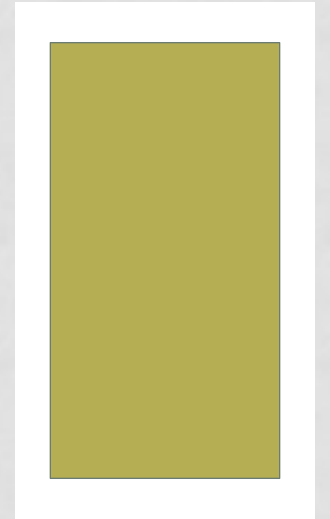
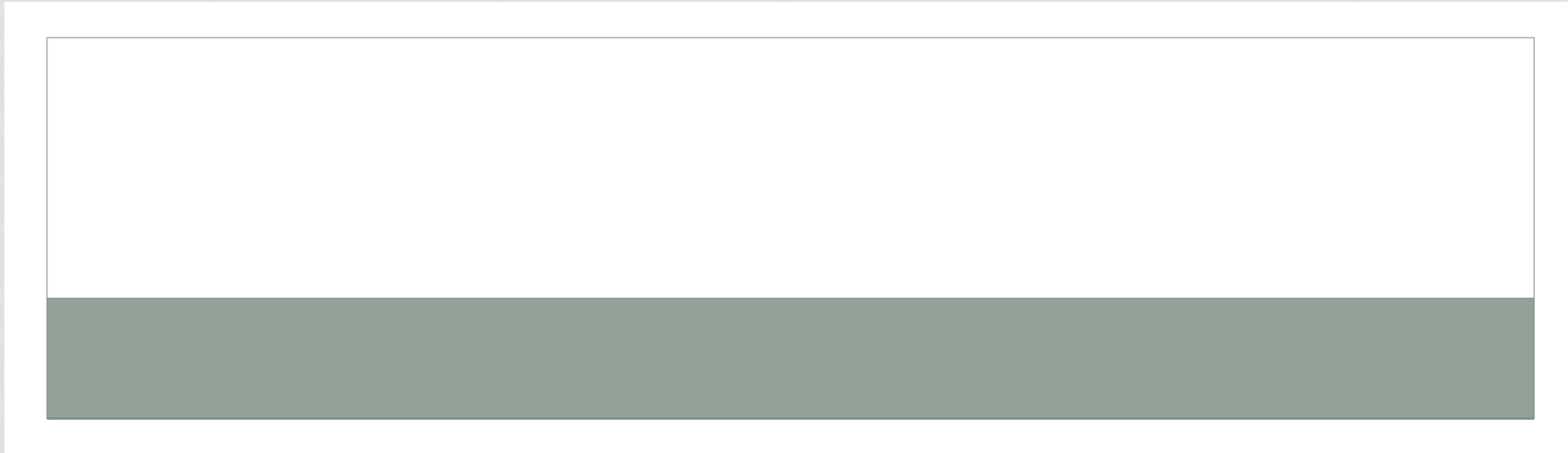


DE-BROGLIE HYPOTHESIS
DAVISSON-GERMER EXPERIMENT
PHASE VELOCITY
GROUP VELOCITY

B.TECH I-SEM



Light is . . .

- Initially thought to be waves
 - They do things waves do, like diffraction and interference
 - Wavelength – frequency relationship $c = \lambda f$
- Planck, Einstein, Compton showed us they behave like particles (photons)
 - Energy comes in chunks
 - Wave-particle duality: somehow, they behave like both $E = hf$
- Photons also carry momentum
 - Momentum comes in chunks

$$p = E/c = hf/c = h/\lambda$$

$$p\lambda = h$$

Electrons are . . .

- They act like particles
 - Energy, momentum, etc., come in chunks
- They also behave quantum mechanically
- Is it possible they have wave properties as well?

The de Broglie Hypothesis

- Two equations that relate the particle-like and wave-like properties of light

$$E = hf$$

$$\lambda p = h$$

1924 – Louis de Broglie postulated that these relationships apply to electrons as well

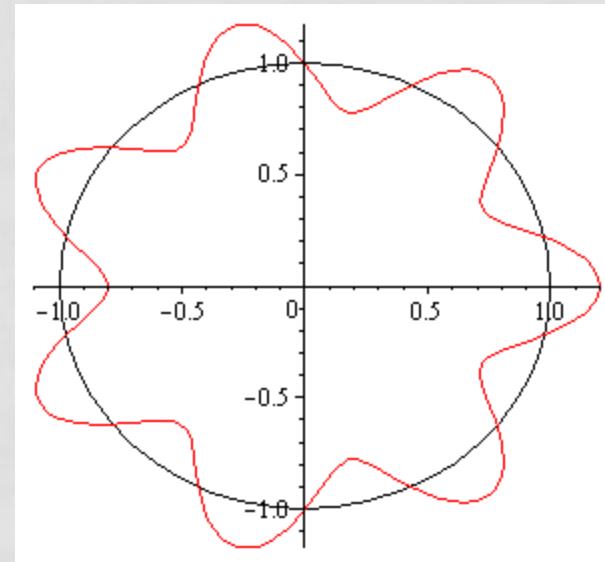
- Implied that it applies to other particles as well
- de Broglie could simply explain the Bohr quantization condition
 - Compare the wavelength of an electron in hydrogen to the circumference of its path

$$L = n\hbar = pr = \frac{hr}{\lambda} = \frac{2\pi\hbar}{\lambda}$$

cancel \square

$$n\lambda = 2\pi r = C$$

Integer number of wavelengths fit around the orbit



Measuring wave properties of electrons

$$\lambda p = h$$

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2c^2}{2mc^2\lambda^2} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})^2 (3.00 \times 10^8 \text{ m/s})^2}{2(0.511 \times 10^6 \text{ eV})\lambda^2}$$
$$= \frac{1.504 \times 10^{-18} \text{ eV} \cdot \text{m}^2}{\lambda^2} = 1.504 \text{ eV} \left(\frac{\text{nm}}{\lambda} \right)^2$$

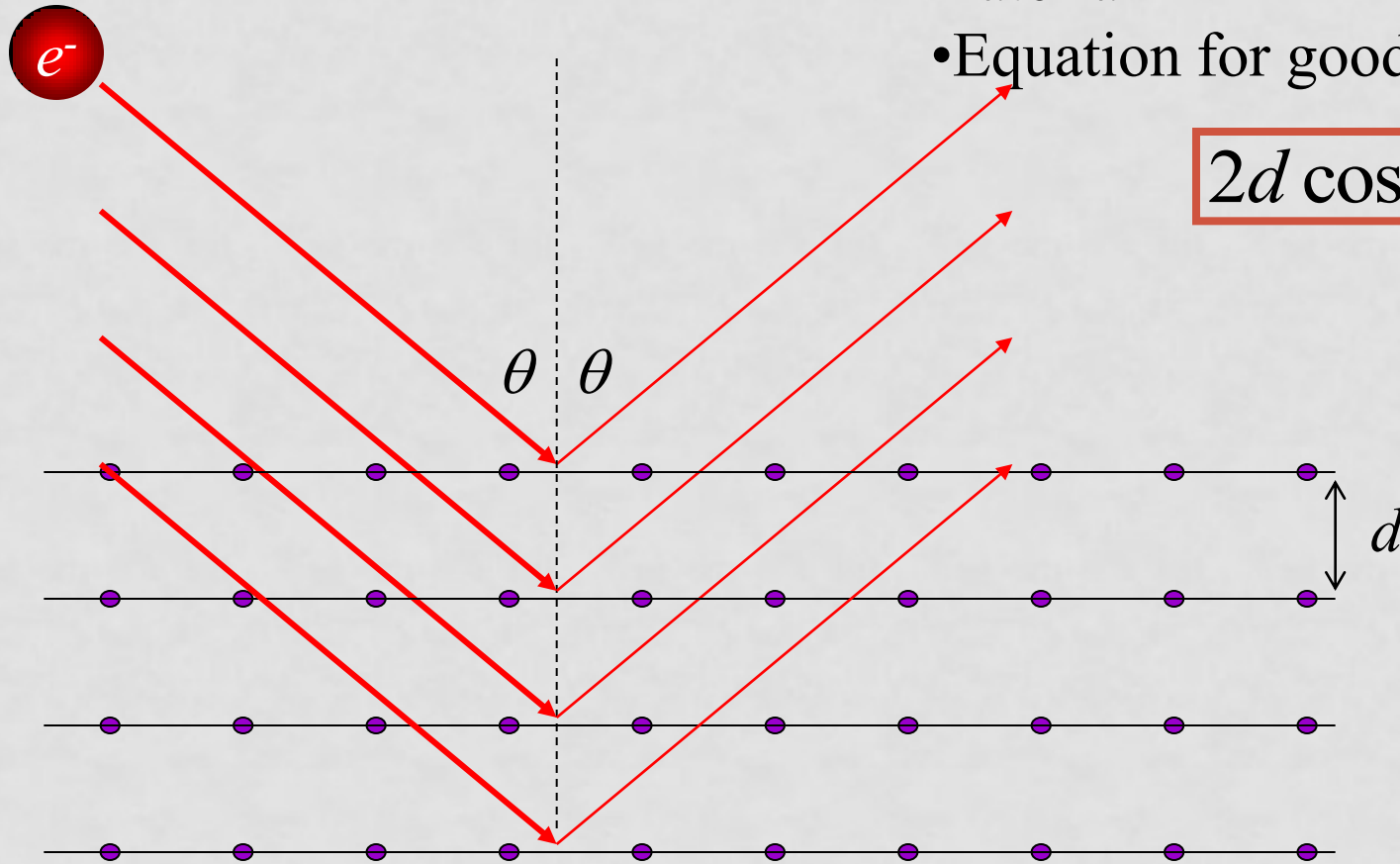
For atomic separations, want distances around 0.3 nm → energies of 10 or so eV

How can we measure these wave properties?

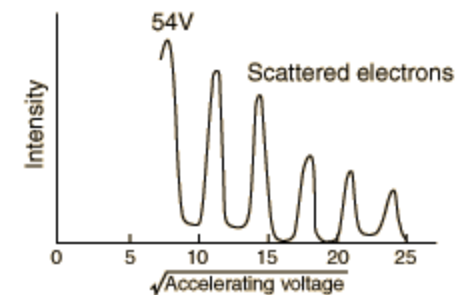
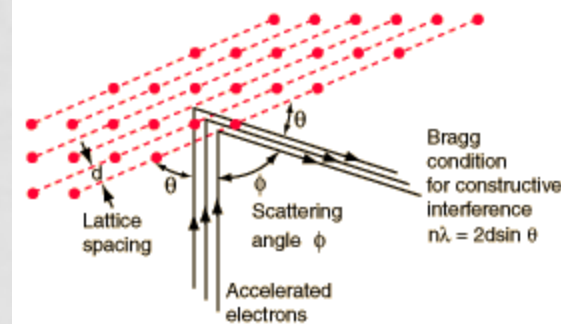
- Scatter off crystals, just like we did for X-rays!
- Complication: electrons change speed inside crystal
 - Work function ϕ increases kinetic energy in the crystal
 - Momentum increases in the crystal
 - Wavelength changes

The Davisson-Germer Experiment

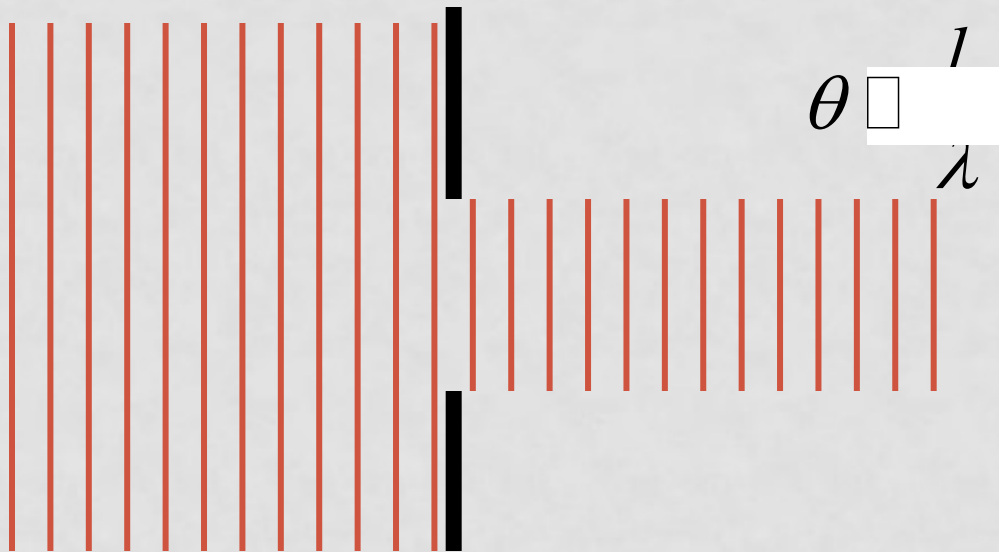
- Same experiment as scattering X-rays, except
- Reflection probability from each layer greater
 - Interference effects are weaker
 - Momentum/wavelength is shifted inside the material
 - Equation for good scattering identical



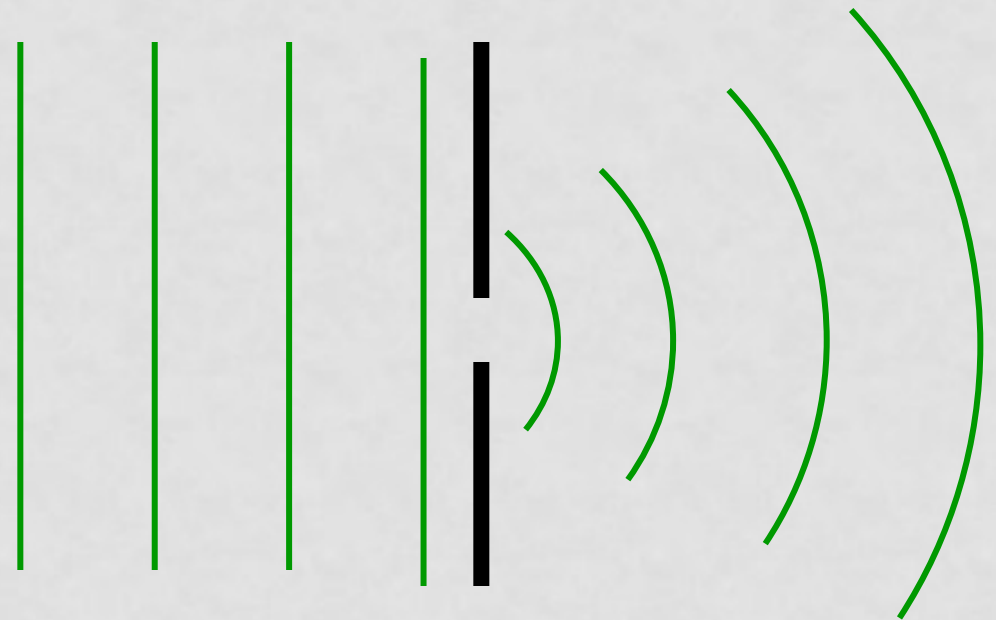
$$2d \cos \theta = m\lambda$$



- Whenever waves encounter a barrier, they get diffracted, their direction changes
- If the barrier is much *larger* than the waves, the waves change direction very little
- If the barrier is much *smaller* than the waves, then the effect is enormous, and the wave diffracts a lot



Light waves through a big hole



Sound waves through a small hole

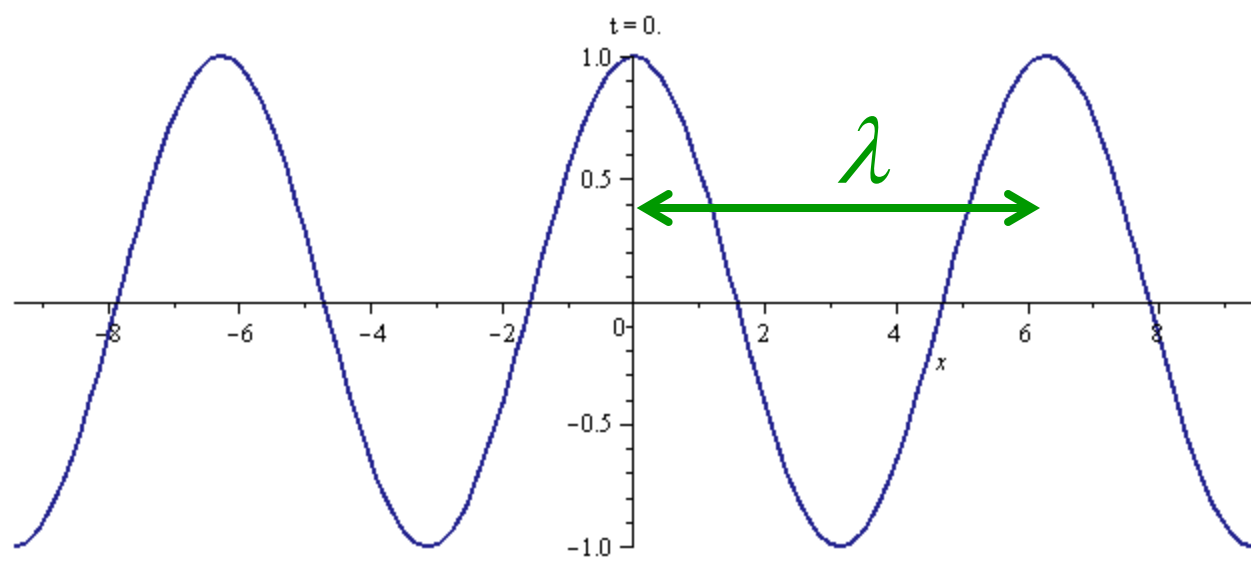
Simple Waves

- cos and sin have periodicity

2π

- If you increase kx by 2π , wave will look the same

- If you increase ωt by 2π , wave will look the same



- Simple waves look like cosines or sines:

- k is called the wave number

- Units of inverse meters

- ω is called the angular frequency

- Units of inverse seconds

- Wavelength λ is how far you have to go in space before it repeats

- Related to wave number k

- Period T is how long you have to wait in time before it repeats

- Related to angular frequency ω

- Frequency f is how many times per second it repeats

- The reciprocal of period

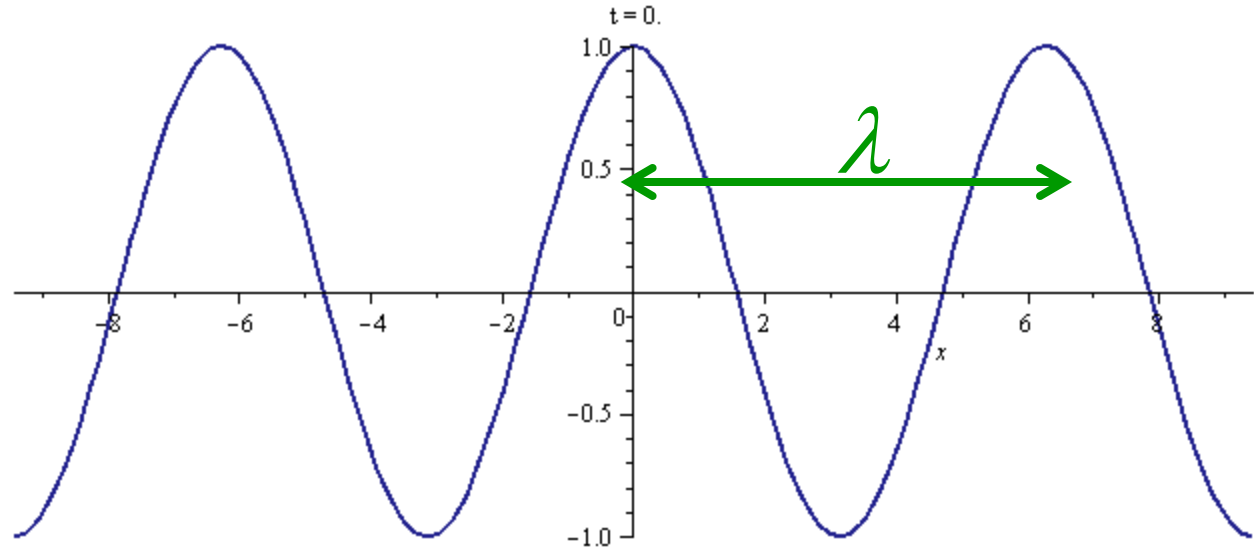
$$\psi(x, t) = A \cos(kx - \omega t)$$

$$\psi(x, t) = A \sin(kx - \omega t)$$

$$\lambda = 2\pi/k$$

$$\omega = 2\pi/T = 2\pi f$$

Phase velocity



$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$k\lambda = 2\pi$$

$$v_p = \lambda f = \frac{\omega}{k}$$

- The wave moves a distance of one wavelength λ in one period T
- From this, we can calculate the *phase velocity* denoted v_p
 - It is how fast the peaks and valleys move

$$v_p = \frac{\lambda}{T} = \lambda f = \frac{2\pi}{k} \frac{\omega}{2\pi} = \frac{\omega}{k}$$

$$v_p = \frac{\omega}{k} = \frac{ck}{k} = c$$

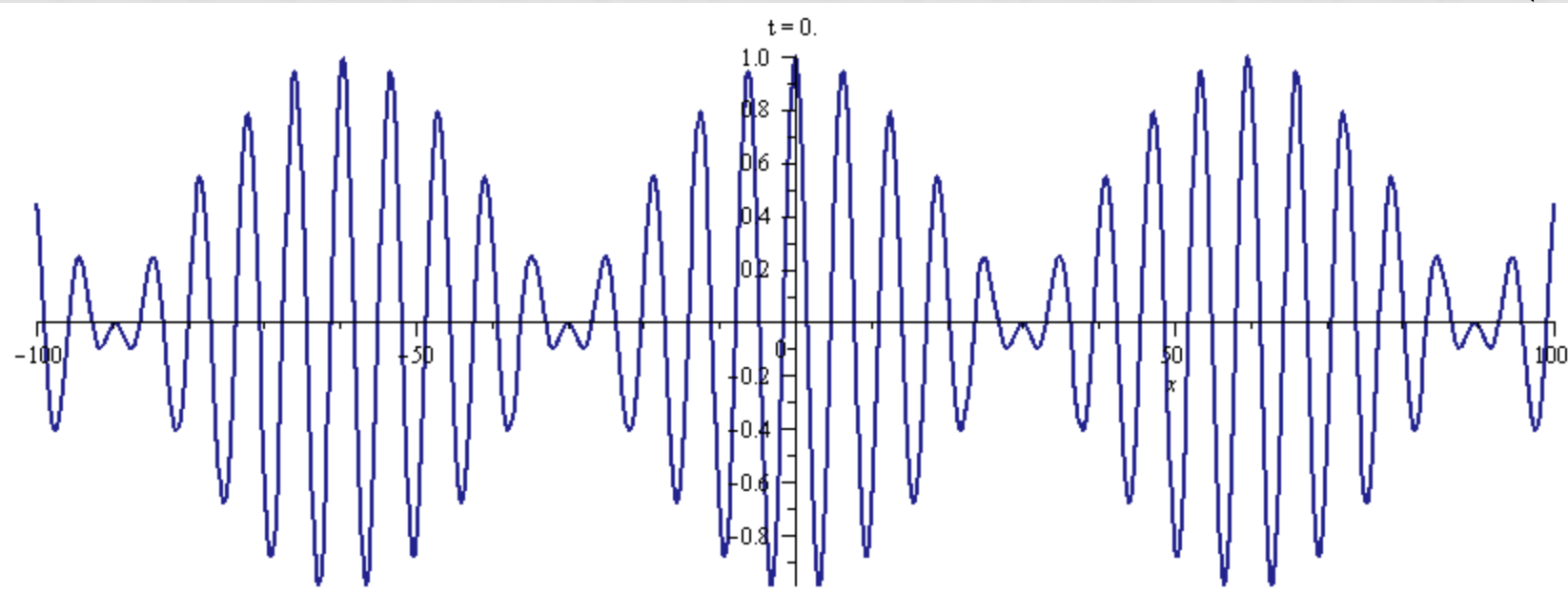
Adding two waves

- Real waves are almost always combinations of multiple wavelengths
- Average these two expressions to get a new wave:

$$\psi(x, t) = \frac{1}{2} \cos(k_1 x - \omega_1 t) + \frac{1}{2} \cos(k_2 x - \omega_2 t)$$

$$\psi_1 = \cos(k_1 x - \omega_1 t)$$

$$\psi_2 = \cos(k_2 x - \omega_2 t)$$



- This wave has two kinds of oscillations:
 - The oscillations at small scales
 - The “lumps” at large scales

Analyzing the sum of two waves:

$$\psi(x, t) = \frac{1}{2} \cos(k_1 x - \omega_1 t) + \frac{1}{2} \cos(k_2 x - \omega_2 t)$$

Need to derive some obscure trig identities:

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

• Average these:

$$\frac{1}{2} \cos(\alpha + \beta) + \frac{1}{2} \cos(\alpha - \beta) = \cos \alpha \cos \beta$$

• Substitute:

$$\alpha = \frac{1}{2}(A + B)$$

$$\beta = \frac{1}{2}(A - B)$$

$$\frac{1}{2} \cos A + \frac{1}{2} \cos B = \cos\left[\frac{1}{2}(A + B)\right] \cos\left[\frac{1}{2}(A - B)\right]$$

Rewrite wave function:

$$\psi(x, t) = \underbrace{\cos(\bar{k}x - \bar{\omega}t)}_{\text{Small scale oscillations}} \underbrace{\cos(\Delta k \cdot x - \Delta \omega \cdot t)}_{\text{Large scale oscillations}}$$

$$\bar{k} = \frac{1}{2}(k_1 + k_2)$$

$$\bar{\omega} = \frac{1}{2}(\omega_1 + \omega_2)$$

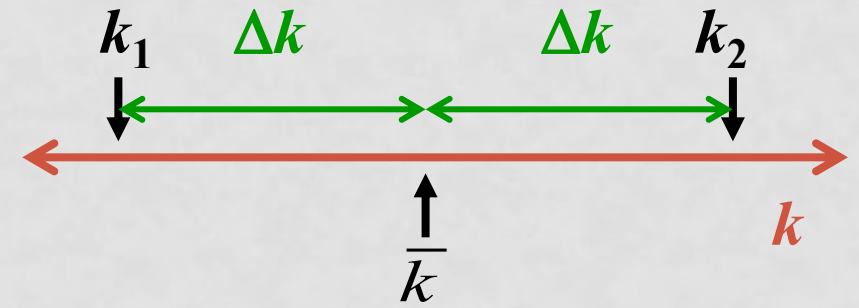
**Small scale
oscillations**

**Large scale
oscillations**

$$\Delta k = \frac{1}{2}(k_1 - k_2)$$

$$\Delta \omega = \frac{1}{2}(\omega_1 - \omega_2)$$

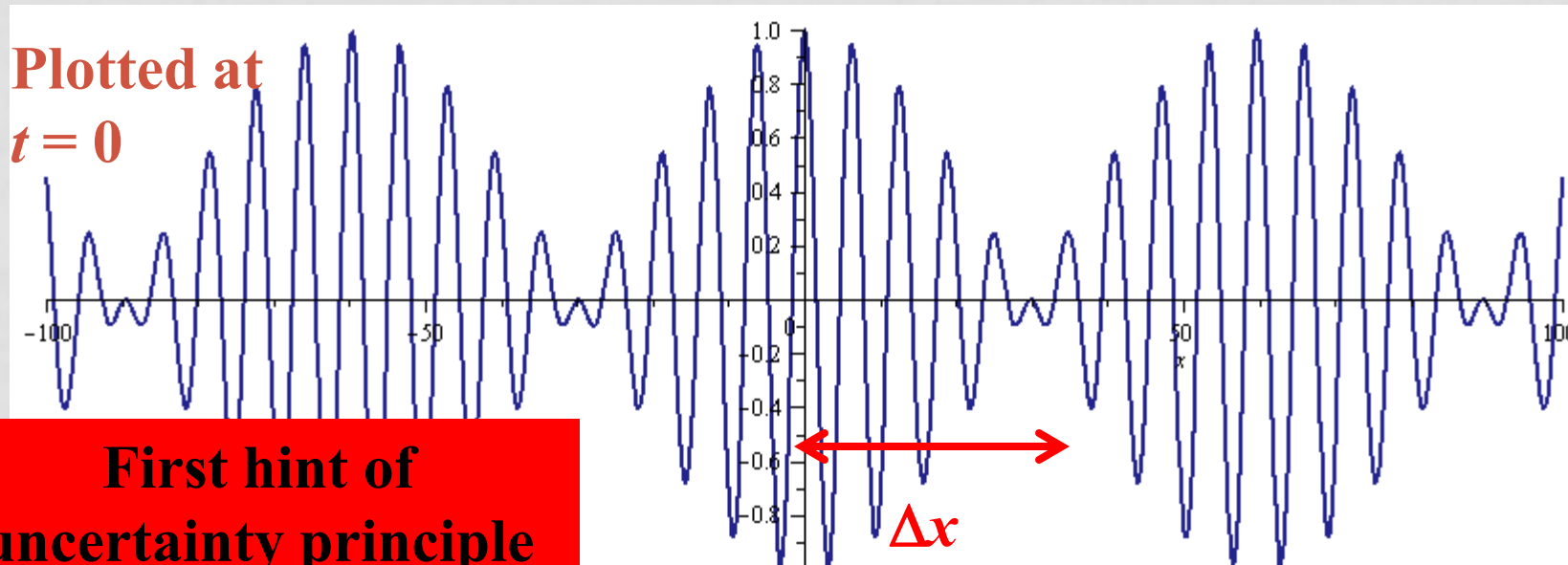
The “uncertainty” of two waves



Our wave is made of two values of k :

- \bar{k} is the average value of these two
- Δk is the amount by which the two values of k actually vary from \bar{k}
 - The value of k is uncertain by an amount Δk

- Each “lump” is spread out in space also
- Define Δx as the distance from the center of a lump to the edge
- The distance is where the cosine vanishes



First hint of uncertainty principle

$$\cos(\Delta k \Delta x) = 0$$

$$\Delta k \Delta x = \frac{1}{2} \pi$$

$$\Delta k \Delta x \square$$

Group Velocity

$$\psi(x, t) = \underbrace{\cos(\bar{k}x - \bar{\omega}t)}_{\text{Small scale oscillations}} \underbrace{\cos(\Delta k \cdot x - \Delta \omega \cdot t)}_{\text{Large scale oscillations}}$$

Small scale
oscillations

Large scale
oscillations

The velocity of little oscillations governed by the first factor

• Leads to the same formula as before for phase velocity:

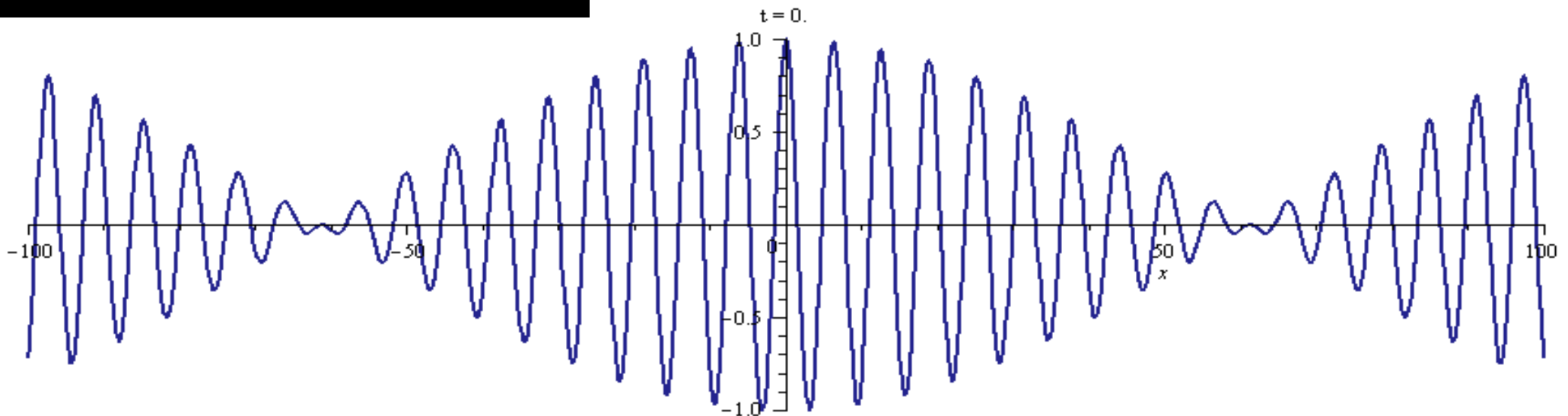
$$v_p = \frac{\bar{\omega}}{\bar{k}}$$

The velocity of big oscillations governed by the second factor

• Leads to a formula for group velocity:

$$v_g = \frac{\Delta \omega}{\Delta k}$$

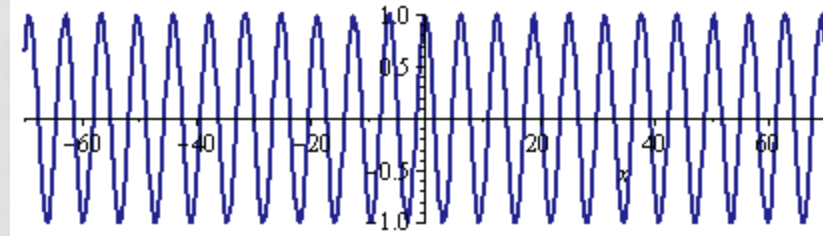
These need not be the same!



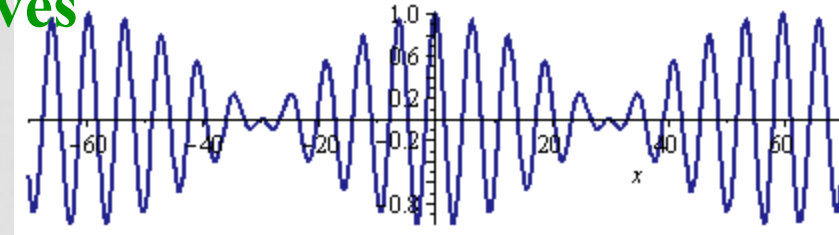
More Waves

- Two waves allow you to create localized “lumps”
- Three waves allow you to start separating these lumps
- More waves lets you get them farther and farther apart
- Infinity waves allows you to make the other lumps disappear to infinity – you have one lump, or a wave packet
- A single lump – a wave packet – looks and acts a lot like a particle

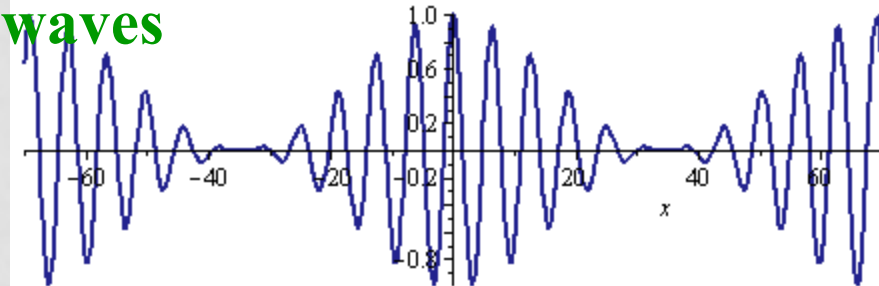
One wave



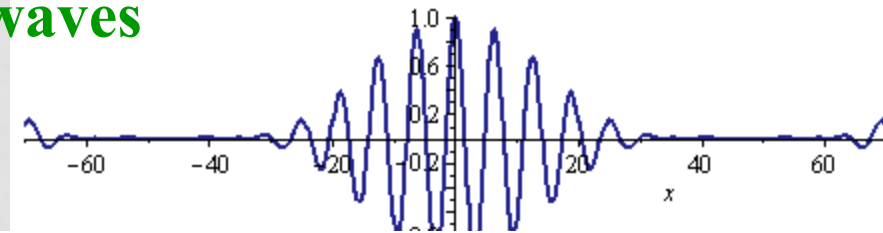
Two waves



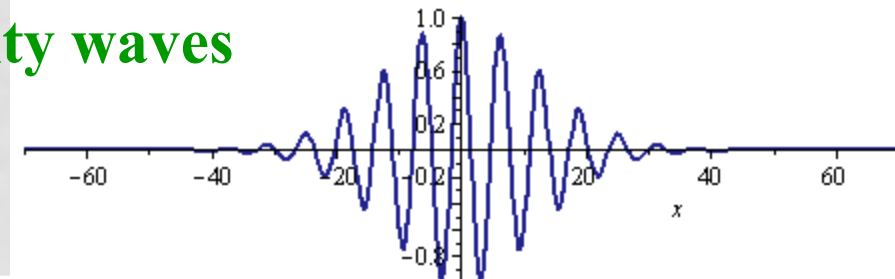
Three waves



Five waves

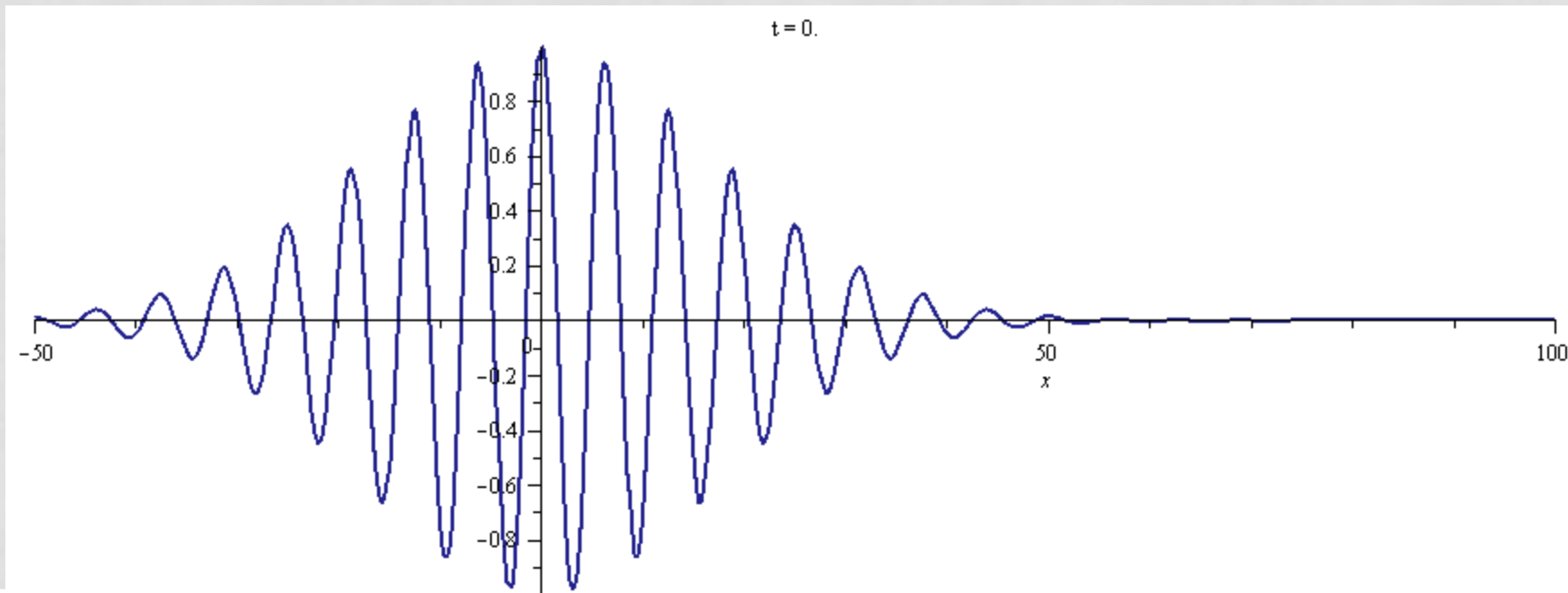
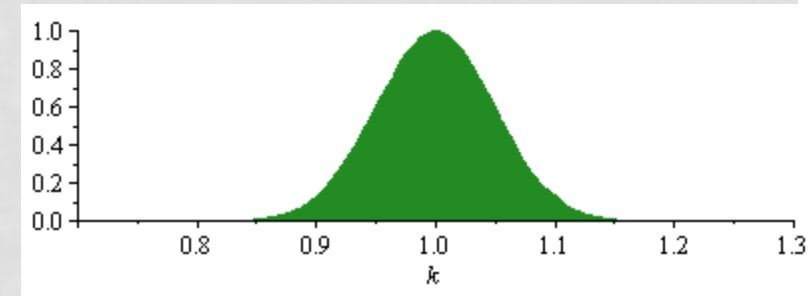


Infinity waves



Wave Packets

- We can combine many waves to separate a “lump” from its neighbors
- With an infinite number of waves, we can make a wave packet
 - Contains continuum of wave numbers k
- Resulting wave travels and mostly stays together, like a particle
- Note both k -values and x -values have a spread Δk and Δx .



Phase and Group velocity

Compare to two wave formulas:

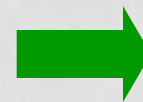
- Phase velocity formula is exactly the same, except we simply rename the average values of k and ω as simply k and ω
- Group velocity now involves very closely spaced values of k (and ω), and therefore we rewrite the differences as . . .

$$v_p = \frac{\bar{\omega}}{\bar{k}}$$

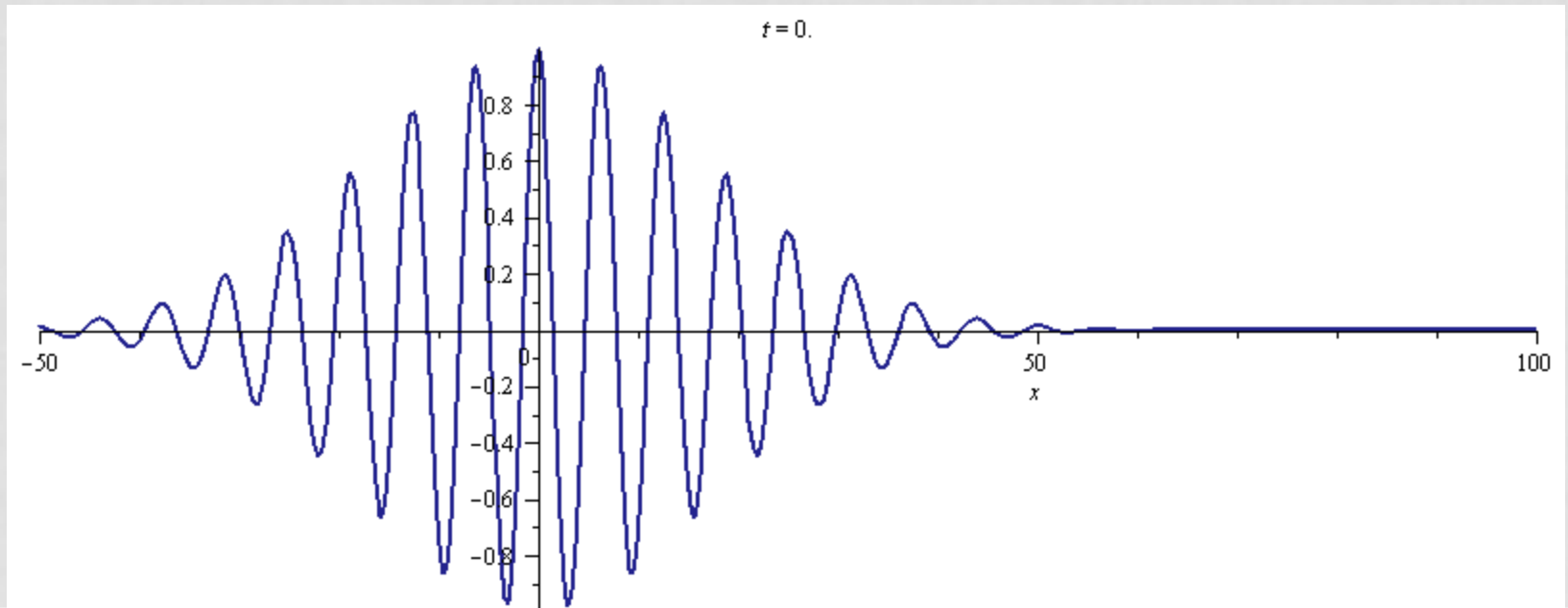


$$v_p = \frac{\omega}{k}$$

$$v_g = \frac{\Delta\omega}{\Delta k}$$



$$v_g = \frac{d\omega}{dk}$$



Phase and Group velocity

$$v_p = \frac{\omega}{k}$$

How to calculate them:

- You need the *dispersion relation*: the relationship between ω and k , with only constants in the formula
- Example: light in vacuum has $\omega = ck$

$$v_p = \frac{\omega}{k} = \frac{ck}{k} = c$$

$$v_g = \frac{d\omega}{dk}$$

$$v_g = \frac{d\omega}{dk} = \frac{d}{dk}(ck) = c$$

Theorem: Group velocity doesn't always equal phase velocity

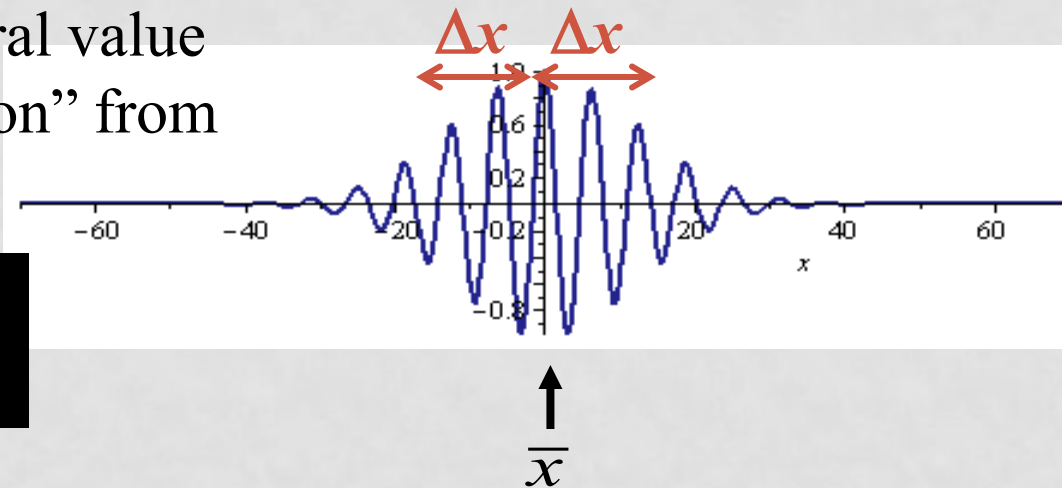
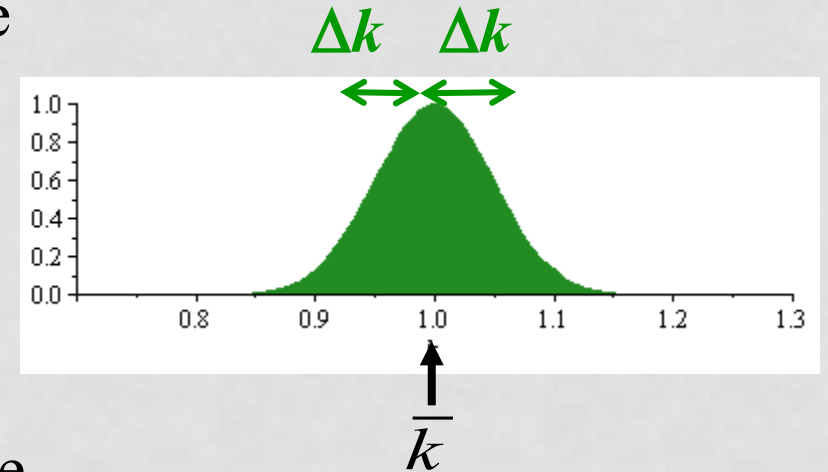
If the dispersion relation is $\omega = Ak^2$, with A a constant, what are the phase and group velocity?

$$\omega = kv_p$$
$$v_g = \frac{d\omega}{dk} = \frac{d}{dk}(v_p k) = v_p + \frac{dv_p}{dk} k \neq v_p$$

$$v_p = \frac{\omega}{k} = \frac{Ak^2}{k} = Ak$$
$$v_g = \frac{d\omega}{dk} = \frac{d}{dk}(Ak^2) = 2Ak$$

The Classical Uncertainty Principle

- The wave number of a wave packet is not exactly one value
 - It can be approximated by giving the central value
 - And the uncertainty, the “standard deviation” from that value
- The position of a wave packet is not exactly one value
 - It can be approximated by giving the central value
 - And the uncertainty, the “standard deviation” from that value



These quantities are related:

• Typically, $\Delta x \Delta k \sim 1$

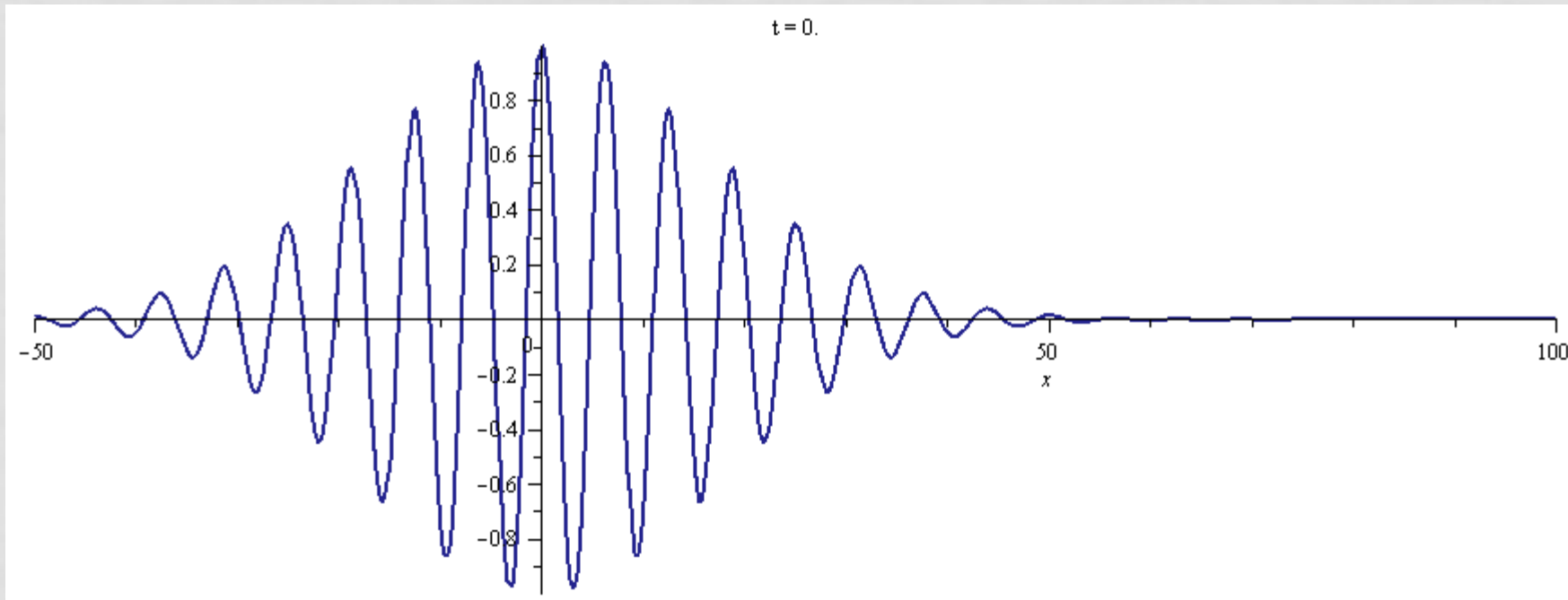
Precise Relation:
(proof hard)

$$\Delta x \Delta k \geq \frac{1}{2}$$

Uncertainty in the Time Domain

Stand and watch a wave go by at one place

- You will see the wave over a period of time Δt
- You will see the wave with a combination of angular frequencies $\Delta\omega$
- The same uncertainty relationship applies in this domain



$$\Delta t \Delta \omega \geq \frac{1}{2}$$