Engg. Mathematics-I

Question Bank

1. If A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 3 & 4 \\ -1 & -3 & -4 \\ 3 & 9 & 12 \end{bmatrix}$, then

(a) Rank of A = Rank of B = 2
(b) Rank of A = Rank of B = 1
(c) Rank of A = 2, Rank of B = 1
(d) Rank of A = 1, Rank of B = 2

2. The eigen values of $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ is

(a) $\pm i$
(b) ± 2
(c) ± 1
(d) ± 3

3. The matrix multiplication $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} [1 \quad 2 \quad -1]$

(a) is not defined
(b) equals $[-1]$
(c) equals $\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$
(d) is not invertible

4. Which of the following option is false:

(a) $AB \neq BA$
(b) $I^{100} = I$
(c) $(AB)^{-1} = A^{-1}B^{-1}$
(d) None of these

5. If A = $\begin{bmatrix} 1 & 0 & -1 \\ 8 & 2 & 6 \\ 3 & 3 & 0 \end{bmatrix}$, then rank of A is

(a) 0
(b) 1
(c) 2
(d) 3

6. The given matrix is $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \\ 2 & 3 & 0 \end{bmatrix}$ is

(a) non-singular matrix
(b) singular matrix
(c) identity matrix
(d) null matrix

7. If $u = e^{x^2+y^2+z^2}$, find u_x, u_y, u_z.

Also, prove that $u_{xyz} = 8xyzu$

8. If $u = x + \log y$ and $v = \log y$, verify that $JJ^* = 1$.

9. Determine the points where the function $x^2y + xy^2 - 2xy$ has a maximum or a minimum.

10. Show that $\vec{r} = \frac{x\,\hat{\imath} + y\,\hat{\jmath}}{x^2+y^2}$ is both solenoidal and irrotational.

11. Find the value of $\Gamma\left(\frac{1}{2}\right)\,\Gamma\left(\frac{3}{2}\right)$

12. Find the approximate value of $[(1.96)^4 + 3(2.12)^4]^{1/6}$

13. Define asymptotes with an example.

Also, trace the curve $y^2(a + x) = (a - x)\,x^2$

14. Test whether $u = \frac{x+y}{x-y}$ and $v = \frac{x}{y}$ are functionally dependent,

and if so, find the relation between them.

15. If $y = x^n \log x$, prove that $y_{n+1} = \frac{n!}{x}$

16. If $u = \sec^{-1}\frac{x^5 - y^5}{x^2 + y^2}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\cot u$

17. If $u = x^2ye^z$, where $x = t$, $y = t^2$, $z = \log t$, find $\frac{du}{dt}$ at $t = 2$.

18. Expand $\sin(x + y)$ upto the second degree terms.

19. Expand y^x in powers of $(x - 1)$ and $(y - 1)$ upto the second degree terms.

20. Expand $\log(1 + \sin x)$ in ascending powers of x upto x^4.

21. Find the area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ using double integration.

22. Evaluate $\int_0^1 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz\, dx\, dy\, dz$

23. Express $\int_0^1 \frac{x}{\sqrt{1-x^4}}\, dx$ in terms of beta function.

24. Find the inverse of A by elementary row operations:

$$A = \begin{bmatrix} 4 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

25. Find the rank of the following matrix by reducing it into echelon form:

$$\begin{bmatrix} 1 & 2 & -5 \\ -4 & 1 & -6 \\ 6 & 3 & -4 \end{bmatrix}$$

26. Test the consistency and solve the following system of equations:

$$\begin{aligned} &x - y - 2z = -2 \\ &3x - y + z = 6 \\ &x - 3y - 4z = -4 \end{aligned}$$

27. Find the eigen values of A, where

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

28. Examine the linear dependence of the following vectors. If the vectors are dependent, find the relation between them.

$X_1 = (3, 1, -4)$, $X_2 = (2, 2, -3)$, $X_3 = (0, -4, 1)$

29. Verify Cayley Hamilton Theorem for the following matrix:

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \quad \text{and hence find } A^{-1}.$$

30. Find the rank of the following matrix by reducing it into the normal form:

$$A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$$