DRONACHARYA GROUP OF INSTITUTIONS

QUESTION BANK

Course: B.Tech. SUBJECT: Engineering Mathematics-III SEMESTER: III

COMPLEX ANALYSIS

1. State and prove necessary and sufficient condition for $f(z)$ to be analytic (C-R equations).
2. Show that the complex variable function $f(z)=|z|^2$ is differentiable only at the origin.
3. Show that the real and imaginary parts of the function $w = log\ z$ satisfy the Cauchy – Riemann equations when z is not zero.
4. Show that the function $z|z|$ is not analytic anywhere
5. Show that the function $f(z)=e^{-z^{-4}}, z\neq 0$ and $f(0)=0$ is not analytic at $z=0$ although Cauchy-Riemann equations are satisfied at the point.
6. Show that the function $f(z)$ defined by $f(z)=\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, z\neq 0$ and $f(0)=0$ is continuous and the Cauchy-Riemann equations are satisfied at the origin. Yet f'(0) does not exist.
7. Show that the function defined by $f(z)=\sqrt{|xy|}$ satisfies Cauchy – Riemann equation at the origin but is not analytic at that point.
8. Show that the function $u=\dfrac{1}{2}\log(x^2+y^2)$ is harmonic. Find its harmonic conjugate.
9. Show that the function x^2-y^2+2y which is harmonic remains harmonic under the transformation $z=w^3$.
10. If $\emptyset$ and φ are functions of x and y satisfying Laplace's equation, show that $s+it$ is analytic, where $s=\dfrac{d\emptyset}{dy}-\dfrac{d\varphi}{dx}$ and t= $\dfrac{d\emptyset}{dx}+\dfrac{d\varphi}{dy}$.
11. Prove that an analytic function with constant modulus is also constant.
12. If $f(z)=u+iv$ is an analytic function of z and $u-v=\dfrac{\cos x+\sin x-e^{-y}}{2\cos x-2\cosh y}$; Prove that $f(z)=\dfrac{1}{2}\left[1-\cot\dfrac{z}{2}\right]$ when $f\left(\dfrac{\pi}{2}\right)=0$.
13. If $f(z)$ is regular function of z, show that $\left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$.
14. Evaluate the line integral $\int_c z^2dz$ where C is the boundary of z triangle with the vertices $0,1+i,-1+i$ clockwise.
15. State and derive Cauchy integral theorem and derive Cauchy integral formula.

16. Evaluate the integral $\int_c \frac{3z^2+7z+1}{z+1}\,dz$, where C is the circle $|z| = \frac{1}{2}$ clockwise.

17. Use Cauchy integral formula to evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\,dz$, where C is the circle $|z| = 3$.

18. State & prove Taylor Series and Laurent Series.

19. Expand $\frac{1}{z^2-3z+2}$ in the region (a) $|z| < 1$ (b) $|z| > 2$ (c) $1 < |z| < 2$.

20. Find out the zeros and discuss the nature of the singularities of $f(z) = \frac{z-2}{z^2}\sin\left(\frac{1}{z-1}\right)$.

21. Evaluate $\oint_c \left[\frac{3z^2+z+1}{(z-1)(z+3)}\right] dz \quad c = |z| = 2$, using Cauchy's residue theorem.

22. Show that: $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}\,d\theta = \frac{\pi}{6}$.

23. Apply calculus of residue to prove that: $\int_0^{2\pi} \frac{\cos 2\theta}{1-2a\cos\theta+a^2}\,d\theta = \frac{2\pi a^2}{1-a^2}\ (a^2<1)$.

24. Using the complex variable techniques, evaluate the integral

(a) $\int_{-\infty}^{\infty} \frac{dx}{x^4+1}$

(b) $\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2+1)(x^2+4)}\,dx$

25. Evaluate $\int_0^{\infty} \frac{\cos mx}{(x^2+1)}\,dx$

NUMERICAL TECHNIQUES

1. Discuss the Rate of convergence for the Newton-Raphson method.
2. Use Newton-Raphson method to find the real root (correct to four decimal places) of the following equations:

   (a) $x^4 - x - 10 = 0$ Near $x = 2$ (b) $3x - \cos x - 1 = 0$
3. Use by Newton-Raphon formula to find $\sqrt[3]{18}$ correct to two decimals, assuming 2.5 as the initial approximation.
4. Discuss the Rate of convergence for the Newton Raphson method and Regula-Falsi method.
5. Use Regula-Falsi method to find the real root (correct to four decimal places) of the following equations: (a) $x^3 - 5x - 7 = 0, (2,3)$ (b) $xe^x = \cos x, (0,1)$.
6. Use Newton iterative method to find the real root (correct to four decimal places) of the following equations: (a) $x \log_{10} x = 1.2$ (b) $(48)^{1/3}$.
7. Prove that the following two sequences, both has convergence of the second order with the same limit $\sqrt{a}$, $x_{n+1} = \frac{1}{2} x_n \left(1 + \frac{a}{x_n^2}\right) and\ x_{n+1} = \frac{1}{2} x_n \left(3 - \frac{a}{x_n^2}\right)$.
8. Define the shift operator, forward and backward difference operators, the central difference operator and the average operator. Establish

   $$\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right) = 2\left(1 + \frac{1}{2}\Delta\right)(1+\Delta)^{\frac{1}{2}}$$

   $$\Delta(1+\Delta)^{-\frac{1}{2}} = \nabla(1-\nabla)^{-\frac{1}{2}}$$ where, all the above notations have usual meanings.

   $$\mu = \sqrt{1 + \frac{1}{4}\delta^2}$$
9. Prove that

   (a) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

   (b) $\Delta^2 = h^2D^2 - h^3D^3 + \frac{7}{12}h^4D^4 + ...$

   (c) $hD = \Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4}..........$, provided $|\Delta| < 1$.

10. Prove that $\left(\frac{\Delta^2}{E}\right)e^x . \frac{Ee^x}{\Delta^2 e^x} = e^x$, the difference of interval being h.
11. Evaluate $\Delta\left(\frac{2^x}{(x+1)!}\right)$.
12. From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age of 46.

Age	45	50	55	60	65
Premium(Rs.)	114.84	96.16	83.32	74.48	68.48

13. Estimate the population for 1964 and 1966 from the following data:

x	1961	1962	1963	1964	1965
y	200	……	260	……	350

14. The Population of a city was as given. Estimate the population for the year 1925.

Year	1891	1901	1911	1921	1931
Population (in thousand)	46	66	81	93	101

15. Develop the divided-difference table from the data given below and obtain the interpolation polynomial $f(x)$:

x	1	3	5	7	11
$f(x)$	5	11	17	23	29

Also, find the value of $f(19.5)$.

16. Apply Gauss-Seidal method to find the solution of
    $(i)\, 20x+y-2z=17, 3x+20y-z=-18, 2x-3y+20z=25$
    $(ii)\, 4x+2y+13z=24, 3x+9y-2z=11, 4x-4y+3z=-8$
17. Use the Crout's method to solve the following system:
    $(i)\, x+y+z=6, x+2y+3z=14, x-2y+3z=6$
    $(ii)\, x+y+z=1, 3x+y-3z=5, x-2y-5z=10$
18. The table given below reveals the velocity '$\boldsymbol{v}$' of a body during the time t specified. Find its acceleration at t=1.1 :

t	1.0	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.4	60.8

19. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by taking $h=\frac{1}{6}$, using (i) Simpson's $\frac{1}{3}$ rule (ii) Trapezoidal rule and compare with the exact result.
20. A motorbike starts from rest , its velocity v in km/hour is given in time t as :

t	2	4	6	8	10	12	14	16	18	20
v	10	18	25	29	32	20	11	5	2	0

Estimate the approximate distance covered by motorbike in 20 minutes.

21. A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and given in the following table. Using Simpson's $\frac{1}{3}$ rule finds the velocity of the rocket at $t = 80$ seconds.

t	0	10	20	30	40	50	60	70	80
$f(cm/\sec^2)$	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

22. A river is 80m wide. The depth d of the river at a distance x from one bank is given by the table:

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find approximately the area of cross-section of the river using Simpson's $\frac{1}{3}$ rule.

23. Using Picard's method of successive approximation $\frac{dy}{dx} = \frac{x^2}{y^2+1}, \quad y(0) = 0$. Obtain $y(0.25), y(0.5) \,\&\, y(1)$ correct to three decimal places.

24. Apply Picard's method to find the third approximation of the solution of $\frac{dy}{dx} = x + y^2, \quad y(0) = 1$

25. Use R-K method of fourth order to find the numerical solution at $x = 0.8$ for $\frac{dy}{dx} = \sqrt{x+y},\ y(0.4) = 0.41$. Assume the step length (0.2).

LINEAR ALGEBRA

1. Show that the vectors X_1 =(1,2,3), X_2=(3,-1,4) and X_3=(4,1,7) are linearly dependent.
2. Examine the following vectors for linear dependence and find the relation if it exists.
   X_1=(1,2,4) , X_2= (2,-1,3), X_3=(0,1,2), X_4=(-3,7,2).
3. Examine for linear dependence and find the relation if possible.
   X1= (1,0,2,1), X2= (3,1,2,1), X3=(4,6,2,-4) and (-6,0,-3,-4).
4. Find whether the vectors are linearly dependent or independent.
   V_1=(1,2,1), V_2=(3,1,5), V_3=(3,-4,7).
5. Prove that the set (1,x,1+x+x^2) is linearly independent set of the vectors in the vector space of all polynomials over the real number field.
6. Solve if the vector (2,-5,3) in the subspace of R3 spanned by the vectors (1,-3,2), (2,-4,1), (1,-5,7).
7. Let R be the field of real numbers. Which of the following are the subspaces of $V_{3(}$R) ?
   (I) W_1 = { (x, 2y,3z) : x,y,z ∈ R}
   (II) W_2 = { (x, x,x) : x ∈ R}
   (III) W_3 = { (x, y,z) : x,y,z ∈ R}
8. If V is a set of all (n x n) matrices over any field F, then a set w of all (n xn) symmetric matrices forms a vector subspace of V(F).
9. Let V be a vector space of all real valued funtions over R. Then show that the solution set W of the differential equation where y =f(x) is a subspace of V.

   $$2\frac{d^2y1}{dx^2} - 9\frac{dy}{dx} +2y = 0$$

10. Prove that the intersection of two subspaces of a vector space is a subspace of the same but union of two subspaces may not be a subspace.
11. Show that the vectors (1,0,0), (1,1,0) and (1,1,1) form a basis for R^3.
12. Show that the set S= {(1,0,0), (1,1,0), (1,1,1), (0,1,0)} is not a basis set.
13. Show that the vectors form a basis of V_3(F) : { (1,2,1). (2,1,0), (1,-1,2)}
14. Prove that the vectors X_1=(1,0,-1), X_2= (1,2,1) , X_3 = (0,-3,2) forms a basis of V_3(R).
15. Show that the following vectors form a basis of R_3. Express each of the standard basis vectors e_i , i =1,2,3 as linear combination of the above basis vectors.
    S= {(1,2,1), (2,1,0), (1,-1,2)}.
16. Find the coordinate vector V= (3,-5,2) relative to the basis of e_1 =(1,1,1), e_2= (0,2,3), e_3= (0,2,-1).
17. If V_3(R) is a vector space then show that S={ (0,1,-1), (1,1,0), (1,0,2)} is a basis of V_3 and hence find the coordinates of the vector (1,0,-1) with respect to the basis.
18. Determine the null space for the following matrix : $\begin{bmatrix} -3 & 0 \\ 2 & -4 \end{bmatrix}$.
19. Determine the basis and null space for the following matrix :
    $$\begin{bmatrix} 1 & 2 & -3 & 2 & -3 \\ 1 & -1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 1 & -1 \\ 1 & 2 & 5 & -6 & -3 \end{bmatrix}.$$

20. Determine a basis for the null space , row space, column space and rank of the matrix.

$$\begin{bmatrix} 1 & 1 & 1 & 3 & 4 \\ 2 & 3 & 2 & 6 & 8 \\ 4 & 7 & 4 & 12 & 16 \\ 5 & 11 & 6 & 15 & 20 \end{bmatrix}.$$

21. Define inner product spaces, orthogonal and orthonormal vectors.
22. Let X_1 = (1,2,1), X_2= (2,1,4), X_3= (3,-2,-1) in R3 then,
    (i) Show that they form an orthogonal set under the standard Euclidean inner product for R^3 but not orthonormal set.
    (ii) Convert them into the set of vectors that will form an orthonormal set of vectors under the standard Euclidean inner product for R^3.
23. State and prove Schwarz Inequality.
24. Construct orthonormal set of vectors form the set :
    X_1 =(1,2,1), X_2=(2,1,4), X_3= (4,5,6).
25. Let R^3 have the Euclidean inner product . Use the Gram Schmidt process to transform the basis vector u_1= (1,1,1), u_2 =(-1,1,0), u_3= (1,2,1) into orthogonal basis $\{v_1,v_2,v_3\}$.
26. Obtain the orthogonal basis for P_2, the space of all real polynomials of degree atmost 2, the inner product being defined by (P1,P2)= $\int_0^1 P1(t)P2(t)dt$.
27. Orthonormalise the set of linearly independent vectors X1 =(1,0,1,1), X2= (-1,0,-1,1), X3= (0,-1,1,1) of R^4 with the standard inner product.
28. If p =p(x) $=p_0 + p_1x + p_2x^2$ and q=q(x) $=q_0 + q_1x+q_2x^2$, the inner product is defined by
    (p,q) $=p_0q_0+p_1q_1+p_2q_2$ for the vectors
    X_1= $1+2x+3x^2$, X_2 $=3+5x+5x^2$, X_3=$2+x+8x^2$.
    Find the orthonormal vectors.