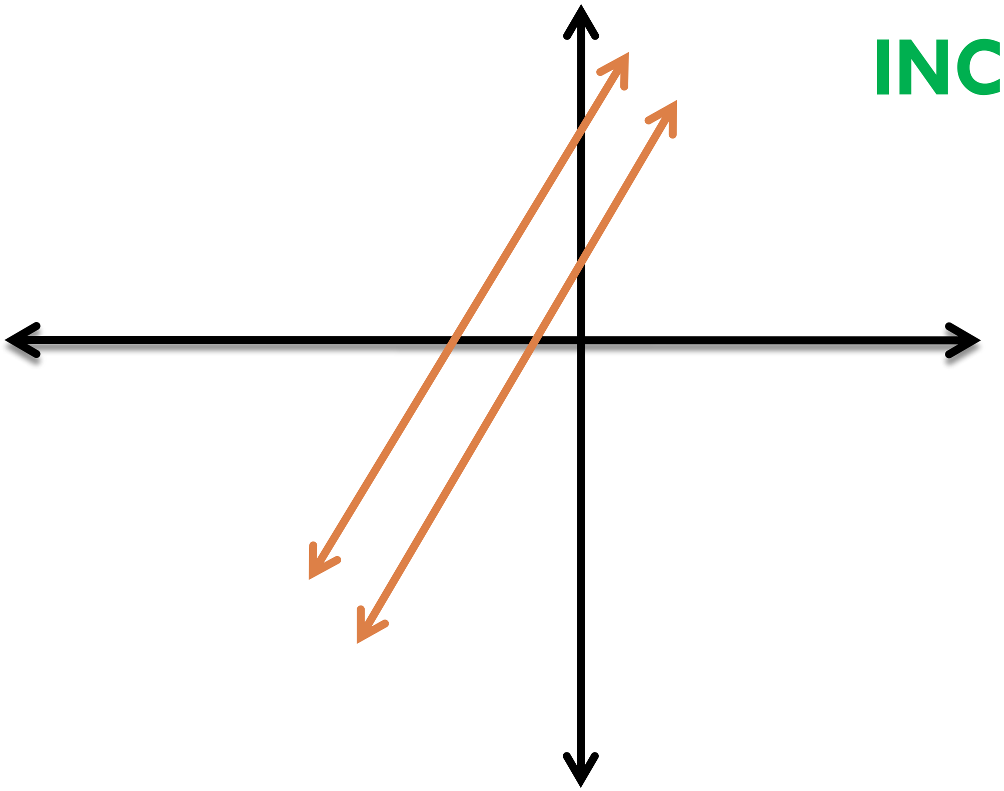
SUMMARY

Properties of a Linear System of Two Equations $y = m_1x + b_1$ and $y = m_2 + b_2$.			
DESCRIPTION	Slopes and y- intercepts	Graphs	Solutions
CONSISTENT	$m_1 \neq m_2$	Intersect in one point	One
DEPENDENT	$m_1 = m_2$ *and* $b_1 = b_2$	Coincide	Infinite number
INCONSISTENT	$m_1 = m_2$ *and* $b_1 \neq b_2$	Parallel	None

Use the graph of each system to classify it as INCONSISTENT, CONSISTENT, DEPENDENT.



INCONSISTENT

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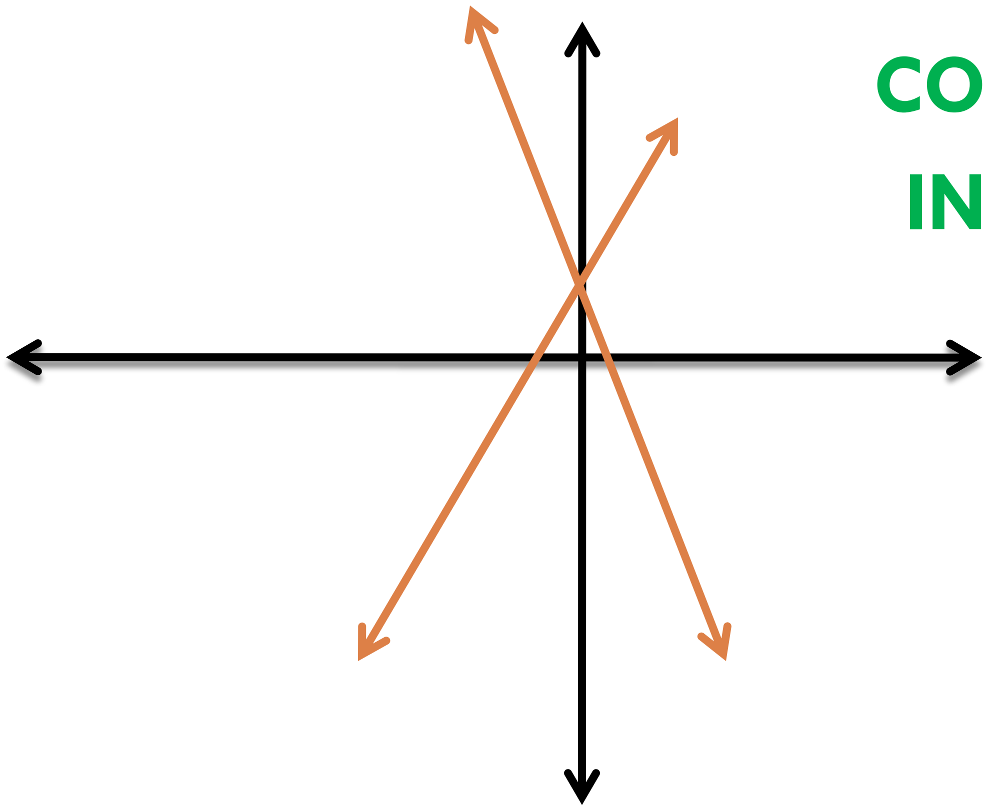

CONSISTENT

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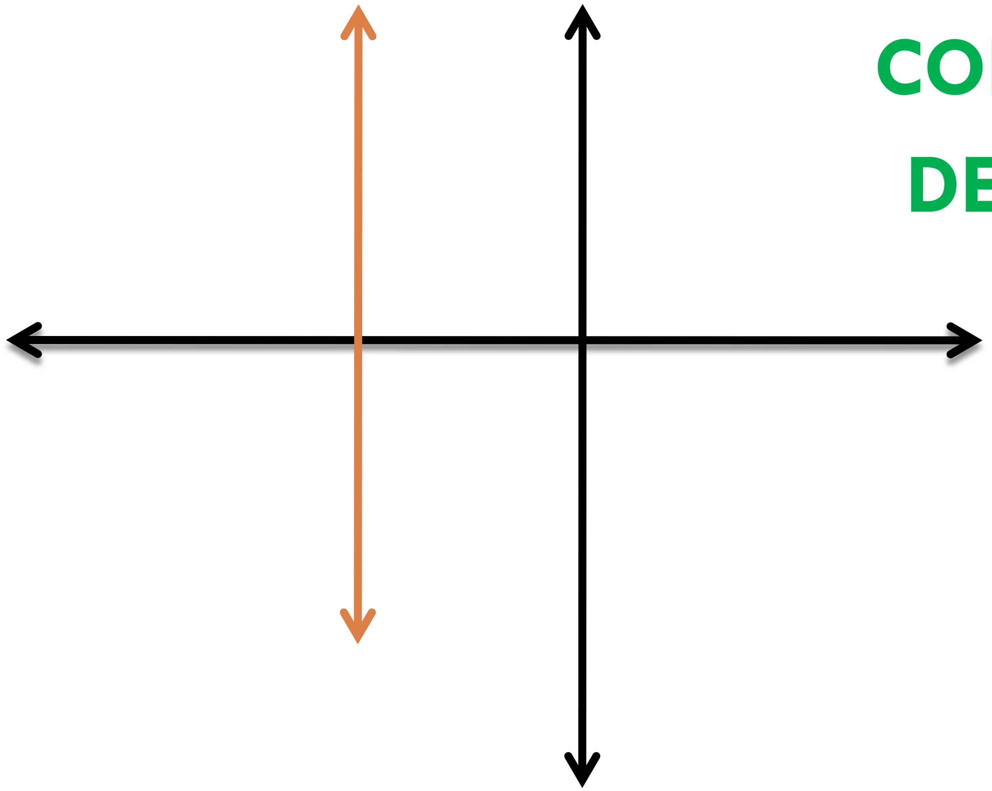
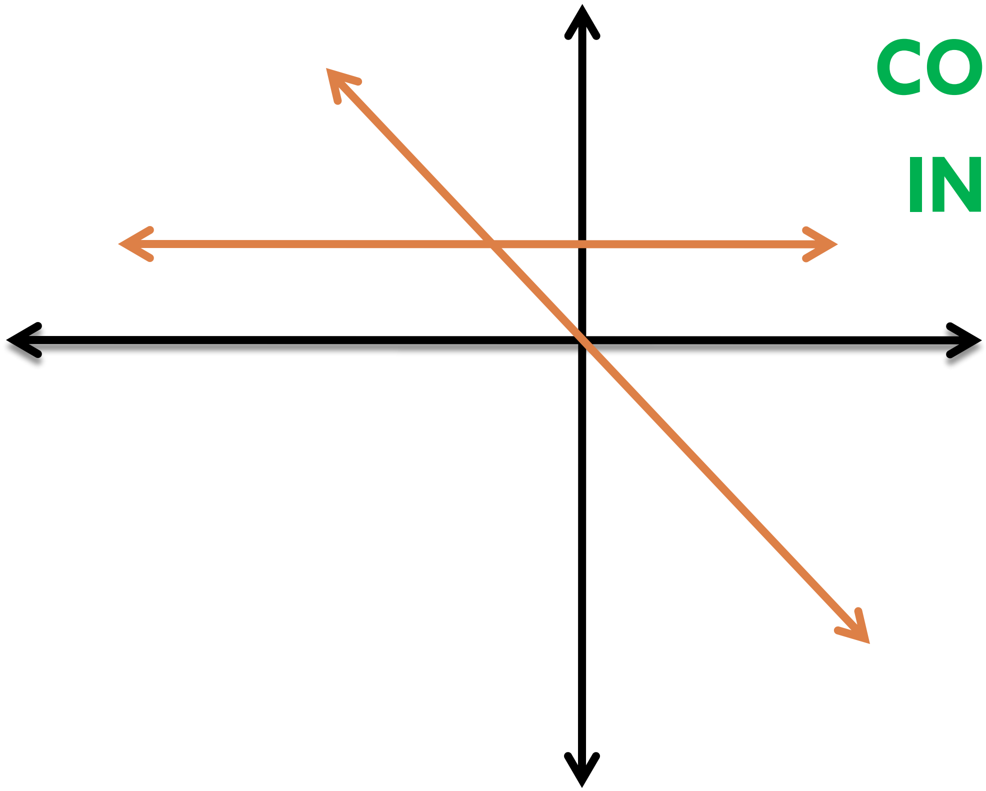


CONSISTENT

DEPENDENT

Use the graph of each system to classify it as [INCONSISTENT, CONSISTENT, DEPENDENT.]



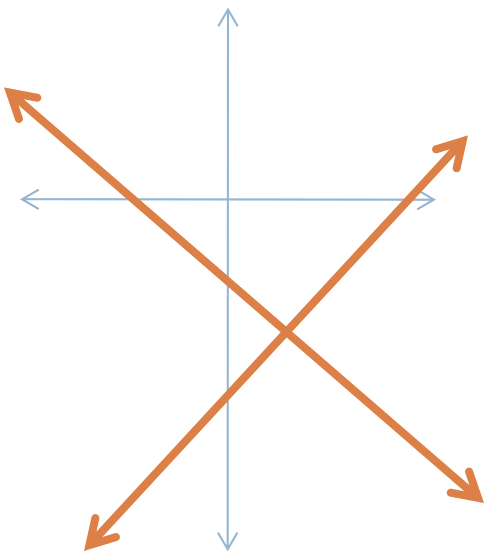
CONSISTENT

INDEPENDENT

OTHER WAY OF DETERMINING WHETHER THE SYSTEMS ARE CONSISTENT,INCONSSITENT, or DEPENDENT.

- Given $a_1 x + b_1 y = c_1$ and $a_2 x + b_2 y = c_2$.
- The system is DEPENDENT if
- $a_1 : a_2 = b_1 : b_2 = c_1 : c_2$
- One equation is a multiple to the other.
- Graphically, the lines coincide.

CONSISTENT & INDEPENDENT



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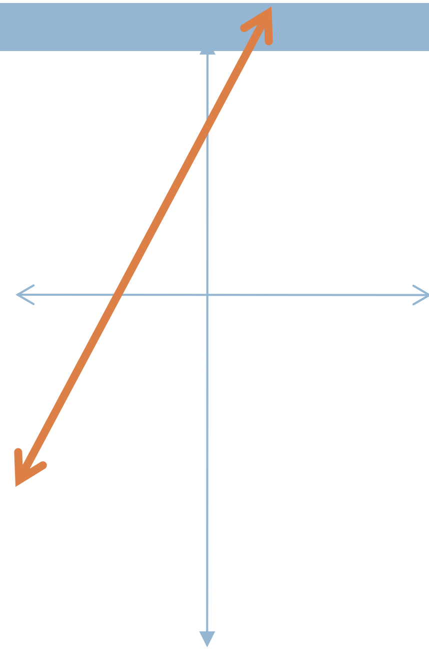

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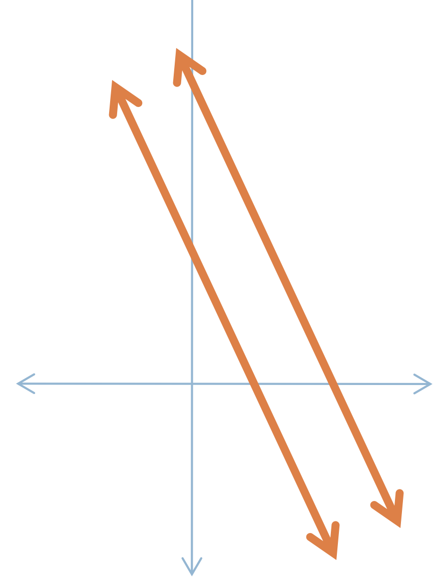

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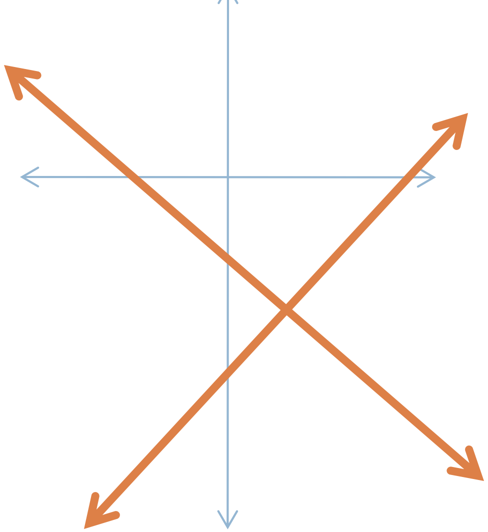
- Given $a_1 x + b_1 y = c_1$ and $a_2 x + b_2 y = c_2$.
- The system is INCONSISTENT if
- $a_1 : a_2 = b_1 : b_2 \neq c_1 : c_2$
- Graphically, the lines are parallel.

CONSISTENT & INDEPENDENT



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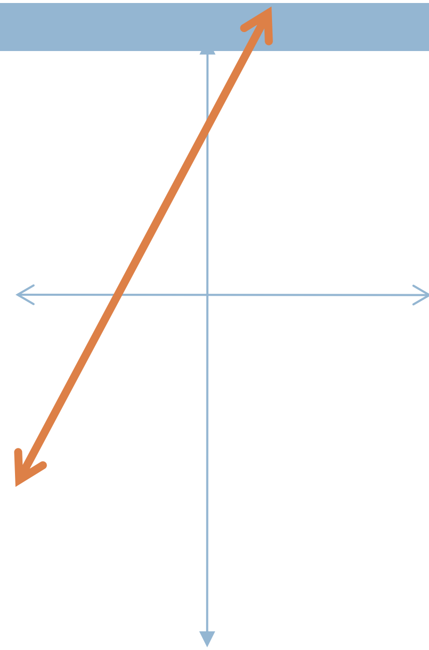

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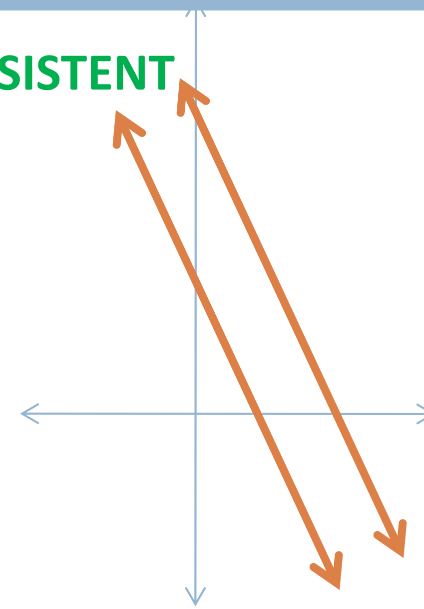

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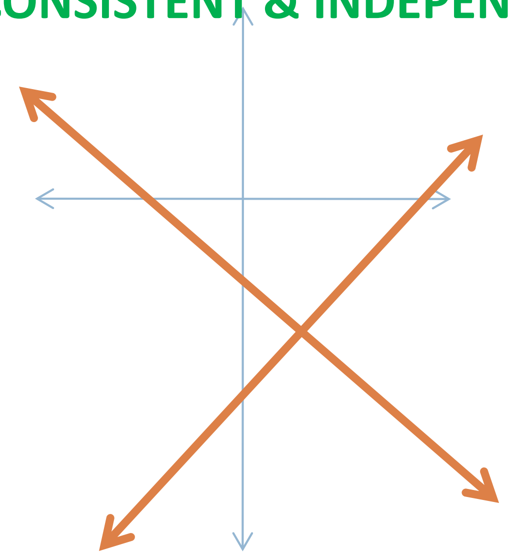
- Given $a_1 x + b_1 y = c_1$ and $a_2 x + b_2 y = c_2$.
- The system is CONSISTENT if neither holds.
- $a_1 : a_2 = b_1 : b_2 = c_1 : c_2$
- $a_1 : a_2 \neq b_1 : b_2$
- Graphically, the lines are intersect.

CONSISTENT & INDEPENDENT



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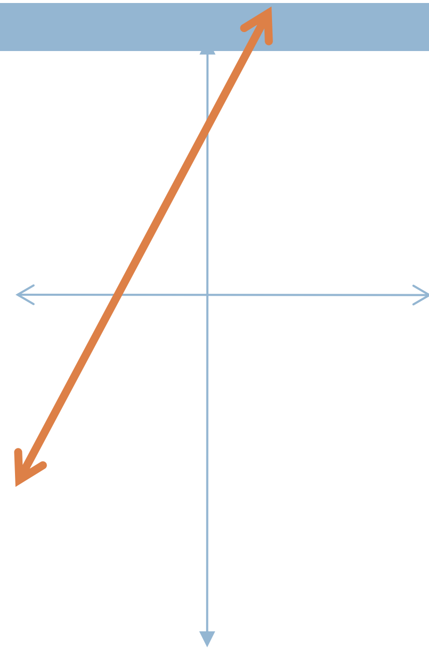

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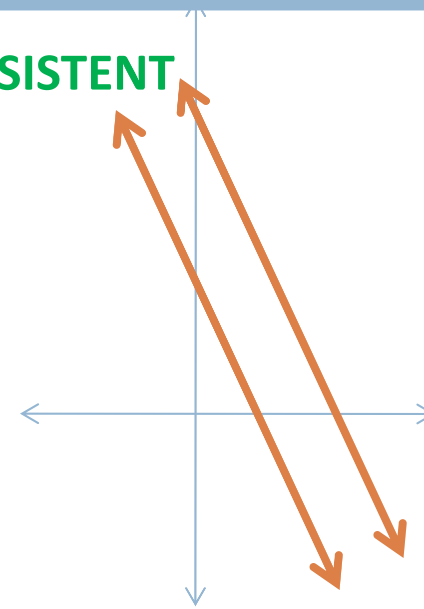

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