Gradient

- If $\phi(x,y,z)$ is a scalar function of three variables and ϕ is differentiable, the gradient of ϕ is defined as

$$\text{grad } \phi = \nabla \phi = \frac{\partial \phi}{\partial x} \underset{\sim}{i} + \frac{\partial \phi}{\partial y} \underset{\sim}{j} + \frac{\partial \phi}{\partial z} \underset{\sim}{k} .$$

$* \phi$ is a scalar function

$* \nabla \phi$ is a vector function

Example:

If $\phi = x^2yz^3 + xy^2z^2$, determine grad ϕ at P = (1,3,2).

Solution

Given $\phi = x^2yz^3 + xy^2z^2$, hence

$$\frac{\partial \phi}{\partial x} = 2xyz^3 + y^2z^2$$

$$\frac{\partial \phi}{\partial y} = x^2z^3 + 2xyz^2$$

$$\frac{\partial \phi}{\partial z} = 3x^2yz^2 + 2xy^2z$$

Therefore,

$$\nabla\phi = \frac{\partial\phi}{\partial x}\underset{\sim}{i} + \frac{\partial\phi}{\partial y}\underset{\sim}{j} + \frac{\partial\phi}{\partial z}\underset{\sim}{k}$$

$$= (2xyz^3 + y^2z^2)\underset{\sim}{i} + (x^2z^3 + 2xyz^2)\underset{\sim}{j}$$

$$+ (3x^2yz^2 + 2xy^2z)\underset{\sim}{k}.$$

At $P = (1,3,2)$, we have

$$\nabla\phi = (2(1)(3)(2)^3 + (3)^2(2)^2)\underset{\sim}{i} + ((1)^2(2)^3 + 2(1)(3)(2)^2)\underset{\sim}{j}$$

$$+ (3(1)^2(3)(2)^2 + 2(1)(3)^2(2))\underset{\sim}{k}.$$

$$= 84\underset{\sim}{i} + 32\underset{\sim}{j} + 72\underset{\sim}{k}.$$

Exercise :

If $\phi = x^3yz + xy^2z^3$,

determine grad ϕ at point $P = (1,2,3)$.

Solution

Given $\phi = x^3yz + xy^2z^3$, then

$$\frac{\partial\phi}{\partial \mathrm{x}} = \dots$$

$$\frac{\partial\phi}{\partial \mathrm{y}} = \dots$$

$$\frac{\partial\phi}{\partial \mathrm{z}} = \dots$$

$$\therefore \ \mathrm{Grad}\,\phi = \nabla\phi = \dots$$

At P = (1,2,3), $\quad \nabla\phi = 126\,\underset{\sim}{i} + 111\,\underset{\sim}{j} + 110\,\underset{\sim}{k}$.

Grad Properties

If A and B are two scalars, then

1) $\nabla(A+B) = \nabla A + \nabla B$
2) $\nabla(AB) = A(\nabla B) + B(\nabla A)$