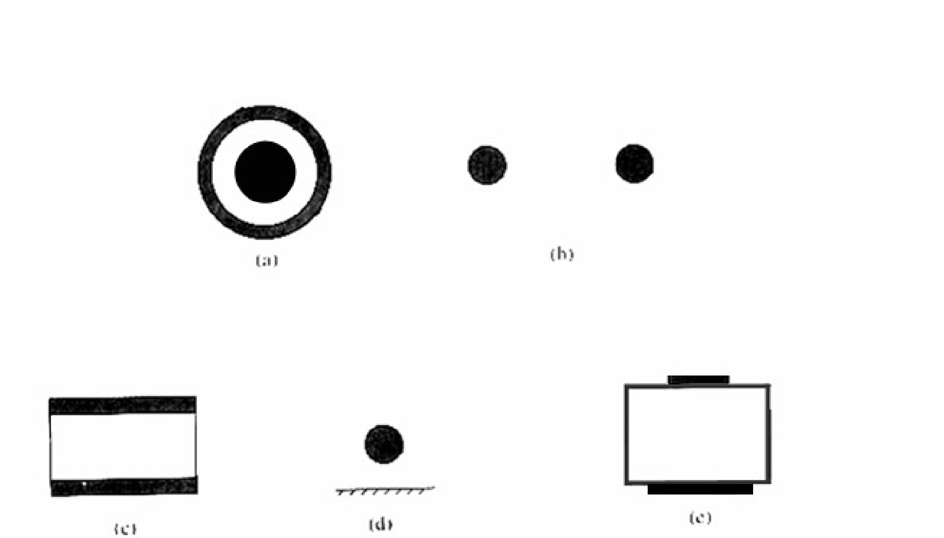
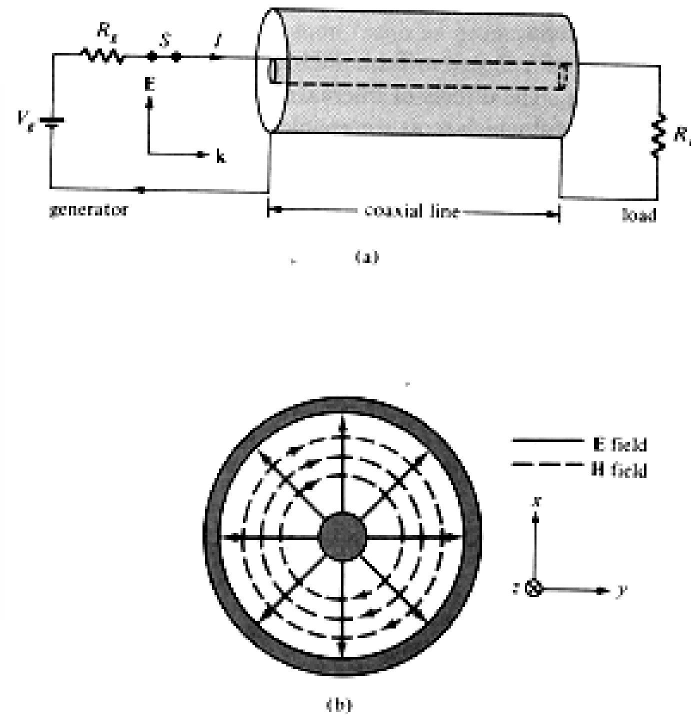
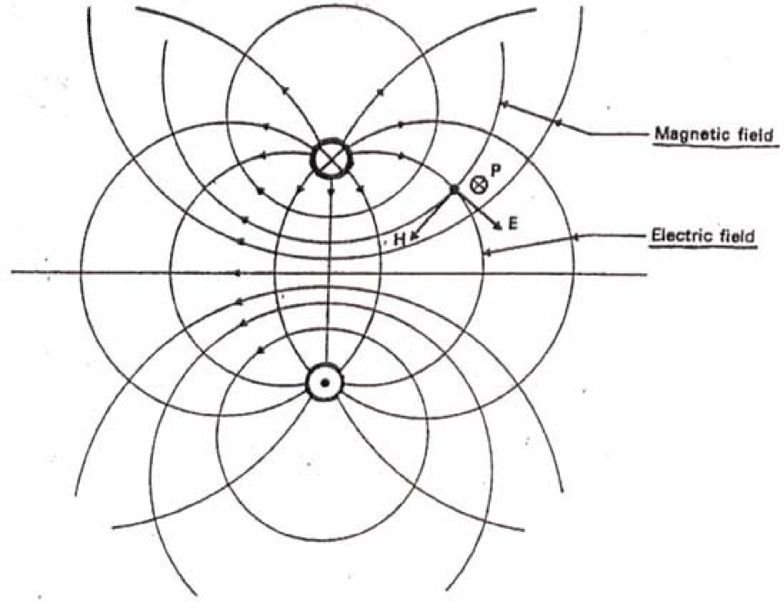
Transmission Lines



Cross-sectional view of typical transmission lines (a) coaxial line, (b) two-wire line, (c) planar line, (d) wire above conducting plane, (e) microstrip line.

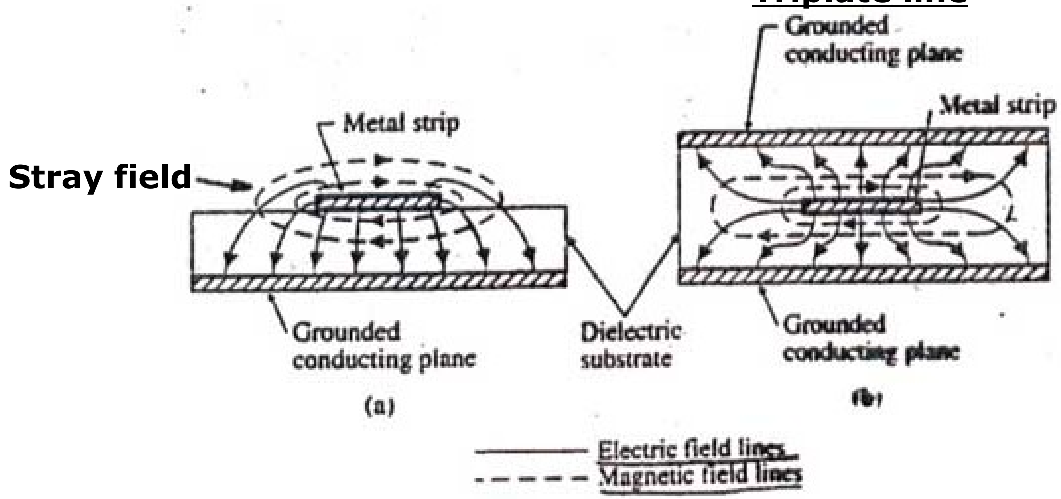


(a) Coaxial line connecting the generator to the load; (b) **E** and **H** fields on the coaxial line



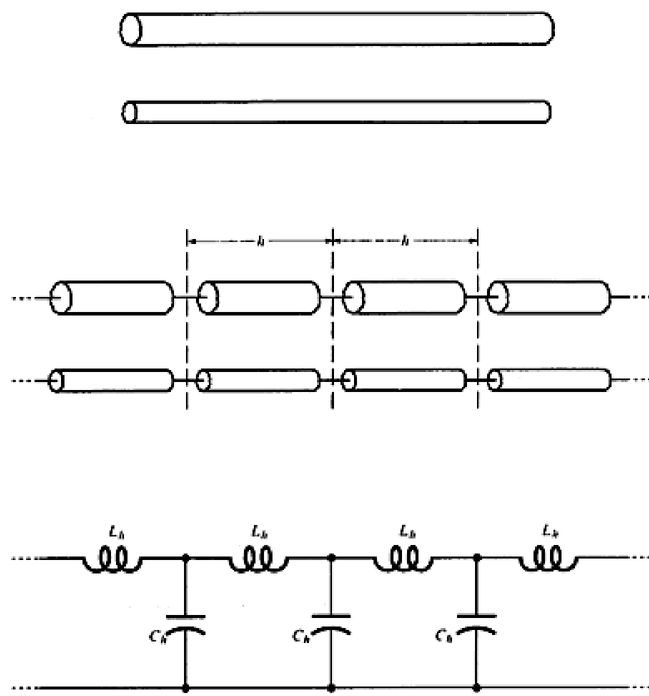
Electric and magnetic fields around single-phase transmission line

Triplate line



Transmission Lines

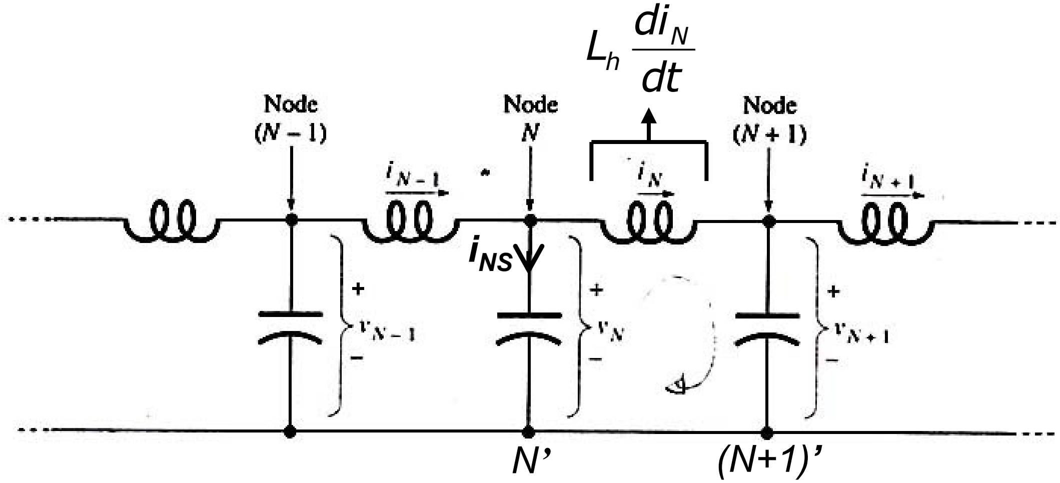
Transmission Line Equations for a Lossless Line



The transmission line consists of two parallel and uniform conuductors, not necessarily identical.

$$L_h = Lh \qquad C_h = Ch$$

Where L and C are the inductance and capacitance <u>per unit length</u> of the line, respectively.



Definitions of currents and voltages for the lumped-circuit transmission-line model.

By applying Kirchhoff's voltage law to *N - (N + 1) - (N + 1)′ - N′* loop, we obtain

$$L_h \frac{di_N}{dt} = v_N - v_{N+1}$$

$$L \frac{di_N}{dt} = -\frac{v_{N+1} - v_N}{h}$$

If node *N* is at the position *z*, node *(N +1)* is at position *z* + *h*, and $i_N = i(z)$

$$L \frac{d}{dt} i(z) = -\frac{v(z+h) - v(z)}{h}$$

Since *h* is an arbitrary small distance, we can let h approach zero

$$L\frac{\partial}{\partial t}i(z) = -\lim_{h\to 0}\left[\frac{v(z+h)-v(z)}{h}\right]$$

$$L\frac{\partial}{\partial t}i(z) = -\frac{\partial}{\partial z}v(z)$$

Applying Kirchhoff's current law to node *N* we get

$$i_{NS} = C_h\frac{dV_N}{dt} = i_{N-1} - i_N$$

from which

$$C\frac{\partial}{\partial t}v(z) = -\frac{\partial}{\partial z}i(z)$$

All cross-sectional information about the particular line is contained in L and C

$$\left.\begin{aligned} L\frac{\partial i}{\partial t} &= -\frac{\partial V}{\partial z} \\ C\frac{\partial V}{\partial t} &= -\frac{\partial i}{\partial z} \end{aligned}\right\}$$

Telegrapher's Equations

$$L\frac{\partial^2 i}{\partial t \partial z} = -\frac{\partial^2 v}{\partial z^2} \qquad C\frac{\partial^2 v}{\partial t^2} = -\frac{\partial^2 i}{\partial z \partial t}$$

$$-LC\frac{\partial^2 v}{\partial t^2} = -\frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial^2 v}{\partial t^2} - \frac{1}{LC}\frac{\partial^2 v}{\partial z^2} = 0$$

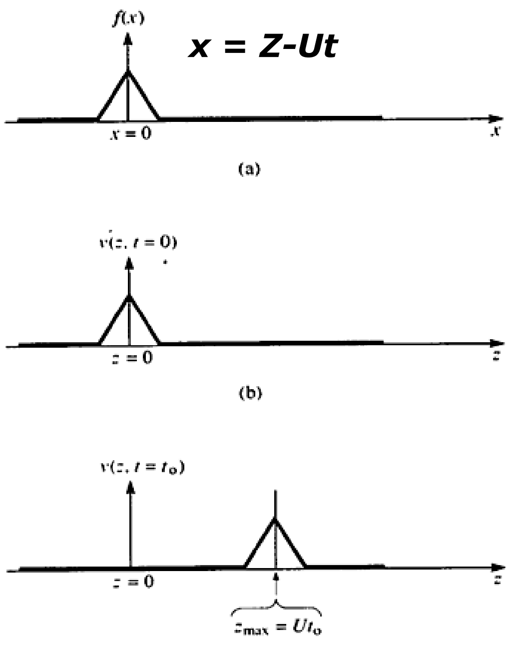
Wave Equation

Waves on the Lossless Transmission Line

Roughly speaking, a wave is a disturbance that moves away from its source as time passes. Suppose that the voltage on a transmission line as a function of position z and time t has the form

$$V(z,t) = f(z-Ut) \qquad\qquad U = \text{const}$$

This is the same function as *f(z)*, but shifted to the right a distance of *Ut* along the z axis. The displacement increases as time increases. The velocity of motion is *U*.



f(x) has its maximum where $x = z - Ut = 0$, and the position of maximum Z_{max} at $t = t_o$ is given by $Z_{max} = Ut_o$

Any function of the argument *(z-Ut)* keeps its shape and moves as a unit in the +z direction. For example, let *f(x)* be the triangular function shown in (a). Then at time t=0 $f(z-Ut)=f(z)$ is the function of *z* shown in (b). At a later time t_o , $f(z-Ut)=f(z-Ut_o)$ is the function of *z* shown in (c). Note that the pulse is moving to the right with velocity *U*.

The function V(z,t) = f(z-Ut) describes undistorted propogation in the +z direction and represents a solution of the wave equation for a lossless transmission line:

$$\frac{\partial^2 v}{\partial t^2} - \frac{1}{LC}\frac{\partial^2 v}{\partial z^2} = 0$$

Wave Equation

$$\frac{\partial^2 v}{\partial t^2} = U^2 f''(z - Ut) \qquad \frac{\partial^2 v}{\partial z^2} = f''(z - Ut)$$

$$U^2 - \frac{1}{LC} = 0$$

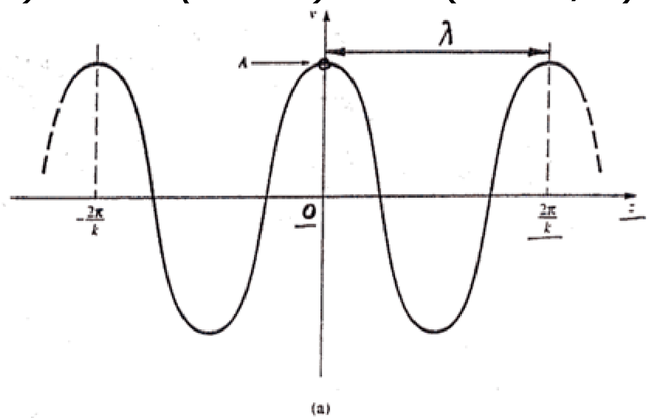
$$U = \frac{1}{\sqrt{LC}}$$

The wave equation is satisfied provided that $U = \frac{1}{\sqrt{LC}}$

The leftward-traveling wave v(z,t) = f(z+Ut) is also a solution.

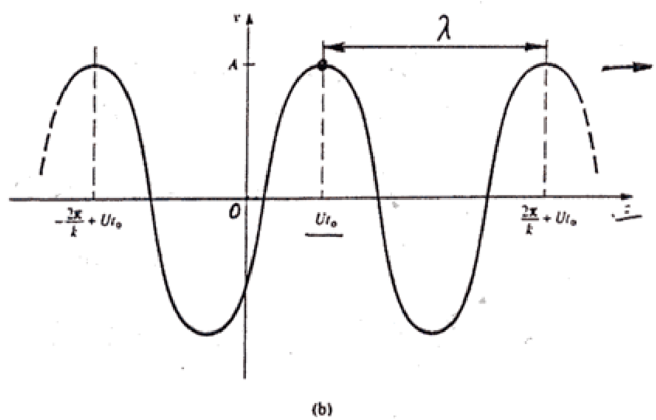
$v(z,t)=A\cos(kz-\omega t)$ $\quad$ $(U= \omega/k)$

t = 0



An important special case is that in which the function f is a sinusoid. Fig (a) shows the function $v(z,t)=A\cos(kz-\omega t)$ as it appears if photographed with a flash camera at time t=0. In (b) it is seen at the later time t_o

$t = t_o$



The wavelength of the wave is defined as the distance between the maxima at any fixed instant of time. V(z,t) has maxima when its argument (kz-ωt) is zero,

±2π, ±4π, etc. At t = 0, there is a maximum at z = 0. The next one occurs when kz = 2π , or z = 2π / k.

$$\longrightarrow \lambda= 2\pi / k$$

$$U = 2\pi f / k = \lambda f$$

The separation of time and space dependence for sinusoidal (time – harmonic) waves is achieved by the use of phasors.

Phasors are the complex quantities (in polar form) representing the magnitude and the phase of sinusoidal functions. Phasors are independent of time.

$$\underbrace{f(t)}_{\text{Time-harmonic function expressed as a cosine wave}} = \underbrace{\mathrm{Re}}_{\text{“The real part of”}}[\underbrace{\underline{f}}_{\text{Phasor}}\,\underbrace{e^{j\omega t}}_{\text{Time Factor}}]$$

For a wave $v(z,t) = A\cos(kz - \omega t)$ moving in the +z direction,

$$v(z,t) = A\cos(kz - \omega t) = A\cos(-kz + \omega t) = \mathrm{Re}[Ae^{-jkz}e^{j\omega t}]$$

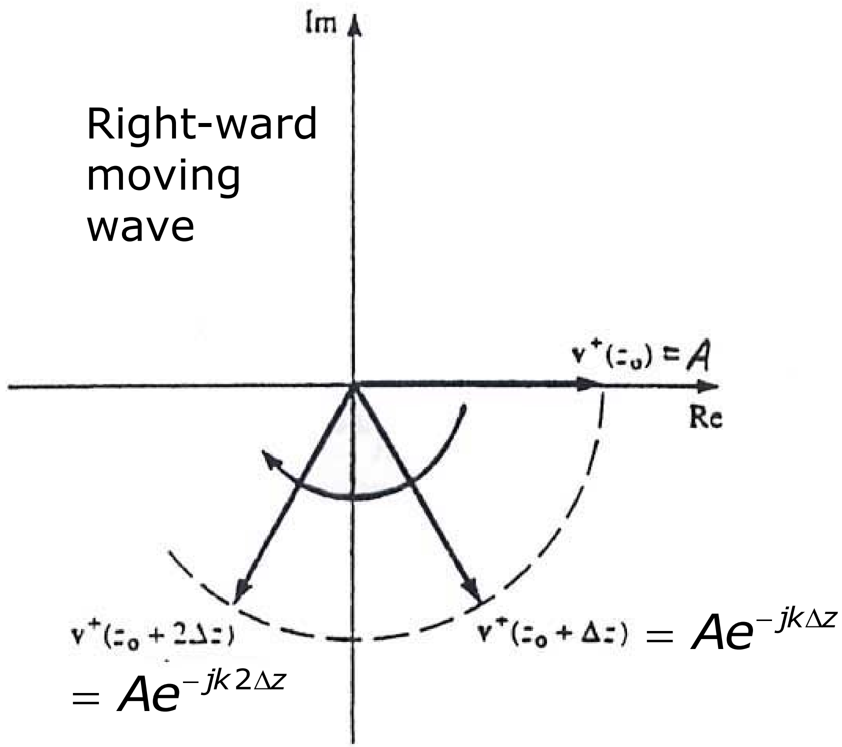
The phasor representing this positive – going wave is $\underline{v}^{+}(z) = Ae^{-jkz}$

For a wave $v(z,t) = A\cos(kz + \omega t)$ moving to the left,

$$v(z,t) = A\cos(kz + \omega t) = \mathrm{Re}[Ae^{jkz}e^{j\omega t}]$$

$$\underline{v}^{-}(z) = Ae^{jkz}$$

A = const at all z since we are dealing with a lossless line. However, the phase does vary with z.



For the <u>leftward – moving</u> wave, the phasor would rotate in the <u>counter – clockwise</u> direction.

Characteristic Impedance

The positive - going voltage wave:

(A=constant, Φ = constant) $v^+ = A\cos(\omega t + \phi - kz)$ Instantaneous voltage

$\underline{v}^+(z) = Ae^{-jkz}e^{j\phi}$ Voltage phasor

The second telegrapher's equation in phasor form

$$Lj\omega \underline{i}(z) = -\frac{\partial}{\partial z}\underline{v}(z)$$

For the positive – going wave,

$$Lj\omega \underline{i}^+(z) = -(-jkAe^{-jkz}e^{j\phi}) = jk\underline{v}^+(z)$$

$$\frac{\underline{v}^+(z)}{\underline{i}^+(z)} = \frac{\omega L}{k} = Z_o$$ Characteristic impedance (independent of position)

Since $k = \frac{\omega}{U}$ and $U = \frac{1}{\sqrt{LC}}$

$Z_o = \sqrt{L/C}$ - real number (50-400Ω)

Characteristic Impedance continued

For a <u>negative – going</u> wave,

$$\frac{\underline{v}^-}{\underline{i}^-} = -Z_o$$

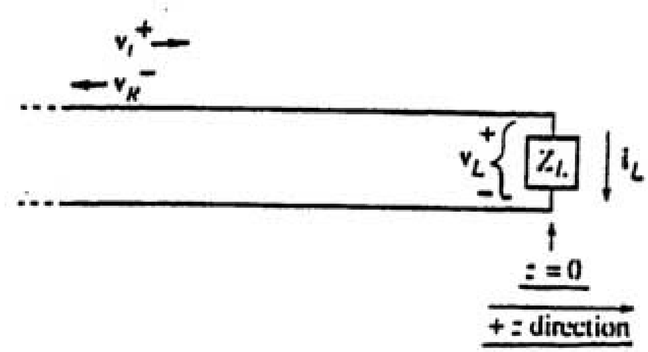
Power transmitted by a single wave

$$P^+ = \tfrac{1}{2}\mathrm{Re}\left[\underline{v}^+(\underline{i}^+)^*\right] = \tfrac{1}{2}\frac{\left|\underline{v}^+\right|^2}{Z_o}$$

(average power; the instantaneous power oscillates at twice the fundamental frequency)

$$\sqrt{\underline{v}^+(\underline{v}^+)^*} = \left|\underline{v}^+\right| \qquad \underline{i}^+ = \frac{\underline{v}^+}{Z_o}$$

Reflection and Transmission



At z = 0, $$\underline{v}(z=0)=\underline{v}_i^+(z=0)+\underline{v}_r^-(z=0)$$

$$\underline{i}(z=0)=\underline{i}_i^+(z=0)+\underline{i}_r^-(z=0)$$

$$=\frac{1}{Z_o}[\underline{v}_i^+(z=0)-\underline{v}_r^-(z=0)]$$

$$\underline{v}(z=0)=\underline{v}_L \qquad \underline{i}(z=0)=\underline{i}_L \qquad \frac{\underline{v}_L}{\underline{i}_L}=Z_L$$

$$\frac{\underline{v}(z=0)}{\underline{i}(z=0)}=Z_o\frac{\underline{v}_i^+(z=0)+\underline{v}_r^-(z=0)}{\underline{v}_i^+(z=0)-\underline{v}_r^-(z=0)}=Z_L$$

Assuming that the incident wave $\underline{v}_i^+$ is known and solving for $\underline{v}_r^-$, we obtain

$$\underline{v}_r^-(z=0)=\frac{Z_L-Z_o}{Z_L+Z_o}\underline{v}_i^+(z=0)$$

$$\frac{\underline{v}_r^-(z=0)}{\underline{v}_i^+(z=0)}=\frac{Z_L-Z_o}{Z_L+Z_o}=\rho_o$$ Load's Reflection Coefficient

Reflection and Transmission continued

Example Suppose $\underline{Z}_L = \infty$ (open circuit). Find the distribution of the voltage on the line if the incident wave is $\underline{v}_i^+(z) = Ae^{-jkz}$

Assume that A is real (Φ = 0)

$$\rho_o = 1$$

The total voltage on the line is: $\underline{v}(z) = \underline{v}_i^+(z) + \underline{v}_r^-(z)$

$$\underline{v}_r^-(z) = \underline{v}_r^-(z=0)e^{jkz} = \rho_o \underline{v}_i^+(z=0)e^{jkz} = Ae^{jkz}$$

$$\underline{v}(z) = Ae^{-jkz} + Ae^{jkz}$$

$$= 2A\cos kz \qquad (\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2})$$

$$\underline{v}(z) = 0 \text{ at } z = \frac{2N+1}{2k}\pi \quad (kz = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2} ...)$$

The instantaneous voltage is: $v(z,t) = \mathrm{Re}[\underline{v}(z)e^{j\omega t}] = 2A\cos kz \cos\omega t$

At $z = \frac{2N+1}{2k}\pi$ $v(z,t) = 0$ at all times.

The total voltage is the sum of the two waves of equal amplitude moving in opposite directions. The positions of zero total voltage stand still. This phenomenon is referred to as a standing wave.

In the case of a single traveling wave, $v(z,t) = A\cos(kz - \omega t)$, there are positions where the voltage vanishes, but these positions move at the velocity of the wave $U = \omega / k$

Reflection and Transmission continued

If $Z_L = 0$ (short circuit), $\rho_o = \dfrac{Z_L - Z_o}{Z_L + Z_o} = -1$ and $\underline{v}(z) = Ae^{-jkz} - Ae^{jkz} = -2jA\sin kz$

$$(\sin\alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j})$$

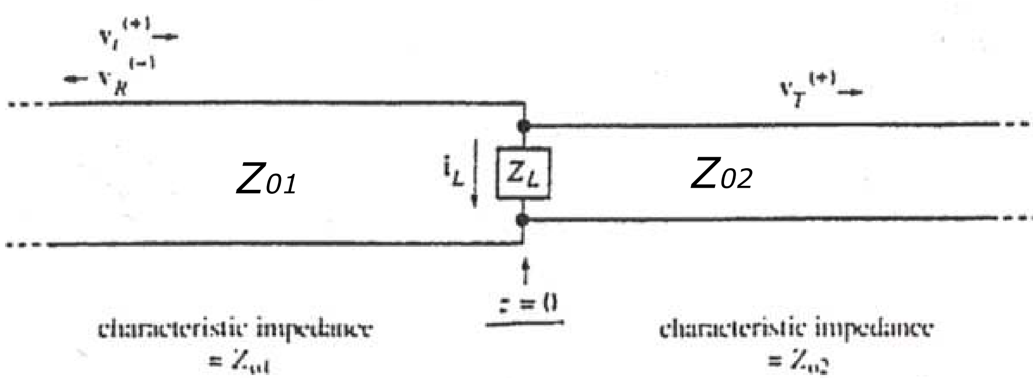
Again we have a standing wave but with the nulls at $z = \dfrac{N\pi}{k}$

If $Z_L = R_L = nZ_o$ (resistive), $\rho_o = \dfrac{(n-1)}{(n+1)}$ $(|\rho_o| \le 1)$

When n=1 ($R_L = Z_o$, i.e. the line is <u>terminated in its characteristic impedance)</u>,

the reflected wave vanishes $\rho_o = 0$

Suppose that one more transmission line is connected at the load terminals (z=0)



Reflection and Transmission continued

The voltage at z=0, if we approach from the left, is $\underline{v}_i^+(z=0)+\underline{v}_r^-(z=0)$. If we approach from the right, it is $\underline{v}_t^+(z=0)$. Thus we can write:

$$\underline{v}_i^+(z=0)+\underline{v}_r^-(z=0)=\underline{v}_t^+(z=0)$$

Applying Kirchhoff's current law we get $\underline{i}_i^+(z=0)+\underline{i}_r^-(z=0)=\underline{i}_t^+(z=0)+\underline{i}_L$

Further, $\underline{i}_L=\dfrac{\underline{v}_t^+(z=0)}{Z_L}$ $\quad \underline{v}_L=\underline{v}_t^+(z=0)$

The currents $\underline{i}_i^+$, $\underline{i}_r^-$, and $\underline{i}_t^+$ can be expressed in term of $\underline{v}_i^+$, $\underline{v}_r^-$, and $\underline{v}_t^+$,respectively:

$$\underline{i}_i^+=\frac{\underline{v}_i^+}{Z_{01}} \qquad \underline{i}_r^-=-\frac{\underline{v}_r^-}{Z_{01}} \qquad \underline{i}_t^+=\frac{\underline{v}_t^+}{Z_{02}}$$

Now, assuming that $\underline{v}_i^+(z=0)$ is known, we can find $\underline{v}_r^-(z=0)$ and $\underline{v}_t^+(z=0)$

$$\boxed{\rho_o=\frac{\underline{v}_r^-(z=0)}{\underline{v}_i^+(z=0)}=\frac{Z_{11}-Z_{01}}{Z_{11}+Z_{01}}} \quad \boxed{\tau=\frac{\underline{v}_t^+(z=0)}{\underline{v}_i^+(z=0)}=\frac{2Z_{11}}{Z_{11}+Z_{01}}} \quad (Z_{11}=Z_L\,||\,Z_{02})$$

(<u>Reflection Coefficient</u>) (<u>Transmission Coefficient</u>)

Standing-Wave Ratio

(lossless transmission line)

The total phasor voltage as a function of position on a line connected to a load at z=0 is $\underline{v}(z) = \underline{v}^+(z) + \underline{v}^-(z) = v_i e^{-jkz} + v_r e^{jkz}$

The magnitude of the reflected voltage phasor is

$$v_r = \underline{v}^-(z=0) = \rho_o \underline{v}^+(z=0) = \rho_o v_i \qquad \rho_o = |\rho_o| e^{j\phi_R}$$

$$\underline{v}(z) = v_i(e^{-jkz} + \rho_o e^{jkz}) \qquad \underline{v}^*(z) = v_i(e^{jkz} + \rho_o^* e^{-jkz})$$

$$\underbrace{|\underline{v}(z)|} = \sqrt{\underline{v}(z)\underline{v}^*(z)} = v_i[1 + \rho_o e^{2jkz} + \rho_o^* e^{-2jkz} + |\rho_o|^2]^{\frac{1}{2}}$$

$$= v_i[1 + |\rho_o|^2 + 2\,\mathrm{Re}(\rho_o e^{2jkz})]^{\frac{1}{2}}$$

$$= v_i[1 + |\rho_o|^2 + 2|\rho_o|\,\mathrm{Re}(e^{2jkz + j\phi_R})]^{\frac{1}{2}}$$

$$= v_i[1 + |\rho_o|^2 + 2|\rho_o|\cos(2kz + \phi_R)]^{\frac{1}{2}}$$

The amplitude of voltage as a function of z

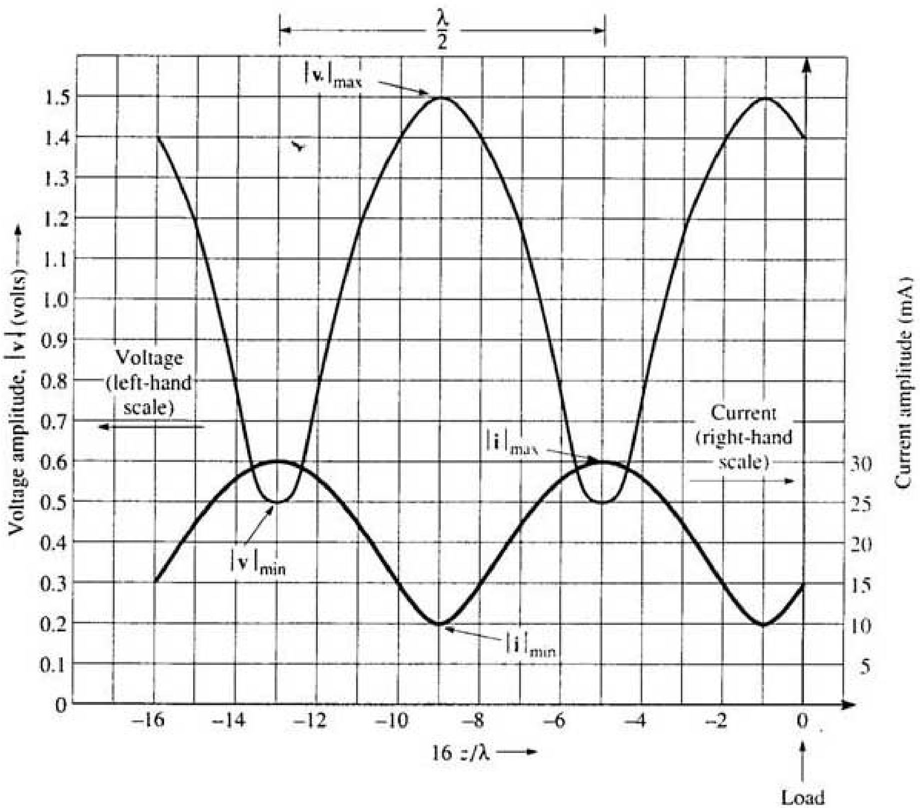
At any position, the instantaneous voltage on the line is a sinusoidal function of time, with the amplitude $|\underline{v}(z)|$ given by the above expression. The amplitude regularly increases and decreases as the cosine function varies. The positions of voltage amplitude maxima and minima are stationary (independent of time). This phenomenon is referred to as a standing wave.

Standing-Wave Ratio continued

(losseless transmission line)

In the special case of $\rho_o = 0$, the reflected wave vanishes and there is only a single traveling wave moving to the right. In this case the voltage amplitude is independent of position ("flat" voltage profile).

If there are two (or more) traveling waves on the line, they will interact to produce a standing wave.



$$V_i = 1\,V$$

$$|\rho_o| = 0.5$$

$$\phi_R = 45^\circ$$

$$Z_o = 50\,\Omega$$

Standing-Wave Ratio continued

(losseless transmission line)

$$\left|\underline{V}\right| = v_i\left[\ 1 + \left|\rho_o\right|^2 + 2\left|\rho_o\right|\cos(2kz + \phi_R)\right]^{\frac{1}{2}}$$

$$\left|\underline{V}\right|_{\max} = v_i\left[1 + \left|\rho_o\right|^2 + 2\left|\rho_o\right|\right]^{\frac{1}{2}} = v_i(1 + \left|\rho_o\right|) \qquad (2kz + \phi_R = 2N\pi)$$

$$\left|\underline{V}\right|_{\min} = v_i\left[1 + \left|\rho_o\right|^2 - 2\left|\rho_o\right|\right]^{\frac{1}{2}} = v_i(1 - \left|\rho_o\right|) \qquad (2kz + \phi_R = (2\underbrace{N}_{\text{an integer}}+1)\pi)$$

The standing-wave ratio (*SWR)* is defined as

$$SWR = \frac{\left|\underline{V}\right|_{\max}}{\left|\underline{V}\right|_{\min}} = \frac{1 + \left|\rho_o\right|}{1 - \left|\rho_o\right|} \geq 1 \qquad SWR = 1 \text{ when } \left|\rho_o\right| = 0$$

For two adjacent maxima at, say, N=1 and N=0 we can write

$$2kz_{N=1} + \phi_R = 2\pi$$

$$2kz_{N=0} + \phi_R = 0$$

$$\Delta z = z_{N=1} - z_{N=0} = \frac{\pi}{k} = \frac{\pi}{\frac{2\pi}{\lambda}} = \frac{\lambda}{2}$$

Voltage maxima and minima repeat every half wavelength.

Transmission Line Equations for a Lossy Line

(sinusoidal waves)

From Kirchhoff's laws in their phasor form, we have

$$\underline{i}_N(j\omega L_h + R_h) = \underline{v}_N - \underline{v}_{N+1}$$

$$\underline{v}_N(j\omega C_h + G_h) = \underline{i}_{N-1} - \underline{i}_N$$

Proceeding as before (for a lossless lines), we obtain the phasor form of the telegrapher equations,

$$\underline{i}(z)(j\omega L + R) = -\frac{\partial \underline{v}}{\partial z}$$

$$\underline{v}(z)(j\omega C + G) = -\frac{\partial \underline{i}}{\partial z}$$

where L, R, C, and G are, respectively, the series inductance, series resistance, shunt capacitance, and shunt conductance per unit length.

The corresponding (voltage) wave equation is

$$\frac{\partial^2 \underline{v}}{\partial z^2} - (j\omega C + G)(j\omega L + R)\underline{v} = 0 \qquad \left(\frac{\partial^2 \underline{v}}{\partial z^2} - \gamma^2 \underline{v} = 0\right)$$

The two solutions of the wave equation are

$$v^+ = A^+ e^{-\gamma z} \quad \longrightarrow +z$$

$$v^- = A^- e^{\gamma z} \quad -z \longleftarrow$$

where A^+ and A^- are constants describing the wave's amplitude and phase and γ is the propagation constant.

Transmission Line Equations for a Lossy Line continued

(sinusoidal waves)

The propagation constant of a lossy transmission line is

(complex number) $$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

where α and β are real numbers.

Inserting R=0, G=0 (lossless line) we obtain

$$\gamma = j\omega\sqrt{LC} = j\beta = jk \quad (\alpha = 0)$$

For the positive-going wave

$$\underline{v}^{+} = A\underbrace{e^{-j\beta z}}\underbrace{e^{-\alpha z}}$$

Causes negative phase shift (phasor rotates clockwise as z increases)

Causes attenuation (amplitude becomes smaller exponentially as z increases)

Transmission Line Equations for a Lossy Line continued

(sinusoidal waves)

A phase shift of βz equal to 2π corresponds to the wave travel distance *z* equal to the wavelength λ :

$$\beta\lambda = 2\pi \longrightarrow \boxed{\lambda = \frac{2\pi}{\beta}}$$

β is the phase constant (measured in radians per meter)

α is the attenuation constant (measured in Nepers per meter)

α^{-1} is the attenuation length (amplitude decreases 1/e over $z = \alpha^{-1}$)

In general, nonlinear functions of ω

The corresponding instantaneous voltage is

$$v(z,t) = A\cos(\beta z - \omega t)e^{-\alpha z}$$

(A is assumed to be real)

The position of a maximum is given by

$$\beta z_{max} = \omega t \qquad z_{max} = \frac{\omega}{\beta} t$$

As t increases, the maximum moves to the right with velocity

$$\frac{dz_{max}}{dt} = \frac{\omega}{\beta}$$ - <u>Phase Velocity</u> (U_p)

Dispersion

In general, the phase velocity U_p is a function of frequency; that is, a signal containing many frequencies tends to become 'dispersed' (some parts of the signal arrived sooner and others later.)

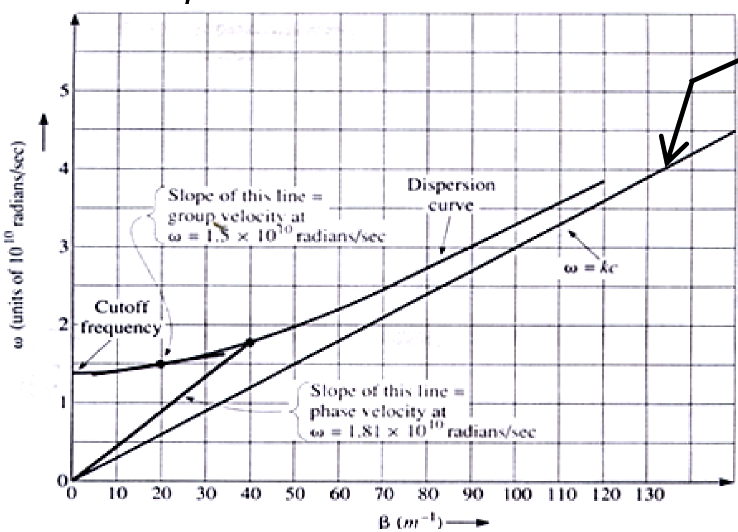
Up is independent of frequency for (1) lossless lines (*R=0, G=0*) and (2) distortionless lines (*R/L=G/C*) because for those lines *β* is a linear function of *ω.*

$$U_p = U_G = c$$

Cut-off frequency

$$\omega_o = 1.35 \times 10^{10}\, rad/s$$

$$\left(U_p = \infty; U_G = 0\right)$$



Example of dispersion diagram for an arbitrary system that is characterized by $U_p > c$

U_p at any frequency is equal to the slope of a line drawn from the origin to the corresponding point on the graph. For $\omega = 1.35 x 10^{10}$ radians/second $Up = \infty$. In general, U_p can be either greater or less then c.

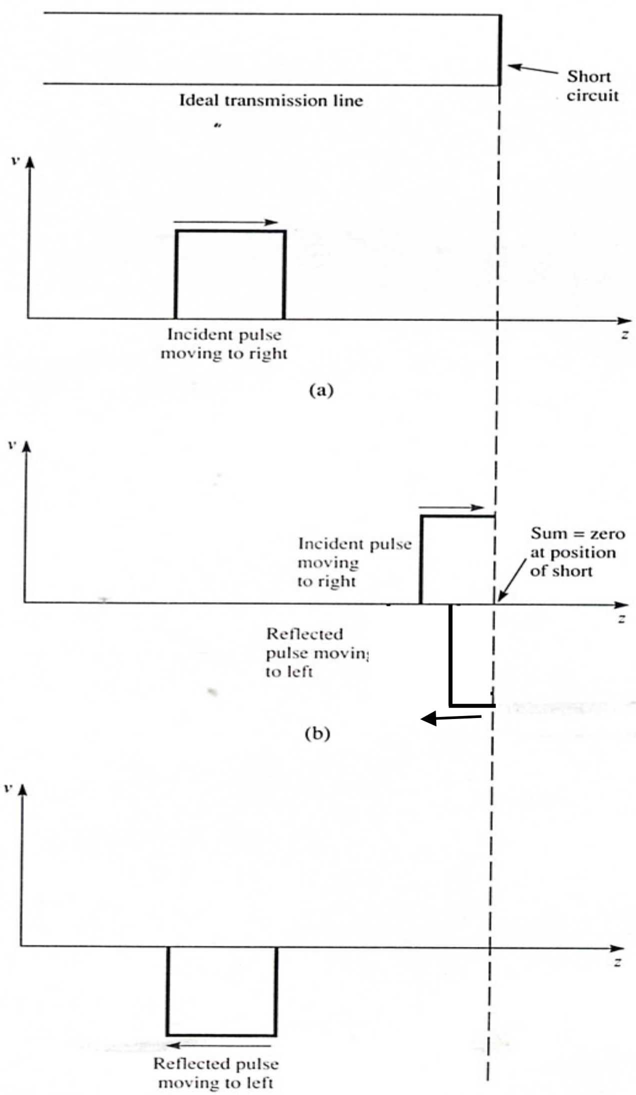
Information in a wave travels at a different velocity known as the group velocity

$$U_G = \frac{d\omega}{d\beta}$$

U_G is equal to the slope of the tangent to the *ω-β* curve at the frequency in question (for $\omega = \omega_o$ $U_G = 0$ for this particular system). U_G always remains less then c.

Non-Sinusoidal Waves

(lossless transmission line)



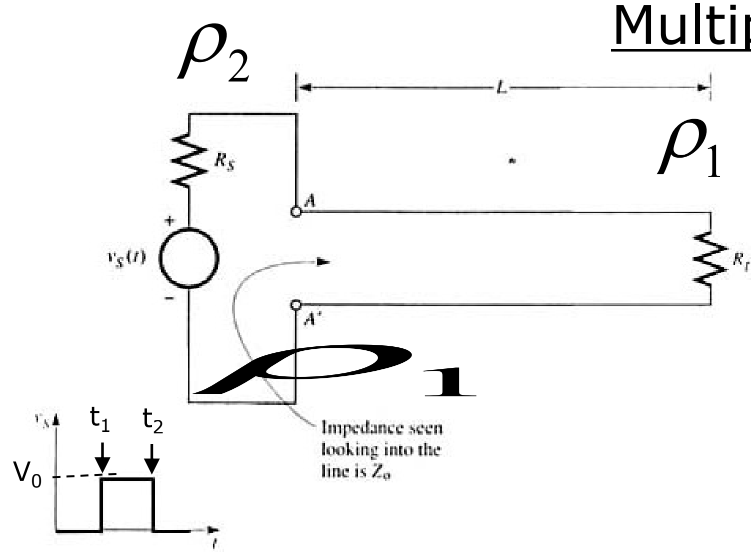
Reflection of a rectangular pulse of a short circuit.

(a) Shows the incident pulse moving to the right.

In (b) it is striking the short-circuit termination, note that the sum of the incident and the reflected voltages must always be zero at that position.

In (c) the reflected pulse is moving to the left.

Multiple Reflections



$$V_1 = \frac{Z_o}{Z_o + R_S} V_o \qquad \longrightarrow +z$$

$$\rho_1 = \frac{R_L - Z_o}{R_L + Z_o} \qquad V_2 = \rho_1 V_1 \qquad -z \longleftarrow$$

$$\rho_2 = \frac{R_S - Z_o}{R_S + Z_o} \qquad V_3 = \rho_2 V_2 \qquad \longrightarrow +z$$

Example Suppose t2 = ∞ (an infinitely long pulse or a step function) and $R_L = R_S$, so that $\rho_1 = \rho_2 = \rho_0$. Find the total voltage on the line after a very long time.

The initial (incident) wave moving to the right has amplitude

$$V_1 = \frac{Z_o}{Z_o + R_S} V_o$$

The first reflected wave moving to the left has amplitude

$$V_2 = \rho_0 V_1 \qquad \text{where} \qquad \rho_0 = \frac{R_S - Z_o}{R_S + Z_o}$$

Multiple Reflections continued

The second reflected wave moving to the right has amplitude

$$V_3 = \rho_0{}^2 V_1;\ V_4 = \rho_0{}^3 V_1 \quad \text{and so on}$$

The total voltage at $t \to \infty$ is given by the infinite series

$$V_T = V_1 + \rho_0 V_1 + \rho_0{}^2 V_1 + \rho_0{}^3 V_1 + \ldots = V_1(1 + \rho_0 + \rho_0{}^2 + \rho_0{}^3 + \ldots) = V_1(\frac{1}{1-\rho_0})$$

Inserting the values of ρ_0 and V_1 we find that $V_T = \dfrac{V_0}{2}$ (simply results from the voltage divider of *Rs* and *RL,* as if the line were not there.

Lattice (bounce) diagram

This is a space/time diagram which is used to keep track of multiple reflections.

Ideal voltage source

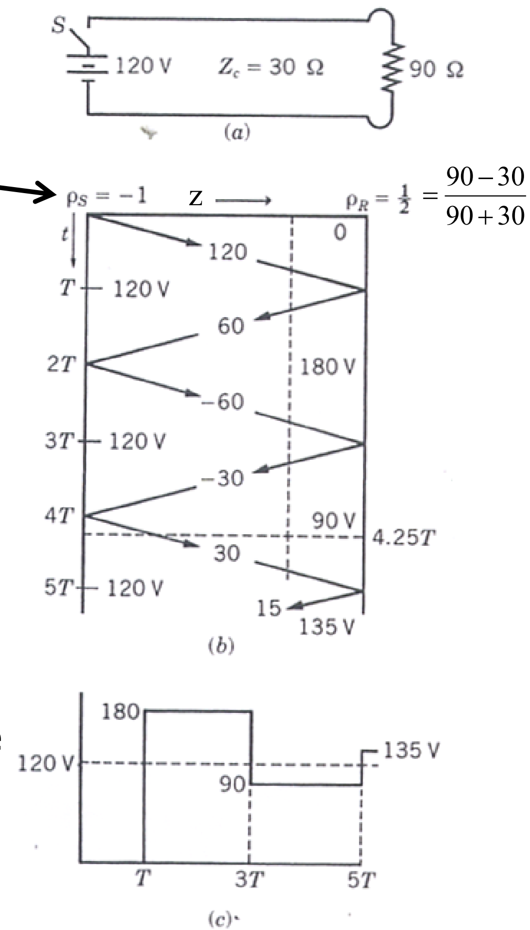


Figure 5.14 Circuit diagram, lattice diagram, and plot of voltage versus time for Example 5.6 where the receiving-end resistance is 90 Ω.

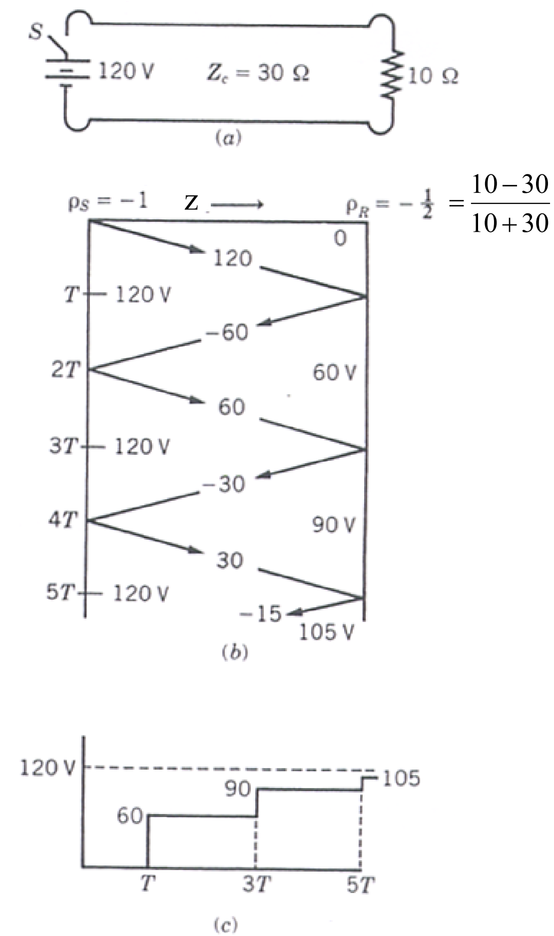


Figure 5.15 Circuit diagram, lattice diagram, and plot of voltage versus time when the receiving-end resistance for Example 5.6 is changed to 10 Ω.

Voltage at the receiving end

$$T = \frac{l}{U}$$

Points to Remember

1. In this chapter we have surveyed several different types of waves on transmission lines. It is important that these different cases not be confused. When approaching a transmission-line problem, the student should begin by asking, "Are the waves in this problem sinusoidal, or rectangular pulses? Is the line ideal, or does it have losses?" Then the proper approach to the problem can be taken.

$$k = \omega\sqrt{LC}$$
$$U_P = 1/\sqrt{LC}$$
$$Z_o = \sqrt{L/C}$$

2. The ideal lossless line supports waves of any shape (sinusoidal or non-sinusoidal), and transmits them without distortion. The velocity of these waves is $(LC)^{-1/2}$. The ratio of the voltage to current is $Z_o = \sqrt{L/C}$, provided that only one wave is present. Sinusoidal waves are treated using phasor analysis. (A common error is that of attempting to analyze *non*-sinusoidal waves with phasors. Beware! This makes no sense at all.)

$$k = \beta - j\alpha$$
$$U_P = \omega / \beta$$
$$U_G = d\omega / d\beta$$

3. When the line contains series resistance and or shunt conductance it is said to be *lossy*. Lossy lines no longer exhibit undistorted propagation; hence a rectangular pulse launched on such a line will not remain rectangular, instead evolving into irregular, messy shapes. However, sinusoidal waves, because of their unique mathematical properties, do continue to be sinusoidal on lossy lines. The presence of losses changes the velocity of propagation and causes the wave to be attenuated (become smaller) as it travels.

Points to Remember continued

4. For lines other than the simple ideal lossless lines, the velocity of propagation usually is a function of frequency. This velocity, the speed of voltage maxima on the line, is properly called the phase velocity U_p. The change of U_p with frequency is called dispersion. The velocity with which information travels on the line is not U_p, but a different velocity, known as the group velocity U_G. The phase velocity is given by $U_P = \omega / \beta$. However $U_G = d\omega / d\beta$

5. Examples of non-sinusoidal waves are short rectangular pulses, and also infinitely long rectangular pulses, which are the same as step functions. Problems involving sudden voltage steps differ from sinusoidal problems, just as in ordinary circuits, problems involving transients differ from the sinusoidal steady state. Pulse problems are usually approached by superposition; that is, one tracks the pulses that propagate back and forth, adding up the waves to obtain the total voltage at any place and time.

Points to Remember continued

(Lossless line)

$$\tau = \frac{2Z_{11}}{Z_{11} + Z_{01}}$$

$$\rho_o = \frac{Z_L - Z_o}{Z_L + Z_o}$$

6. All kinds of waves are reflected at discontinuities in the line. If the line continues beyond the discontinuity, a portion of the wave is transmitted as well. The reflected and transmitted waves are described by the reflection coefficient and the transmission coefficient. For sinusoidal waves there is a simple formula giving the reflection coefficient for any load impedance Z_L. For non-sinusoidal waves, the same formula can be used, but only if the load impedance is purely resistive. Otherwise the reflected wave has a different shape from the incident wave, and a reflection coefficient cannot be meaningfully defined.

(Bounce diagram)

7. In the case of non-sinusoidal waves, it is sometimes necessary to add up the contributions of many reflected waves bouncing back and forth on the line. However, for sinusoidal steady-state problems, it is only necessary to consider *two* waves, one moving to the right and the other to the left.

Points to Remember continued

(Sinusoidal Waves)

8. When both an incident and reflected wave are simultaneously present on a transmission line, a *standing wave* is said to be present. This means that a stationary pattern of voltage maxima and minima is present. The ratio of the maximum voltage to the minimum voltage is called the *standing-wave ratio* (SWR). The positions of the voltage maxima are determined by the phase angle of the load's reflection coefficient, and the spacing between each pair of adjacent maxima is $\lambda/2$ (and not λ, as one might think). Positions of maximum voltage are positions of minimum current, and vice versa.

(Sinusoidal Waves)

9. The impedance $Z(z)$ at any point on a line is defined as the ratio of the total voltage phasor to the total current phasor at the point z. If a standing wave is present, the impedance will be a periodic function of position along the line, with period $\lambda/2$. Note that this impedance is different from the "characteristic impedance" Z_o, which is a constant that depends only on the construction of the line.