MAGNETIC MATERIALS

PARAMAGNETISM

Paramagnetism occurs in those substances where the individual atoms, ions or molecules posses a permanent magnetic dipole moments.

The permanent magnetic moment results from the following contributions:

- The spin or intrinsic moments of the electrons.

- The orbital motion of the electrons.

- The spin magnetic moment of the nucleus.

Examples of paramagnetic materials:

- Metals.

- Atoms, and molecules possessing an odd number of electrons, viz., free Na atoms, gaseous nitric oxide (NO) etc.

- Free atoms or ions with a partly filled inner shell: Transition elements, rare earth and actinide elements. Mn^{2+}, Gd^{3+}, U^{4+} etc.

- A few compounds with an even number of electrons including molecular oxygen.

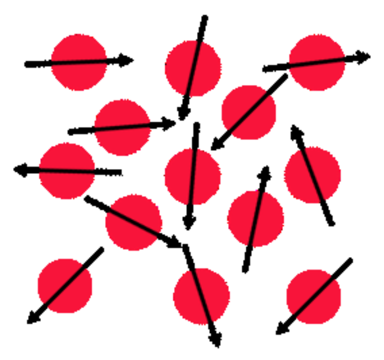
CLASSICAL THEORY OF PARAMAGNETISM

Let us consider a medium containing N magnetic dipoles per unit volume each with moment μ.

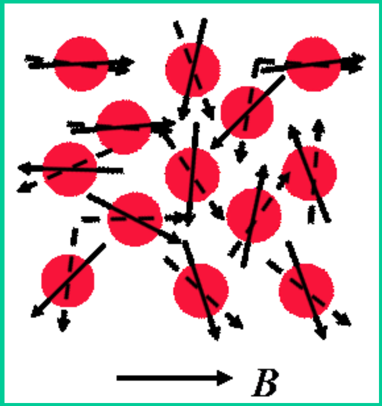
In presence of magnetic field, potential energy of magnetic dipole

$$V = -\vec{\mu}.\vec{B} = -\mu B \cos\theta$$

Where, θ is angle between magnetic moment and the field.



B =0, M=0

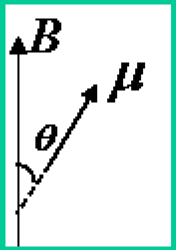


B ≠0, M≠0

$$V = -\mu B \text{ (minimum) when } \theta = 0$$

It shows that dipoles tend to line up with the field. The effect of temperature, however, is to randomize the directions of dipoles. The effect of these two competing processes is that some magnetization is produced.

Suppose field B is applied along z-axis, then θ is angle made by dipole with z-axis. The probability of finding the dipole along the θ direction is



$$f(\theta) = e^{-\frac{V}{kT}} = e^{\frac{\mu B \cos\theta}{kT}}$$

$f(\theta)$ is the Boltzmann factor which indicates that dipole is more likely to lie along the field than in any other direction.

The average value of μ_z is given as

$$\overline{\mu}_z = \frac{\int \mu_z f(\theta) d\Omega}{\int f(\theta) d\Omega}$$

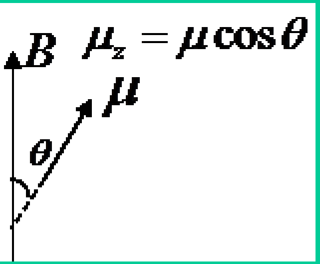
Where, integration is carried out over the solid angle, whose element is $d\Omega$. The integration thus takes into account all the possible orientations of the dipoles.

Substituting $\mu_z = \mu \cos\theta$ and $d\Omega = 2\pi \sin\theta \, d\theta$

$$\bar{\mu}_z = \frac{\int_0^{\pi} \mu \cos\theta \, \cancel{2\pi} \sin\theta \, e^{\frac{\mu B \cos\theta}{kT}} d\theta}{\int_0^{\pi} \cancel{2\pi} \sin\theta \, e^{\frac{\mu B \cos\theta}{kT}} d\theta} = \frac{\int_0^{\pi} \mu \cos\theta \sin\theta \, e^{\frac{\mu B \cos\theta}{kT}} d\theta}{\int_0^{\pi} \sin\theta \, e^{\frac{\mu B \cos\theta}{kT}} d\theta}$$

$$\bar{\mu}_z = \frac{\mu \int_0^{\pi} \cos\theta \sin\theta \, e^{a\cos\theta} d\theta}{\int_0^{\pi} \sin\theta \, e^{a\cos\theta} d\theta}$$

$$\text{Let } \frac{\mu B}{kT} = a$$



Let $\cos\theta = x$, then $\sin\theta \, d\theta = -\,dx$ and Limits -1 to +1

$$\bar{\mu}_z = \frac{\mu \int_{-1}^{+1} x e^{ax} dx}{\int_{-1}^{+1} e^{ax} dx} \Rightarrow \bar{\mu}_z = \mu \left[\frac{e^{a} + e^{-a}}{e^{a} - e^{-a}} - \frac{1}{a} \right]$$

$$\bar{\mu}_z = \mu\left[\frac{e^a + e^{-a}}{e^a - e^{-a}} - \frac{1}{a}\right] = \mu\left[\coth(a) - \frac{1}{a}\right]$$

coth(a)

Langevin function, L(a)

$$\Rightarrow \vec{\mu}_z = \mu \mathrm{L(a)} \qquad [a = \frac{\mu B}{kT}]$$

$$L(a) = \frac{a}{3} - \frac{a^3}{45} + \frac{2a^5}{945} - \cdots$$

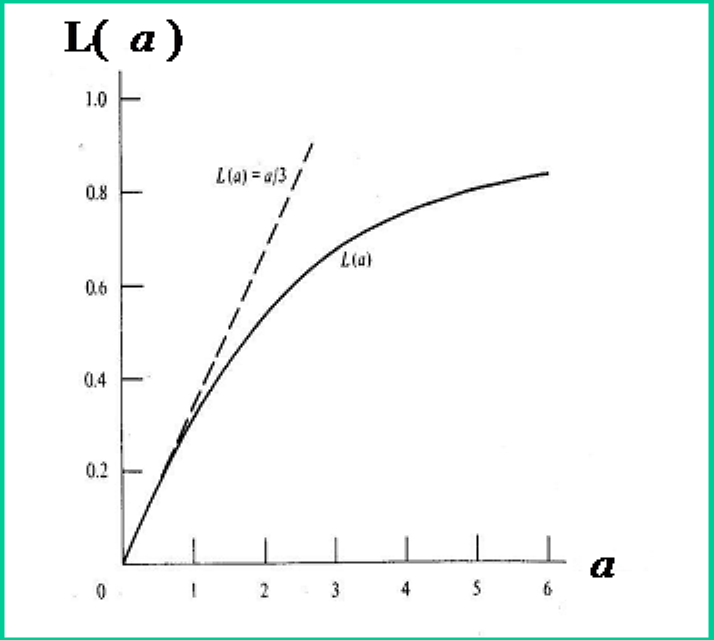
In most practical situations a<<1, therefore,

$$L(a) \approx \frac{a}{3} \quad \Rightarrow \bar{\mu}_z = \mu\frac{\mathrm{a}}{3} = \frac{\mu^2 B}{3kT}$$

The magnetization is given as

$$M = N\bar{\mu}_z = \frac{N\mu^2 B}{3kT}$$

(N = Number of dipoles per unit volume)



Variation of L(a) with a.

$$M = N\overline{\mu}_z = \frac{N\mu^2 B}{3kT} = \frac{N\mu^2 \mu_0 H}{3kT}$$

$$\Rightarrow \frac{M}{H} = \chi = \frac{N\mu_0\mu^2}{3kT}$$

This equation is known as CURIE LAW. The susceptibility is referred as Langevin paramagnetic susceptibility. Further, contrary to the diamagnetism, paramagnetic susceptibility is inversely proportional to T

Above equation is written in a simplified form as:

$$\chi = \frac{C}{T} \qquad \text{where, } C = \frac{N\mu_0\mu^2}{3k}$$

Curie constant