Introduction to Electromagnetic Fields; Maxwell's Equations; Electromagnetic Fields in Materials; Electrostatics: Coulomb's Law, Electric Field, Discrete and Continuous Charge Distributions; Electrostatic Potential

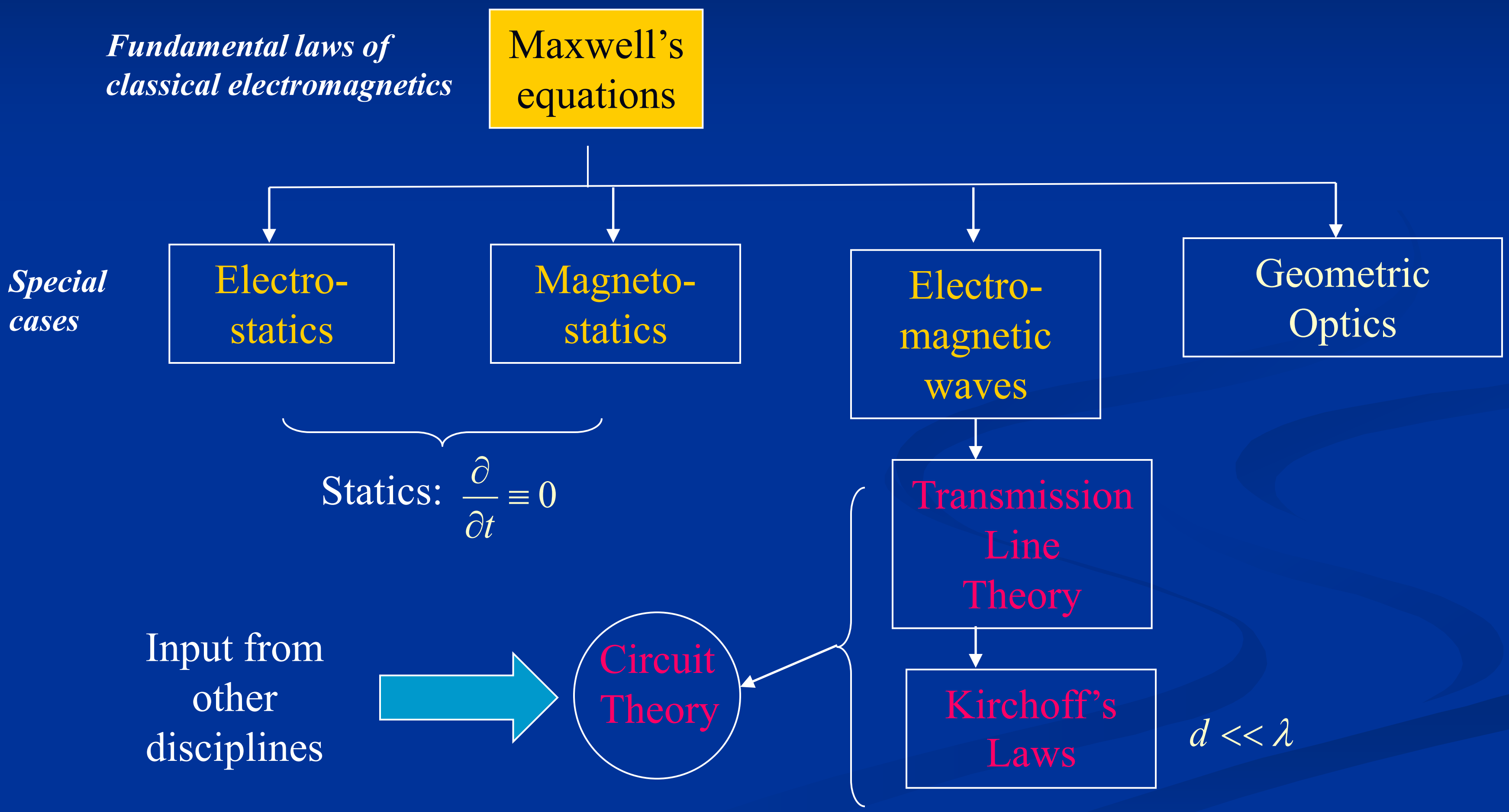
Lecture 2 Objectives

- To provide an overview of classical electromagnetics, Maxwell's equations, electromagnetic fields in materials, and phasor concepts.
- To begin our study of electrostatics with Coulomb's law; definition of electric field; computation of electric field from discrete and continuous charge distributions; and scalar electric potential.

Introduction to Electromagnetic Fields

- Electromagnetics is the study of the effect of charges at rest and charges in motion.
- Some special cases of electromagnetics:
  - Electrostatics: charges at rest
  - Magnetostatics: charges in steady motion (DC)
  - Electromagnetic waves: waves excited by charges in time-varying motion

Introduction to Electromagnetic Fields

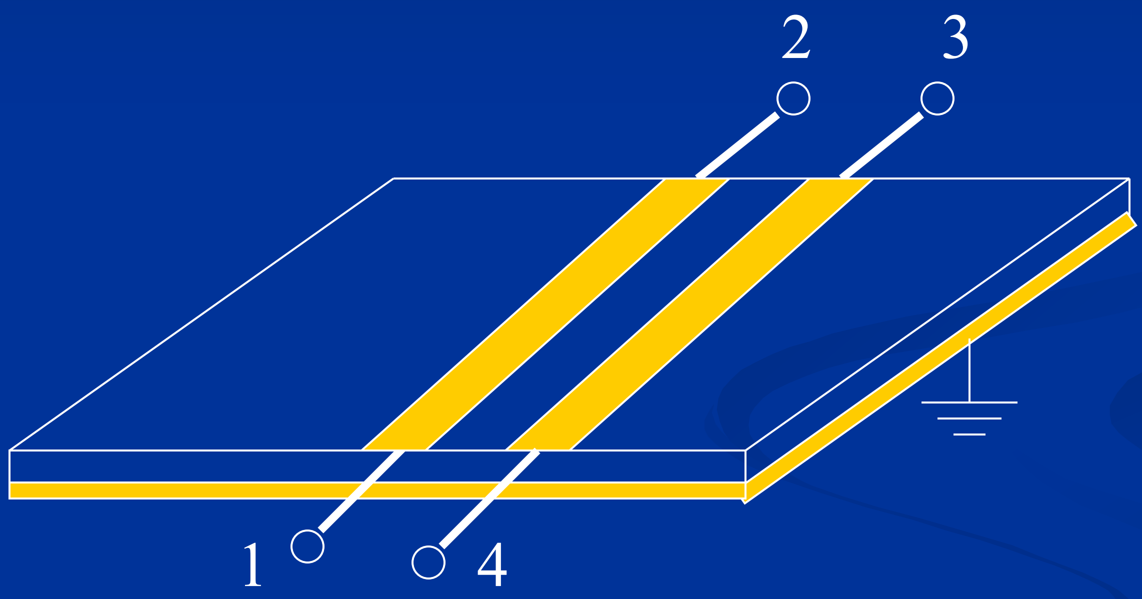


Introduction to Electromagnetic Fields

- transmitter and receiver are connected by a “field.”

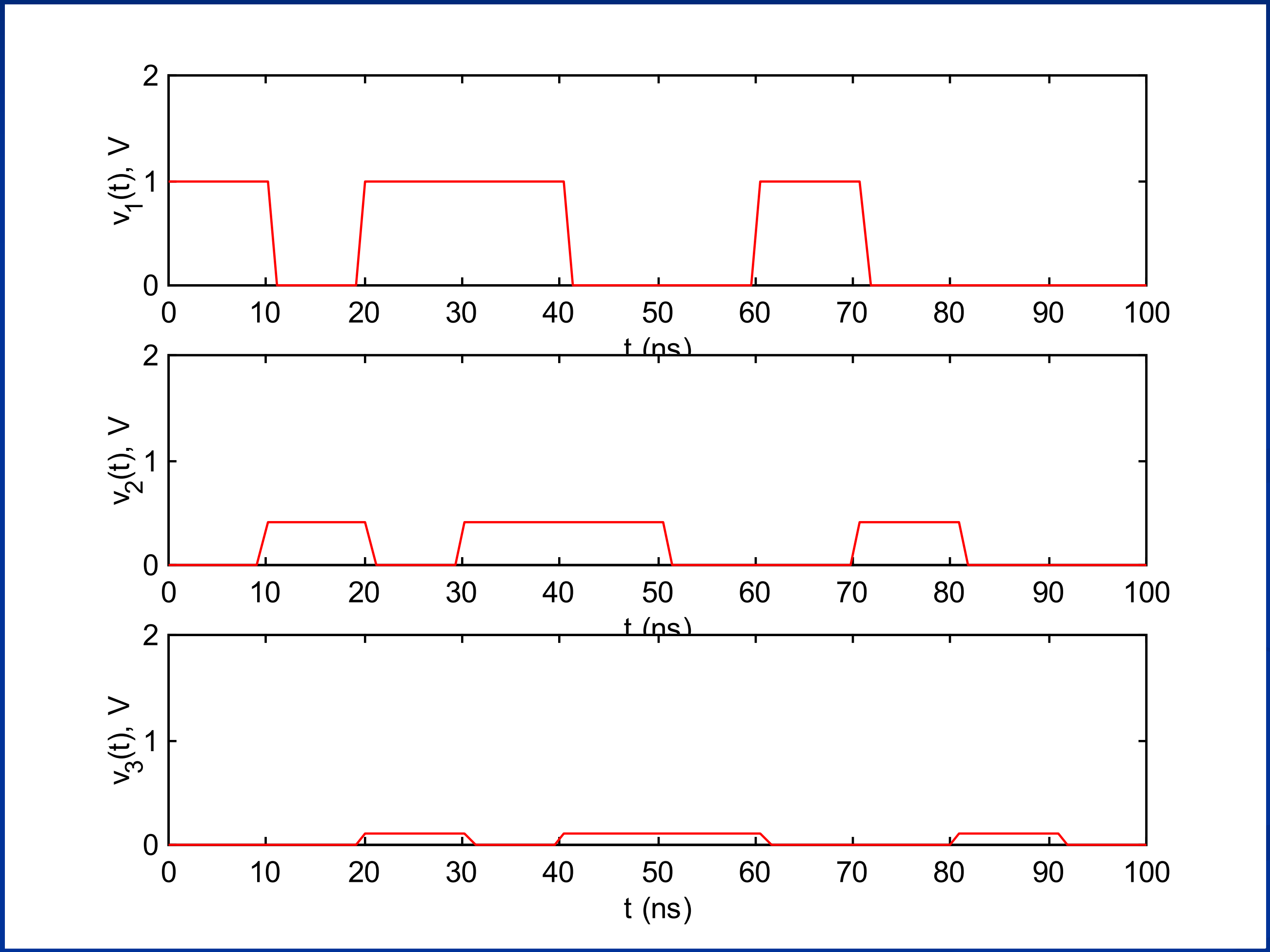
Introduction to Electromagnetic Fields

High-speed, high-density digital circuits:



- consider an interconnect between points "1" and "2"

Introduction to Electromagnetic Fields



- Propagation delay
- Electromagnetic coupling
- Substrate modes

Introduction to Electromagnetic Fields

- When an event in one place has an effect on something at a different location, we talk about the events as being connected by a "field".
- A ***field*** is a spatial distribution of a quantity; in general, it can be either *scalar* or *vector* in nature.

Introduction to Electromagnetic Fields

- Electric and magnetic fields:
  - Are vector fields with three spatial components.
  - Vary as a function of position in 3D space as well as time.
  - Are governed by partial differential equations derived from Maxwell's equations.

Introduction to Electromagnetic Fields

- A ***scalar*** is a quantity having only an amplitude (and possibly phase).

  Examples: voltage, current, charge, energy, temperature

- A ***vector*** is a quantity having direction in addition to amplitude (and possibly phase).

  Examples: velocity, acceleration, force

Introduction to Electromagnetic Fields

- Fundamental vector field quantities in electromagnetics:
  - Electric field intensity $\left(\underline{E}\right)$

    units = volts per meter ($V/m = kg\ m/A/s^3$)
  - Electric flux density (electric displacement) $\left(\underline{D}\right)$

    units = coulombs per square meter ($C/m^2 = A\ s\ /m^2$)
  - Magnetic field intensity $\left(\underline{H}\right)$

    units = amps per meter (A/m)
  - Magnetic flux density $\left(\underline{B}\right)$

    units = teslas = webers per square meter ($T = Wb/\ m^2 = kg/A/s^3$)

Introduction to Electromagnetic Fields

- Universal constants in electromagnetics:
  - Velocity of an electromagnetic wave (e.g., light) in free space (perfect vacuum)

$$c \approx 3 \times 10^{8} \text{ m/s}$$

  - Permeability of free space

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

  - Permittivity of free space:

$$\varepsilon_0 \approx 8.854 \times 10^{-12} \text{ F/m}$$

  - Intrinsic impedance of free space:

$$\eta_0 \approx 120\pi \ \Omega$$

Introduction to Electromagnetic Fields

- Relationships involving the universal constants:

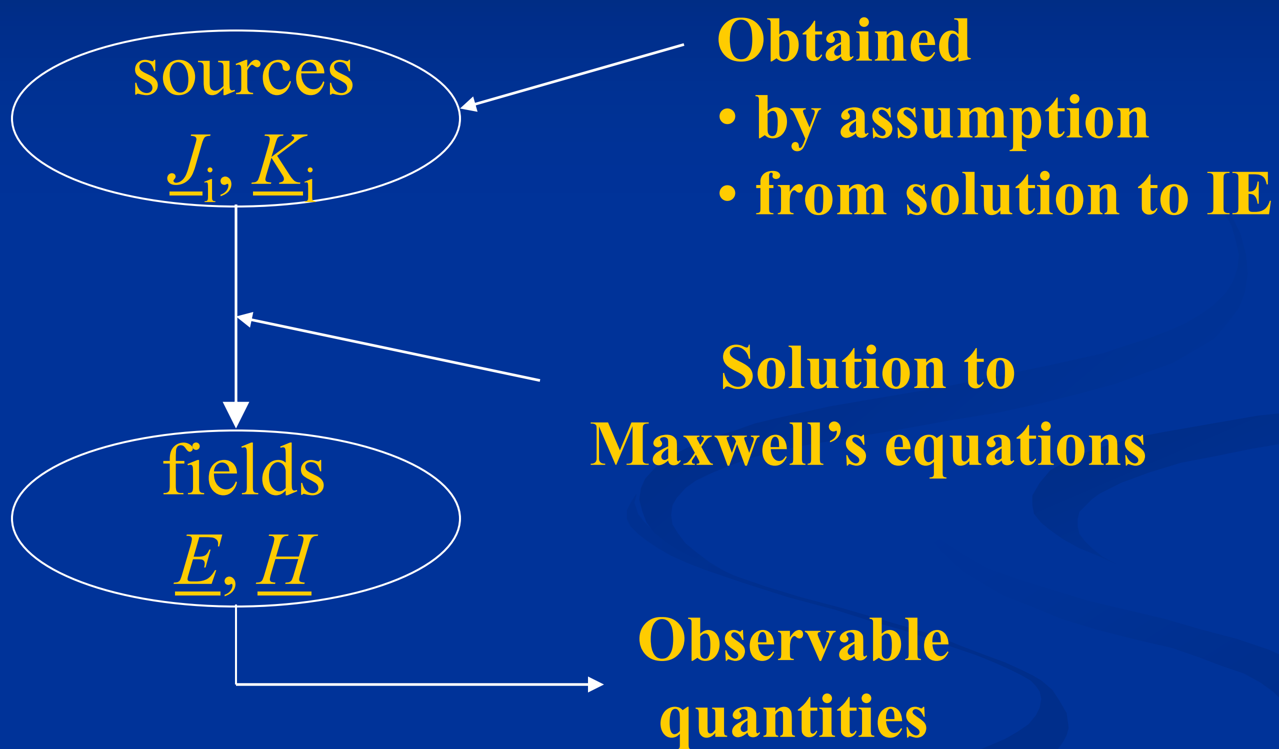
$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

In free space:

$$\underline{B} = \mu_0 \underline{H}$$

$$\underline{D} = \varepsilon_0 \underline{E}$$

Introduction to Electromagnetic Fields



Maxwell's Equations

- ***Maxwell's equations in integral form*** are the fundamental postulates of classical electromagnetics - all classical electromagnetic phenomena are explained by these equations.
- Electromagnetic phenomena include electrostatics, magnetostatics, electromagnetostatics and electromagnetic wave propagation.
- The differential equations and boundary conditions that we use to formulate and solve EM problems are all derived from *Maxwell's equations in integral form*.

Maxwell's Equations

- Various ***equivalence principles*** consistent with Maxwell's equations allow us to replace more complicated electric current and charge distributions with ***equivalent magnetic sources***.
- These *equivalent magnetic sources* can be treated by a generalization of Maxwell's equations.

Maxwell's Equations in Integral Form (Generalized to Include Equivalent Magnetic Sources)

$$\oint_C \underline{E} \cdot d\underline{l} = -\frac{d}{dt}\int_S \underline{B} \cdot d\underline{S} - \int_S \underline{K}_c \cdot d\underline{S} - \int_S \underline{K}_i \cdot d\underline{S}$$

$$\oint_C \underline{H} \cdot d\underline{l} = \frac{d}{dt}\int_S \underline{D} \cdot d\underline{S} + \int_S \underline{J}_c \cdot d\underline{S} + \int_S \underline{J}_i \cdot d\underline{S}$$

$$\oint_S \underline{D} \cdot d\underline{S} = \int_V q_{ev} dv$$

$$\oint_S \underline{B} \cdot d\underline{S} = \int_V q_{mv} dv$$

Adding the fictitious magnetic source terms is equivalent to living in a universe where magnetic monopoles (charges) exist.

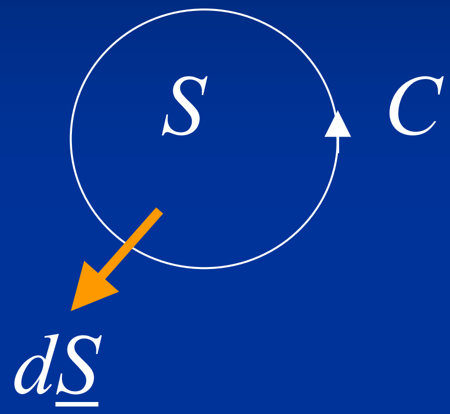
Continuity Equation in Integral Form (Generalized to Include Equivalent Magnetic Sources)

$$\oint_S \underline{J} \cdot d\,\underline{s} = -\frac{\partial}{\partial t}\int_V q_{ev}\, dv$$

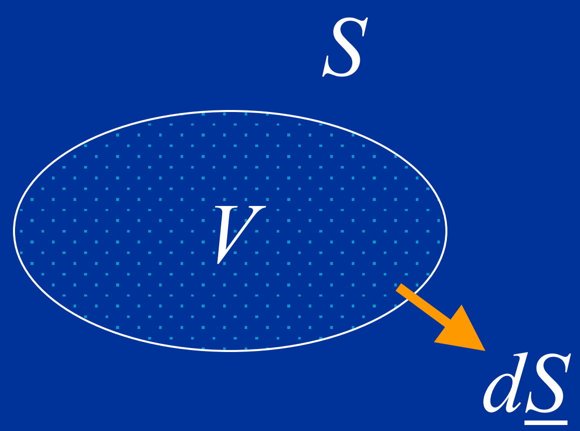
$$\oint_S \underline{K} \cdot d\,\underline{s} = -\frac{\partial}{\partial t}\int_V q_{mv}\, dv$$

- The *continuity equations* are <u>implicit</u> in Maxwell's equations.

Contour, Surface and Volume Conventions



- open surface *S* bounded by closed contour *C*
- *d*<u>*S*</u> in direction given by RH rule



- volume *V* bounded by closed surface *S*
- *d*<u>*S*</u> in direction outward from *V*

Electric Current and Charge Densities

- $\boldsymbol{J}_{\mathrm{c}}$ = (electric) conduction current density (A/m^2)
- $\boldsymbol{J}_{\mathrm{i}}$ = (electric) impressed current density (A/m^2)
- q_{ev} = (electric) charge density (C/m^3)

Magnetic Current and Charge Densities

- $\boldsymbol{K}_c$ = magnetic conduction current density (V/m^2)
- $\boldsymbol{K}_i$ = magnetic impressed current density (V/m^2)
- q_{mv} = magnetic charge density (Wb/m^3)

Maxwell's Equations - Sources and Responses

- Sources of EM field:
  - $\boldsymbol{K}_i$, $\boldsymbol{J}_i$, q_{ev}, q_{mv}

- Responses to EM field:
  - $\boldsymbol{E}$, $\boldsymbol{H}$, $\boldsymbol{D}$, $\boldsymbol{B}$, $\boldsymbol{J}_c$, $\boldsymbol{K}_c$

Maxwell's Equations in Differential Form (Generalized to Include Equivalent Magnetic Sources)

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} - \underline{K}_c - \underline{K}_i$$

$$\nabla \times \underline{H} = \frac{\partial \underline{D}}{\partial t} + \underline{J}_c + \underline{J}_i$$

$$\nabla \cdot \underline{D} = q_{ev}$$

$$\nabla \cdot \underline{B} = q_{mv}$$

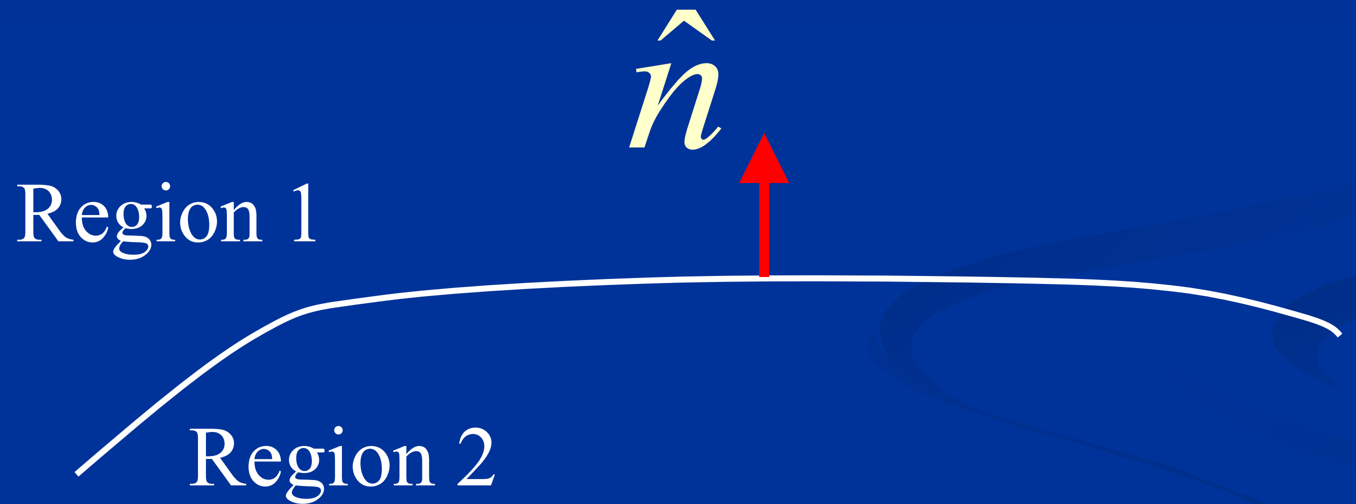
Continuity Equation in Differential Form (Generalized to Include Equivalent Magnetic Sources)

$$\nabla \cdot \underline{J} = -\frac{\partial q_{ev}}{\partial t}$$

$$\nabla \cdot \underline{K} = -\frac{\partial q_{mv}}{\partial t}$$

- The *continuity equations* are implicit in Maxwell's equations.

Electromagnetic Boundary Conditions



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Electromagnetic Boundary Conditions

$$\hat{n} \times \left(\underline{E}_1 - \underline{E}_2 \right) = -\underline{K}_S$$

$$\hat{n} \times \left(\underline{H}_1 - \underline{H}_2 \right) = \underline{J}_S$$

$$\hat{n} \cdot \left(\underline{D}_1 - \underline{D}_2 \right) = q_{es}$$

$$\hat{n} \cdot \left(\underline{B}_1 - \underline{B}_2 \right) = q_{ms}$$

Surface Current and Charge Densities

- Can be either *sources of* or *responses to* EM field.
- Units:
  - $\boldsymbol{K}_s$ - V/m
  - $\boldsymbol{J}_s$ - A/m
  - q_{es} - C/m^2
  - q_{ms} - W/m^2

Electromagnetic Fields in Materials

- In time-varying electromagnetics, we consider ***E*** and ***H*** to be the "primary" responses, and attempt to write the "secondary" responses ***D***, ***B***, $\mathbf{J}_c$, and $\mathbf{K}_c$ in terms of ***E*** and ***H***.
- The relationships between the "primary" and "secondary" responses depends on the *medium* in which the field exists.
- The relationships between the "primary" and "secondary" responses are called ***constitutive relationships***.

Electromagnetic Fields in Materials

- Most general *constitutive relationships*:

$$\underline{D} = \underline{D}(\underline{E}, \underline{H})$$

$$\underline{B} = \underline{B}(\underline{E}, \underline{H})$$

$$\underline{J}_c = \underline{J}_c(\underline{E}, \underline{H})$$

$$\underline{K}_c = \underline{K}_c(\underline{E}, \underline{H})$$

Electromagnetic Fields in Materials

- In free space, we have:

$$\underline{D} = \varepsilon_0 \underline{E}$$

$$\underline{B} = \mu_0 \underline{H}$$

$$\underline{J}_c = 0$$

$$\underline{K}_c = 0$$

Electromagnetic Fields in Materials

- In a ***simple medium***, we have:

$$\underline{D} = \varepsilon \underline{E}$$

$$\underline{B} = \mu \underline{H}$$

$$\underline{J}_c = \sigma \underline{E}$$

$$\underline{K}_c = \sigma_m \underline{H}$$

- ***linear*** (independent of field strength)
- ***isotropic*** (independent of position within the medium)
- ***homogeneous*** (independent of direction)
- ***time-invariant*** (independent of time)
- ***non-dispersive*** (independent of frequency)

Electromagnetic Fields in Materials

- ε = permittivity = $\varepsilon_r\varepsilon_0$ (F/m)
- μ = permeability = $\mu_r\mu_0$ (H/m)
- σ = electric conductivity = $\varepsilon_r\varepsilon_0$ (S/m)
- σ_m = magnetic conductivity = $\varepsilon_r\varepsilon_0$ (Ω/m)

Phasor Representation of a Time-Harmonic Field

- A ***phasor*** is a complex number representing the amplitude and phase of a sinusoid of known frequency.

phasor

$$A\cos(\omega t + \theta) \Leftrightarrow Ae^{j\theta}$$

time domain

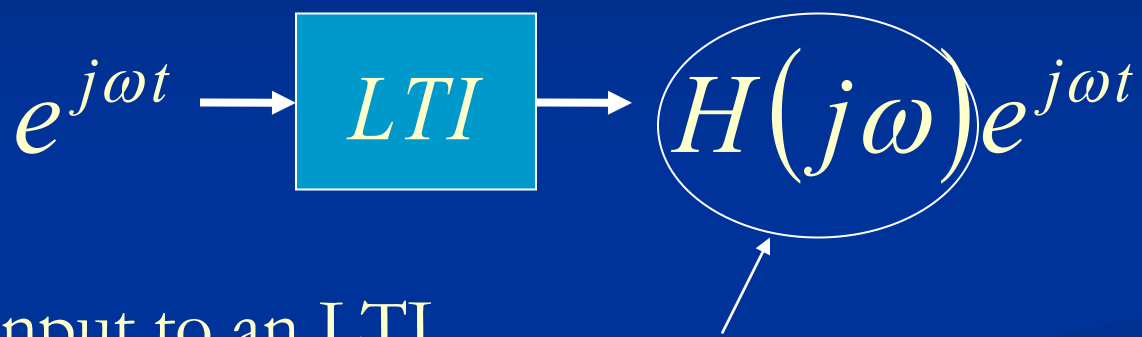
frequency domain

Phasor Representation of a Time-Harmonic Field

- ***Phasors*** are an extremely important concept in the study of classical electromagnetics, circuit theory, and communications systems.
- Maxwell's equations in simple media, circuits comprising linear devices, and many components of communications systems can all be represented as ***linear time-invariant*** (***LTI***) systems. (Formal definition of these later in the course …)
- The eigenfunctions of any LTI system are the complex exponentials of the form:

$$e^{j\omega t}$$

Phasor Representation of a Time-Harmonic Field



A complex constant (for fixed ω); as a function of ω gives the frequency response of the LTI system.

- If the input to an LTI system is a sinusoid of frequency ω, then the output is also a sinusoid of frequency ω (with different amplitude and phase).

Phasor Representation of a Time-Harmonic Field

- The amplitude and phase of a sinusoidal function can also depend on position, and the sinusoid can also be a vector function:

$$\hat{a}_A A(\underline{r}) \cos\left(\omega t - \theta(\underline{r})\right) \Leftrightarrow \hat{a}_A A(\underline{r}) e^{j\theta(\underline{r})}$$

Phasor Representation of a Time-Harmonic Field

- Given the phasor (frequency-domain) representation of a time-harmonic vector field, the time-domain representation of the vector field is obtained using the recipe:

$$\underline{E}\left(\underline{r},t\right)=\mathrm{Re}\left\{\underline{E}\left(\underline{r}\right)e^{j\omega t}\right\}$$

Phasor Representation of a Time-Harmonic Field

- ***Phasors*** can be used provided all of the media in the problem are ***linear*** ⇒ no frequency conversion.
- When phasors are used, integro-differential operators in time become algebraic operations in frequency, e.g.:

$$\frac{\partial \underline{E}(\underline{r},t)}{\partial t} \Leftrightarrow j\omega \underline{E}(\underline{r})$$

Time-Harmonic Maxwell's Equations

- If the sources are time-harmonic (sinusoidal), and all media are linear, then the electromagnetic fields are sinusoids of the same frequency as the sources.
- In this case, we can simplify matters by using Maxwell's equations in the *frequency-domain*.
- Maxwell's equations in the frequency-domain are relationships between the phasor representations of the fields.

Maxwell's Equations in Differential Form for Time-Harmonic Fields

$$\nabla \times \underline{E} = -j\omega \underline{B} - \underline{K}_c - \underline{K}_i$$

$$\nabla \times \underline{H} = j\omega \underline{D} + \underline{J}_c + \underline{J}_i$$

$$\nabla \cdot \underline{D} = q_{ev}$$

$$\nabla \cdot \underline{B} = q_{mv}$$

Maxwell's Equations in Differential Form for Time-Harmonic Fields in Simple Medium

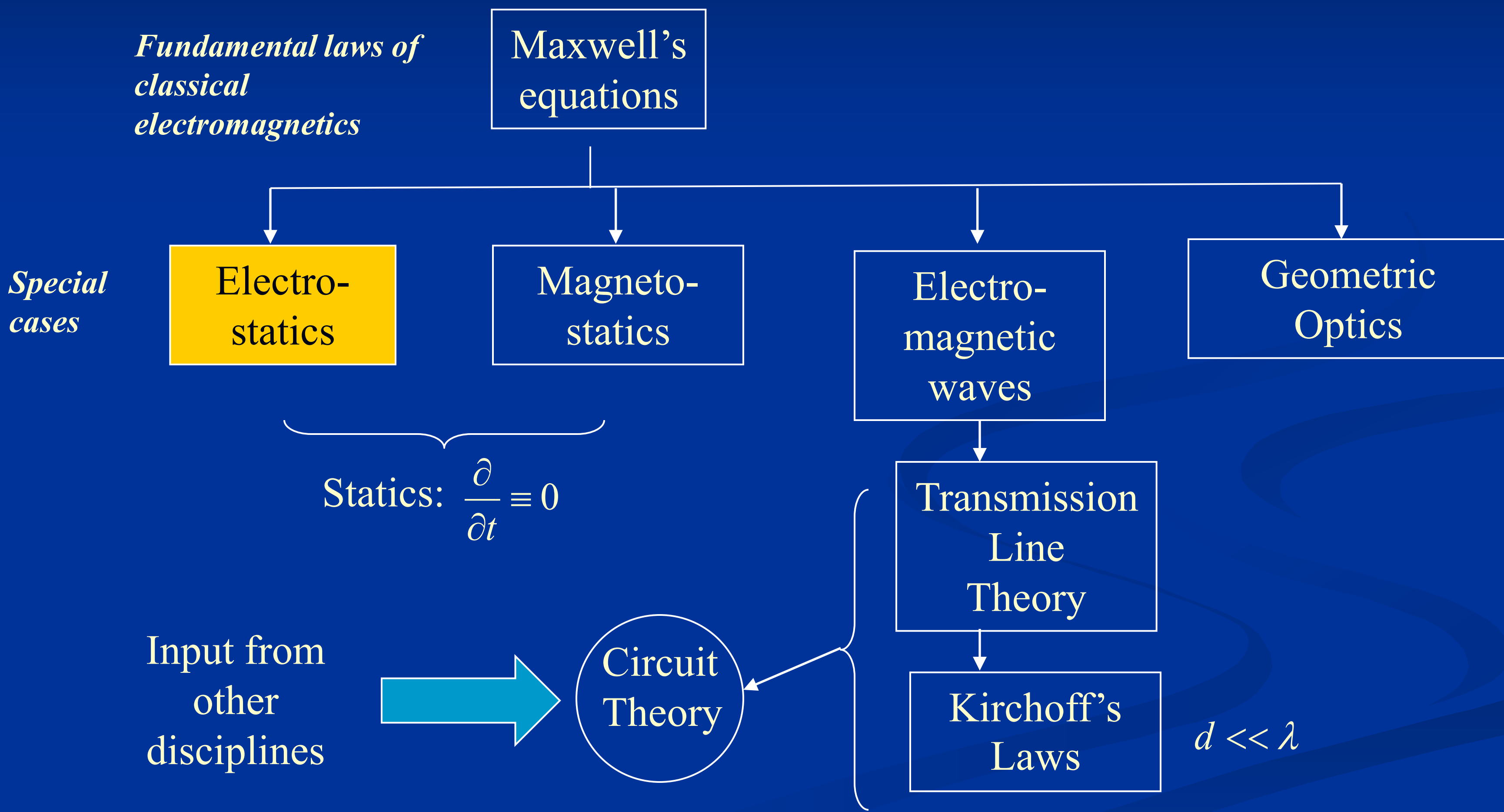
$$\nabla \times \underline{E} = -\left(j\omega\mu + \sigma_m \right) \underline{H} - \underline{K}_i$$

$$\nabla \times \underline{H} = \left(j\omega\varepsilon + \sigma \right) \underline{E} + \underline{J}_i$$

$$\nabla \cdot \underline{E} = \frac{q_{ev}}{\varepsilon}$$

$$\nabla \cdot \underline{H} = \frac{q_{mv}}{\mu}$$

Electrostatics as a Special Case of Electromagnetics



Electrostatics

- ***Electrostatics*** is the branch of electromagnetics dealing with the effects of electric charges at rest.
- The fundamental law of *electrostatics* is ***Coulomb's law***.

Electric Charge

- Electrical phenomena caused by friction are part of our everyday lives, and can be understood in terms of ***electrical charge***.
- The effects of ***electrical charge*** can be observed in the attraction/repulsion of various objects when "charged."
- Charge comes in two varieties called "positive" and "negative."

Electric Charge

- Objects carrying a net positive charge attract those carrying a net negative charge and repel those carrying a net positive charge.
- Objects carrying a net negative charge attract those carrying a net positive charge and repel those carrying a net negative charge.
- On an atomic scale, electrons are negatively charged and nuclei are positively charged.

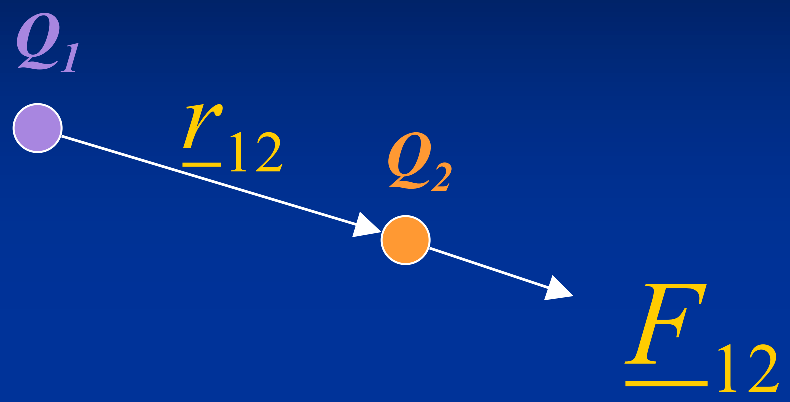
Electric Charge

- Electric charge is inherently quantized such that the charge on any object is an integer multiple of the smallest unit of charge which is the magnitude of the electron charge
  $e = 1.602 \times 10^{-19}$ C.
- On the macroscopic level, we can assume that charge is "continuous."

Coulomb's Law

- ***Coulomb's law*** is the "law of action" between charged bodies.
- ***Coulomb's law*** gives the electric force between two ***point charges*** in an otherwise empty universe.
- A ***point charge*** is a charge that occupies a region of space which is negligibly small compared to the distance between the point charge and any other object.

Coulomb's Law



Unit vector in direction of R_{12}

Force due to Q_1 acting on Q_2

$$\underline{F}_{12} = \hat{a}_{R_{12}} \frac{Q_1 Q_2}{4\pi \varepsilon_0 r_{12}^2}$$

Coulomb's Law

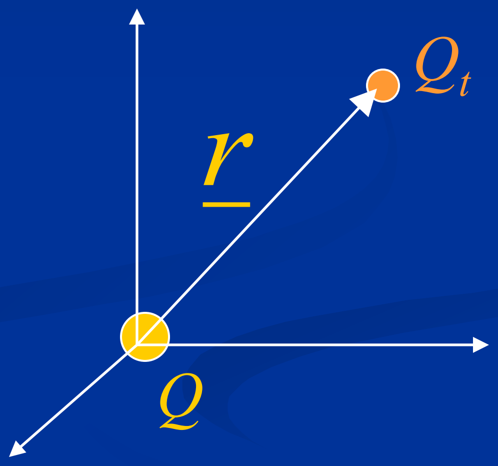
- The force on Q_1 due to Q_2 is equal in magnitude but opposite in direction to the force on Q_2 due to Q_1.

$$\overline{F}_{21} = -\overline{F}_{12}$$

Electric Field

- Consider a point charge Q placed at the <u>origin</u> of a coordinate system in an otherwise empty universe.
- A test charge Q_t brought near Q experiences a force:

$$\underline{F}_{Q_t} = \hat{a}_r \frac{Q Q_t}{4\pi\varepsilon_0 r^2}$$



Electric Field

- The existence of the force on Q_t can be attributed to an ***electric field*** produced by Q.
- The ***electric field*** produced by Q at a point in space can be defined as the force per unit charge acting on a test charge Q_t placed at that point.

$$\overline{E} = \lim_{Q_t \to 0} \frac{\overline{F}_{Q_t}}{Q_t}$$

Electric Field

- The electric field describes the effect of a stationary charge on other charges and is an abstract "action-at-a-distance" concept, very similar to the concept of a gravity field.
- The basic units of electric field are *newtons per coulomb*.
- In practice, we usually use *volts per meter*.

Electric Field

- For a point charge at the <u>origin</u>, the electric field at any point is given by

$$\overline{E}(r) = \hat{a}_r \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{Q\underline{r}}{4\pi\varepsilon_0 r^3}$$

Electric Field

- For a point charge located at a point *P'* described by a position vector $\underline{r}'$

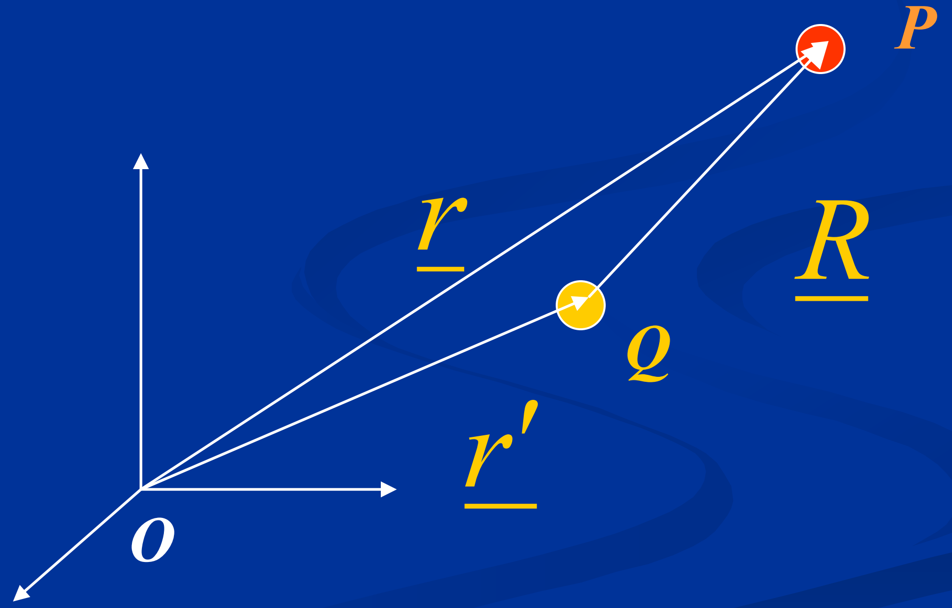
the electric field at *P* is given by

$$\underline{E}(\underline{r}) = \frac{Q\underline{R}}{4\pi\varepsilon_0 R^3}$$

where

$$\underline{R} = \underline{r} - \underline{r}'$$

$$R = |\underline{r} - \underline{r}'|$$



Electric Field

- In electromagnetics, it is very popular to describe the source in terms of *primed coordinates*, and the observation point in terms of *unprimed coordinates*.
- As we shall see, for continuous source distributions we shall need to integrate over the source coordinates.

Electric Field

- Using the principal of ***superposition***, the electric field at a point arising from multiple point charges may be evaluated as

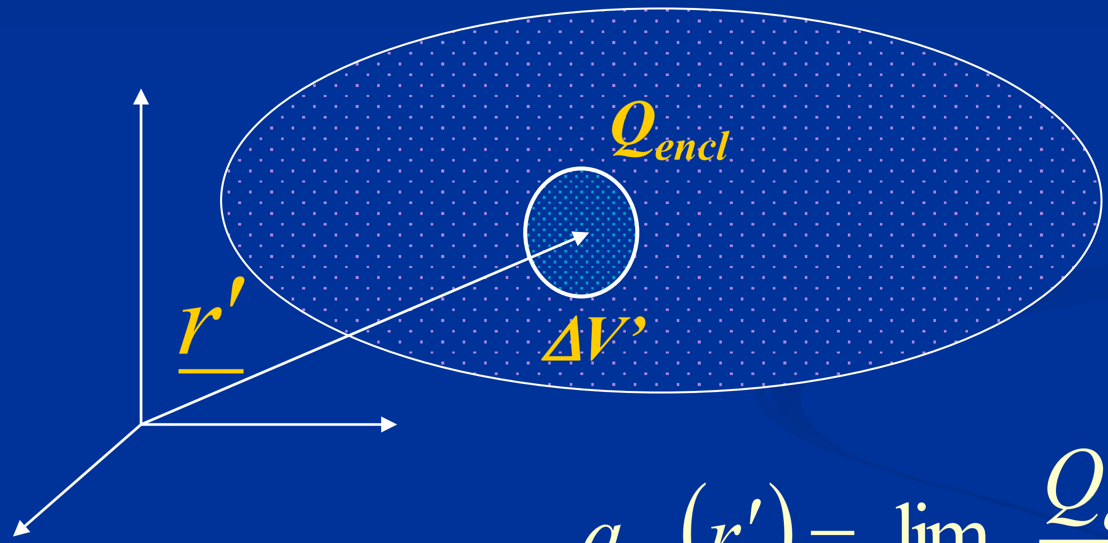
$$\underline{E}(\underline{r}) = \sum_{k=1}^{n} \frac{Q_k \underline{R}_k}{4\pi\varepsilon_0 R_k^{\ 3}}$$

Continuous Distributions of Charge

- Charge can occur as
  - *point charges* (C)
  - *volume charges* (C/m^3) $\Leftarrow$ **most general**
  - *surface charges* (C/m^2)
  - *line charges* (C/m)

Continuous Distributions of Charge

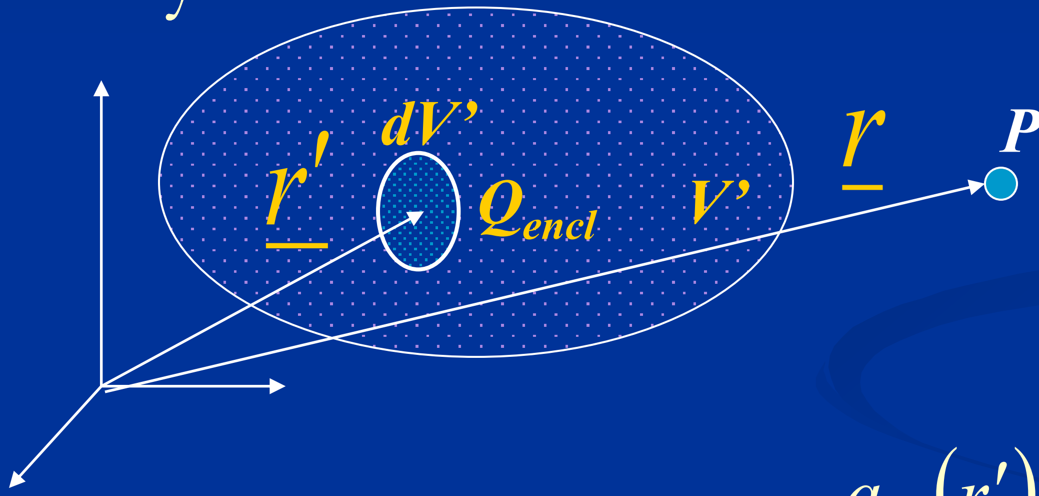
- Volume charge density



$$q_{ev}\left(\underline{r}'\right) = \lim_{\Delta V \to 0} \frac{Q_{encl}}{\Delta V'}$$

Continuous Distributions of Charge

- Electric field due to volume charge density



$$d\,\underline{E}(\underline{r}) = \frac{q_{ev}(\underline{r}')dv'\,\underline{R}}{4\pi\varepsilon_0 R^3}$$

Electric Field Due to Volume Charge Density

$$\underline{E}(\underline{r}) = \frac{1}{4\pi\varepsilon_0} \int_{V'} \frac{q_{ev}(\underline{r}')\underline{R}}{R^3} dv'$$

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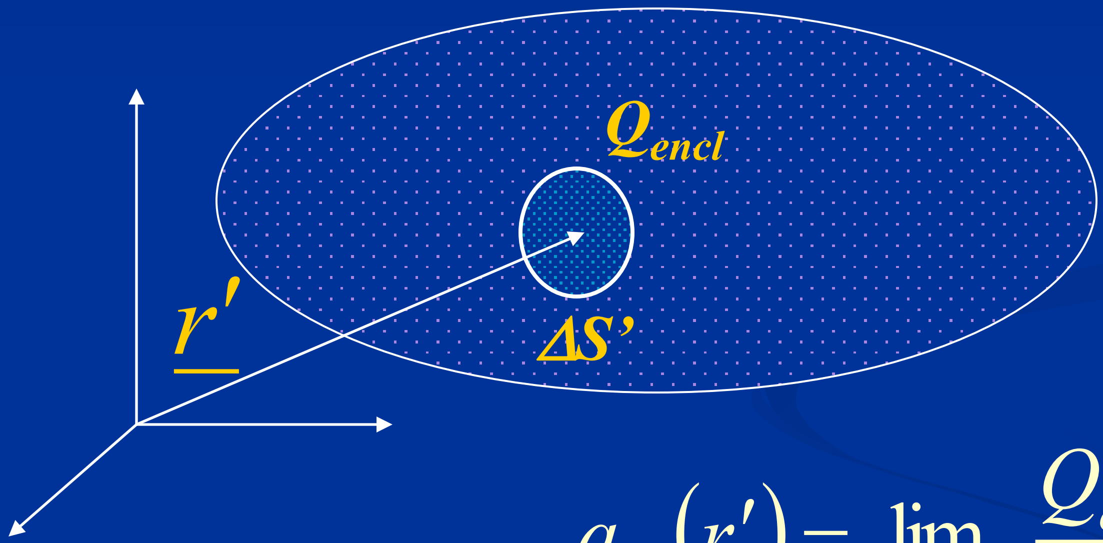
Continuous Distributions of Charge

- Surface charge density

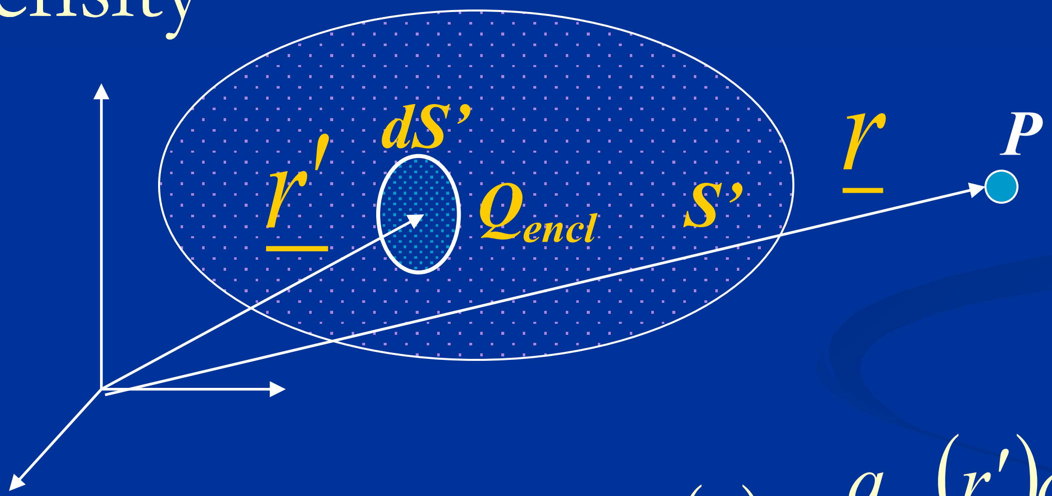


$$q_{es}\left(\underline{r}'\right) = \lim_{\Delta S' \to 0} \frac{Q_{encl}}{\Delta S'}$$

Lecture 2

Continuous Distributions of Charge

- Electric field due to surface charge density



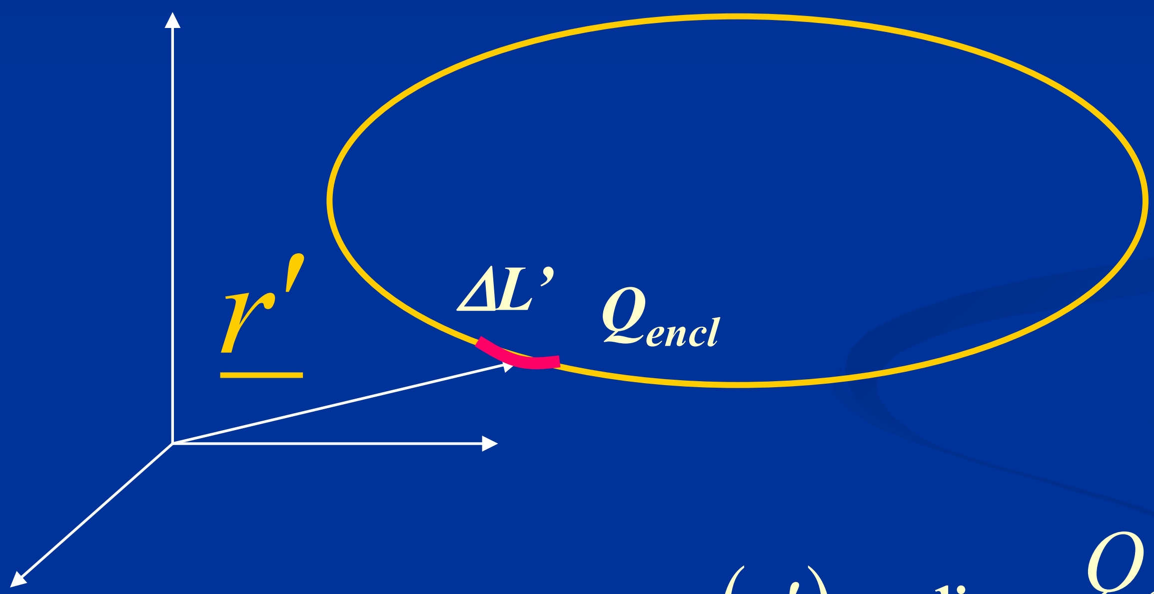
$$d\,\underline{E}\left(\underline{r}\right)=\frac{q_{es}\left(\underline{r}'\right)ds'\,\underline{R}}{4\pi\varepsilon_0 R^3}$$

Electric Field Due to Surface Charge Density

$$\underline{E}\left(\underline{r}\right) = \frac{1}{4\pi\varepsilon_0} \int_{S'} \frac{q_{es}\left(\underline{r}'\right)\underline{R}}{R^3}\, ds'$$

Continuous Distributions of Charge

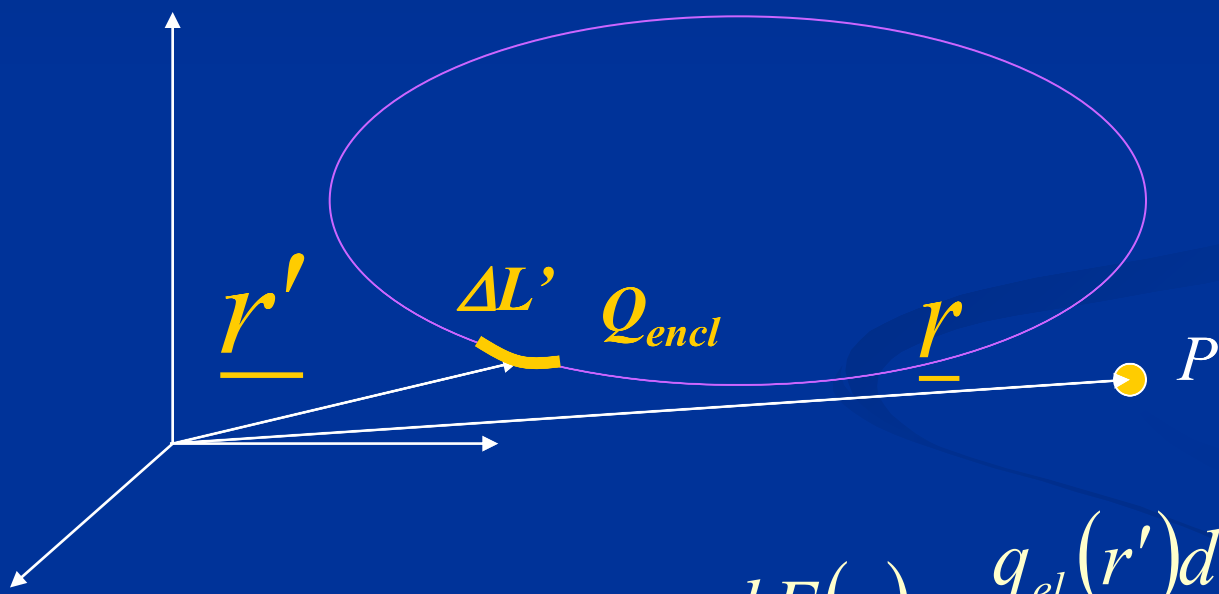
- Line charge density



$$q_{el}\left(\underline{r}'\right) = \lim_{\Delta L' \to 0} \frac{Q_{encl}}{\Delta L'}$$

Continuous Distributions of Charge

- Electric field due to line charge density



$$d\,\underline{E}(\underline{r}) = \frac{q_{el}(\underline{r}')dl'\,\underline{R}}{4\pi\varepsilon_0 R^3}$$

Lecture 2

Electric Field Due to Line Charge Density

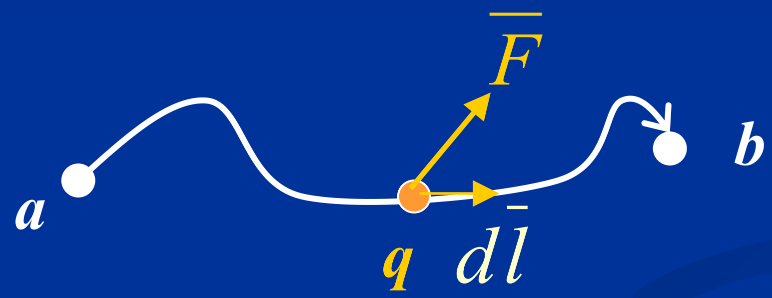
$$\underline{E}\left(\underline{r}\right)=\frac{1}{4\pi\varepsilon_0}\int_{L'}\frac{q_{el}\left(\underline{r}'\right)\underline{R}}{R^3}\,dl'$$

Electrostatic Potential

- An electric field is a *force field*.
- If a body being acted on by a force is moved from one point to another, then ***work*** is done.
- The concept of ***scalar electric potential*** provides a measure of the work done in moving charged bodies in an electrostatic field.

Electrostatic Potential

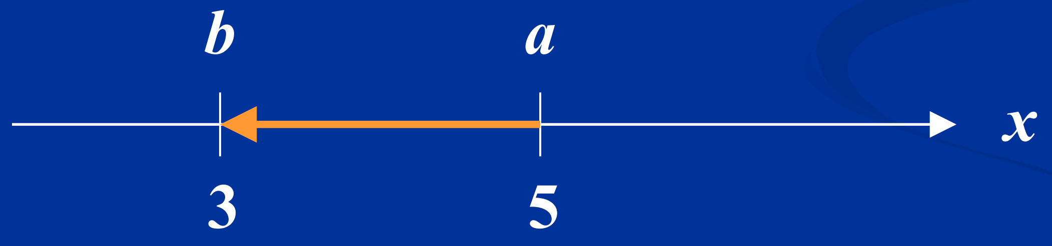
- The work done in moving a test charge from one point to another in a region of electric field:



$$W_{a\to b} = -\int_a^b \underline{F}\cdot d\underline{l} = -q\int_a^b \underline{E}\cdot d\underline{l}$$

Electrostatic Potential

- In evaluating line integrals, it is customary to take the *dl* in the direction of increasing coordinate value so that the manner in which the path of integration is traversed is unambiguously determined by the limits of integration.



$$W_{a\to b} = -q\int_{5}^{3} \underline{E} \bullet \hat{a}_x \, dx$$

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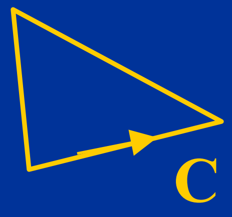
Electrostatic Potential

- The electrostatic field is ***conservative***:
  - The value of the line integral depends only on the end points and is independent of the path taken.
  - The value of the line integral around any closed path is zero.



$$\oint_C \underline{E} \cdot d\underline{l} = 0$$

Electrostatic Potential

- The work done per unit charge in moving a test charge from point *a* to point *b* is the ***electrostatic potential difference*** between the two points:

$$V_{ab} \equiv \frac{W_{a\to b}}{q} = -\int_a^b \underline{E} \cdot d\underline{l}$$

electrostatic potential difference
Units are volts.

Electrostatic Potential

- Since the electrostatic field is conservative we can write

$$V_{ab} = -\int_a^b \underline{E} \bullet d\underline{l} = -\int_a^{P_0} \underline{E} \bullet d\underline{l} - \int_{P_0}^b \underline{E} \bullet d\underline{l}$$

$$= -\int_{P_0}^b \underline{E} \bullet d\underline{l} - \left(-\int_{P_0}^a \underline{E} \bullet d\underline{l} \right)$$

$$= V(b) - V(a)$$

Electrostatic Potential

- Thus the ***electrostatic potential*** V is a scalar field that is defined at every point in space.
- In particular the value of the *electrostatic potential* at any point P is given by

$$V(\underline{r}) = -\int_{P_0}^{P} \underline{E} \bullet d\underline{l}$$

P_0 ← **reference point**

Electrostatic Potential

- The ***reference point*** (P_0) is where the potential is zero (analogous to ***ground*** in a circuit).
- Often the reference is taken to be at infinity so that the potential of a point in space is defined as

$$V(\underline{r}) = -\int_{\infty}^{P} \underline{E} \bullet d\underline{l}$$

Electrostatic Potential and Electric Field

- The work done in moving a point charge from point *a* to point *b* can be written as

$$W_{a\to b} = Q V_{ab} = Q\left\{V(b) - V(a)\right\}$$

$$= -Q\int_a^b \underline{E} \bullet d\underline{l}$$

Lecture 2

Electrostatic Potential and Electric Field

- Along a short path of length Δl we have

$$\Delta W = Q\Delta V = -Q\underline{E}\cdot\Delta\underline{l}$$

or

$$\Delta V = -\underline{E}\cdot\Delta\underline{l}$$

Electrostatic Potential and Electric Field

- Along an incremental path of length *dl* we have

$$dV = -\underline{E} \cdot d\underline{l}$$

- Recall from the definition of *directional derivative*:

$$dV = \nabla V \cdot d\underline{l}$$

Electrostatic Potential and Electric Field

- Thus:

$$\underline{E} = -\nabla V$$

the "del" or "nabla" operator