Divergence of a Vector

If $\underset{\sim}{A} = a_x \underset{\sim}{i} + a_y \underset{\sim}{j} + a_z \underset{\sim}{k}$, the divergence of $\underset{\sim}{A}$ is defined as

$$div\, \underset{\sim}{A} = \nabla . \underset{\sim}{A}$$

$$= \left(\frac{\partial}{\partial x} \underset{\sim}{i} + \frac{\partial}{\partial y} \underset{\sim}{j} + \frac{\partial}{\partial z} \underset{\sim}{k} \right) . (a_x \underset{\sim}{i} + a_y \underset{\sim}{j} + a_z \underset{\sim}{k})$$

$$\Rightarrow div\, \underset{\sim}{A} = \nabla . \underset{\sim}{A} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}.$$

Example :

If $\underset{\sim}{A} = x^2 y\,\underset{\sim}{i} - xyz\,\underset{\sim}{j} + yz^2\,\underset{\sim}{k}$,

determine $div\,\underset{\sim}{A}$ at point (1,2,3).

Answer

$$div\,\underset{\sim}{A} = \nabla.\underset{\sim}{A} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

$$= 2xy - xz + 2yz.$$

At point (1,2,3),

$$div\,\underset{\sim}{A} = 2(1)(2) - (1)(3) + 2(2)(3)$$

$$= 13.$$

Exercise :

If $\underset{\sim}{A} = x^3y^2\,\underset{\sim}{i} + xy^2z\,\underset{\sim}{j} - yz^3\,\underset{\sim}{k}$,

determine $div\,\underset{\sim}{A}$ at point (3,2,1).

Answer $$div\,\underset{\sim}{A} = \nabla.\underset{\sim}{A} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

$$= \ldots$$

At point (3,2,1),

$$div\,\underset{\sim}{A} = \ldots$$

$$= 114.$$

Remarks

$\underset{\sim}{A}$ is a vector function, but $div\,\underset{\sim}{A}$ is a scalar function.

If $div\,\underset{\sim}{A} = 0$, vector $\underset{\sim}{A}$ is called *solenoid vector*.