Engg. Mathematics-II

Question Bank

Q1. Solve:

$$\frac{dy}{dx} + \frac{y}{x} = \frac{-y^2}{x^2}$$

Q2. Solve:

$$y \log y \frac{dx}{dy} + x = \log y$$

Q3. Solve:

$$\frac{dy}{dx} - \frac{y}{x+1} = e^x(x+1)$$

Q4. Solve the differential equation:

$$\frac{d^2y}{dx^2} + y = 0 \text{ given that } y(0) = 2 \text{ and } y\left(\frac{\pi}{2}\right) = -2$$

Q5. Solve the differential equation:

$$\frac{d^4y}{dx^4} - \frac{d^2y}{dx^2} = 0$$

Q6. Solve the differential equation:

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$$

Q7. Solve:

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = \sin 3x + \cos 2x$$

Q8. Solve:

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = e^{-x}$$

Q9. Solve:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y = e^{2x} + 3\cos 4x$$

Q10. Solve:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \cos x$$

Q11. Solve:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x$$

Q12. Solve:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^3 e^{2x}$$

Q13. Solve the following simultaneous equations:

$$\frac{d^2y}{dt^2} = x \quad \text{and} \quad \frac{d^2x}{dt^2} = y$$

Q14. Solve the following simultaneous equations:

$$\frac{dx}{dt} - y = \sin t \qquad \text{and} \qquad \frac{dy}{dt} - x = \cos t$$

Q15. Apply the method of Variation of Parameters to solve:

$$\frac{d^2y}{dx^2} + y = \tan x$$

Q16. Obtain a Fourier series to represent $f(x) = x^2$ in the interval of $(0, 2\pi)$ and hence deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$$

Q17. Find the Fourier series to represent the function

$$f(x) = \begin{cases} -1 & \text{when } -\pi < x < 0 \\ 1 & \text{when } 0 < x < \pi \end{cases}$$

Q18. Obtain the Fourier series expansion of

$$f(x) = \left(\frac{\pi - x}{2}\right) \quad \text{for } 0 < x < 2$$

Q19. Expand $f(x) = x$ as a half range

(i) sine series in $0 < x < 2$

(ii) cosine series in $0 < x < 2$

Q20. Solve:

$$\frac{\partial^3 z}{\partial x^3} - \frac{\partial^3 z}{\partial y^3} = x^3 y^3$$

Q21. Solve the linear partial differential equation:

$$r - 2s + t = \sin(2x + 3y)$$

Q22. If $L(\cos^2 t) = \frac{p^2+2}{p(p^2+4)}$, find $L(\cos^2 at)$.

Q23. Find the Laplace transform of

$$\int_0^t \int_0^t \int_0^t \cos at \; dt \; dt \; dt$$

Q24. Evaluate:

$$\int_0^\infty \frac{e^{-t} \sin\sqrt{3}t}{t} \, dt$$

Q25. Express the function in unit step function and obtain its Laplace transform.

$$F(t) = \begin{cases} t-1 \;, & 1 < t < 2 \\ 3-t \;, & 2 < t < 3 \end{cases}$$

Q26. Draw the graph and find the Laplace transform of the triangular wave function of period $2c$ given by

$$F(t) = \begin{cases} t \;, & 0 < t \leq c \\ 2c-t \;, & c < t \leq 2c \end{cases}$$

Q27. Find $L\{F(t)\}$, where $F(t)$ is defined by

$$F(t) = \begin{cases} 1 \;, & 0 \leq t < 2 \\ -1 \;, & 2 \leq t < 4 \end{cases} \quad \text{and} \quad F(t+4) = F(t)$$

Q28. Find the inverse Laplace transform of $\frac{2p+1}{p^2-4}$.

Q29. Use convolution theorem to evaluate:

$$L^{-1}\left\{\frac{p^2}{(p^2+a^2)(p^2+b^2)}\right\}$$

Q30. Solve the following differential equation using Laplace transform.

$$\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - y = t^2e^t \quad \text{where} \quad y(0) = 1, \left(\frac{dy}{dt}\right)_{t=0} = 0, \left(\frac{d^2y}{dt^2}\right)_{t=0} = -2.$$