LORENTZ TRANSFORMATION

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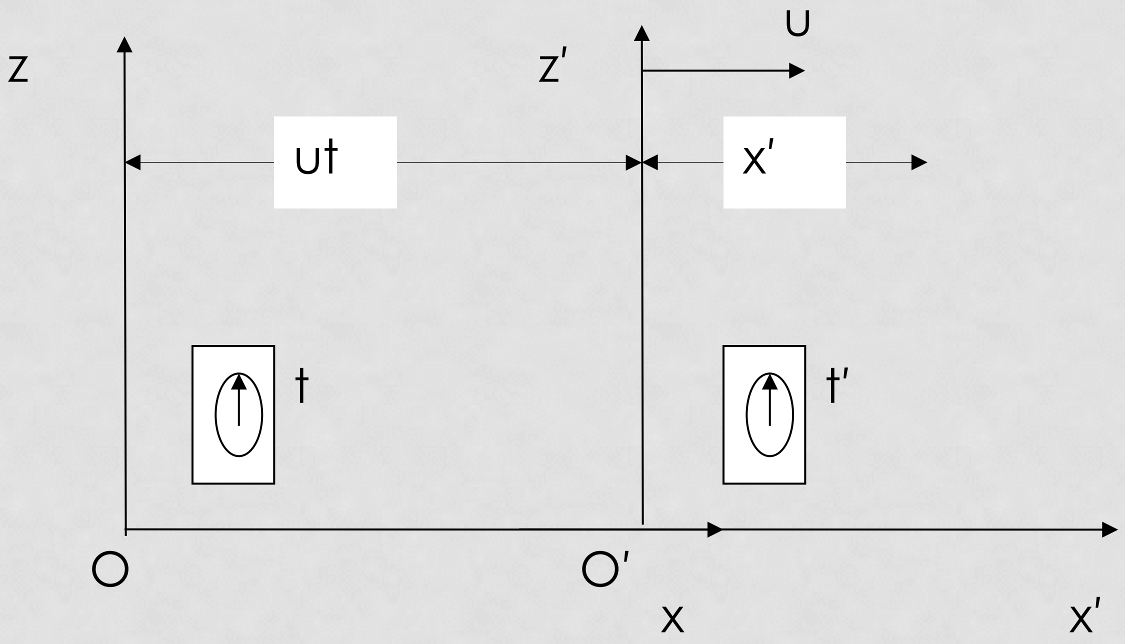
- Again consider the transformation problem.
- The required transformation consists of equations allowing us to calculate the primed set of numbers in terms of the unprimed set or vice versa.
- The Lorentz transforms replace the Galilean transforms of position and time.

$$(x', y', z'; t')$$

$$(x, y, z; t)$$

LORENTZ TRANSFORMATION

- The Lorentz transformations will be proved at a later.
- Again consider the case,



LORENTZ TRANSFORMATION

- The Lorentz transformations for position and time are:

$$x = \left(x' + vt'\right)\gamma$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$y = y' \quad z = z'$$

LORENTZ TRANSFORMATION

- The inverse of these equations give:

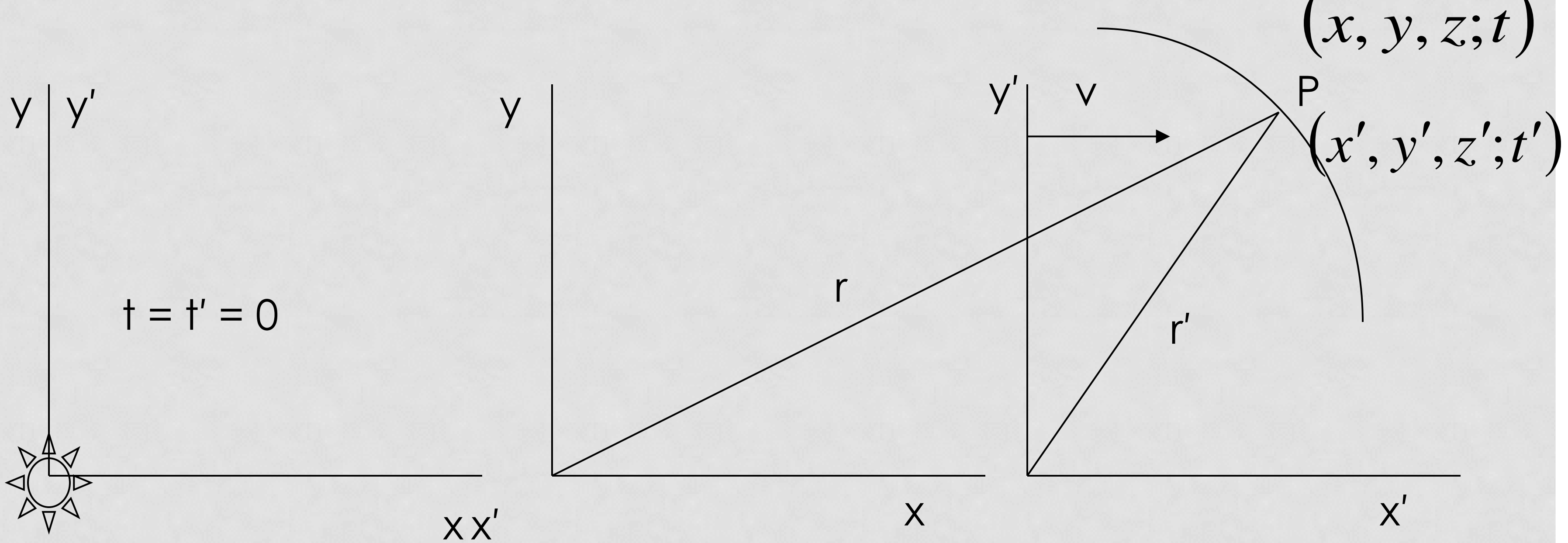
$$x' = (x - vt)\gamma$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$y' = y \qquad z' = z$$

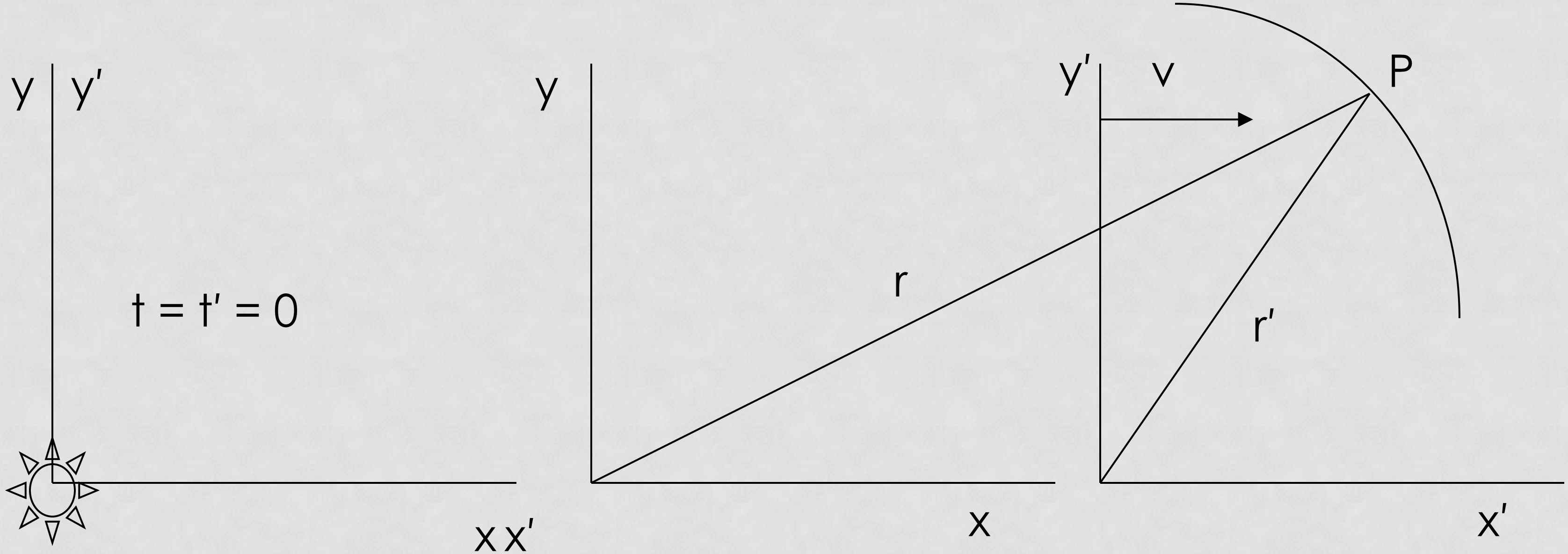
LORENTZ TRANSFORMATION

- The transformation equations are valid for all speeds < c.
- Consider a flash bulb attached to S′ that goes off,



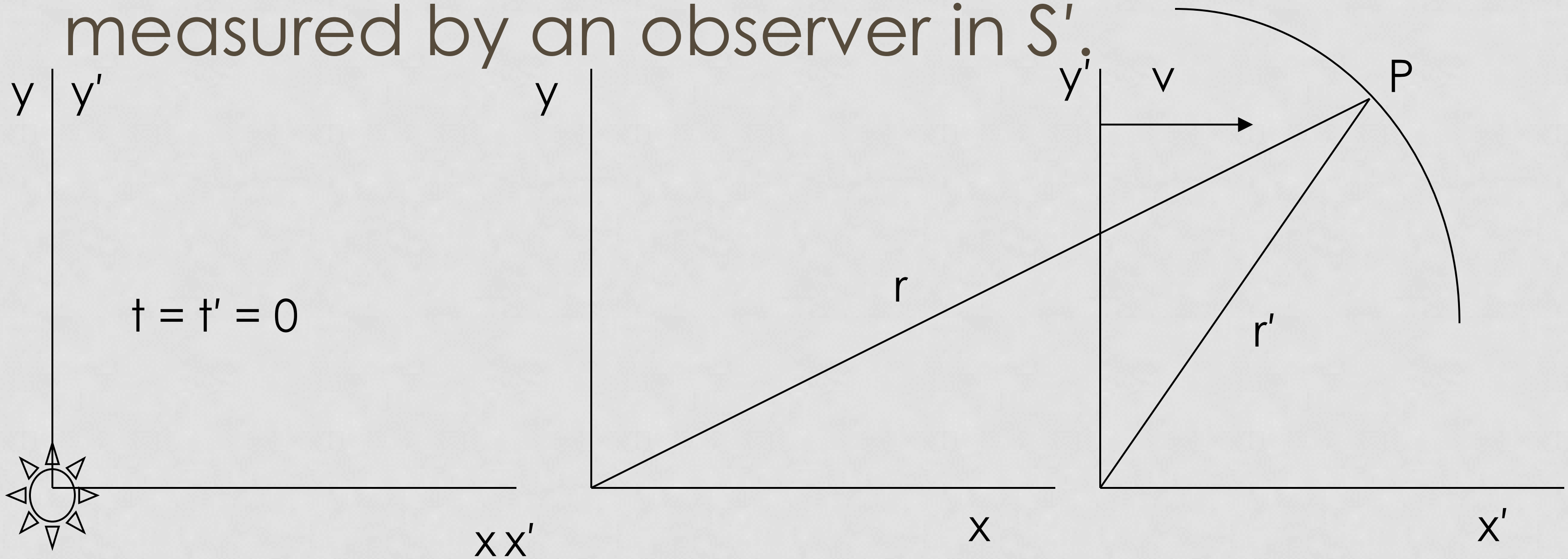
LORENTZ TRANSFORMATION

- At the instance it goes off the two frames coincide. At some later time the wavefront is at some point P.



LORENTZ TRANSFORMATION

- r: distance to a point on the wavefront measured by an observer in S.
- r′: distance to a point on the wavefront measured by an observer in S′.

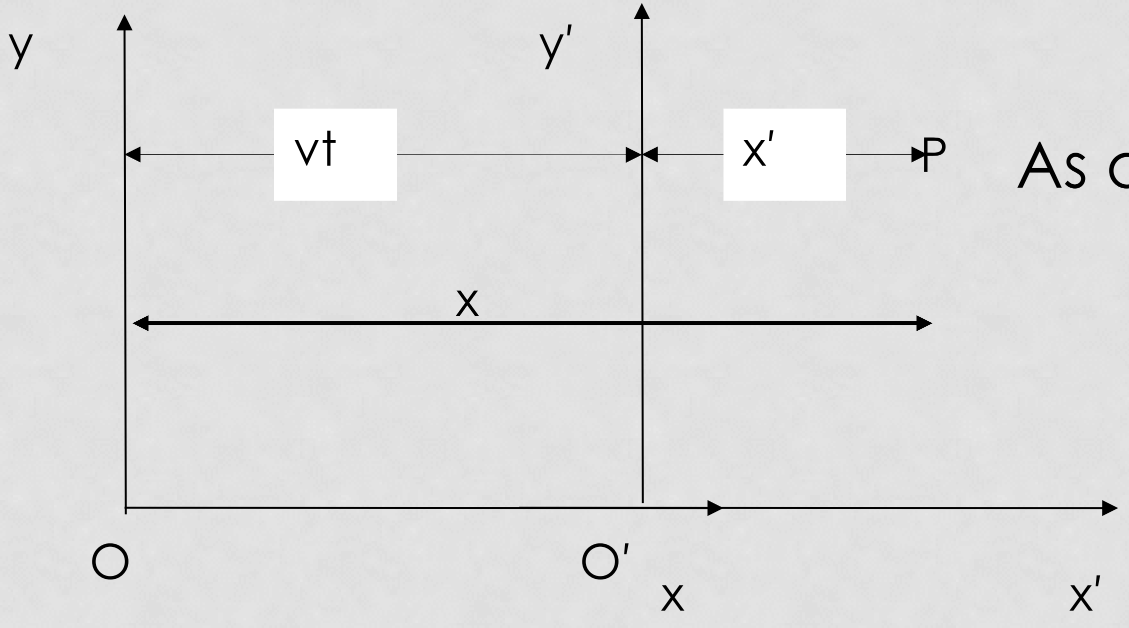


LORENTZ TRANSFORMATION

- $r=ct$ $\quad ..(1)$ Stationary Frame
- $r'=ct'$ $\quad ..(2)$ Moving Frame

LORENTZ TRANSFORMATION

- For simplicity, the general problem is stated so that the motion of P is along the x-x′ axis.



As a result: $y=y'$; $z=z'$

LORENTZ TRANSFORMATION

- Radius of a sphere is $r^2 = x^2 + y^2 + z^2$ in the S frame and similarly $(r')^2 = (x')^2 + (y')^2 + (z')^2$ in the S′ frame.

$$x^2 + y^2 + z^2 = c^2 t^2 \qquad ..(3)$$

$$(x')^2 + (y')^2 + (z')^2 = c^2 (t')^2 \qquad ..(4)$$

LORENTZ TRANSFORMATION

- Substituting is $y = y'; z = z'$ into the previous equations and subtracting we get that,

$$x^2 - (x')^2 = c^2t^2 - c^2(t')^2$$

$$x^2 - c^2t^2 = (x')^2 - c^2(t')^2 \qquad ..(5)$$

LORENTZ TRANSFORMATION

- We know that in the stationary frame, the distance travelled is given by

$$x = vt \qquad ..(6)$$

- In the stationary frame, the distance travelled is

$$x' = 0 \qquad ..(7)$$

LORENTZ TRANSFORMATION

- We know that in the stationary frame, the distance travelled is given by

$$x = vt \qquad ..(6)$$

- In the stationary frame, the distance travelled is
- Using equations 5,6,7 we can show that,

$$x' = 0 \qquad ..(7)$$

$$x' = (x - vt)\gamma \quad ..(8) \qquad t' = \left(t - \frac{v}{c^2}x\right)\gamma \quad ..(9)$$

LORENTZ TRANSFORMATION

- Summary:

$$x' = \left(x - vt\right)\gamma$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$y' = y \qquad z' = z$$

LORENTZ TRANSFORMATION

- Summary:

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- The Lorentz transformations can be verified by substituting equations 8,9 into the RHS of equation 5.

LORENTZ TRANSFORMATION

- To produce the Lorentz transformations for primed frame to the unprimed frame we substitute v with – v.

$$x = (x' + vt')\gamma$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$y = y' \quad z = z'$$

LORENTZ TRANSFORMATION

- For $v << c$, the Lorentz transformations reduce to the Galilean transformations. When $v << c$; v/c <<1 and
  - $\dfrac{v^2}{c^2} << 1$

LORENTZ TRANSFORMATION

- Solution:
- The question requires us to transform from the unprimed to the primed! Therefore use,

$$x' = (x - vt)\gamma$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$y' = y \qquad z' = z$$