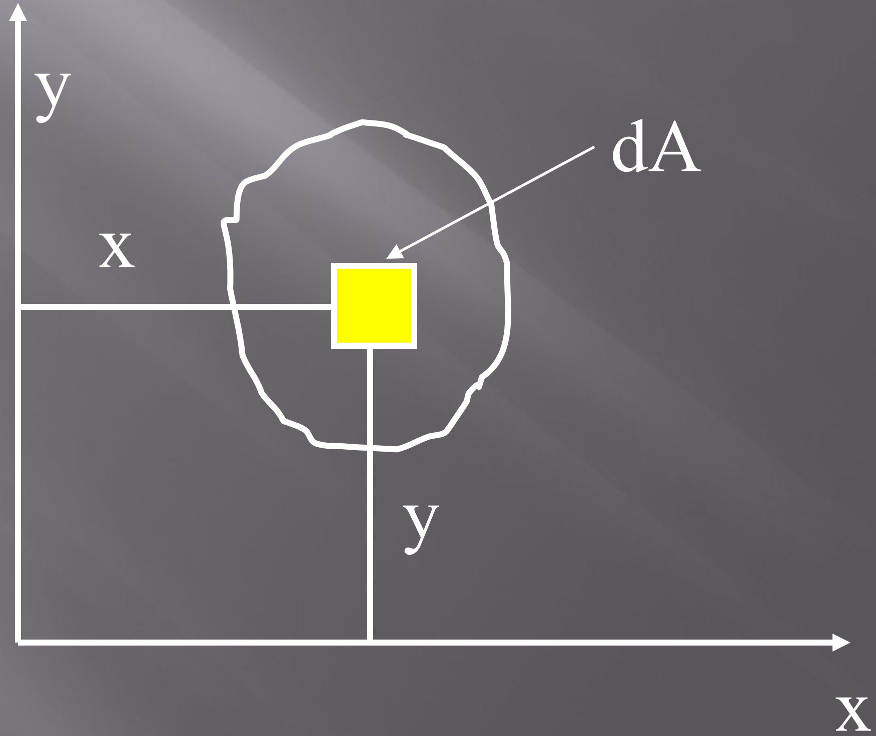
MOMENT OF INERTIA

Moment of Inertia:

The product of the elemental area and square of the perpendicular distance between the centroid of area and the axis of reference is the "Moment of Inertia" about the reference axis.

$I_{xx} = \int dA.\ y^2$

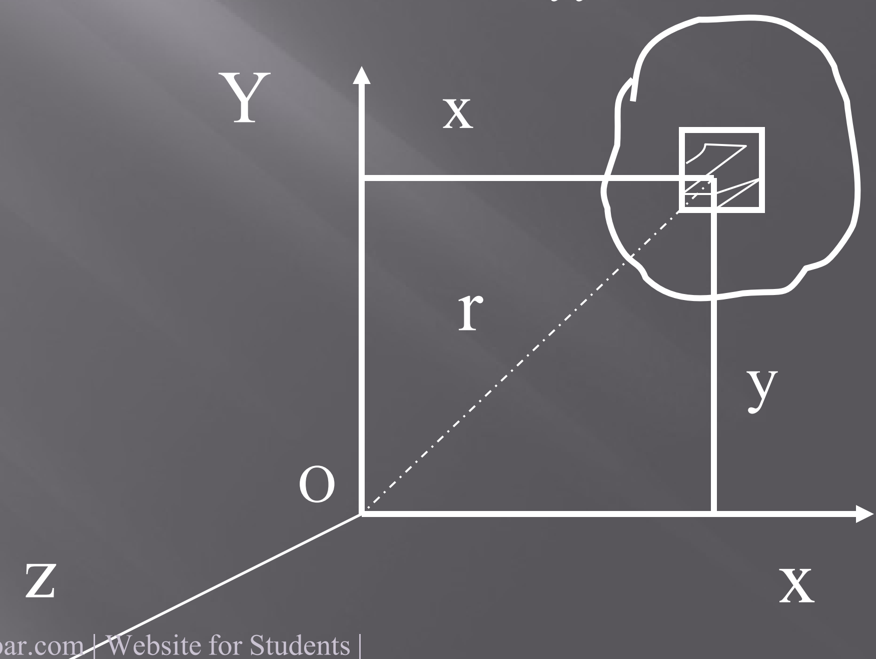
$I_{yy} = \int dA.\ x^2$



It is also called second moment of area because first moment of elemental area is dA.y and dA.x; and if it is again multiplied by the distance,we get second moment of elemental area as (dA.y)y and (dA.x)x.

Polar moment of Inertia (Perpendicular Axes theorem)

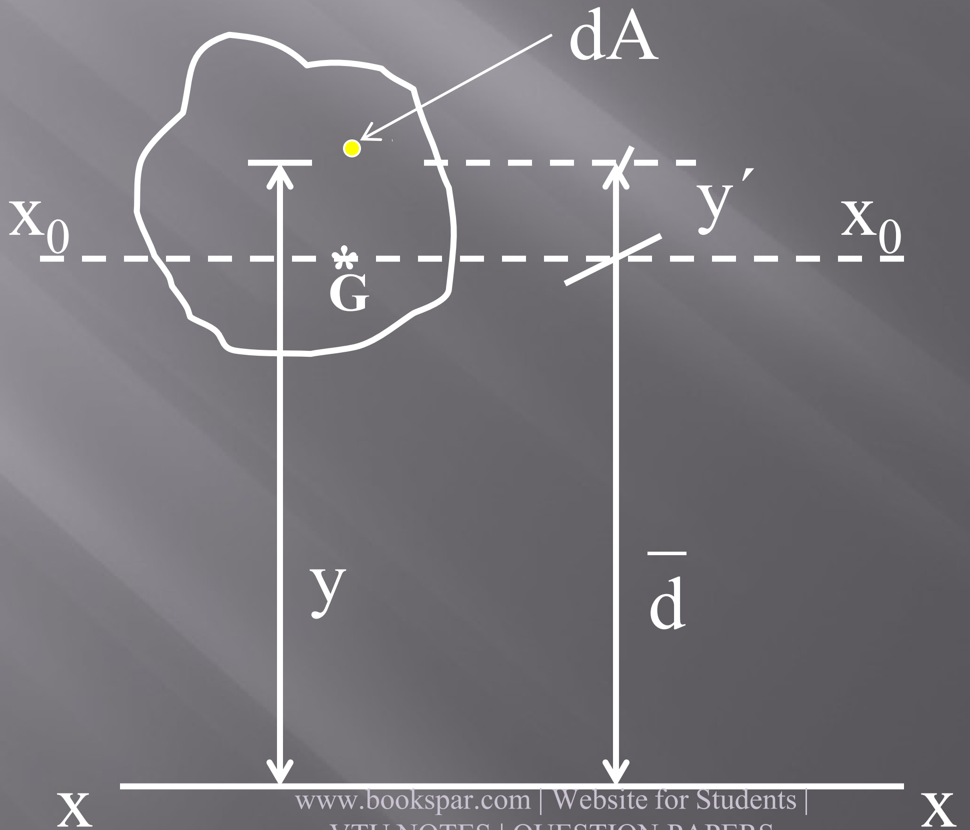
The moment of inertia of an area about an axis perpendicular to the plane of the area is called "Polar Moment of Inertia" and it is denoted by symbol I_{zz} or J or I_p. The moment of inertia of an area in xy plane w.r.to z. axis is $I_{zz} = I_p = J = \int r^2 dA = \int (x^2 + y^2)\, dA = \int x^2 dA + \int y^2 dA = I_{xx} + I_{yy}$



PERPENDICULAR AXIS THEOREM

Hence polar M.I. for an area w.r.t. an axis perpendicular to its plane of area is equal to the sum of the M.I. about any two mutually perpendicular axes in its plane, passing through the point of intersection of the polar axis and the area.

Parallel Axis Theorem



$Ixx = \int dA.\ y2$

$= \int dA\ (\bar{d} + y')^2$

$= \int dA\ (\bar{d}^2 + y'^2 + \bar{2}dy')$

$= \int dA.\ \bar{d}^2 + \int dAy'^2 + \int 2d.dAy'$

$d^2 \int dA = A.(\bar{d})^2$

$\int dA.\ y'2 = Ix_0x_0$

$2\bar{d}\ \int dAy' = 0$

(since Ist moment of area about centroidal axis = 0)

$\therefore I_{x\,x} = I_{x_0\ x_0} + A\bar{d}^2$

Hence, moment of inertia of any area about an axis xx is equal to the M.I. about parallel centroidal axis plus the product of the total area and square of the distance between the two axes.

Radius of Gyration

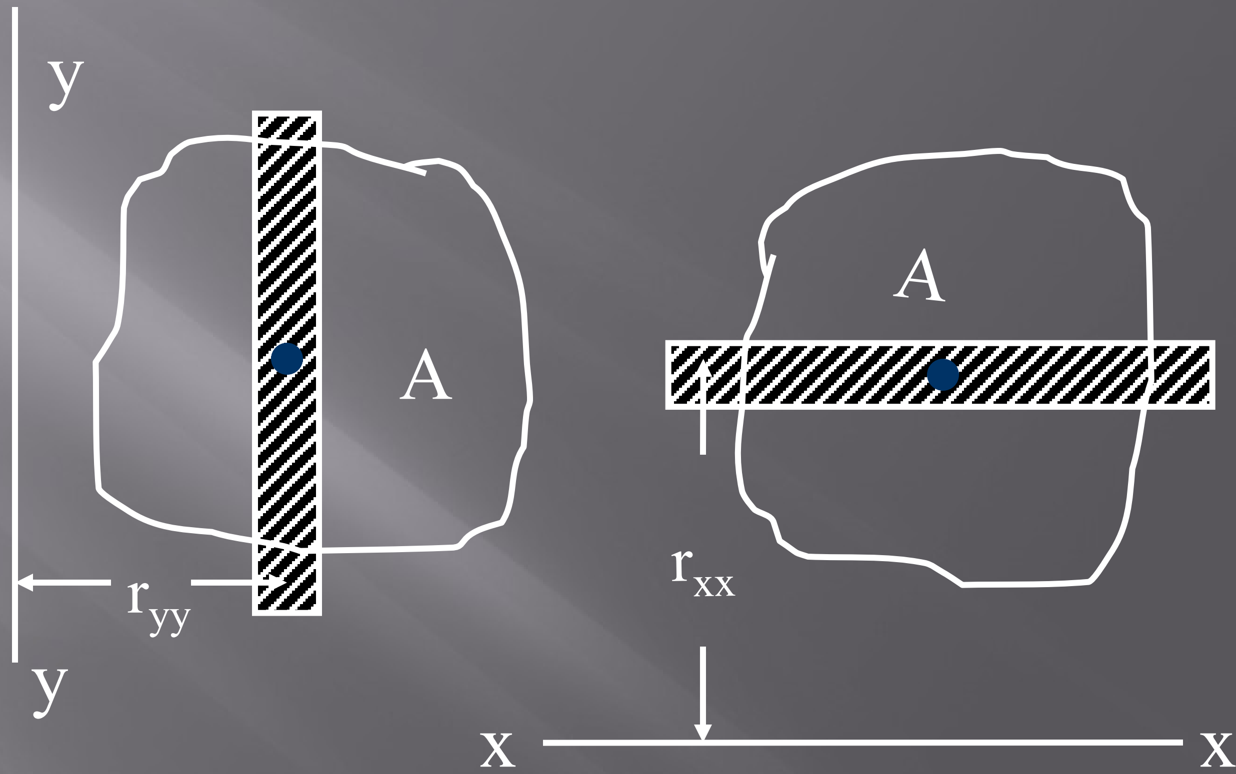
It is the perpendicular distance at which the whole area may be assumed to be concentrated, yielding the same second moment of the area above the axis under consideration.

$$I_{yy} = A.r_{yy}^2$$

$$I_{xx} = A.r_{xx}^2$$

$$\therefore r_{yy} = \sqrt{I_{yy}/A}$$

And $r_{xx} = \sqrt{I_{xx}/A}$



r_{xx} and r_{yy} are called the radii of gyration

MOMENT OF INERTIA BY DIRECT INTEGRATION

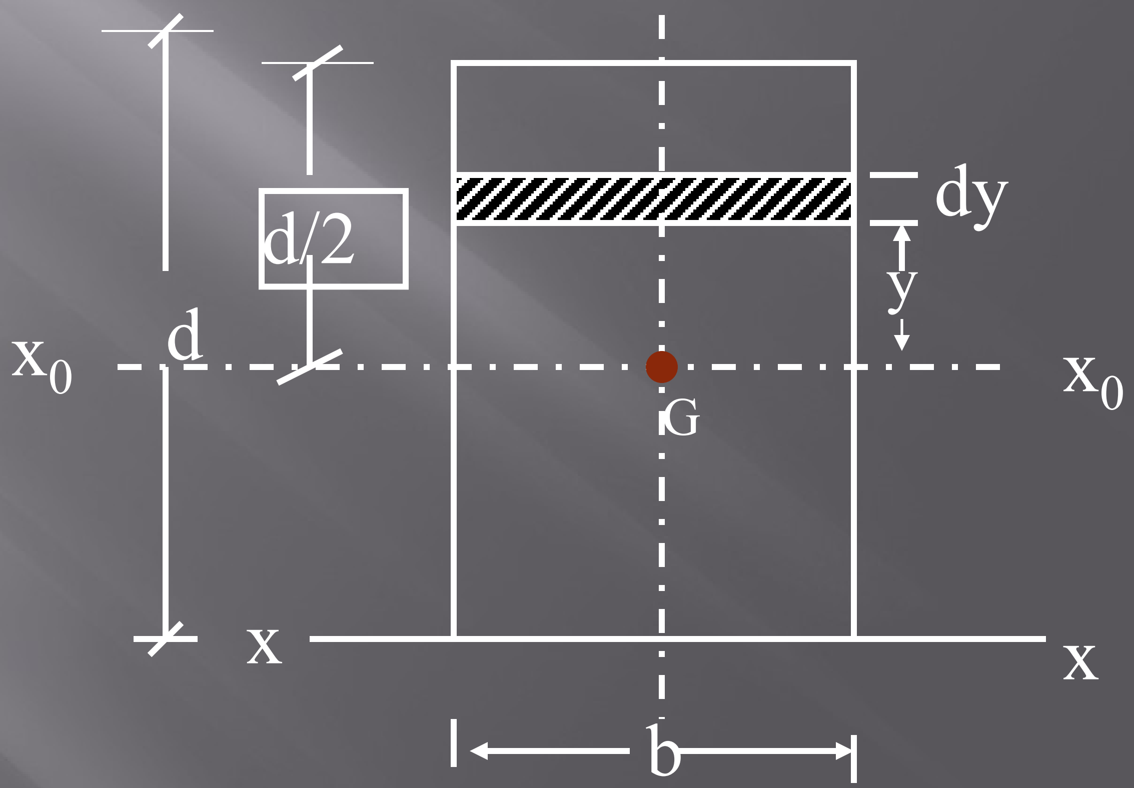
M.I. about its horizontal centroidal axis :

RECTANGLE :

$$I_{XoXo} = \int_{-d/2}^{+d/2} dA y^2$$

$$= \int_{-d/2}^{+d/2} (b.dy) y^2$$

$$= bd^3/12$$



About its base

$$I_{XX} = I_{XoXo} + A(\bar{d})^2$$

Where $\bar{d} = d/2$, the distance between axes xx and $x_o x_o$

$$= bd^3/12 + (bd)(d/2)^2$$

$$= bd^3/12 + bd^3/4 = bd^3/3$$

(2) TRIANGLE :

(a) M.I. about its base :

$I_{xx} = \int dA.y^2 = \int (x.dy)y^2$

From similar triangles

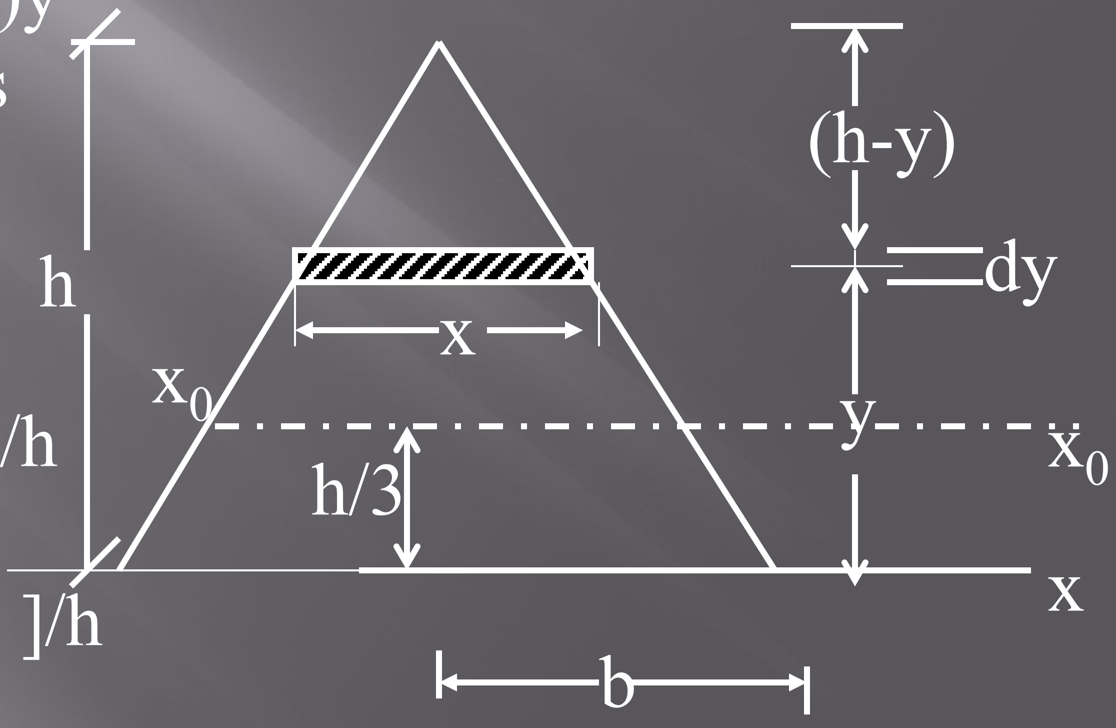
$b/h = x/(h-y)$

$\therefore\ x = b\ .\ (h-y)/h$

$I_{xx} = \int_0^h (b\ .\ (h-y)y^2.dy)/h$

$= b[\ h\ (y^3/3) - y^4/4\ ]/h$

$= bh^3/12$



(b) Moment of inertia about its centroidal axis:

$$I_{xx} = I_{x_0x_0} + A\bar{d}^2$$

$$I_{x_0x_0} = I_{xx} - A\bar{d}^2$$

$$= bh^3/12 - bh/2 \,.\, (h/3)^2 = bh^3/36$$

3. CIRCULAR AREA:

$$I_{x_0x_0} = \int dA \cdot y^2$$

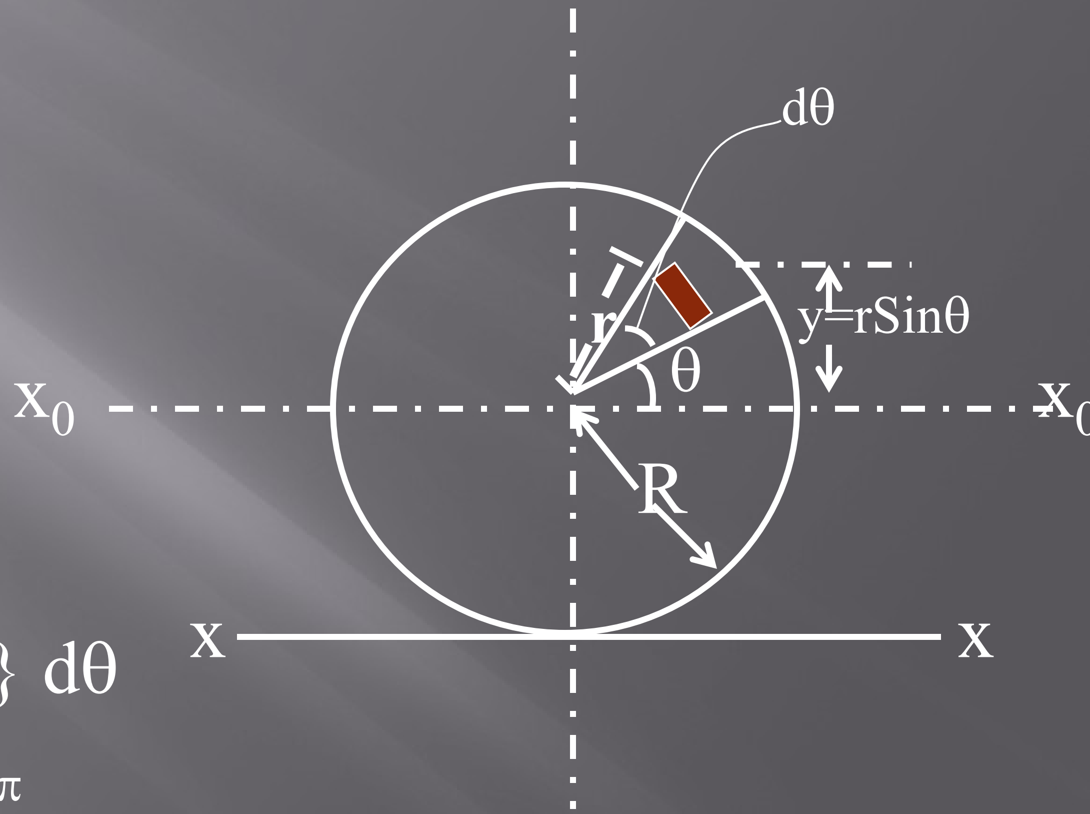
$$= \int_0^R \int_0^{2\pi} (x.d\theta.dr)\, r^2 Sin^2\theta$$

$$= \int_0^R \int_0^{2\pi} r^3.dr\, Sin^2\theta\, d\theta$$

$$= \int_0^R r^3\, dr \int_0^{2\pi} \{(1 - Cos2\theta)/2\}\, d\theta$$

$$= [r^4/4]_0^R \; [\theta/2 - Sin2\theta/4]_0^{2\pi}$$

$$= R^4/4[\pi - 0] = \pi R^4/4$$



$$I_{XoXo} = \pi R^4/4 = \pi D^4/64$$

4. SEMI CIRCULAR AREA:

$$I_{xx} = \int dA \,.\, y^2$$

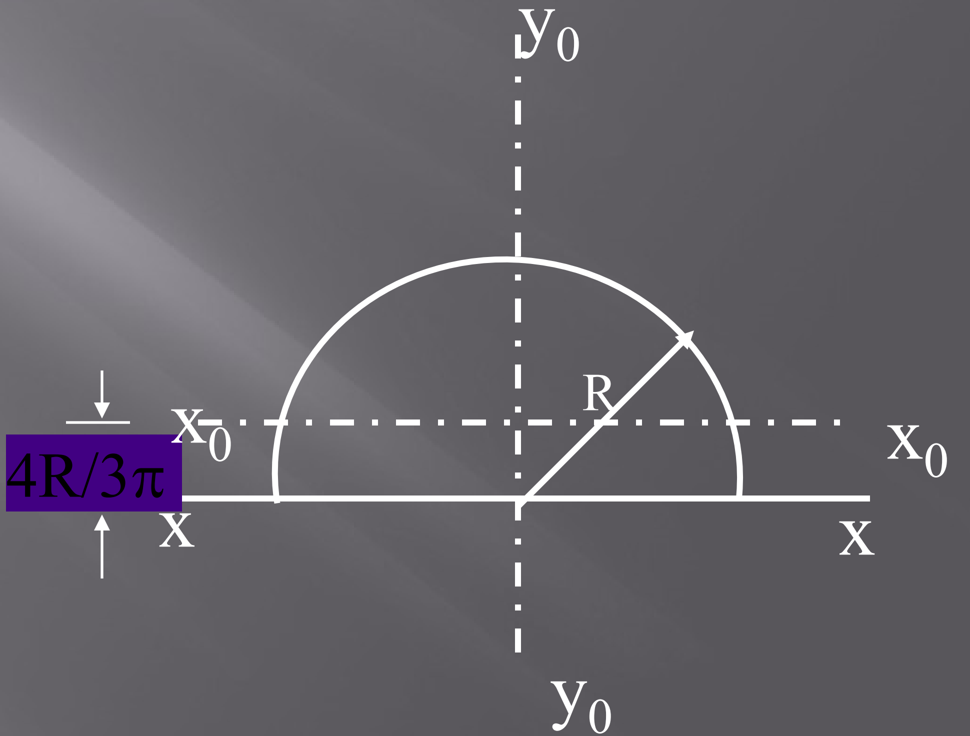
$$= \int_0^R \int_0^\pi (r.d\theta.dr)\, r^2 Sin^2\theta$$

$$= \int_0^R r^3.dr \int_0^\pi Sin^2\theta \, d\theta$$

$$= \int_0^R \int_0^\pi r^3 \, dr \, (1- Cos2\theta)/2) \, d\theta$$

$$= [R^4/4] \quad [\theta/2 - Sin2\theta/4]_0^\pi$$

$$= R^4/4[\pi/2 - 0] = \pi R^4/8$$



About horizontal centroidal axis:

$$I_{xx} = I_{x_0x_0} + A(\overline{d})^2$$

$$I_{x_0x_0} = I_{xx} - A(\overline{d})^2$$

$$= \pi R^4/8 - \pi R^2/2 \, . \, (4R/3\pi)^2$$

$$I_{x_0x_0} = 0.11R^4$$

QUARTER CIRCLE:

$$I_{xx} = I_{yy}$$

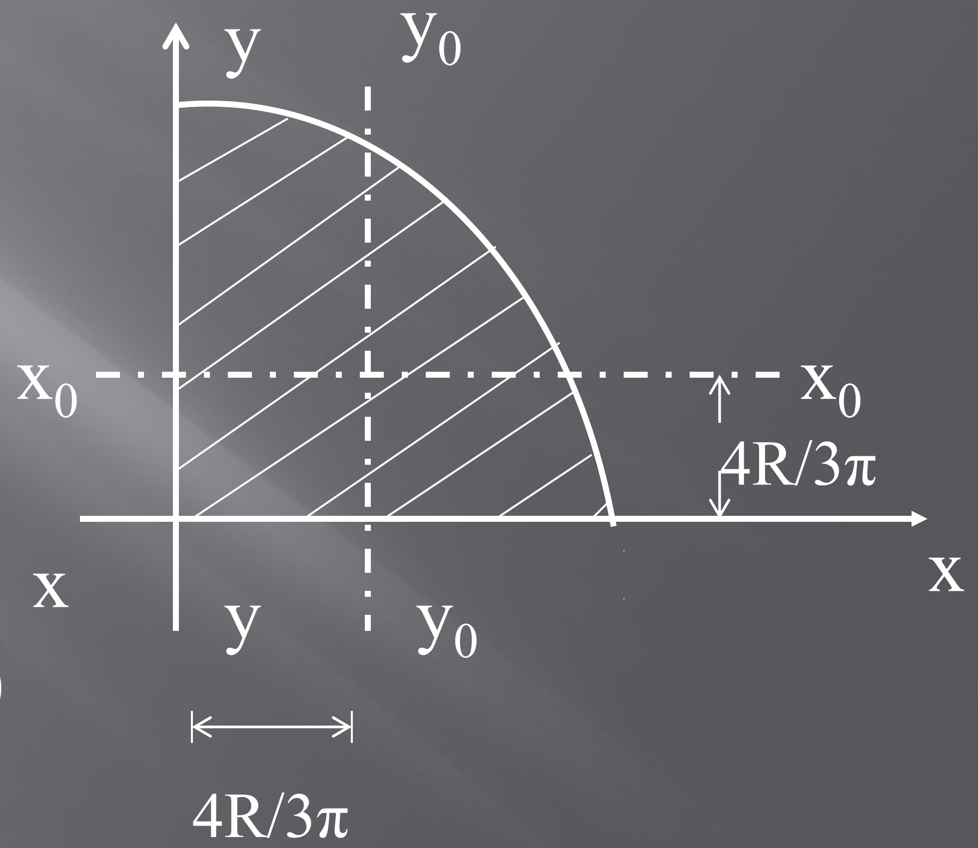
$$I_{xx} = \int_0^R \int_0^{\pi/2} (r.d\theta.dr).\ r^2 Sin^2\theta$$

$$= \int_0^R r^3.dr \int_0^{\pi/2} Sin^2\theta\ d\theta$$

$$= \int_0^R r3\ dr \int_0^{\pi/2} (1- Cos2\theta)/2)\ d\theta$$

$$= [R^4/4] \quad [\theta/2 - (Sin2\ \theta)/4]_0^{\pi/2}$$

$$= R^4\ (\pi/16 - 0) = \pi R^4/16$$



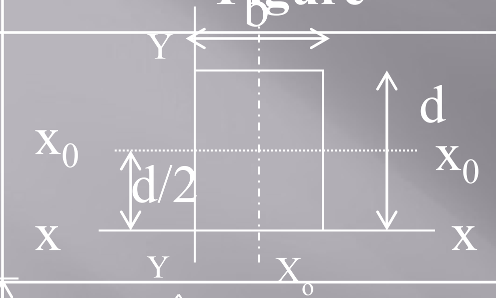
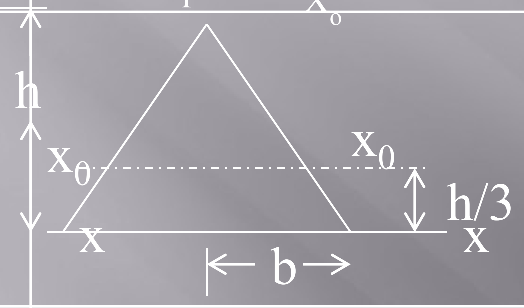
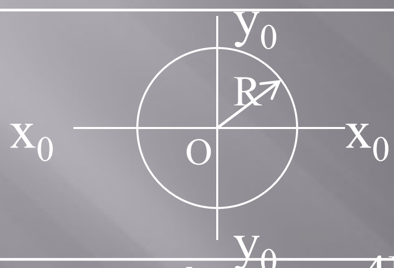
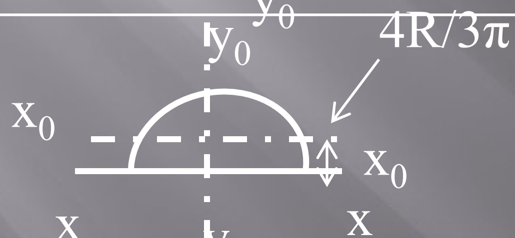
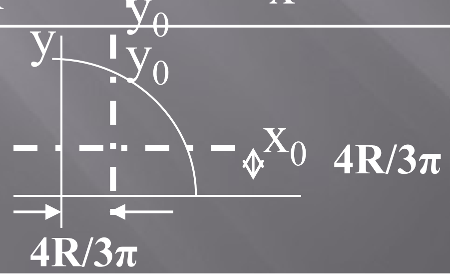
Moment of inertia about Centroidal axis,

$$I_{x_0x_0} = I_{xx} - A\bar{d}^2$$

$$= \pi R^4/16 - \pi R^2 . (0.424R)^2$$

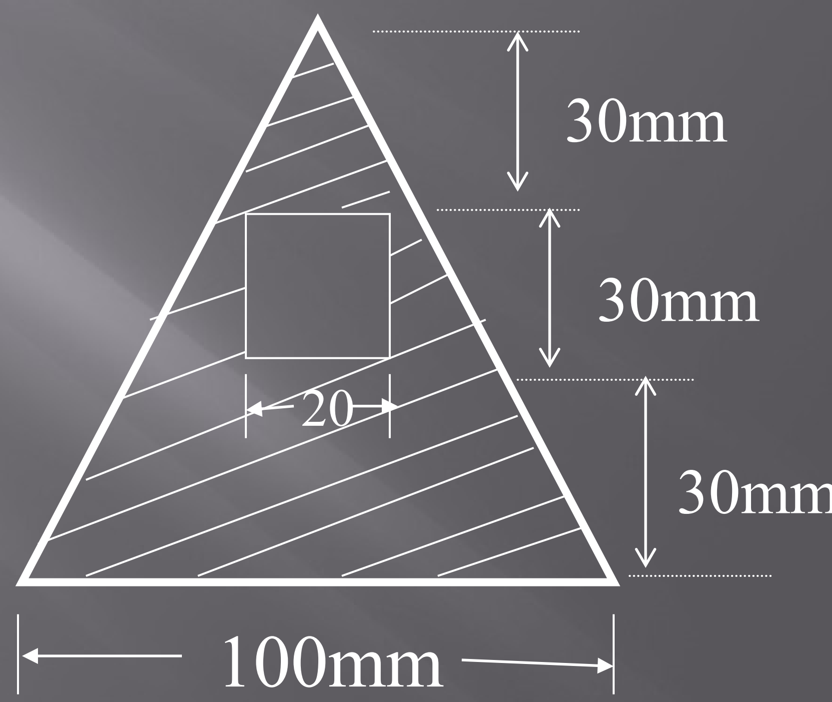
$$= 0.055R^4$$

The following table indicates the final values of M.I. about X and Y axes for different geometrical figures.

Sl.No	Figure	$I_{x_0-x_0}$	$I_{y_0-y_0}$	I_{xx}	I_{yy}
1		$bd^3/12$	-	$bd^3/3$	-
2		$bh^3/36$	-	$bh^3/12$	-
3		$\pi R^4/4$	$\pi R^4/4$	-	-
4		$0.11R^4$	$\pi R^4/8$	$\pi R^4/8$	-
5		$0.055R^4$	$0.055R^4$	$\pi R^4/16$	$\pi R^4/16$

EXERCISE PROBLEMS ON M.I.

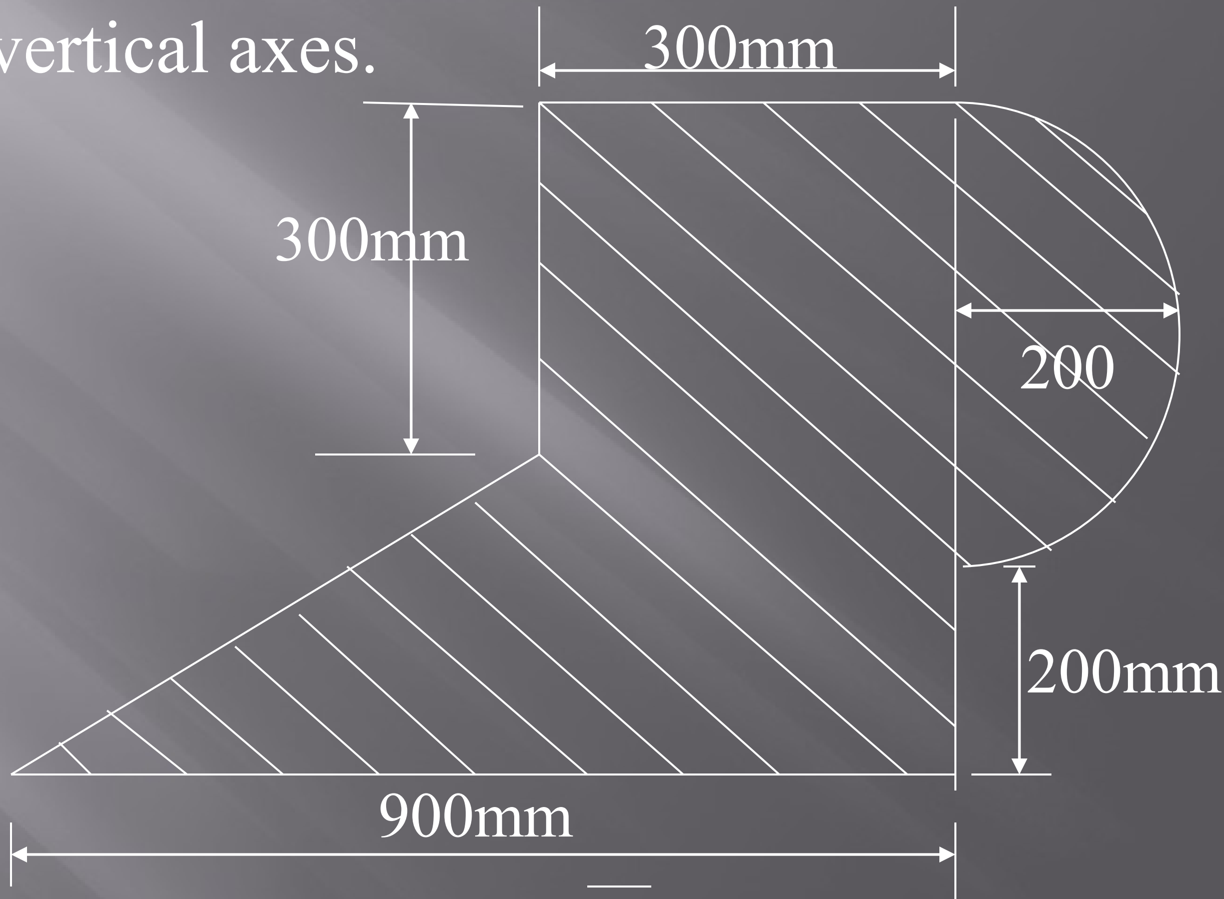
Q.1. Determine the moment of inertia about the centroidal axes.



[Ans: $\overline{Y} = 27.69\text{mm}$ $I_{xx} = 1.801 \times 10^6 \text{mm}^4$

$I_{yy} = 1.855 \times 10^6 \text{mm}^4$]

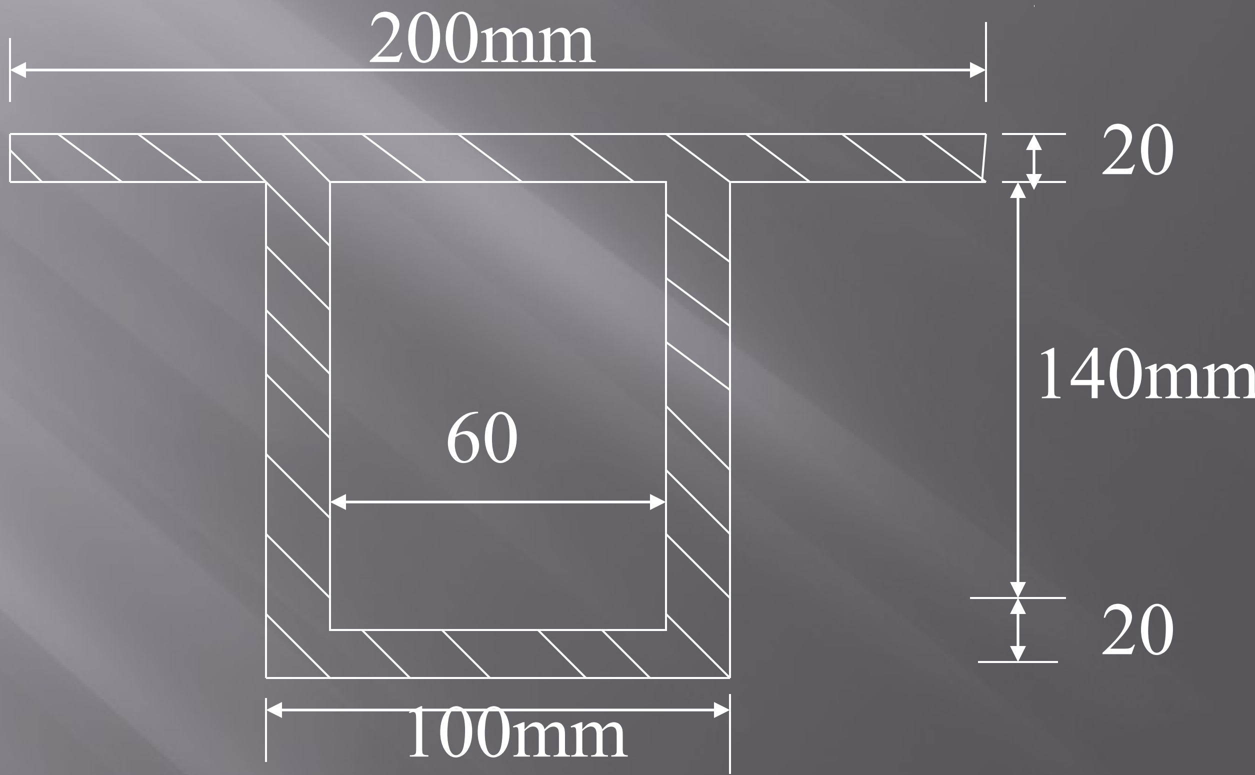
Q.2. Determine second moment of area about the centroidal horizontal and vertical axes.



[Ans: $\bar{X} = 99.7$mm from A, $\bar{Y} = 265$ mm

$I_{xx} = 10.29 \times 10^9 mm^4$, $I_{yy} = 16.97 \times 10^9 mm^4$]

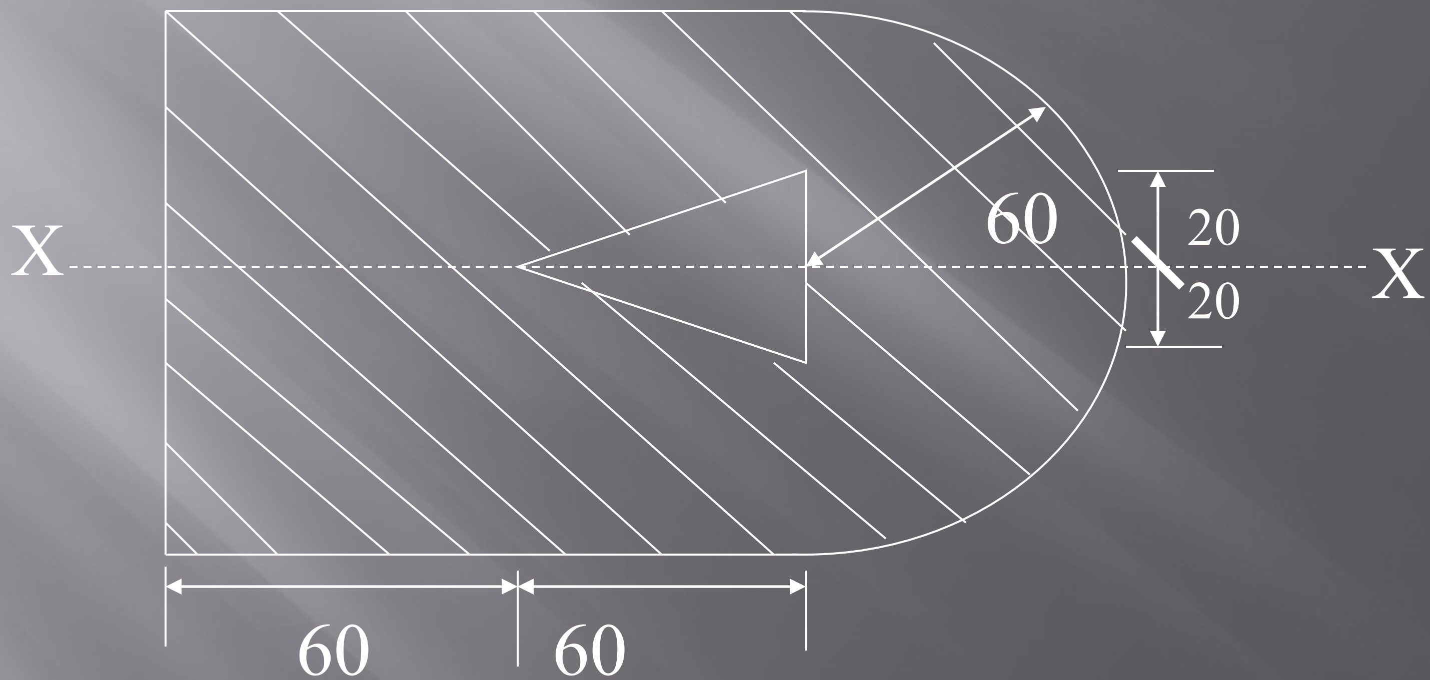
Q.3. Determine M.I. Of the built up section about the horizontal and vertical centroidal axes and the radii of gyration.



[Ans: $I_{xx} = 45.54 \times 10^6 mm^4$, $I_{yy} = 24.15 \times 10^6 mm^4$

$r_{xx} = 62.66mm$, $r_{yy} = 45.63mm$]

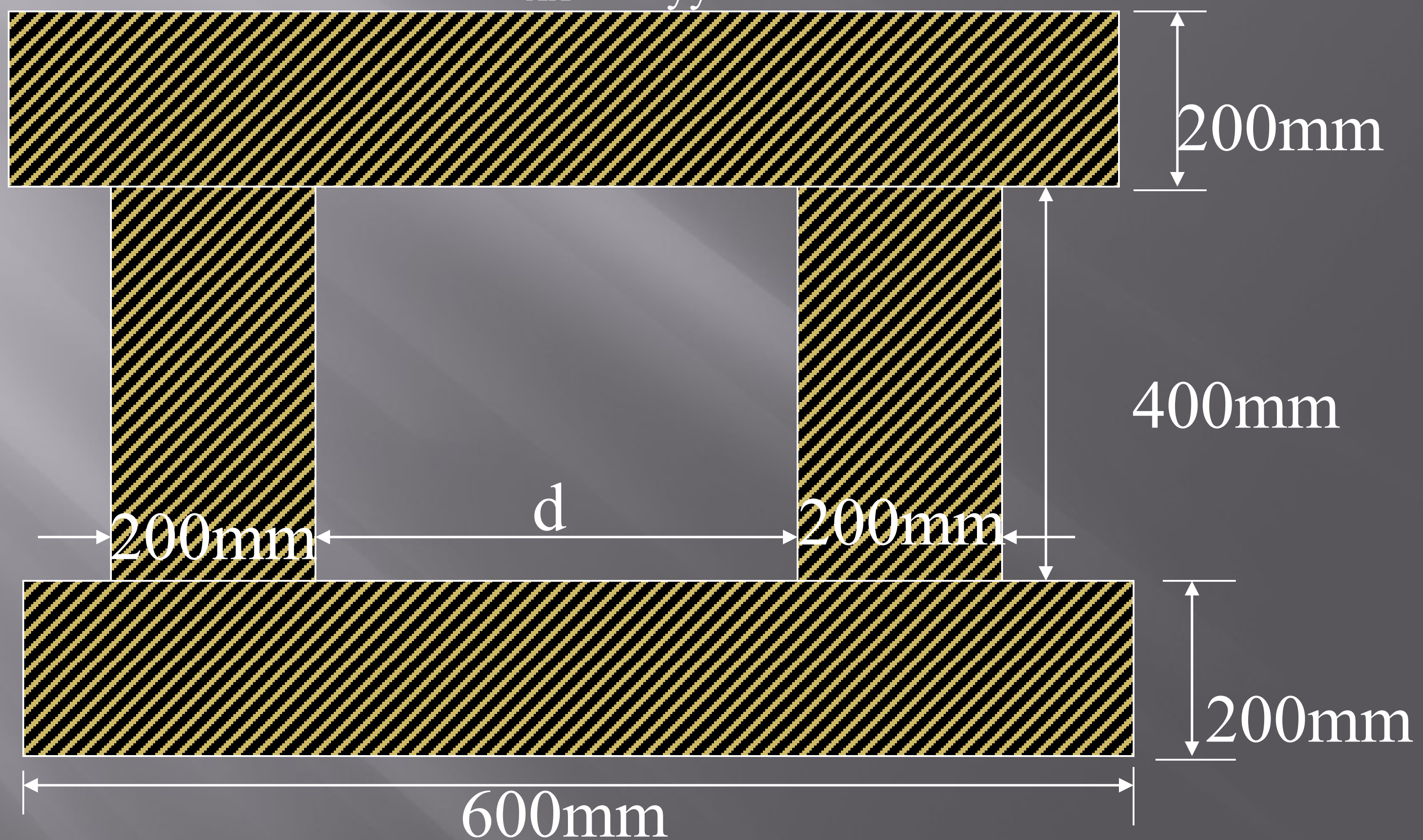
Q.4. Determine the horizontal and vertical centroidal M.I. Of the shaded portion of the figure.



[Ans: $\overline{X} = 83.1mm$

$I_{xx} = 2228.94 \times 10^4 mm^4$, $I_{yy} = 4789.61 \times 10^4 mm^4$]

Q.5. Determine the spacing of the symmetrically placed vertical blocks such that $\bar{I}_{xx} = \bar{I}_{yy}$ for the shaded area.



[Ans: d/2 = 223.9mm d=447.8mm]

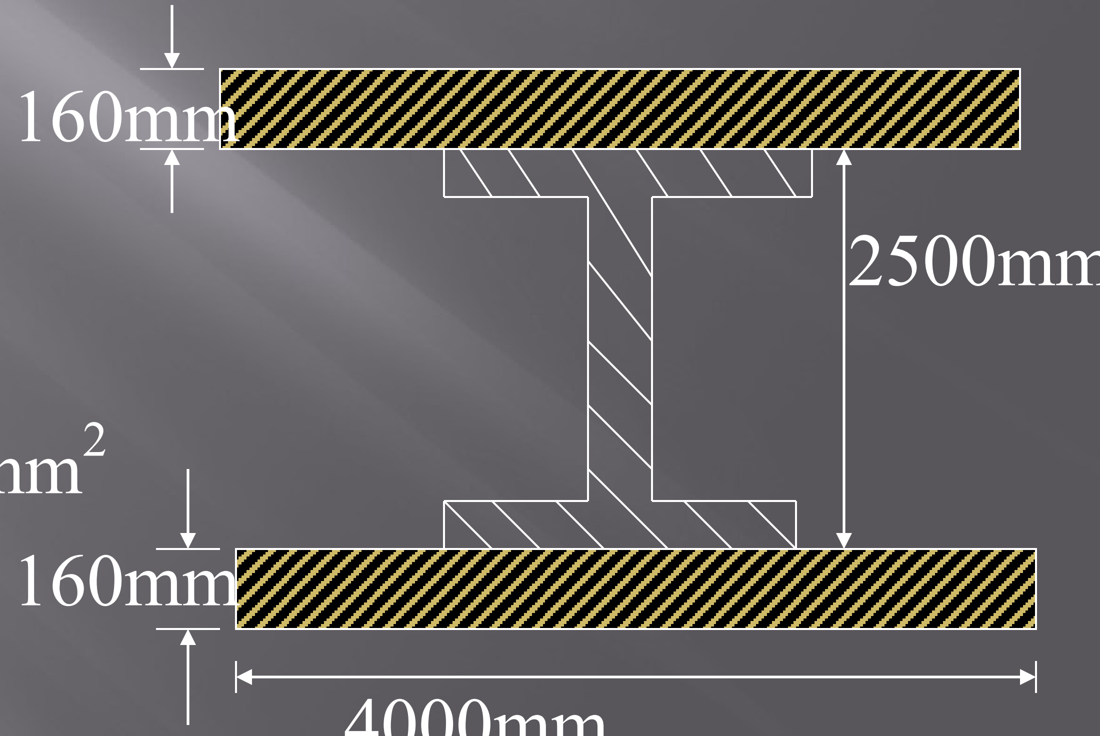
Q.6. Find the horizontal and vertical centroidal moment of inertia of the section shown in Fig. built up with R.S.J. (I-Section) 250 x 250 and two plates 400 x 16 mm each attached one to each.

Properties of I section are

$I_{xx} = 7983.9 \times 10^4 mm^4$

$I_{yy} = 2011.7 \times 10^4 mm^4$

Cross sectional area=6971mm^2



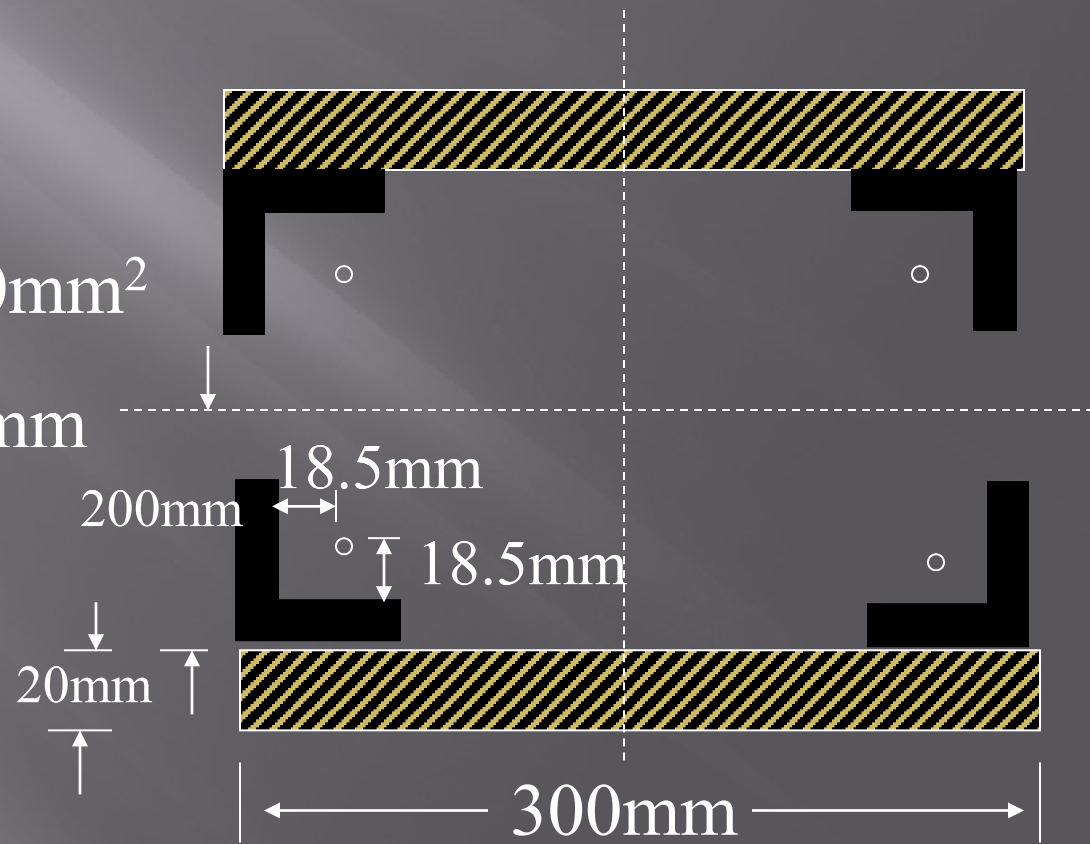
[Ans: $\overline{I}_{xx} = 30.653 \times 10^7 mm^4$, $I_{yy} = 19.078 \times 10^7 mm^4$]

Q.7. Find the horizontal and vertical centroidal moment of inertia of built up section shown in Figure. The section consists of 4 symmetrically placed ISA 60 x 60 with two plates 300 x 20 mm^2.

Properties of ISA

Cross sectional area = 4400mm^2

$I_{xx} = I_{yy}$; $C_{xx} = C_{yy}$ = 18.5mm



[Ans: $\bar{I}_{xx}$ = 111.078 x 10^7 mm^4; $\bar{I}_{yy}$ = 39.574 x 10^{7}mm^4]

Q.8. The R.S. Channel section ISAIC 300 are placed back to back with required to keep them in place. Determine the clear distance d between them so that $I_{xx} = I_{yy}$ for the composite section.

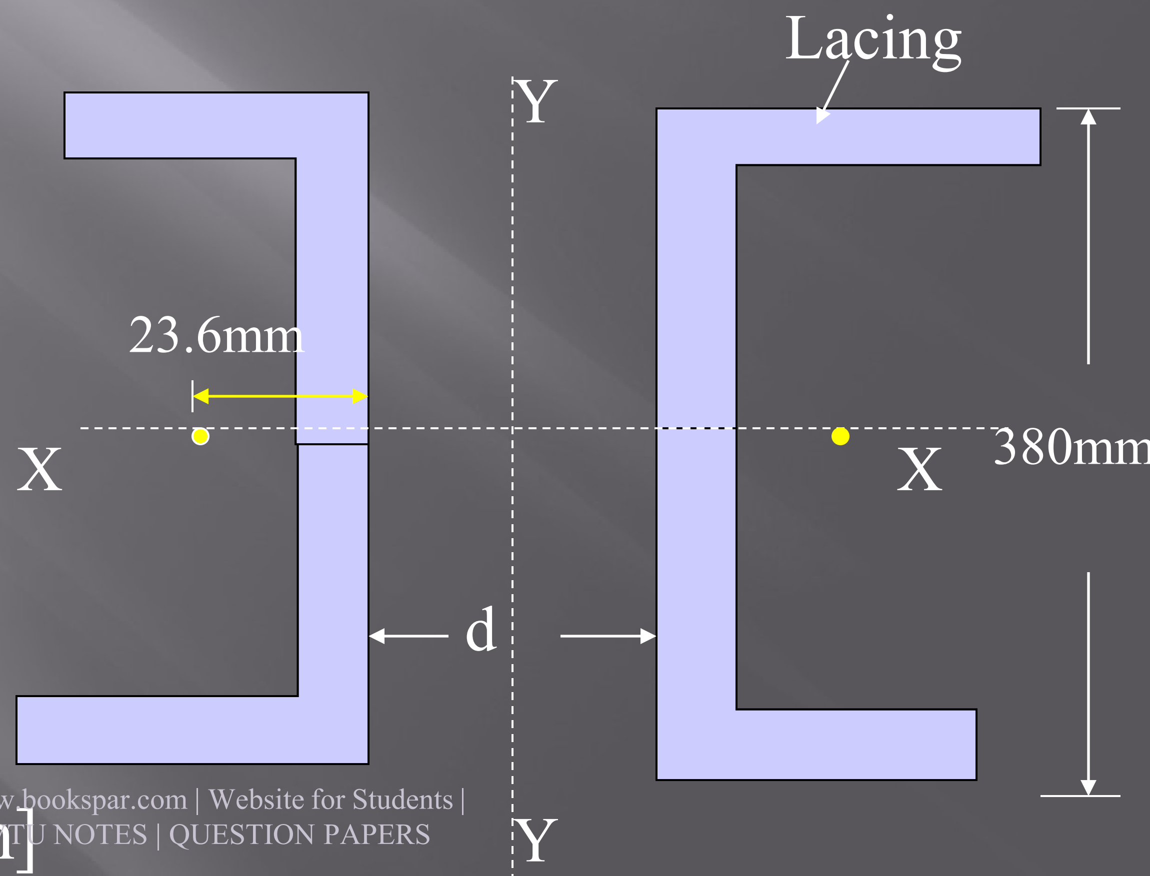
Properties of ISMC300

C/S Area = 4564mm^2

$I_{xx} = 6362.6 \times 10^4 mm^4$

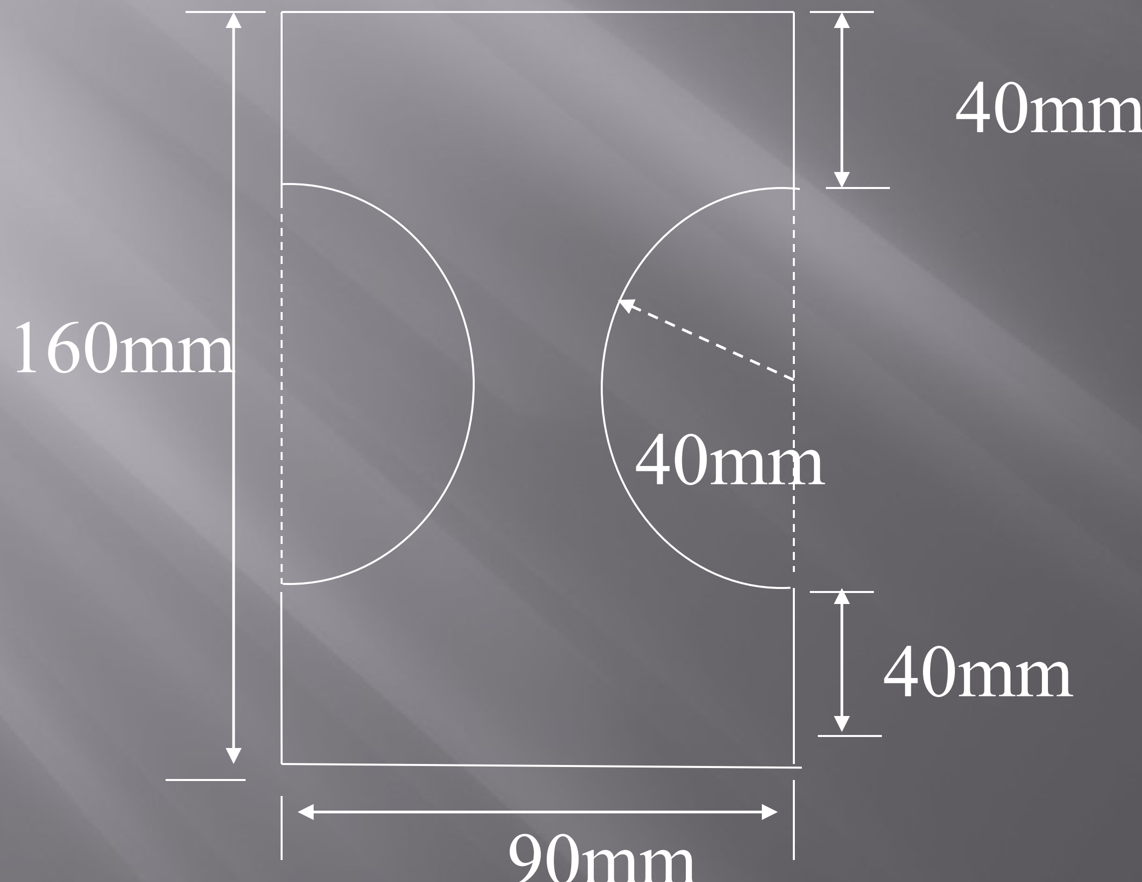
$I_{yy} = 310.8 \times 10^4 mm^4$

$C_{yy} = 23.6mm$



[Ans: d = 183.1mm]

Q9. Determine horizontal and vertical centroidal M.I. for the section shown in figure.



[Ans: $\bar{I}_{xx}$ = 2870.43 x 10^4 mm^4, $\bar{I}_{yy}$ = 521.64 x 10^4mm^4]